

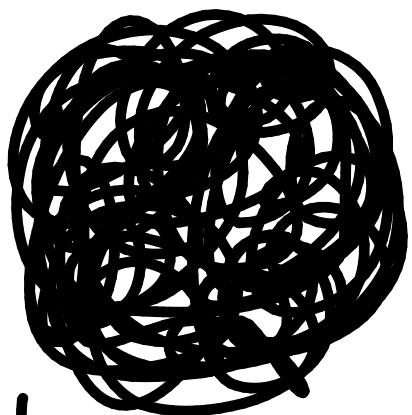
Unitarity and Entropy

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× Gute Regional Meeting
in String Theory

Sep. 15, 2019

Bekenstein - Bremerman Bound



$$I \leq R \leq |$$

$$S \leq M R$$

In gravity is
saturated by black holes.
Bekenstein entropy:

$$S_{BH} = M_{Pl} R = (RM_p)^2$$

* What is physical meaning of bound?

* What is physical meaning of Area-Law?


$$S = (R M_p)^2$$

The bound has (mostly)
been discussed in gravity.

What happens beyond
gravity?

In renormalizable
theories?

Our main results:

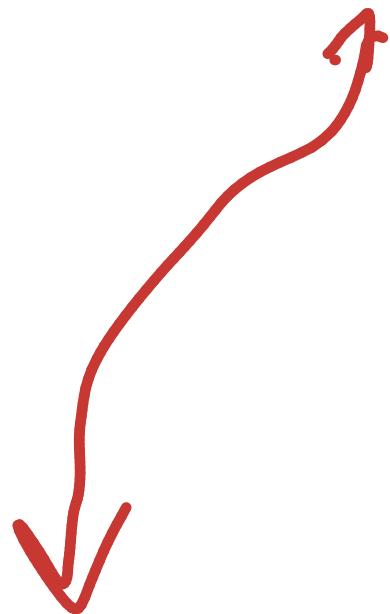
* Bekenstein bound is saturated when theory saturates the bound on unitarity.

* Simultaneously the entropy assumes the area law:

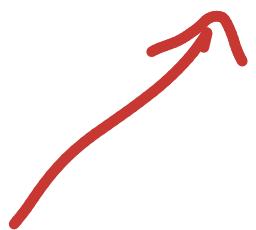
$$S = M_R = (R_f)^2$$

\uparrow scale

Bekenstein = Unitality = Area



$$S_{\max} = \frac{1}{g^2} = \text{Area}$$



Quantum
Coupling

We shall demonstrate

for:

① 't Hooft - Polyakov

monopole:

$$S = M_m R_m = \frac{1}{g_{\text{gauge}}^2} = (R_m b)^2$$

② Baryon:

$$S = M_B R_B = \frac{1}{g_{\text{QCD}}^2} = (R_B f_\pi)^2$$

③ QCD-Instanton

$$S_{\text{inst}} = \frac{1}{g_{\text{QCD}}^2} = (R_{\text{inst}})^2$$

In all cases at the saturation point

$$S = \frac{\text{Area}}{4G}$$

Goldstone coupling.

Monopole. $SO(3)$ gauge symmetry Higgsed by

$$\phi^\alpha \quad (\alpha = 1, 2, 3)$$

$$\mathcal{L} = \partial_\mu \phi^\alpha \partial^\mu \phi^\alpha$$

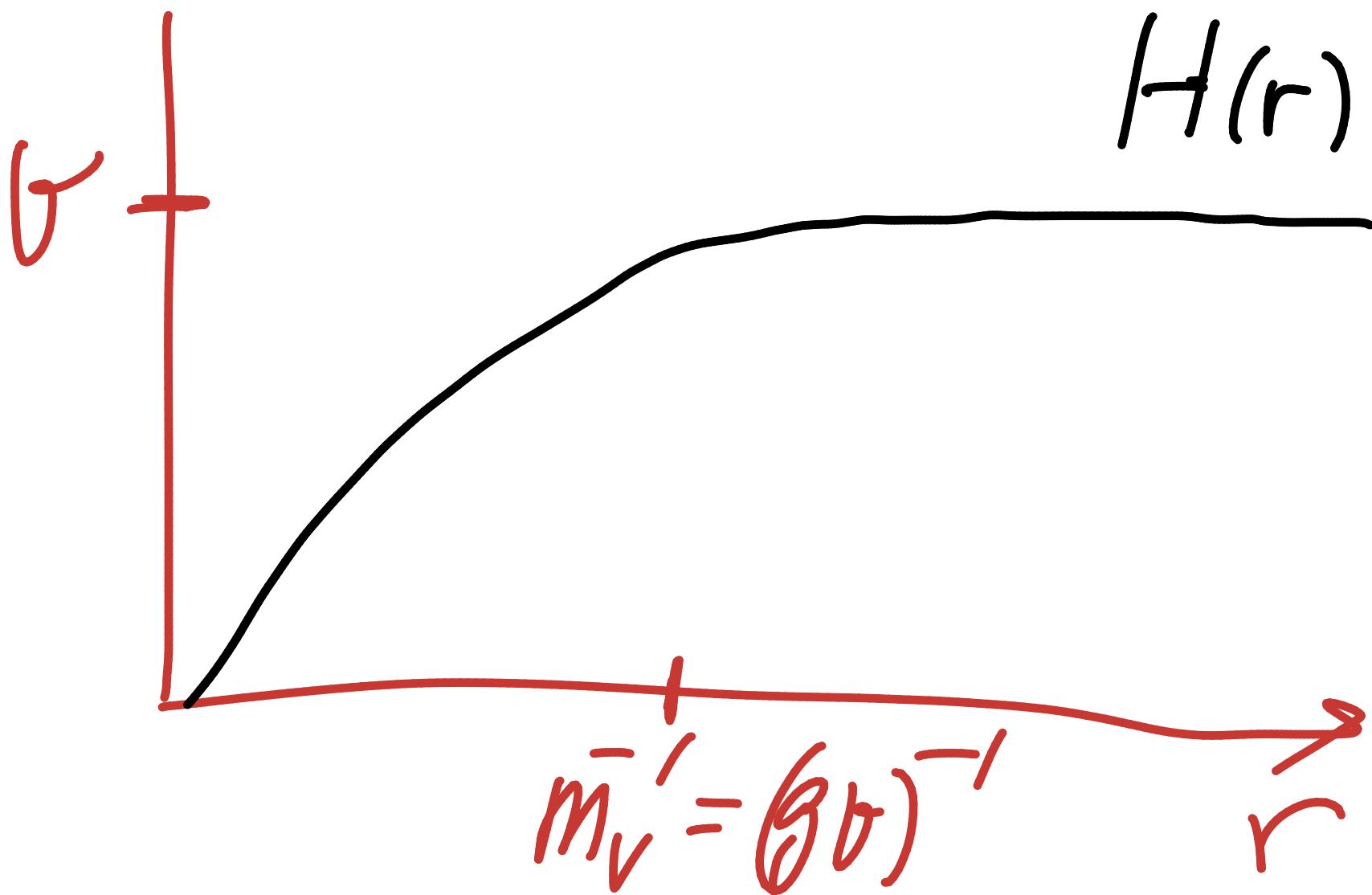
$$- \lambda^2 \left(\phi^\alpha \phi^\alpha - V^2 \right)^2$$

$$- F_{\mu\nu}^a \tilde{F}^{a\mu\nu}$$

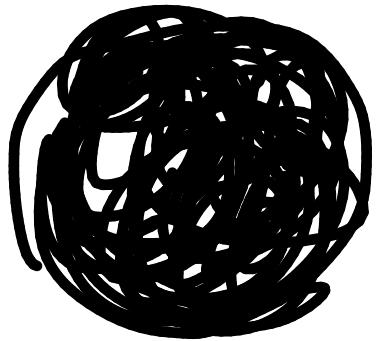
Monopole:

$$\phi^a = \frac{x^a}{r} H(r)$$

$$A_\mu^a = \epsilon_{0a\mu\nu} \frac{x^\nu}{gr^2} F(r)$$



Monopole mass and size



$$R_m = (M_V)^{-1} = (g_V)^{-1}$$

$$\ll R_m \quad M_m = \frac{M_V}{g^2}$$

Entropy bound on monopole:

$$S' \leq M_m R_m = \frac{1}{g^2}$$

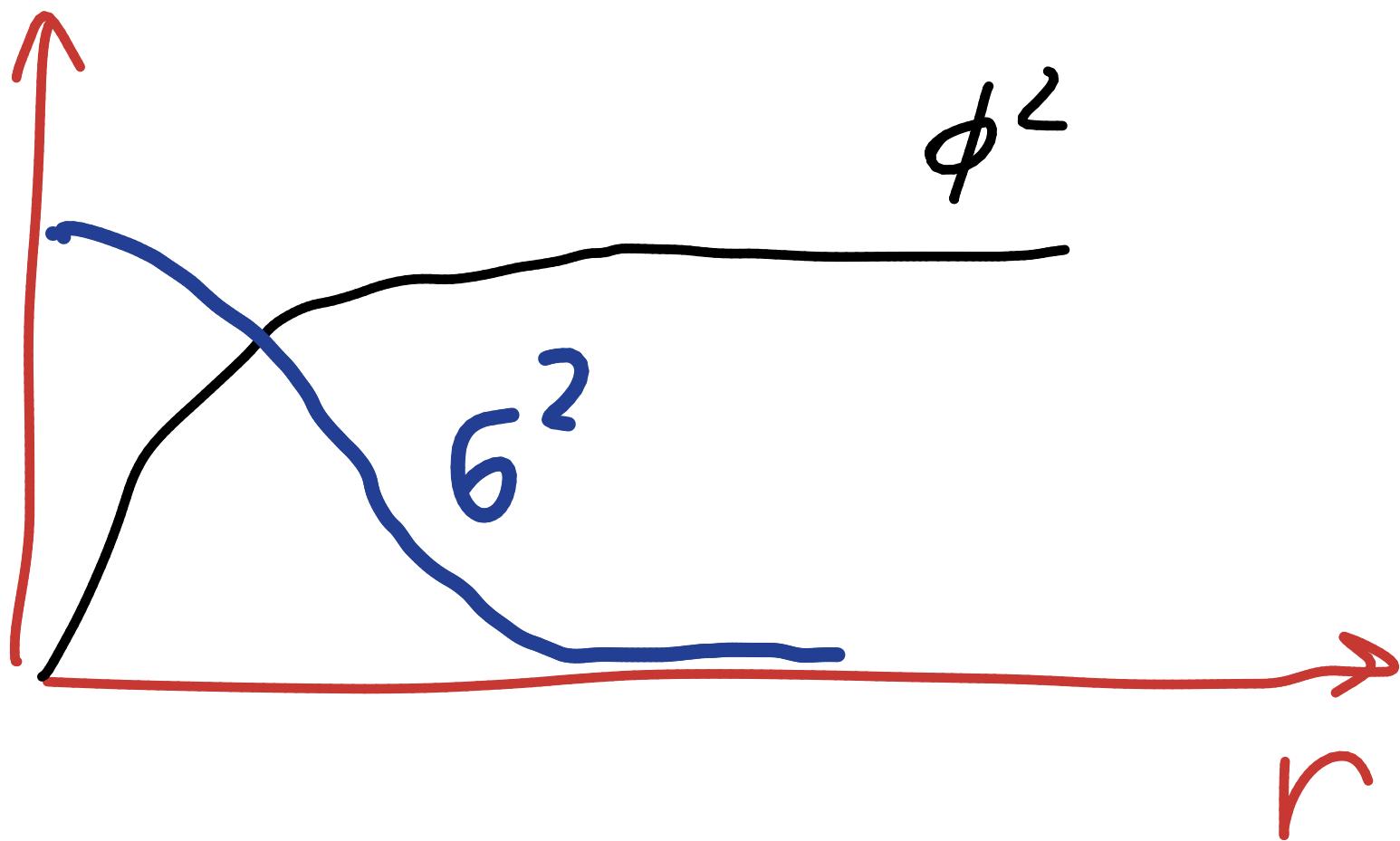
Can it be saturated?

Entropy from Goldstones

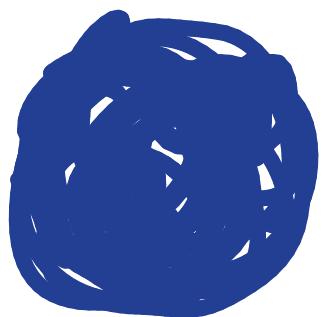
$\zeta_\alpha \quad \zeta = 1, 2, \dots N$
 $\zeta_\alpha \leftarrow \text{scalar}$

$SO(N)$ - global symmetry
spontaneous breaking
is monopole:

$$\begin{aligned} L &= g_1 \zeta_\alpha \partial^\mu \zeta_\alpha \\ &- (g^2 \phi^2 - m^2) \bar{\zeta}_\alpha \zeta_\alpha \\ &- g_5^2 (\zeta_\alpha \zeta_\alpha)^2 \end{aligned}$$



$\downarrow \langle \sigma \rangle \neq 0$



$$SO(N) \rightarrow SO(N-1)$$

$\sim N$ localized
Goldstones!

Number of degenerate
micro-states

$$n_{st} \sim \binom{2N}{N} \sim 2^{2N}$$

Monopole entropy:

$$S_{\text{mon}} = \ln(n_{st}) \sim N$$

Unitarity bound:

$$g^2 N \leq 1$$

For $N = \frac{1}{g^2}$ entropy bound is saturated

by area-law!

$$S_{\text{min}} = N = \frac{1}{g^2} = M_m R_m = (R_m b)^2$$

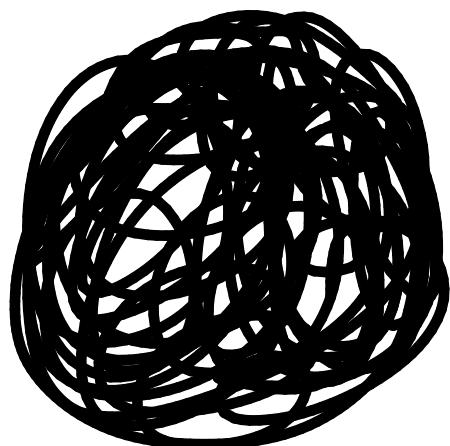
The same is achieved by coupling to fermion flavors:

$$\phi^a \gamma^b \gamma^c$$

$$_{\alpha} \not= \delta^{abc}$$

$$\alpha = 1, 2 \dots N$$

$$SO(N)$$



$\sim N$ localized

fermion zero modes!

Again:

$$n_{st} \sim 2^N$$

Entropy: $S_{mon} \sim \kappa$

Unitarity bound

$$g^2 N \leq 1$$

$$S = N_{mon} = \frac{1}{g^2} = M_m R_m = (R_m g)^2$$

Baryons in $SU(N_c)$
QCD with
 N -flavors of quarks.

't Hooft limit

$$N_c \rightarrow \infty$$

$$g^2 N_c = \text{fixed}$$

$$\Lambda = \text{fixed}$$

Spontaneous chiral symmetry breaking

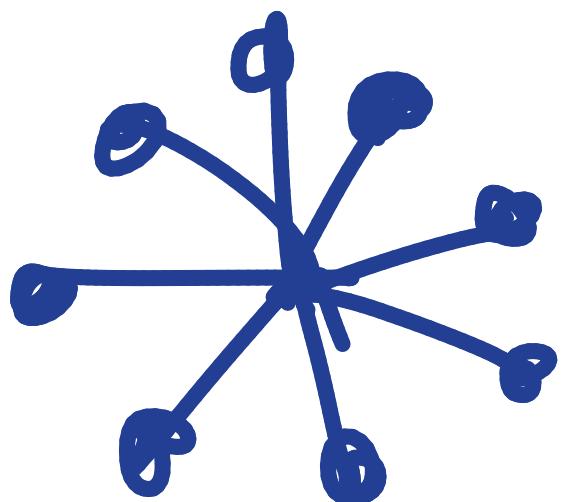
$$U(N)_L \otimes U(N)_R \rightarrow U(N)_F$$

$N^2 - 1$ Goldstones (pions)
+ γ' -meson

Pion decay constant:

$$f_\pi = \sqrt{N_c} \Lambda$$

Baryons (Witten)



N_c quarks

$$[\leftarrow R_B \sim \bar{\Delta}^{-1} \rightarrow]$$

Mass: $M_B \sim N_c$

Entropy bound:

$$S_{MAX} = M_B R_B = N_c$$

I_S saturated at
the unitarity bound:

$$N_c \sim N \sim \frac{1}{g^2}$$

Baryon entropy:

$$S_B = \ln \left(\frac{N_c + N}{N_c} \right) \sim N$$

Thus, at the unitarity bound we have:

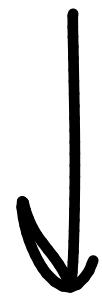
$$S_B = M_B R_B = \frac{1}{g^2} = N = \\ = (R_B f_\pi)^2$$

Area law,

Conclusions:

The universal phenomenon:

$$B.P. = \text{unitarity} = \text{Area}$$



$$\text{Bound} = \frac{1}{\text{coupling}} = \text{Area}$$

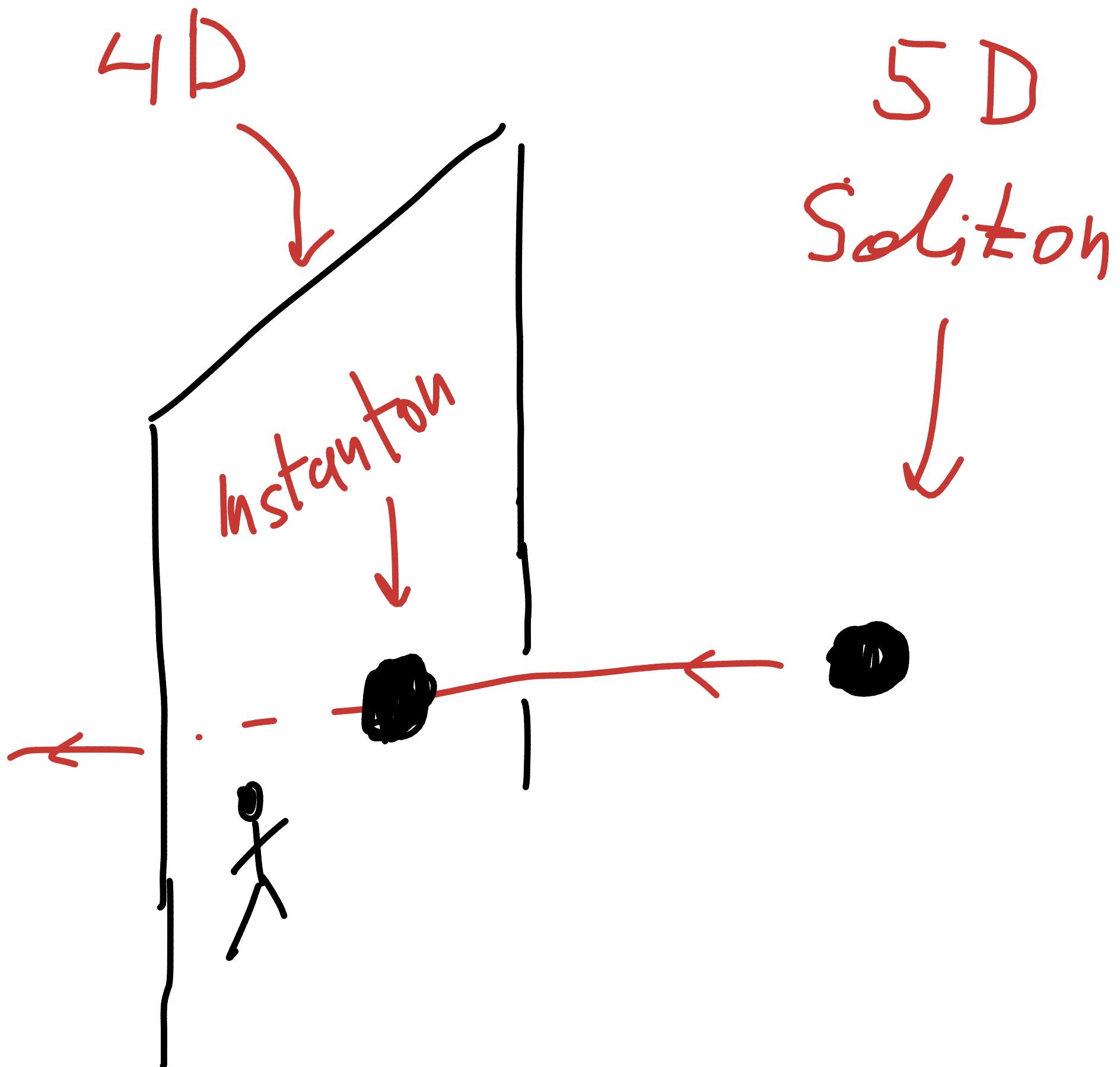
Instances

I shall assign entropy
to an instanton:

D-instanton \longleftrightarrow D+1-soliton

process \downarrow object

$S_{\text{inst}} \longleftrightarrow S_{\text{soliton}}$



Instanton = tunnelling
soliton

Example:

QCD-instanton = 5D-monopole

$$L_5 = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

$$\text{Monopole: } F_{\mu\nu}^a = -\frac{4\eta^a}{g} \frac{R^2}{(x^2 + R^2)^2}$$

$$a = 1, 2, 3 \quad \mu\nu = 0, 1, 2, 3, 4$$

topological charge:

$$Q = \frac{g^2}{32\pi^2} \int \epsilon^{\alpha\beta\gamma\delta} r^\delta F_{\alpha\beta}^a F_{\gamma\delta}^a = 1$$

Mass

$$M_{\text{Mon}} = \frac{8\pi^2}{g^2}$$

Notice: In 5D

$$[g^2] = (\text{Energy})^{-1}$$



Perturbative coupling:

$$\alpha \equiv g^2 E$$

Pert. Unitarity cut off:

$$\Lambda = \frac{1}{g^2}$$

Entropy $SU(2) \subset SU(N)$

tHooft coupling

$$\lambda_t \equiv N \frac{g^2 E}{8\pi^2}$$

Moduli space $\frac{SU(N)}{SO(N-2) \cdot U(1)}$

Number of Bosonic zero
modes (Goldstones)?

$$\simeq 4N$$

Micro-state count.

Effective Hamiltonian:

$$\hat{H} = \chi \left[\sum_{j=1}^{4N} \hat{a}_j^\dagger \cdot \hat{a}_j - N_{\text{mon}} \right]$$

where

$$N_{\text{mon}} = RQ = \frac{R 32 \pi^2}{g^2}$$

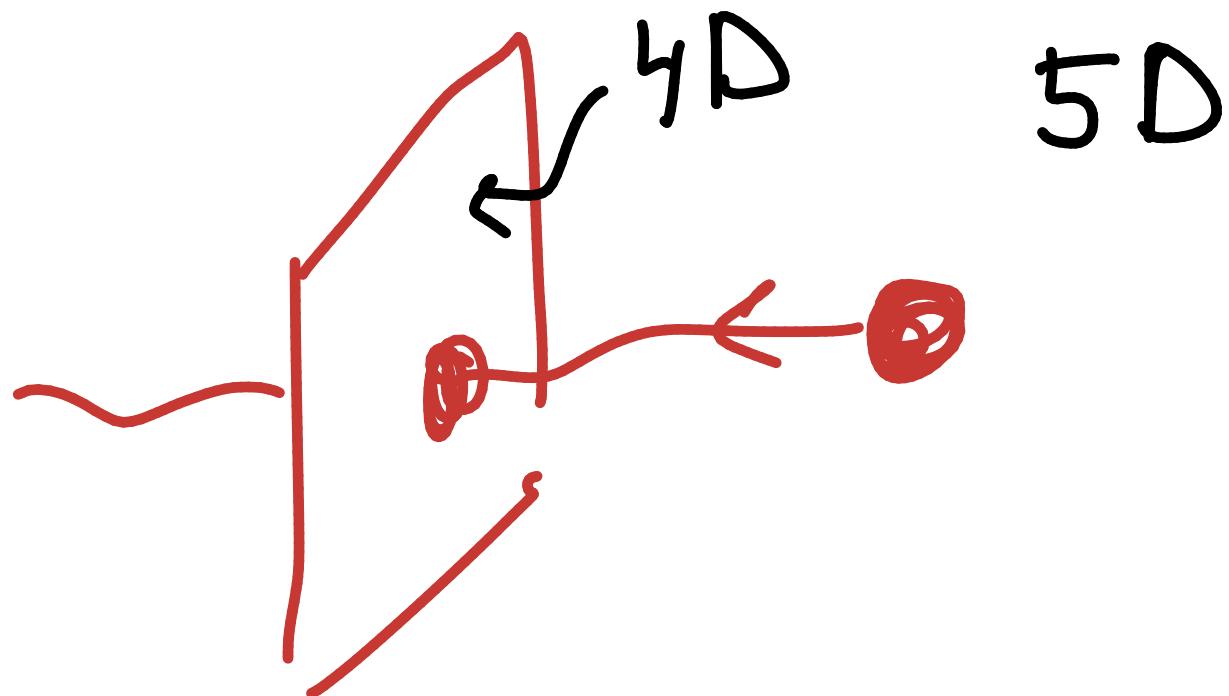
Then, number of micro-states:

$$N_{\text{st}} \approx \binom{N_{\text{mon}} + 4N}{N_{\text{mon}}}$$

Notice, for $\lambda_t \ll 1$

$$h_{st} \sim \frac{1}{(N!)^4} \left(\frac{8\pi^2 R}{g^2} e^{\lambda_t} \right)^{4N}$$

Matches 4D - Instanton
transition rate!



Bekenstein Bound is
saturated for $\lambda_t \approx 1$:

$$S_{\text{mon}} = k \ln(N_{\text{st}}) \approx \frac{64\pi^2}{g^2} R \ln(2)$$

$$\approx 1.3 \left(\frac{4\pi}{3} M_{\text{mon}} \cdot R \right)$$

$$S_{\max}$$

Thus, $S_{\text{mon}}(\lambda_t=1) = S_{\text{max}}$.

And entropy = Area

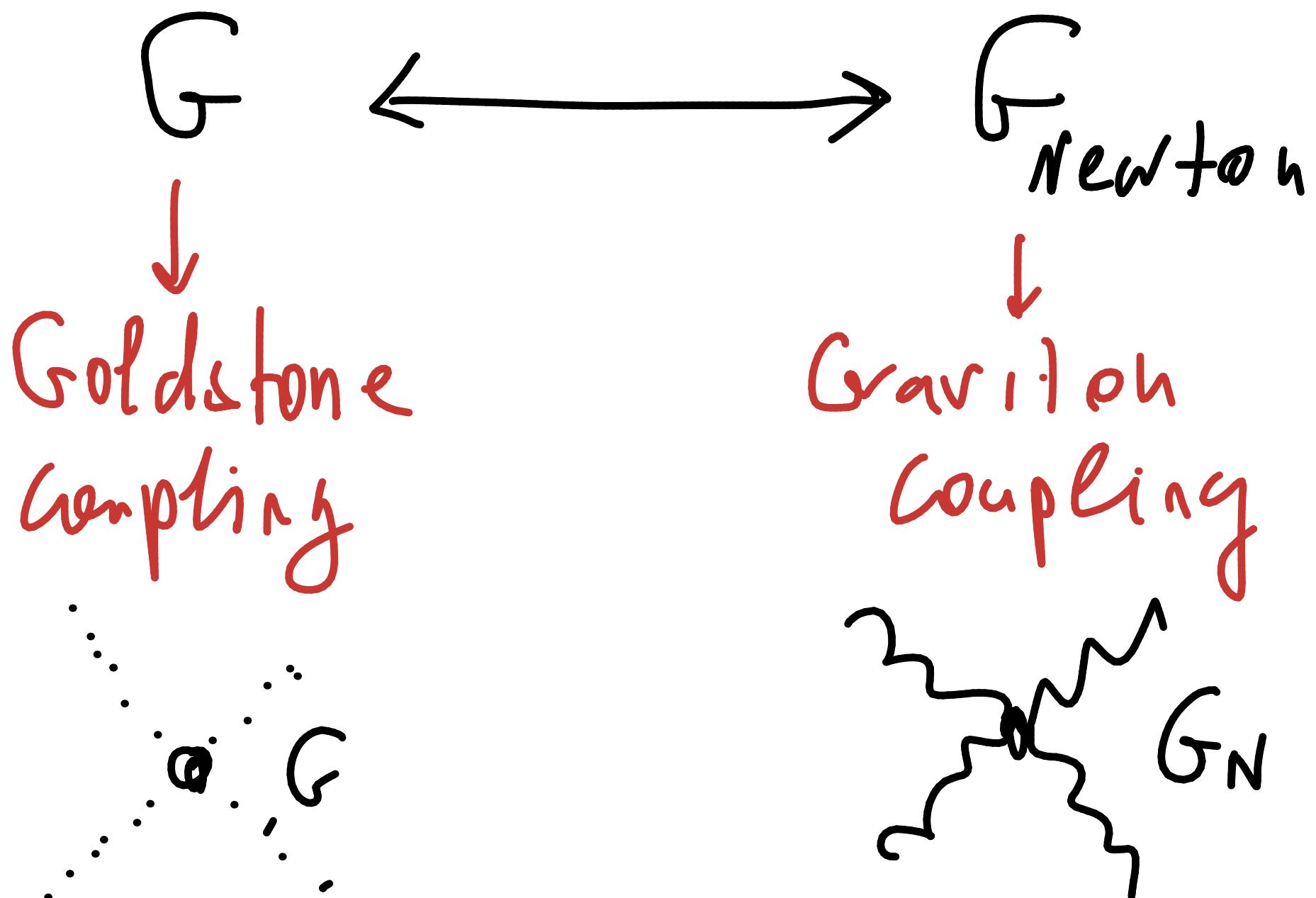
$$S_{\text{mon}} = \frac{A}{4G} = \frac{k\pi^2 R^3}{4G}$$

$$G \equiv \frac{3}{64\pi} f^2 R^2 \equiv f^{-3}$$

Goldstone coupling.

$$S_{\text{M}\ell\ell} = \frac{k}{4G}$$

Exactly as for Black
Hole!



Exactly same results
for Instantons in

4D QCD :

$$\alpha_5 = \frac{R}{g^2} \quad \longleftrightarrow \quad \alpha_{QCD} = \frac{1}{g^2(R)}$$

Instanton entropy

Saturates bound for $\lambda_T = 1$

$$S_{\text{ins.}} = \frac{64\pi^2}{g^2} \ln(2)$$

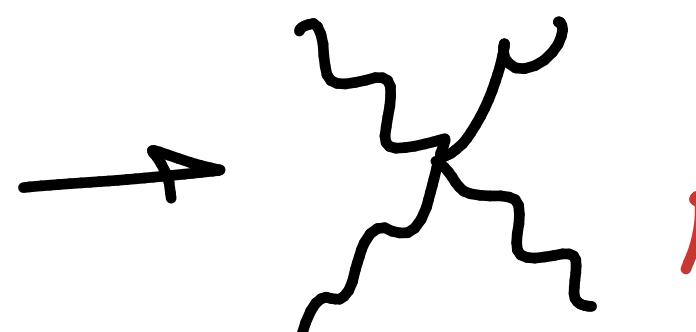
And at the saturation point we get the Area!

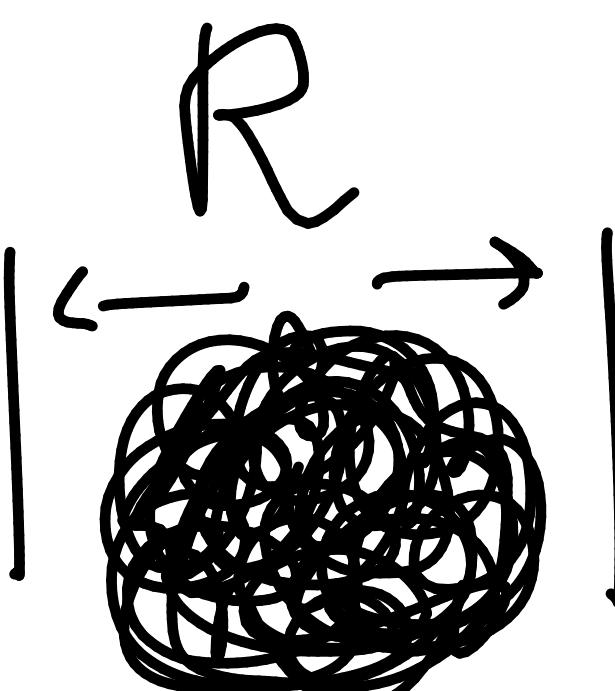
$$S_{\text{Inst}} = \frac{\pi}{4G} = \frac{(4\pi R^2)}{4f^{-2}}$$

$$G = \frac{g^2 R^2}{64\pi \hbar c} = f^{-2}$$

↑ Goldstone coupling!

Conclusions

In a theory with 4-point coupling λ → 

a non-perturbative entity
of size R 

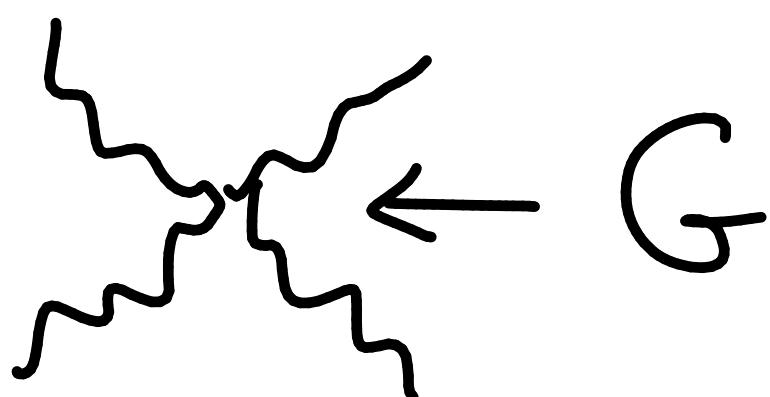
Saturates Bekenstein
entropy bound in the following
way →

* When theory saturates perturbative unitarity.

* At saturation point:

$$P_{\max} = \frac{\text{Area}}{4G} = \frac{1}{\alpha(R)}$$

4-point coupling of
bosons (pions, gravitons, . . .)



* It is meaningful
to talk about instanton
entropy. At instanton
saturates entropy bound
for $\lambda_L = 1$ and

$$S_{\text{inst}} = \frac{1}{\alpha} = \frac{\text{Area}}{4G}$$

 Oldstone warping

Can confinement in QCD (at least for large N_c) be understood as preventive mechanism against violation of Bekenstein bound by free colored states?

Entropy of a free quark

$$S_Q = \ln(2N_c N_f) \quad ?$$

$$S_{\max} = M_Q R \quad ?$$