GEOMETRY FROM MODULAR CHAOS

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THE LIFE OF QUANTUM INFORMATION

1

storage

Quantum systems store information in their entanglement pattern, e.g. quantum error correcting codes.

2

propagation

Entanglement assists the efficient transport of information through quantum teleportation.

3

processing

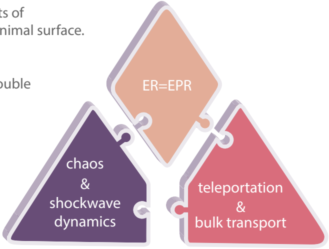
Unitarity and locality impose bounds on the allowed speed of information processing, e.g. maximal chaos bound.

THE GRAVITY-INFORMATION TRIANGLE

1) Features of **entanglement** are mapped to aspects of **geometry**, e.g. Entanglement entropy = Area of minimal surface.

2) The **teleportation** protocol in the thermofield double state of two CFTs holographically **propagates** the excitation through a **bulk traversable wormhole**.

3) The **scrambling** of CFT operators due to **chaotic dynamics** descibes the exponential **blueshifting of the infalling bulk particle** due to the black hole's gravitational well.



THE PLAN

Chaos in the entanglement pattern

I will present a chaos bound characterizing the modular flow of a QFT subregion.

Maximal modular chaos in holography

Saturation of modular chaos bound will allow us to construct bulk $T_{\mu\nu}$ lightray operators from CFT data.

Bulk locality & curvature

Bulk curvature and local Poincare algerba is extracted from chaos commutators.

MODULAR HAMILTONIAN ESSENTIALS I

Consider a state $|\psi\rangle$ and the algebra $\mathcal{A}(R)$ of observables localized in spacetime region R.

The **modular Hamiltonian** H_{mod} is a Hermitian operator generating an automorphism of $\mathcal{A}(R)$.

It encodes the **pattern of entanglement** between degrees of freedom in R and its complement.

In **lattice systems** it is defined as:

$$H_{\text{mod}} = -\log \rho(R) + \log \rho(R^c)$$

H_{mod} is an unbounded operator. However:

$$||e^{iH_{\text{mod}}s} O|\psi\rangle|| < \infty$$

for: $O \in \mathcal{A}(R)$

$$-\frac{1}{2} \leq \text{Im}[s] \leq 0$$

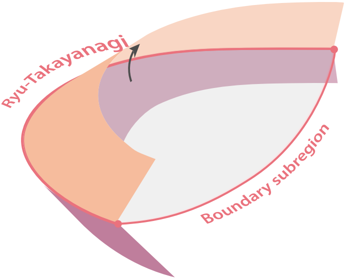
For two subregions $R_1 \subset R_2$ one can prove that:

1) $H_{\text{mod}}(R_1) \leq H_{\text{mod}}(R_2)$

2) $||e^{-iH_{\text{mod}}(R_2)s} e^{iH_{\text{mod}}(R_1)s}|| \leq 1$

in the complex strip: $-\frac{1}{2} \leq \text{Im}[s] \leq 0$

MODULAR HAMILTONIAN ESSENTIALS II



Jafferis-Lewkowycz-Maldacena-Suh:

In holographic CFTs, the modular Hamiltonian of a subregion is equal to the modular Hamiltonian for the bulk degrees of freedom in the entanglement wedge:

$$H_{\rm mod}^{\rm bulk} = H_{\rm mod}^{\rm CFT}$$

In a small neighborhood of the RT surface, generates homogeneous boosts along the orthogonal 2-d plane.

$$H_{\rm mod}^{\rm bulk} \to \epsilon^{\alpha\beta} x_\beta \, \partial_\alpha + \mathcal{O}(x^2)$$

A BOUND ON MODULAR CHAOS

Suppose we infinitesimally perturb the modular Hamiltonian of a subalgebra $H_{\rm mod} \to H_{\rm mod} + \delta H_{\rm mod}$.

This can be done either by a state perturbation $|\psi\rangle \to |\psi\rangle + \delta|\psi\rangle$ or by a shape deformation $R \to R + \delta R$.

The modular chaos bound states that the matrix elements of $\delta H_{\rm mod}(s)$ cannot grow under modular flow faster than exponentially with exponent 2π.

$$\frac{d}{ds} \log |\langle \chi_i | \delta H_{\rm mod}(s) | \chi_j \rangle| \leq 2\pi \quad \text{for:} \quad s \gg 1$$

where: $\delta H_{\rm mod}(s) = e^{-iH_{\rm mod}s} \delta H_{\rm mod} e^{iH_{\rm mod}s}$

MODULAR SCRAMBLING MODES

The modular chaos bound picks out a special set of **operators that saturate** the allowed exponential growth.

They can be extracted from the **large modular parameter limit** of $\delta H_{\mathrm{mod}}(s)$

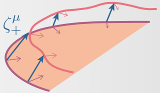
$$G_+ = \frac{1}{2\pi} \lim_{s \to +\Lambda} e^{-2\pi s} \delta H_{\mathrm{mod}}(s)$$

$$G_- = -\frac{1}{2\pi} \lim_{s \to -\Lambda} e^{2\pi s} \delta H_{\mathrm{mod}}(s)$$

In holographic theories, $\Lambda \sim \log N^2$ and the modular scrambling modes are stress-tensor lightray operators

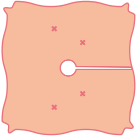
$$G_\pm = \int_{-\ell_K}^{\ell_K} dx^\pm \int_{RT} d^d y \sqrt{\gamma}\, \zeta^\mu_{(\pm)}(y^i, x^\pm)\, T_{\mu\pm}$$

generating parallel transport of the RT surface.

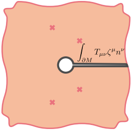


ROAD MAP OF THE DERIVATION

$\rho^{(R)} \propto$



$\delta\rho^{(R)} \propto$



$$\delta H^{(R)}_{\text{mod}} = -\delta \log \rho^{(R)}$$

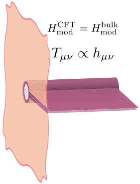
$$= \lim_{b\to 0} \int_{-\infty}^{+\infty} \frac{ds}{4\sinh^2(\frac{s+i\epsilon}{2})} \int_{\partial M} \zeta^\mu n^\nu e^{-iH_{\text{mod}}s} : T_{\mu\nu} : e^{iH_{\text{mod}}s}$$

Euclidean path integral for density matrix

Variation of ρ = $T_{\mu\nu}$ insertion along the cut

Baker-Campbell-Hausdorff

ROAD MAP
OF THE DERIVATION



$$\delta H_{\mathrm{mod}}^{(R)} = \int_0^{+\infty} ds\, e^{-2\pi s}\, e^{-iH_{\mathrm{mod}}s} T_{+\mu}\zeta^{\mu} e^{iH_{\mathrm{mod}}s} - \int_{-\infty}^{0} ds\, e^{2\pi s}\, e^{-iH_{\mathrm{mod}}s} T_{-\mu}\zeta^{\mu} e^{iH_{\mathrm{mod}}s}$$

$$= \int_0^{\ell_K} dx^{+} T_{+\mu}\zeta^{\mu}_{(+)} - \int_{-\ell_K}^{0} dx^{-} T_{-\mu}\zeta^{\mu}_{(-)} + \text{non local}$$

$$e^{-2\pi|s|}\delta H_{\mathrm{mod}}(s) \sim \int_{-\ell_K}^{\ell_K} dx^{\pm} T_{\pm\mu}\zeta^{\mu}_{(\pm)}$$

where: $\ell_K \sim 1/|K_{ij|\pm}|$

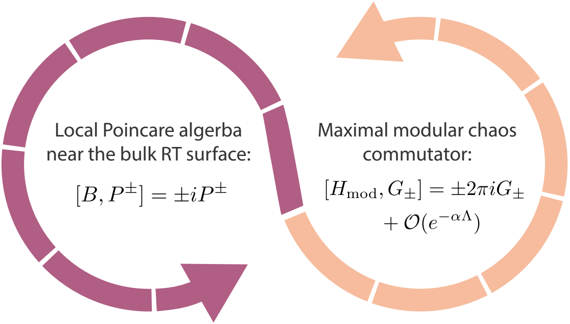
the non-locality scale of the bulk modular H.

Holographic dictionary & gravitational "Gauss' law"

Split local and non-local part of modular integral

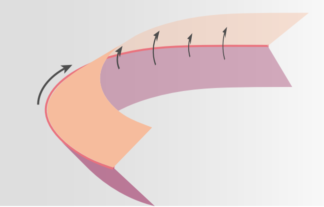
Extract maximally scrambling piece

POINCARE FROM LYAPUNOV



CURVATURE FROM CHAOS COMMUTATORS

$$[G_+, G_-] \in \text{zero mode algebra} \to \text{large diffeo } \xi^\mu$$



The commutator of scrambling modes is a modular zero mode:

$$[H_{\text{mod}}, [G_+, G_-]] = 0$$

In holographic theories it is an element of the RT surface symmetry algebra that preserve the surface and commute with modular boosts:

$$\xi^\mu \partial \mu = \omega(y) \epsilon^{\alpha\beta} x_\beta \, \partial_\alpha + \zeta^i(y) \partial_i + \mathcal{O}(x^2)$$

The Noether charge of the large diffeomorphism generated by the commutator of the scrambling modes associated to two infinitesimal shape deformations of the modular Hamiltonian measures the bulk curvature:

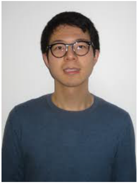
$$Q_{\text{Noether}} = \int_{RT} \sqrt{\gamma} \, \epsilon_{\alpha\beta} \delta x_1^\alpha \delta x_2^\mu \Big(R^\alpha_\alpha + K_{\alpha|ij} K^{\alpha|ij} \Big)$$

OPEN QUESTIONS

- Can the modular chaos bound be proved in full generallity?
- Gravitation as effective theory for scrambling modes?
- Is there a more local version of the story in the bulk?
  - Is modular chaos the right principle for extracting the local Poincare symmetry algebra in an observer's neighborhood?
  - Can we use it to translate ourselves inside a black hole?

IN COLLABORATION WITH

Jan de Boer & L.L. *in progress*



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A Modular Sewing Kit for
Entanglement Wedges
arXiv: 1903.04493