N=8 no-scale models

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A fruitful and enjoyable collaboration

(started many kilograms ago, for both of us)

12 papers (with > 1250 citations) in 1987-2005

continuing friendship and scientific interactions

Main theme of our collaboration: effective supergravities from string compactifications

Aspects touched: susy breaking, mass sum rules, string threshold effects & dualities, gaugings from fluxes, dynamical determination of parameters

N=1 no-scale models: virtues/problems

[Cremmer-Ferrara-Kounnas-Nanopoulos 193; Ellis-Lahanas-N-Tamvakis 1984; EKN 1984; ...; Polchinski 1985; ...; K-Pavel-FZ 1994; ... ; ATLAS and CMS collaborations 2012]

- $V_0 \geq 0$ in a finite region of field space with broken SUSY
  ➔ classical Minkowski background without fine-tuning

- Susy-breaking scale classically flat direction: might generate $v \sim m_{susy} << M_P$ by perturbative quantum corrections, hopefully with a realistic $\langle V \rangle \sim v^8/M_P^4$

- UV sensitivity makes hierarchies uncalculable

  $\mathrm{Str}\, M^2 \neq 0$ ➔ quadratic ➔ v & $\langle V \rangle$

  $\mathrm{Str}\, M^4 \neq 0$ ➔ logarithmic ➔ $\langle V \rangle$

- Increasing tension of $m_{susy} \sim v$ with LHC data

N=8 no-scale models

N=1 no-scale sugra not enough to explain hierarchies
Must look for some extra, crucial missing ingredient:
in superstrings we have not been able to identify it yet

In the absence of better ideas, study the most constrained no-scale models, those with N=8, even if not realistic, to understand first what are their calculable properties

Will describe in the following some recent results from:

- G.Dall'Agata, F.Z., JHEP09(2012)078 [ArXiV:1205.4711]
- F.Catino, G.D., G.Inverso, F.Z., paper to appear soon

N=8 supergravity: field content

unique multiplet: $2^8 = 256 = 128_B + 128_F$ d.o.f.

$\|+2\rangle$:	1 graviton
$\|+3/2, i\rangle = Q_i \|+2\rangle$:	8 gravitini
$\|+1, [ij]\rangle = Q_i Q_j \|+2\rangle$:	28 vectors
$\|+1/2, [ijk]\rangle = Q_i Q_j Q_k \|+2\rangle$:	56 fermions
$\|0, [ijkl]\rangle = Q_i Q_j Q_k Q_l \|+2\rangle$:	70 scalars

and similarly for the CPT conjugates of negative helicity, ending with $|-2\rangle$

The ungauged theory

[De Wit-Freedman 1977; Cremmer-Julia 1978+1979]

Scalar fields parameterize $E_{7(7)}/SU(8)$ manifold

$E_{7(7)}$ duality group, invariance of B.I. & E.O.M.

Gauge group $U(1)^{28}$, no charged fields

Minkowski background with exact N=8
is solution of the classical E.O.M.
Vanishing potential, all fields massless

Remarkable UV properties: on-shell finiteness up to 4 loops, perhaps to all perturbative orders?

The gauged theories

A subgroup G of $E_{7(7)}$ is made local (dim ≤ 28)

Theory fully determined by embedding tensor

[DeWit-Samtleben-Trigiante 2003+2007]

gauge generators → $X_M = \Theta_M{}^\alpha t_\alpha$ ← E_7 generators

M=1,…,56 counts electric and magnetic vectors

$$[X_M, X_N] = -X_{MN}{}^P X_P \qquad X_{MN}{}^P = \Theta_M{}^\alpha (t_\alpha)_N{}^P$$

Supersymmetry → linear constraints on Θ

Gauge invariance → quadratic constraints on Θ

Effects of gauging

Gauge coupling g deformation parameter:

$$\partial_\mu \longrightarrow D_\mu \equiv \partial_\mu - g\, A_\mu^M X_M$$

Scalar potential and mass terms are generated

Possibility of partial or total SUSY breaking

Critical points with

positive, zero or negative energy

but no locally stable dS vacuum found so far

The Cremmer-Scherk-Schwarz gauging

Until 2011, the only known explicit gauging leading to classically stable N=0 Minkowski vacua was the one found by Cremmer-Scherk-Schwarz (1979):

- Positive semidefinite potential (no-scale model)
- Gauge group $U(1) \ltimes T^{24}$ broken to U(1) [xU(1)^3]
- Four independent mass parameters:
  U(1) charge matrices in Sp(8,R) [Ferrara-Zumino 1979]
- Str M^2 = Str M^4 = Str M^6 = 0 , Str $M^8 \neq 0$
- One-loop finite, $V_1 < 0$ [Sezgin-Van Nieuwenhuizen 1982]
- Flux compactification of D=11 sugra with geometrical and non-geometrical fluxes
  [Scherk-Schwarz 1979; FC-GD-GI-FZ 2012, to appear]

The focus of our study

Consider gaugings leading to classical Minkowski vacua
especially those with fully broken (N=0) supersymmetry

Are there other candidates besides the CSS gauging and vacua?
If so, what are the common features of different possible choices?

Study the 1-loop corrections to the theory
around the classical N=0 Minkowski vacua
(no arguments from ungauged theory):

1-loop finiteness? 1-loop stable Mink or dS vacua?

A general result [GD-FZ, 2012]

At **any** classical Minkowski vacuum of gauged N=8

$$Str\, M^0 = Str\, M^2 = Str\, M^4 = 0$$

Skip here all technicalities. Our proof makes use of:

1. Critical point condition
2. Vanishing vacuum energy
3. Quadratic constraints

Implication: one-loop effective potential V_1 is finite

$$V_1 = \frac{1}{64\pi^2} Str\left(M^4 \log M^2\right)$$

(provided that no tachyons in classical spectrum) and scales in moduli space as classical potential

A recent development [GD-GI 2011]

New way of generating gaugings and vacua, including N=0 Minkowski, but also others

- Exploit duality transformations to stay at the "origin" of field space

- Solve simple quadratic constraints on Θ to identify consistent gaugings, gauge group, vacuum energy, mass spectrum

An important application [FC-GD-GI-FZ to appear]

Consider the gauging+vacuum (no parameters)

SO(6,2) ➔ SO(6) x SO(2)

In a N=1 truncation, this corresponds to :

$$K = -\log Y \qquad Y = (S + S)\prod_{A=1}^{3}(T_A + \overline{T_A})\prod_{A=1}^{3}(U_A + \overline{U_A})$$

W = -1 - S T1 T2 T3 + i S T1 U1 + i T2 T3 U1 + i S T2 U2 + i T1 T3 U2 + T1 T2 U1 U2 + S T3 U1 U2 + i T1 T2 U3 + i S T3 U3 + S T2 U1 U3 + T1 T3 U1 U3 + S T1 U2 U3 + T2 T3 U2 U3 - i U1 U2 U3 - i S T1 T2 T3 U1 U2 U3

$\langle S\rangle = \langle T_1\rangle = \langle T_2\rangle = \langle T_3\rangle = \langle U_1\rangle = \langle U_2\rangle = \langle U_3\rangle = 1$

An important application [FC-GD-GI-FZ to appear]

3 complex flat directions, parameterizing $[SU(1,1)/U(1)]^3$

In the N=1 truncation, they are (U_1,U_2,U_3)

In N=8: 6 real **moduli** $(x_1,x_2,x_3, e_1,e_2,e_3)$

(+ overall scaling modulus absorbable in g)

All known gaugings with Minkowski vacua:

(singular) limits in this moduli space.

Example: CSS with 4 parameters is $(x_1,x_2,x_3) \rightarrow 0$, g $\rightarrow$ 0

gravitino mass ratios (e_1,e_2,e_3) freeze into **parameters**

Generic features emerging
(in models built so far)
[GD-FZ 2012; FC-GD-GI-FZ to appear]

- Possible tachyonic instabilities along flat directions, but finite regions of stability
- Minimal unbroken gauge group $U(1)^4$ [in SU(8)]
- At least 6 massless scalars in the spectrum (4 would-be Goldstone + 2 additional ones)
- Fermion masses (3/2 & ½) are all Dirac-like: maximum of 4 gravitino mass eigenvalues

An intriguing pattern of the spectrum

Supercharges Q_i (i=1,...,8) transform non-trivially under at most 4 unbroken U(1) factors in SU(8)

Charge vectors: $\vec{q}_i \equiv (q_i^1, \ldots, q_i^n)$

8 supercharges Q_i always come in pairs:

$$\vec{q}_1 = -\vec{q}_2 \quad \vec{q}_3 = -\vec{q}_4 \quad \vec{q}_5 = -\vec{q}_6 \quad \vec{q}_7 = -\vec{q}_8$$

Neutral graviton |±2>: $\sum_{i=1}^{8} \vec{q}_i = \vec{0}$

Charges of all other states fully determined:

|±3/2,i>: $\pm q_i$ |±1,[ij]>: $\pm(q_i+q_j)$

|±1/2,[ijk]>: $\pm(q_i+q_i+q_k)$ |0,[ijkl]>: $q_i+q_j+q_k+q_l$

Spectrum controlled by U(1) charges

$$
\begin{aligned}
|2\rangle : \quad & M^2 = 0 \,, \\
|3/2\,,\, i\rangle : \quad & M_i^2 = (\vec{q}_i)^2 \,, \\
|1\,,\, [ij]\rangle : \quad & M_{ij}^2 = (\vec{q}_i + \vec{q}_j)^2 \,, \\
|1/2\,,\, [ijk]\rangle : \quad & M_{ijk}^2 = (\vec{q}_i + \vec{q}_j + \vec{q}_k)^2 \,, \\
|0\,,\, [ijkl]\rangle : \quad & M_{ijkl}^2 = (\vec{q}_i + \vec{q}_j + \vec{q}_k + \vec{q}_l)^2
\end{aligned}
$$

Scalar products of (field-dep.) charge vectors taken with suitable **field-dependent real diagonal metric**:

$$
\vec{q}_i \cdot \vec{q}_j = \sum_{A=1}^{n} q_i^A \, q_j^A \, \mu_A^2
$$

Not necessarily positive definite (when so, absence of tachyons in the classical spectrum guaranteed)

Example 1: CSS gauging [single unbroken U(1)]

$$q_1 = -q_2 = e_1\,, \quad q_3 = -q_4 = e_2\,, \quad q_5 = -q_6 = e_3\,, \quad q_7 = -q_8 = e_4$$

$\mu^2 = \phi^2$ (universal modulus giving scale of all masses)

Example 2: SO(6,2) gauging [unbroken $U(1)^4$]

$$\vec{q}_1 = -\vec{q}_2 = (+1,+1,+1,+1)\,, \quad \vec{q}_3 = -\vec{q}_4 = (+1,+1,-1,-1)\,,$$
$$\vec{q}_5 = -\vec{q}_6 = (+1,-1,+1,-1)\,, \quad \vec{q}_7 = -\vec{q}_8 = (+1,-1,-1,+1)\,,$$

Choosing some of the parameters to avoid tachyons:

$$\mu_1^2 = \frac{(x-y)^2(1+x^2y^2)}{8x^2y^2}\,, \quad \mu_2^2 = \mu_3^2 = 0\,, \quad \mu_4^2 = \frac{(1+x^2y^2)(1+xy^3)^2}{8x^2y^4}$$

[now unbroken $U(1)^2$ x SO(4) and dilaton-like modulus]

Now generalized into a single formalism:

$$\vec{q}_1 = -\vec{q}_2 = m_1(+1,+1,+1,+1)\,, \qquad \vec{q}_3 = -\vec{q}_4 = m_2(+1,+1,-1,-1)\,,$$
$$\vec{q}_5 = -\vec{q}_6 = m_3(+1,-1,+1,-1)\,, \qquad \vec{q}_7 = -\vec{q}_8 = m_4(+1,-1,-1,+1)\,,$$

$$m_1 = g\frac{e_2 e_3}{e_1}\,, \quad m_2 = g\frac{e_1 e_3}{e_2}\,, \quad m_3 = g\frac{e_1 e_2}{e_3}\,, \quad m_4 = g\frac{1}{e_1 e_2 e_3}\,.$$

$$\mu_1^2 = \frac{(x-y)(x-z)(1+x^2yz)}{8x^2yz}\,, \qquad \mu_2^2 = \frac{(y-x)(y-z)(1+xy^2z)}{8xy^2z}\,,$$
$$\mu_3^2 = \frac{(x-z)(y-z)(1+xyz^2)}{8xyz^2}\,, \qquad \mu_4^2 = \frac{(1+x^2yz)(1+xy^2z)(1+xyz^2)}{8x^2y^2z^2}\,.$$

$$x = \frac{x_2 x_3}{x_1}\,, \quad y = \frac{x_1 x_3}{x_2}\,, \quad z = \frac{x_1 x_2}{x_3}\,.$$

Some important consequences

(not valid in general, but for all classical N=0 Minkowski vacua identified so far)

$$Str\, M^6 = 0$$

$$Str\, M^8 = 40320 \sum_{A=1}^{n} \left(\prod_{i=1}^{8} q_i^A \right) \mu_A^8 > 0$$

Empirically, this seems to imply that the one-loop effective potential V_1 is negative definite:

$$V_1 < 0$$

no stable N=0 Minkowski or dS vacua at 1 loop

Summary of conclusions

- Quadratic and quartic supertraces vanish at any classical Minkowski vacuum → **finite one-loop effective potential**

- All known gaugings leading to N=0 Minkowski vacua are connected with SO(6,2) and have always an **unbroken $U(1)^4$ spectrum fixed by 6 moduli via related charges + metric**

- As a consequence, **$\mathrm{Str}\, M^6 = 0$** and **$\mathrm{Str}\, M^8 > 0$**

- In turn, this implies a **negative definite 1-loop potential**,

$$V_1 < 0$$

no locally stable 1-loop Mink or dS vacuum found

Open questions and outlook

- **More N=0 Minkowski or any dS classical vacua?**
- General proof of **unbroken $U(1)^4$** and of **relation between classical spectrum and U(1) charges** (or counterexamples)?
- General proof of the model-dependent results on **$\mathrm{Str}\, M^6$ and $\mathrm{Str}\, M^8$** (or counterexamples)?
- General proof of **relation between $\mathrm{Str}\, M^8 > 0$ and $V_1 < 0$**?
- **Extensions to generalized flux compactifications?**
- **Nature of the obstructions to locally stable dS vacua?**

Happy Birthday Costas!

(also from Giuditta, Alessandro and Federico)