$\mathcal{N} = 1$ conformal dualities

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Regional Meeting

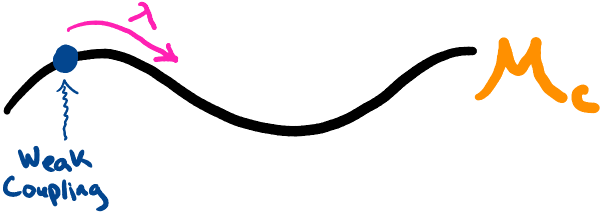
September 16, 2019 - Crete.

Motivation

- $4d$, $\mathcal{N} = 1$, conformal, Lagrangian
- What is the space of $\mathcal{N} = 1$ SCFTs in four dimensions?
- What fraction of the space is captured by conformal Lagrangians?

Guage fields (Gauge group G), chiral fields with R-sharge $\frac{2}{3}$ (Representation $\mathcal{R}$), cubic superpotentials, all β-functions vanish

Conformal Manifolds



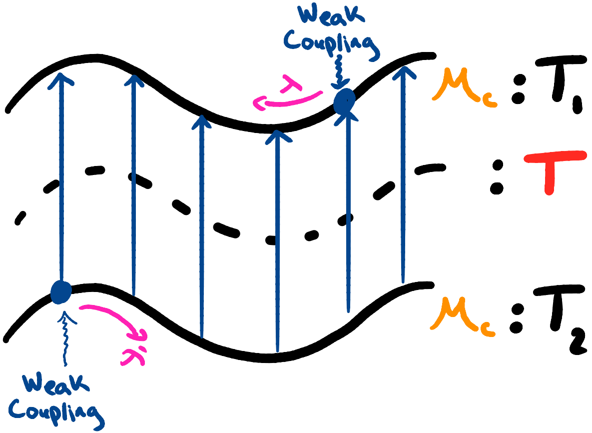
- Example A: $\mathcal{N} = 4$ SYM $G = SU(2)$, $\dim \mathcal{M}_c = 1$
- Example B: $\mathcal{N} = 4$ SYM $G = SU(N > 2)$, $\dim \mathcal{M}_c = 3$
- Example C: $\mathcal{N} = 1$ SQCD $G = SU(3)$, $\dim \mathcal{M}_c = 7$

$\mathcal{M}_c$ invariants

Theories at different points of the conformal manifold are different SCFTs in general. However some properties are invariants of the manifold

- $\dim \mathcal{M}_c$
- G_F: symmetry on a *generic* locus
- 't Hooft anomalies
- conformal anomalies a and c
- protected quantities, indices

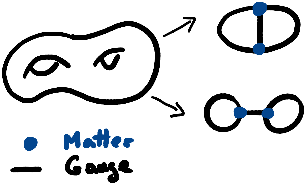
Conformal dualities

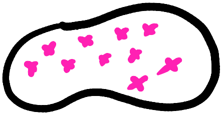


- Example A: $\mathcal{N} = 4$, T_1 has $G = so(2N+1)$, T_2 has $G = usp(2N)$

Geometry of dualities

- Example B: Many of the known dual pairs with extended supersymmetry were understood geometrically as compactifications on equivalent geometries of the same $6d$ theory
- $\mathcal{N}=2$ $N_f=2N$ SQCD is same $(2,0)$ on a four punctured sphere
- Some more examples with minimal supersymmetry have been understood in recent years. For example, $SU(3)$ SQCD with $N_f=9$ can be obtained as a compactification on a sphere with 10 punctures of some $6d$ SCFT and thus has 7 dimensional conformal manifold





Questions

- How ubiquitous are $\mathcal{N} = 1$ conformal dualities?
- Can we construct $\mathcal{N} = 1$ conformal duals to strongly coupled (non-Lagrangian) SCFTs?
- Can these duals shed some light on geometry of dualities?

Algorithm

- Say are given an $\mathcal{N}=1$ SCFT. By this I mean know all the $\mathcal{M}_c$ invariants
- In particular,

$$a = n_v a_v + n_\chi a_\chi \,, \qquad c = n_v c_v + n_\chi c_\chi$$

- Here $a_v = \frac{3}{16}$, $a_\chi = \frac{1}{48}$, $c_v = \frac{1}{8}$, $c_\chi = \frac{1}{24}$
- Step I: Find all conformal gauge theories with $dimG = n_v$ and $dim\mathcal{R} = n_\chi$ – there is a finite number of these
- Step II: Out of these isolate only the ones with matching $\mathcal{M}_c$ invariants
- Step III: Perform any farther checks you can
- You have a putative dual

Plan

Surprizingly, this simple algorithm provides duals for many SCFTs

- $\mathcal{N}=1$ conformal gauge theory duals to $\mathcal{N}=1$ conformal gauge theories with simple G
- $\mathcal{N}=1$ conformal gauge theory duals to $\mathcal{N}=2$ strongly-coupled theories of class $\mathcal{S}$
- $\mathcal{N}=1$ conformal gauge theory duals to compactifications of E-string on genus g Riemann surface

$\mathcal{N} = 1$ (simple) gauge theories

- Organise the assault by n_v
- $n_v = 3$ $G = SU(2)$, no $\mathcal{N} = 1$ SCFTs
- $n_v = 8$ $G = SU(3)$, many $\mathcal{N} = 1$ SCFTs $(2adj + 3f + 3\bar{f}$, $10f + 3\bar{f} + 1\bar{\mathbf{6}}, \cdots)$
- 8 is not divisible by 3; all these models have different $\mathcal{M}_c$ invariants; so can be only self-dual
- $n_v = 10$ $G = USp(4)$, same as $SU(3)$ – many SCFTs but 10 cannot be constracted out of 3 and 8 so only possibility is self-duality
- $n_v = 14$ $F = G_2$, two examples $3 \times \mathbf{7} + \mathbf{27}$ and $12 \times \mathbf{7}$

$$\boxed{14 = 3+3+8}$$

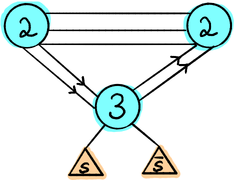
Example A: $G_2 + 3 \times \mathbf{7} + \mathbf{27}$

- Can compute $\mathcal{M}_c$ invariants
- $n_v = 14$ and $n_\chi = 48$
- $\dim \mathcal{M}_c = 3$,
  deduced by computing the Kahler quotient $\{\lambda_i\}/G_{\mathbb{C}}^{free}$
- $G_F = SU(2)$,
  the symmetry at the free point is $G^{free} = SU(3) \times U(1)$ which is broken on a generic point of $\mathcal{M}_c$ to $SU(2)$
- Can compute the supersymmetric indices

Example A: $G_2 + 3 \times \mathbf{7} + \mathbf{27}$: dual

- We are after a dual theory with $n_v = 14$, thus the group is $SU(2) \times SU(2) \times SU(3)$ and $n_\chi = 48$ so that all the β functions vanish

- Here is a solution



- $\dim \mathcal{M}_c = 3$, $G_F = SU(2)$, $TrRSU(2)^2 = -\frac{14}{3}$ 't Hooft anomaly matches, indices match

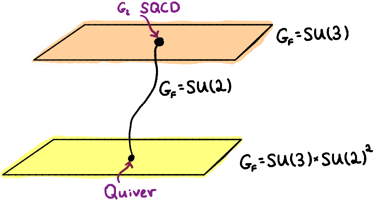
- We have a putative duality

Example A: $G_2 + 3 \times \mathbf{7} + \mathbf{27}$: comments

- We can farther check the duality by studying flows. Giving vev to one of the fundamentals maps to giving vevs to one of the bifundamentals of the two $SU(2)$s.

- In both cases the flow can be explicitly anlayzed and leads to $\mathcal{N} = 2$ SQCD with $G = SU(3)$, one fundamental hypermultiplet, one $\mathbf{6}$ hypermultiplet, and free chiral field

- The structure of the conformal manifold is as follows,

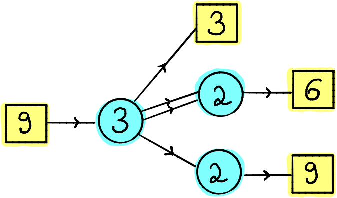


Example B: $G_2 + 12 \times \mathbf{7}$

- $\mathcal{M}_c$ invariant information is,

$n_v = 14$, $n_\chi = 84$, $dim \mathcal{M}_c = 77$, $G_F = \emptyset$, indices

- Match perfectly the following quiver

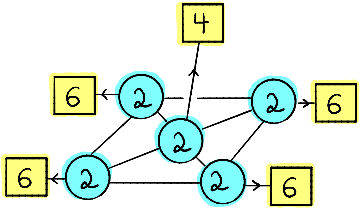


Example C: $SU(4) + 8 \times \mathbf{4} + 8 \times \overline{\mathbf{4}} + 4 \times \mathbf{6}$

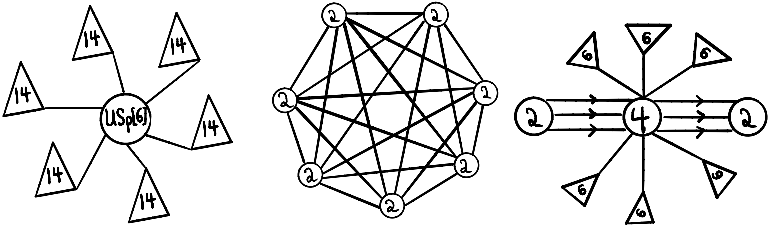
- $\mathcal{M}_c$ invariant information is,

  $n_v = 15$, $n_\chi = 88$, $dim\mathcal{M}_c = 82$, $G_F = U(1)$, 't Hooft anomalies, indices

- Match perfectly the following quiver



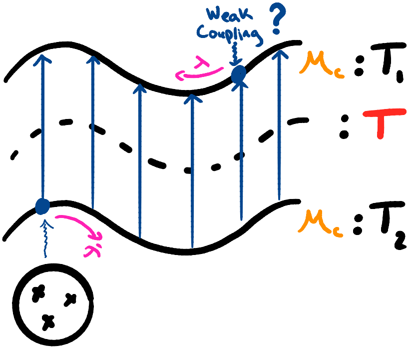
Example D: triality



- $n_v = 21$, $n_\chi = 84$, $dim\mathcal{M}_c = 21$, $G_F = \emptyset$, indices

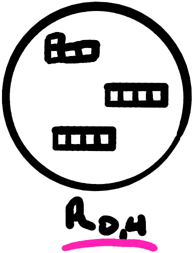
Class $\mathcal{S}$: generalities

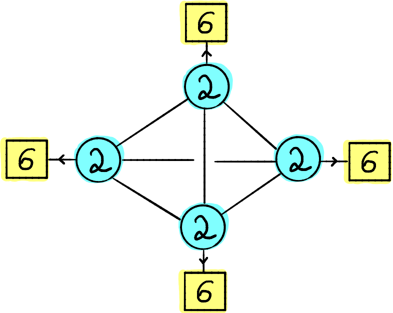
Conformal duals of strongly coupled "non-Lagrangian" SCFTs?



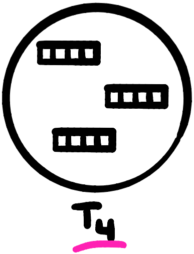
- Need $\mathcal{N} = 2$ SCFT with $\mathcal{N} = 1$ conformal manifold
- Typically this means need to have marginal Higgs and Coulomb operators

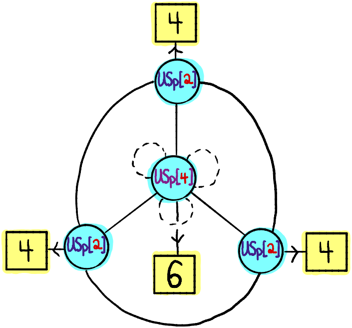
Class $\mathcal{S}$ Example A: $R_{0,4}$





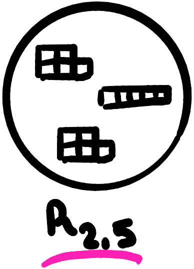
- $n_v = 12$, $n_\chi = 72$, $dim\mathcal{M}_c = 74$, $G_F = \emptyset$, indices

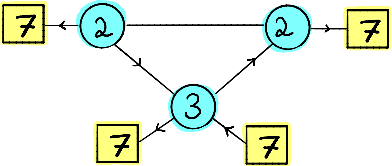




- $n_v = 19$, $n_\chi = 99$, $dim\mathcal{M}_c = 83$, $G_F = \emptyset$, indices

Class $\mathcal{S}$ Example C: $R_{2,5}$

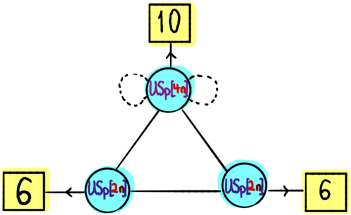




- $n_v = 14$, $n_\chi = 86$, $dim\mathcal{M}_c = 36$, $G_F = \emptyset$, indices

Class $\mathcal{S}$ Example D: even rank MN E_6

- Finally let us mention an example of sequences of theories
- $n_v = N(3N+2)$, $n_\chi = 9N^2 + 30N - 2$



Conformal dual of E-string on genus g

- Consider taking rank one E-string on a genus g surface
- Integrating $6d$ anomaly polynomial on the surface we obtain

$$n_v = 16(g-1)\,, \qquad n_\chi = 81(g-1)$$

- For $g > 1$ we expect,

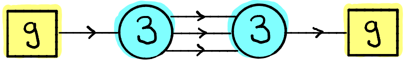
$$dim\mathcal{M}_c = 3g - 3 + 248(g-1) + \delta_{g,2}\,.$$

Here 248 is $dimE_8$, the symmetry group of the $6d$ SCFT

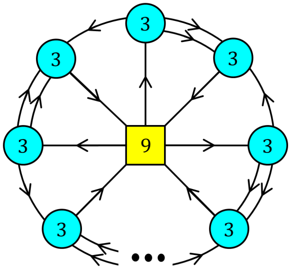
- Indices are known

Conformal dual of E-string on genus g

- Consider $g = 2$, here $n_v = 16$ and $n_\chi = 81$
- Natural guess $16 = 8 + 8$ and the gauge group is $SU(3) \times SU(3)$



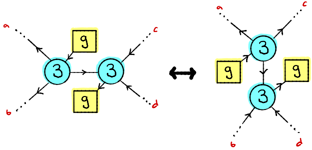
- And for general g we get,



- All $\mathcal{M}_c$ invariant information matches expectations

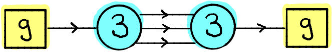
Conformal dual of E-string on genus g: Geometry

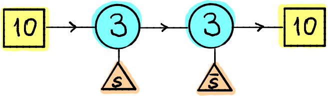
- Note that we have $3g-3$ bifundamentals and $2g-2$ $SU(3)$ gauge groups
- It is tempting to identify then pairs-of -pants with gauge nodes and the tubes with matter
- Note this is opposite to class $\mathcal{S}$ where matter is pairs-of-pants and tubes are gauge groups
- This geometric interpretation implies the following duality move on the quiver



Yet another duality

- The geometric move when applied for genus two implies the following duality





- This is an example of a duality between two theories with not simple gauge groups

- Have checked that the simple duality tests are satisfied

Summary

- Have discussed a simple algorithm to seek for conformal duals of SCFTs
- Surprisingly it provides with many new conformal dualities!!
- Note that symmetry does not play an important role, moreover in many cases on generic points of the conformal manifolds we do not have any symmetry
- Found Lagrangians to strongly coupled SCFTs
- Again a key is that the Lagrangians do not exhibit the full (super)symmetry of the SCFT

Open questions

- Are these sporadic examples or there is an organizing principle?
- Were we lucky with the lower dimension cases or the ubiquity of dualities persists as we increase the dimensions of the groups?
- What is the geometry behind these dualities?
- Look for dualities with strongly coupled ingredients

Thank You!!