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# Holographic duals of 5-dimensional SCFTs on a Riemann surface

arXiv:1807.06031

with Ibrahima Bah and Peter Weck

arXiv:1805.03661

with Daniël Prins and Alessandro Tomasiello

10th Crete Regional Meeting in String Theory

## Compactifying Higher Dimensional Field Theories

- | large classes of theories in lower dimensions
- | their properties admit a description in terms of the geometry and topology of the compact manifold

## (2,0) theory on a Riemann surface

[Maldacena, Núñez '00] [Gaiotto, Maldacena '09] [Bah, Beem, Bobev, Wecht '12]

Class S SCFTs in four dimensions

## (2,0) theory on a Riemann surface

[Maldacena, Núñez '00] [Gaiotto, Maldacena '09] [Bah, Beem, Bobev, Wecht '12]

$$\text{AdS}_5 \times_w \mathcal{M}_6$$

## (2,0) theory on a Riemann surface

[Maldacena, Núñez '00] [Gaiotto, Maldacena '09] [Bah, Beem, Bobev, Wecht '12]

$$S^4 \rightarrow M_6 \downarrow \Sigma_g$$

## (2,0) theory on a Riemann surface

[Maldacena, Núñez '00] [Gaiotto, Maldacena '09] [Bah, Beem, Bobev, Wecht '12]

part of a domain wall geometry which at large distances asymptotes to  
 $\text{AdS}_7 \times S^4$

# D4-D8/O8 brane configuration

[Brandhuber, Oz '99]

$\mathcal{N} = 1$  SUSY  $USp(2N)$  gauge theory  
coupled to

- | 1 hyper in the antisymmetric representation ( $H_a$ )
- |  $N_f$  hypers in the fundamental ( $H_f$ )

| vector multiplet scalar parametrizes  $x_9$  fluctuations



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- | 4  $H_a$  scalars parametrize  $x_5-x_8$  fluctuations

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- |  $H_f$  from D4-D8 open strings

## Global Symmetries

$$SU(2)_R \times SU(2) \times SO(2N_f) \times U(1)_I$$

## Global Symmetries

$SU(2)_R \times SU(2)$   
 $\chi^5 - \chi^8$  rotations

## Global Symmetries

$SU(2)_R$   
 $R$ -symmetry

# Global Symmetries

$$\begin{matrix} \text{SU}(2) \\ H_a \end{matrix}$$

# Global Symmetries

$$\begin{array}{c} \mathrm{SO}(2N_f) \\ H_f \end{array}$$

# Global Symmetries

$$\begin{matrix} U(1)_I \\ j = *Tr(F \wedge F) \end{matrix}$$

| fixed point at strong coupling [Seiberg '96]

|

- | fixed point at strong coupling [Seiberg '96]
- | degrees of freedom scale as  $N^{5/2}$  at large  $N$  [Jafferis, Pufu '12]

$$\text{AdS}_6 \times_w S^4$$

[Brandhuber, Oz '99]

$$ds_{10}^2 = \ell_s^2 \Omega \left[ ds_{\text{AdS}_6}^2 + \frac{4}{9} d\alpha^2 + \cos^2(\alpha) ds_{S^3}^2 \right], \quad \Omega = \frac{18\pi^2 N}{n_0 \sin^{2/3}(\alpha)}$$

$$e^{-4\phi} = \frac{9N n_0^3 \sin^{10/3}(\alpha)}{8\pi^2}, \quad F_4 = \frac{80}{9} \ell_s^3 \pi N \cos^3(\alpha) \sin^{1/3}(\alpha) \text{vol}_{S^3}$$

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$$[-\frac{\pi}{2}, \frac{\pi}{2}]$$

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$$SO(5) \rightarrow SO(4) \equiv SU(2)_R \times SU(2)$$

are there 3D SCFTs produced by  
the 5D SCFT on a Riemann surface?

look for  $\text{AdS}_4$  solutions

# SUSY on $\mathbb{R}^{1,4}$

$$d\epsilon = 0$$

# SUSY on $\mathbb{R}^{1,2} \times \Sigma$

$$\Bigl(d+\tfrac{1}{4}\omega^{ab}\gamma_{ab}\Bigr)\epsilon=0$$

## SUSY on $\mathbb{R}^{1,2} \times \Sigma$

$$\left( d + \underbrace{\frac{1}{4}\omega^{ab}\gamma_{ab} + A_R}_{=0} \right) \epsilon = 0$$

## SUSY on $\mathbb{R}^{1,2} \times \Sigma$

$$\mathrm{U}(1)_{\text{holonomy}} \equiv \mathrm{U}(1)_R \subset \mathrm{SU}(2)_R$$

$$J_h=J_R$$

## SUSY on $\mathbb{R}^{1,2} \times \Sigma$

$$U(1)_{\text{holonomy}} \equiv U(1)_R, \, U(1) \subset SU(2)_R, \, SU(2)$$

$$\mathbf{J}_h = \mathbf{J}_R + z \mathbf{J}$$

$$\mathrm{AdS}_4\times \mathcal{M}_6$$

$$S^4_{\mathsf{U}(1)_R\times\mathsf{U}(1)}\rightarrow\mathcal{M}_6\downarrow\Sigma_g$$

$\mathcal{N} = 2$  supersymmetric  $\text{AdS}_4 \times Y$

## M-theory

[Gabella,Martelli,AP,Sparks '12]

Type IIB

[AP,Solard,Tomasiello '17]

Type IIA

[AP,Prins,Tomasiello '18]

## Background

$$ds_{10}^2 = e^{2A} \left( ds_{AdS_4}^2 + ds_{M_6}^2 \right)$$

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$$ds_{10}^2 = e^{2A} (ds_{AdS_4}^2 + ds_{M_6}^2)$$

+

$$\phi, H, F_{p_{\text{even}}}$$

preserving the symmetries of  $AdS_4$

## Supersymmetry

$$\exists \epsilon_{1,2} : \delta_{\epsilon_{1,2}} \psi = 0 = \delta_{\epsilon_{1,2}} \lambda$$

# Supersymmetry

$$\begin{aligned}\epsilon_1 &= \sum_{I=1}^2 \chi_+^I \otimes \eta_{1+}^I + \sum_{J=1}^2 \chi_-^J \otimes \eta_{1-}^J \\ \epsilon_2 &= \sum_{I=1}^2 \chi_+^I \otimes \eta_{2-}^I + \sum_{J=1}^2 \chi_-^J \otimes \eta_{2+}^J\end{aligned}$$

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$$\nabla_\mu \chi_\pm^I = \frac{1}{2} \gamma_\mu \chi_\mp^I$$

# Supersymmetry

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# Supersymmetry

$$\eta_{1+}^I \quad \eta_{2+}^J$$

$$so(2)_R \simeq u(1)_R$$

R-symmetry

## G-structure

$SO(6)$



$G$

stabilizer

## G-structure

$M_6$  acquires a G-structure characterized by a set of tensors  
constructed as bilinears of  $\{\eta_{1+}^I, \eta_{2+}^J\}$

## G-structure

$$G = \begin{cases} \text{SU}(2) \\ \text{identity} \end{cases}$$

## SU(2)-structure

$$\{w,\, j,\, \omega\}$$

$$\iota_w j = 0 = \iota_w \omega \qquad j \wedge \omega = 0 \qquad j \wedge j = \tfrac{1}{2} \omega \wedge \overline{\omega}$$

## Supersymmetry Equations

$$\Phi_{\pm}^{IJ} \equiv \eta_{1+}^I \bar{\eta}_{2\pm}^J$$

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$$\Phi_{\pm}^{IJ} \propto \sum_p \bar{\eta}_{2\pm}^J \gamma^{m_1 \dots m_p} \eta_{1+}^I \gamma^{m_1 \dots m_p}$$

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+

$$\gamma^{m_1 \dots m_k} \rightarrow dx^{m_1} \wedge \dots \wedge dx^{m_k}$$

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↓

polyforms

$$\Phi_{\pm}^{IJ}(w, j, \omega)$$

## Supersymmetry Equations

$$\delta_{\epsilon_{1,2}} \psi = 0 = \delta_{\epsilon_{1,2}} \lambda$$



constraints on  $\{\eta_{1+}^I, \eta_{2+}^J\}$



constraints on  $\Phi_{\pm}^{IJ}(w, j, \omega)$



constraints on SU(2)-structure  $\{w, j, \omega\}$

## Generic Geometry

$$ds_{M_6}^2 = \overline{w}w(y, \psi) + ds_{M_4}^2(x^i)$$

Base

## Generic Geometry

$$ds_{M_6}^2 = \frac{1}{e^{4A} - y^2} dy^2 + \frac{1}{4}(1 - e^{-4A}y^2)(d\psi + \rho)^2 + ds_{M_4}^2$$



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|  $\partial_\psi$  generates  $U(1)_R$

|

## Generic Geometry

$$ds_{M_6}^2 = \frac{1}{e^{4A} - y^2} dy^2 + \frac{1}{4}(1 - e^{-4A}y^2)(d\psi + \rho)^2 + g_{ij}(y, x^i) dx^i dx^j$$

- |  $\partial_\psi$  generates  $U(1)_R$
- |  $g_{ij}$  conformally Kähler

## Ansatz for $M_4$

$$ds_{M_4}^2 = \underbrace{e^{2W}(dx_1^2 + dx_2^2)}_{\Sigma_g} + e^{2Z}[(d\tau + V_1)^2 + e^{2C}(d\varphi + V_2)^2]$$

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|  $\partial_\varphi$  Killing vector generating second  $U(1)$

| keep  $F_0, F_4, \phi$

## General System

$$ds_{10}^2 = e^{2A}[ds_{AdS_4}^2 + \frac{1}{4}H[(e^{2W}(dx_1^2 + dx_2^2) + 4h^{ij}\eta_i\eta_j + g_{ij}t^it^j)]$$

$$i,j = \pm, \quad \eta_{\pm} = d\phi_{\pm} + \frac{1}{2}*_2d_2\partial_{\pm}D_0, \quad (\partial_{x_1}^2 + \partial_{x_2}^2)D_0 = e^{2W}$$

## Constant Curvature

$$ds_{10}^2 = \frac{H^{-1/2}}{\frac{3^{1/6}}{2}\mu_0^{1/3}F_0^{2/3}}(ds^2(\text{AdS}_4) + e^{2\nu}ds^2(\Sigma_g) + ds^2(\mathcal{M}_4))$$

$$H=\frac{2}{3\mu_0^2+4(1-\mu_0^2)q(\theta)}\qquad q(\theta)=a_+\cos^2\theta+a_-\sin^2\theta$$

## Constant Curvature

$$ds^2(M_4) = \frac{1}{3} \frac{1}{q(\theta)} \frac{d\mu_0^2}{1 - \mu_0^2} + \frac{(1 - \mu_0^2)}{2a_+ a_-} H ds^2(M_3)$$

$$ds^2(M_3) = q(\theta) \left( d\theta - (a_+ + a_-) \frac{\sin(2\theta)}{2q(\theta)} \frac{\mu_0 d\mu_0}{1 - \mu_0^2} \right)^2 + a_- \cos^2(\theta) \eta_+^2 + a_+ \sin^2(\theta) \eta_-^2$$

$$\eta_{\pm} = d\phi_{\pm} - 2(\kappa \pm z)V, \quad \theta \in [0, \pi/2], \quad \mu_0 \in [0, 1]$$

## Free Energy

$$\mathcal{F} = \frac{\pi L_{AdS_4}^2}{2G_4} = \frac{16\pi^3}{(2\pi\ell_s)^8}\int e^{8A - 2\phi} vol(M_6)$$

## Free Energy

$$\mathcal{F}_{g \neq 1} = \frac{8\pi 2(1-g)}{5} \frac{\textcolor{red}{N^{5/2}}}{\kappa n_0^{1/2}} F_z(z, \kappa)$$

$$F_z(z, \kappa) = \frac{(|z^2 - \kappa^2|)^{3/2} (\sqrt{\kappa^2 + 8z^2} - \kappa)}{(14z^2 - \kappa^2 + \kappa\sqrt{\kappa^2 + 8z^2})^{3/2}}$$

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Reproduced by

[Crichigno, Jain, Willett '18] and [Hosseini, Yaakov, Zaffaroni '18]

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