SUPERFIELDS
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Supergravity, black holes and E_7

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Many publications with Costas

	citations
E. Cremmer, S. Ferrara, C. Kounnas and D.V. Nanopoulos, *Naturally Vanishing Cosmological Constant in N=1 Supergravity* Phys. Lett. B133 (1983) 61	532
J.P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, *On loop corrections to string effective field theories: Field dependent gauge couplings and sigma model anomalies* Nucl. Phys. B372 (1992) 145	309
E. Cremmer, C. Kounnas, A. Van Proeyen, J.P. Derendinger, S. Ferrara, B. de Wit and L. Girardello, *Vector Multiplets Coupled to N=2 Supergravity: SuperHiggs Effect, Flat Potentials and Geometric Structure* Nucl. Phys. B250 (1985) 385	263
S. Ferrara, C. Kounnas and M. Porrati, *General Dimensional Reduction of Ten-Dimensional Supergravity and Superstring* Phys. Lett. B181 (1986) 263	230

citations

S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner,
Superstrings with Spontaneously Broken Supersymmetry and their Effective Theories 225
Nucl. Phys. B318 (1989) 75

S. Ferrara, C. Kounnas and F. Zwirner,
Mass formulae and natural hierarchy in string effective supergravities 143
Nucl. Phys. B429 (1994) 589

J.P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner,
All loop gauge couplings from anomaly cancellation in string effective theories 122
Phys. Lett. B271 (1991) 307

S. Ferrara, C. Kounnas, D. Lust and F. Zwirner,
Duality invariant partition functions and automorphic superpotentials for (2,2) string compactifications 102
Nucl. Phys. B365 (1991) 431

One paper in particular (1985):

Cremmer, **Kounnas**, Van Proeyen, Derendinger, Ferrara, De Wit, Girardello

Vector multiplets coupled to N=2 Supergravity : Super-BEH (Brout-Englert-Higgs) mechanism study scalar potential arising from "gauging"

Examples of spontaneous breaking of local SUSY with vanishing cosmological constant

"vanishing flat potentials"

Associated sigma model can be obtained from 5D: geometric structure given by cubic F

to

$$\phi(z^A, \bar{z}^A) = \sum_{\alpha,\beta,\gamma=1}^{n} d_{\alpha\beta\gamma}(f^\alpha(z^A) + \bar{f}^\alpha(\bar{z}^A)) \times (f^\beta(z^B) + \bar{f}^\beta(\bar{z}^B))(f^\gamma(z^C) + \bar{f}^\gamma(\bar{z}^C)) , \quad (5.12)$$

provided that we can invert the mapping $y^\alpha = f^\alpha(z^A)$. For $N=2$ supergravity we are able to give a non-trivial extension for only one of the solutions (4.12) or (4.13):

$$F(X^0, X^A) = i d_{ABC} \frac{X^A X^B X^C}{X^0} , \quad (5.13)$$

$$Y = -\tfrac{1}{4} i d_{ABC}(z^A - \bar{z}^A)(z^B - \bar{z}^B)(z^C - \bar{z}^C) , \quad (5.14)$$

where the d_{ABC} are arbitrary *real* coefficients. With arbitrary gauge coupling con-

d-geometries

4D matter coupled supergravities originating from 5D by Kaluza-Klein dimensional reduction

d_{IJK} is a constant symmetric invariant tensor of the duality group G_5 appearing in the 5D action with Chern-Simons term:

$$e^{-1}\mathcal{L}_{bos} = -\frac{1}{2}R - \frac{1}{4}\mathring{a}_{IJ}F^I_{\mu\nu}F^{J|\mu\nu} + \frac{1}{2}g_{xy}\partial_\mu\varphi^x\partial^\mu\varphi^y + \frac{e^{-1}}{6\sqrt{6}}d_{IJK}\varepsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda,$$

N=2: GUNAYDIN SIERRA TOWNSEND 1983

- n scalars φ^x
- n_v =n+1 vectors $A^I_\mu = \{A^x_\mu, A_\mu\}$
- n_v =n+1 scalars $\lambda^I = \lambda^I(\varphi^x)$

$\rightarrow$ $d_{IJK}\lambda^I\lambda^J\lambda^K = 1$

N=2 scalar manifold σ-model $\frac{G_5}{H_5}$

kinetic term for scalars: $d_{IJK}\lambda^I\partial_\mu\lambda^J\partial^\mu\lambda^K$

kinetic matrix for vectors: $\mathring{a}_{IJ}(\varphi^x)$

5D/4D Reduction : N=2 SG + Vector Multiplets

DE WIT, VAN PROEYEN 1984

$$e^{-1}\mathcal{L}_{BOS} = -\frac{1}{2}R + \mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} F^{\Sigma\mu\nu} + \frac{e^{-1}}{2}\mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} F^{\Sigma}_{\rho\sigma}\,\epsilon^{\mu\nu\rho\sigma}$$

$$+ \frac{1}{2} g_{rs}\,\partial_\mu\phi^r\partial^\mu\phi^s$$

$n_v + 1$ VECTORS $\quad A^{\Lambda}_{\mu} \begin{cases} A^{I}_{\mu} & n_v \\ g_{\mu 5} & 1 \end{cases}$ $\qquad A^I_5 = a^I$

n SCALARS (REAL) $\quad \phi^s \begin{cases} \lambda^I & n_v - 1 \\ g_{55} & 1 \end{cases}$

in N=2 you can form COMPLEX SCALARS

$$\boxed{z^I = a^I - i\lambda^I} \quad n_v$$

the reduced theory is determined by

$$F(X) = \frac{1}{3!} d_{IJK} \frac{X^I X^J X^K}{X^0}$$

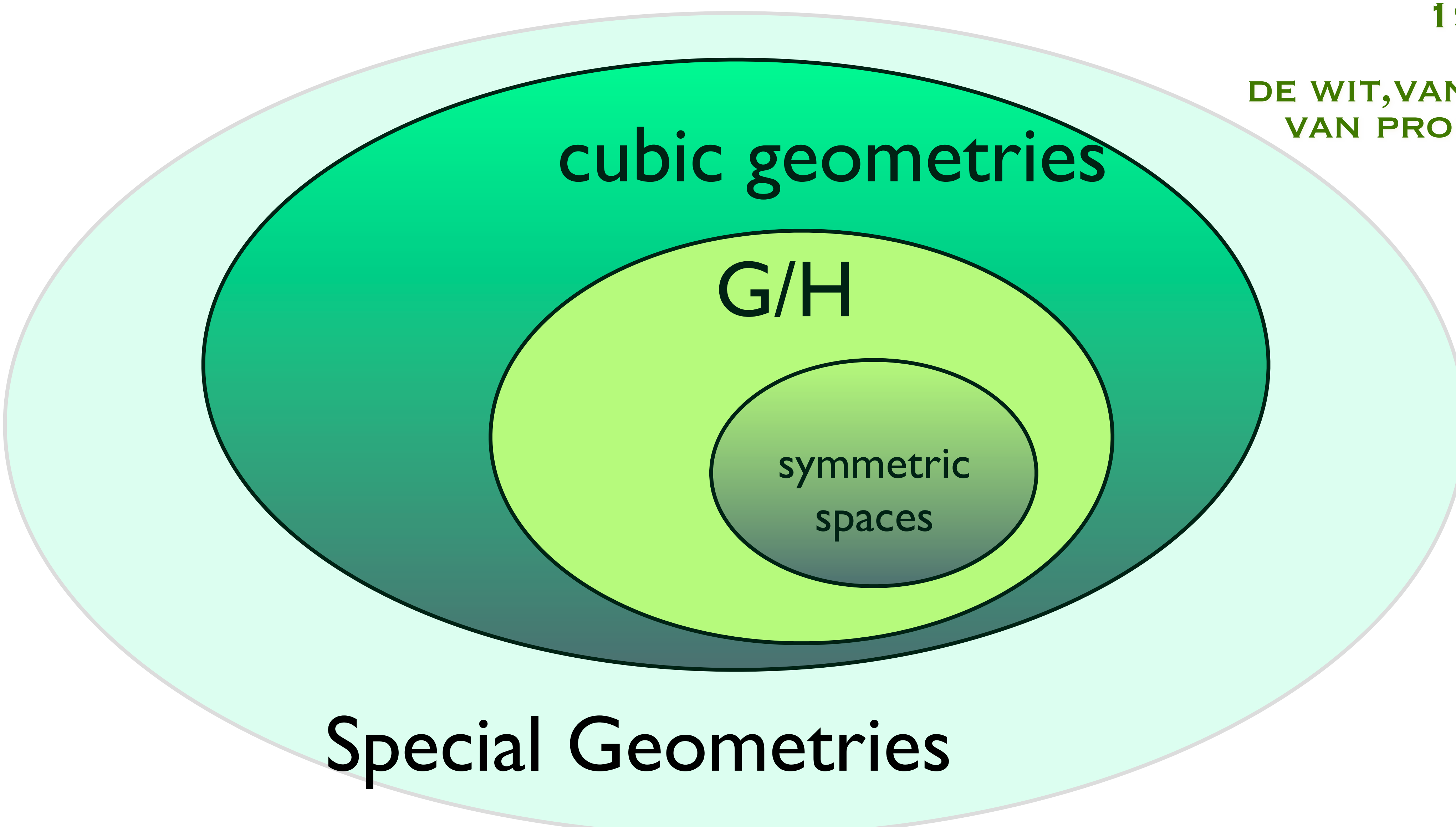
$X^\Lambda(z)$ PARAMETRIZE SPECIAL MANIFOLD

d-spaces for N=2

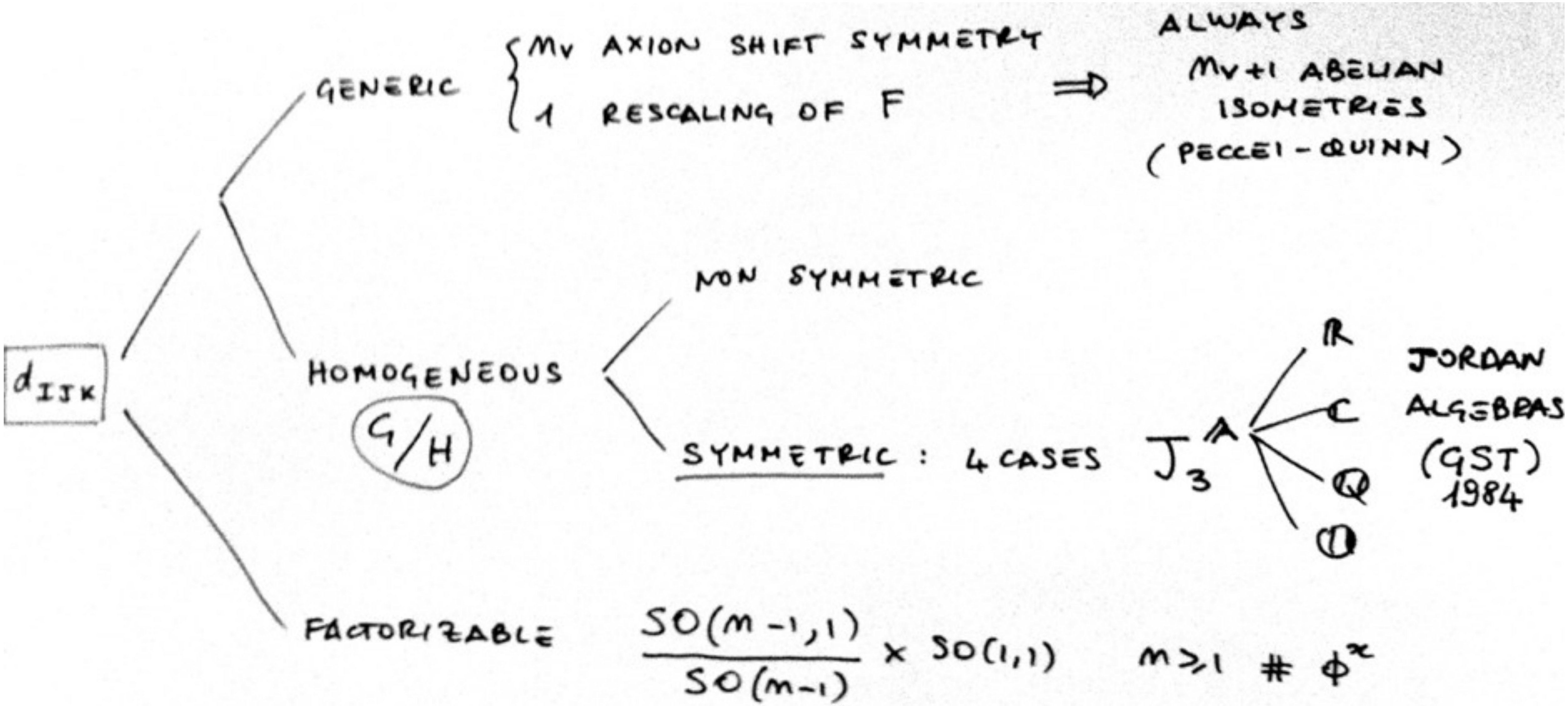
- Classification of homogeneous special manifolds as cosets G/H

DE WIT VAN PROEYEN 1992

DE WIT, VANDERSEYPEN, VAN PROEYEN 1993



Various kinds of d-spaces



SYMMETRIC CASE: $d_{I(JK}\, d_{PQ)R}\, d^{IRL} = \frac{4}{3} \delta^L_{(J}\, d_{KPR)}$

JORDAN ALGEBRAS: COMMUTATIVE BUT NOT ASSOCIATIVE

Counting in 5D/4D reduction for any N>1

#4D vectors = # 5D vectors + 1

axions

#4D scalars = #5D scalars + # 5D vectors + 1

φ^x $\quad a^I = A^I_5$ $\quad \phi = g_{55}$

only in N=2 you have complex scalars

N=8: 28=27+1 vectors

70=42+27+1 scalars

Aim:

- Consider generalized d-geometry structure for $N \geq 2$
- Provide universal parametrization of scalar manifold reflecting 5D origin and display corresponding symplectic structure in axion frame
- Applications to BH
- Hope: useful to understand the structure of quantum corrections N=8,4

RENATA KALLOSH

Symmetric Spaces G/H in Sugra

Scalars live on G/H, charges are in fundamental representation of G

G global symmetry, H local symmetry: "classical" e-m duality, limit of large charges; in full quantum theory the duality is broken to discrete subgroup G(Z)=U-duality

Hull & townsend 1995

N=8: d=4 $\frac{E_{7(7)}}{SU(8)}$ d=5 $\frac{E_{6(6)}}{USp(8)}$

N=4: d=4 $\frac{SU(1,1)}{U(1)} \times \frac{SO(6,n)}{SO(6)\times SO(n)}$ d=5 $SO(1,1) \times \frac{SO(5,n)}{SO(5)\times SO(n)}$

N=2: Special geometry (Very special in d=5) defined by cubic F(X) $F(X) = \frac{1}{3!} d_{ijk} \frac{X^i X^j X^k}{X^0}$

can be lifted to 5d

Two way to use the 5D/4D relation in SG

A) Bottom Up:

take specific geometry of spacetime and solve equations of motion to construct solutions

1) trivial reduction

2) Taub-NUT

B) Top Down:

use symmetry of the theory (geometry, group theory) and extract general features

$$G_5 \Longleftrightarrow G_4 \qquad E_7 \to E_6 \times O(1,1)$$

for symmetric spaces, use invariants, compare attractor solutions and their susy features

GUNAYDIN SF

$$I_3(p) = \frac{1}{3!} d_{ijk} p^i p^j p^k \qquad I_3(q) = \frac{1}{3!} d^{ijk} q^i q^j q^k$$

$$I_4 = -(p^0 q_0 + p^i q_i)^2 + 4[q_0 I_3(p) - p^0 I_3(q) + \frac{\partial I_3(p)}{\partial p} \frac{\partial I_3(q)}{\partial q}]$$

$$I_3 \Longleftrightarrow I_4$$

Menu

- Review Generalized Special Geometry in Symplectic language
- Determine universal representation of coset representative in the axion frame by 5D/4D relation
- Applications to BH

Special Geometry 101

DE WIT VAN PROEYEN 1984

STROMINGER 1990
CASTELLANI D'AURIA SF 1990

$$z^i = X^i/X^0 = a^i - i\,\lambda^i$$

$$R_{i\bar{j}k\bar{l}} = g_{i\bar{j}}g_{k\bar{l}} + g_{i\bar{l}}g_{k\bar{j}} - C_{ikp}\overline{C}_{\bar{j}\bar{l}\bar{p}}g^{p\bar{p}} \qquad C_{ijk} = e^{K(z,\bar{z})}d_{ijk}$$

$$F(X) \equiv \frac{1}{3!}d_{ijk}\frac{X^iX^jX^k}{X^0} \qquad (i = 1, ..., n_V)$$

Large volume limit of Calabi-Yau compactifications, $d_{ijk} = \partial_i\partial_j\partial_k F$
give Yukawa couplings

$$(X^\Lambda(z), F_\Lambda(z)) \qquad F_\Lambda = \partial_\Lambda F(X)$$

Simplectic Sections $Sp(2n_V + 2, R)$

$$e^{K/2}(X^\Lambda(z), F_\Lambda(z)) = (L^\Lambda, M_\Lambda) = V$$

$$D_{\bar{\imath}}V = (\partial_{\bar{\imath}} - \frac{1}{2}\partial_{\bar{\imath}}K)V = 0$$

Covariantly holomorphic sections of a flat bundle

Tool: Generalised Special Geometry

use 2n x 2n complex square matrices for Sp(2n,R), with one vector index and one flat scalar index

$$N = 2 \qquad (L^\Lambda, \overline{D}_{\bar a}\overline{L}^\Lambda; M_\Lambda, \overline{D}_{\bar a}\overline{M}_\Lambda) \qquad g_{i\bar\jmath}e_i^a e_{\bar\jmath}^{\bar b}\delta_{a\bar b}$$

$$A = (0, a) \; ; \Lambda = (0, n_V)$$

$$N \geq 2 \qquad V_A = (f^\Lambda{}_A, h_{\Lambda\, A})$$

generalised symplectic sections

related to the vector kinetic matrix: $h_{\Lambda\, A} = \mathcal{N}_{\Lambda\Sigma} f^\Sigma{}_A$

$$i(f^\dagger h - h^\dagger f) = 1 \; , \qquad f^t h - h^t f = 0$$

symplectic conditions on the sub blocks

Extremal Black Holes & Attractors

- N- extended susy algebra: $\{\mathcal{Q}_{\alpha A}, \mathcal{Q}_{\beta B}\} = \epsilon_{\alpha\beta} Z_{AB}(p,q;\phi)$

$$V_{BH} = -\tfrac{1}{2} Q^T \mathcal{M}(\mathcal{N}) Q = \tfrac{1}{2} Z_{AB} \overline{Z}^{AB} + Z_I Z^I$$

$$\boxed{\partial_\phi V_{BH} = 0}$$

$$\begin{cases} Z_{AB} = -Z_{BA} & \text{central charges} \\ Z_I & \text{matter charges} \end{cases}$$

A,B in SU(N)
I: fundam of matter group when present

$$(N = 2 : Z_{AB} = \epsilon_{AB} Z, \quad Z_I = D_i Z) \qquad Z_{AB} = f^\Lambda{}_{AB} q_\Lambda - h_{AB\Lambda} p^\Lambda$$

$$\begin{cases} Q = (p^\Lambda, q_\Lambda) \\ Z_A =< Q, V_A >= Q^T \Omega V_A = f^\Lambda{}_A q_\Lambda - h_{\Lambda\ A} p^\Lambda \end{cases}$$

$\mathcal{N}(\phi)$ kinetic matrix for vector fields $\quad \Omega = \begin{pmatrix} 0 & -\mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}$

$$Sp(2n, \mathbb{R})$$

- BPS bound: $M_{ADM}(\phi, Q) \geq |z_1(\phi, Q)| \geq \ldots \geq |z_{[N/2]}(\phi, Q)|$

BPS states: M=highest eigenvalue of central charge

Goal:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} f \\ h \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} A - iB \\ C - iD \end{pmatrix}$$

$$\mathbf{L} \equiv \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \sqrt{2} \begin{pmatrix} \mathrm{Re}\, f & -\mathrm{Im}\, f \\ \mathrm{Re}\, h & -\mathrm{Im}\, h \end{pmatrix}$$

$$\mathbf{L}^T \Omega \mathbf{L} = \Omega$$

Compute the generic symplectic representative using the 5D/4D relation, in the axion bases

5D/4D relation - Lesson 1: N=2 Black Holes

- Express the 4D N=2 extremal BH potential for cubic geometries in terms of 5D real special geometry data

- V_{BH} is a polynomial of degree 6 in the axions whose coefficients depend on d_{IJK} and on λ^I

  Study generic attractors for various charge configurations and non trivial axions

- Connect 5D and 4D attractors and their entropies, compare their susy features (BPS and non-BPS orbit stratification)

5D/4D relation - Lesson 2: N=8 Black Holes

CERESOLE, SF, GNECCHI 2009

- Decompose $E_7 \to E_6 \times O(1,1)$ 28=27+1

$$\mathrm{Re}\mathcal{N} = \begin{pmatrix} \frac{d}{3} & -\frac{d_I}{2} \\ -\frac{d_J}{2} & d_{IJ} \end{pmatrix} , \quad \mathrm{Im}\mathcal{N} = \begin{pmatrix} -e^{6\phi} - e^{2\phi} a^I a^J a_{IJ} & a_{IJ} a^J \\ a_{IJ} a^I & -e^{2\phi} a_{IJ} \end{pmatrix}$$

$$d \equiv d_{IJK} a^I a^J a^k , \qquad d_I \equiv d_{IJK} a^J a^k , \qquad d_{IJ} \equiv d_{IJK} a^K$$

- Relate the 5D kinetic vector matrix to the 4D one
- ReN does not depend on 5D scalars
- ImN does not depend on the d-tensor

5D/4D relation - Lesson 2: N=8 Black Holes

- Compute the 28x28 symplectic sections from N in terms of 5D fields:

$$f^{\Lambda}{}_{A} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|c} e^{-3\phi} & 0 \\ \hline e^{-3\phi} a^I & e^{-\phi} (a^{-1/2})^I{}_a \end{array} \right)$$

$$h_{\Lambda\, A} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|c} -e^{-3\phi}\frac{d}{6} - i e^{3\phi} & -\frac{1}{2} e^{-\phi} d_K (a^{-1/2})^K{}_a + i e^{\phi} a^K (a^{1/2})_K{}^a \\ \hline \frac{1}{2} e^{-3\phi} d_I & e^{-\phi} d_{IJ} (a^{-1/2})^J{}_a - i e^{\phi} (a^{1/2})_I{}^a \end{array} \right)$$

$$E(\lambda) \equiv (a^{-1/2})_a{}^J = E_a{}^J$$

square root of 5D vector kinetic matrix
coset representative of 5D scalar manifold

- Interpret this for any d-geometry, N=0,2,4,6,8 by taking appropriate representations!

Note: Freedom on the symplectic sections

- Given

$$\textbf{a)}\ \mathcal{N}_{\Lambda\Sigma} = h_{\Lambda A}(f^{-1})^{A}{}_{\Sigma}\ ,$$

$$\textbf{b)}\ i(\mathbf{f}^{\dagger}\mathbf{h} - \mathbf{h}^{\dagger}\mathbf{f}) = \mathbf{Id}\ ,$$

$$\textbf{c)}\ \mathbf{f}^{T}\mathbf{h} - \mathbf{h}^{T}\mathbf{f} = 0\ .$$

can still perform any unitary transformation:

$$h \to hM\ , \quad f \to fM\ , \qquad MM^{\dagger} = 1$$

BH potential is invariant: $V_{BH} \equiv ZZ^{\dagger}$

Important to connect central charges in N=2 and N=8

$$V_{BH} = \frac{1}{2}(Z_0^e)^2 + \frac{1}{2}\left(Z_m^0\right)^2 + \frac{1}{2}Z_I^e a^{IJ} Z_J^e + \frac{1}{2}Z_m^I a_{IJ} Z_m^J$$

Note: Freedom on the symplectic sections

- In N=2, this unitary M transform the axion basis into the usual symplectic basis of Special Geometry:

$$M = A^{1/2}\hat{M}G^{-1/2}$$

$$A = \left(\begin{array}{c|c} 1 & 0...0 \\ \hline 0 & \\ . & a_{IJ} \\ . & \\ 0 & \end{array}\right), \qquad G = \left(\begin{array}{c|c} 1 & 0...0 \\ \hline 0 & \\ . & g_{IJ} \\ . & \\ 0 & \end{array}\right), \qquad g_{IJ} = \frac{1}{4}e^{-4\phi}a_{I.}$$

$$\hat{M} = \frac{1}{2}\begin{pmatrix} 1 & \partial_{\bar{J}}K \\ -i\lambda^I e^{-2\phi} & e^{-2\phi}\delta^I_{\bar{J}} + ie^{-2\phi}\lambda^I\partial_{\bar{J}}K \end{pmatrix}$$

CERESOLE, SF, GNECCHI
2009

$$Z_0 = \frac{1}{\sqrt{2}}(Z^e_0 + iZ^0_m) \ ,$$

$$Z_I = \frac{1}{\sqrt{2}}(Z^e_I + ia_{IJ}Z^J_m)$$

Main Message:

The 5D and 4D U-duality groups are always related by:

$$G_5 \times SO(1,1) \subset G_4 \subset Sp(2n_V + 2, \mathbb{R})$$

Because of the 5D origin, there is a natural splitting of the 4D scalars, covariant with respect to G_5 :

$$\Phi = \left\{ a^I, \phi, \; \lambda^x \right\}$$

This suggests to look for a symplectic representative of the type:

$$\mathbf{L}\left(a^I, \phi, \lambda^x\right) = \mathcal{A}(a^I)\mathcal{D}(\phi)G(\lambda)$$

partial Iwasawa decomposition into a translation along axions, a dilatation and a G_5 -dependent transformation

The axionic translations

$$\mathcal{A}(a) \equiv e^{T(a)}$$

Computed by ANDRIANOPOLI D'AURIA SF LLEDO 1998 in the context of gaugings of 4D supergravity and Scherk-Shwarz mechanism

$$T(a) = \left(\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ a^J & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -a^I \\ 0 & d_{IJ} & 0 & 0 \end{array}\right)$$

nilpotent of order 3 (relation with nilpotent part of flat connections in Special Geometry):

$$T^4(a) = 0 \Rightarrow \mathcal{A}(a) = \mathbf{Id} + T(a) + \frac{1}{2}T^2(a) + \frac{1}{3!}T^3(a)$$

$$\mathcal{A}(a) = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ a^J & 1 & 0 & 0 \\ \hline -\frac{1}{6}d & -\frac{1}{2}d_I & 1 & -a^I \\ \frac{1}{2}d_J & d_{IJ} & 0 & 1 \end{array}\right)$$

Dilatation and G5 transformation

1-dimensional abelian SO(1,1) factor:

$$\mathcal{D}(\phi) = \left(\begin{array}{cc|cc} e^{-3\phi} & 0 & 0 & 0 \\ 0 & e^{-\phi} & 0 & 0 \\ \hline 0 & 0 & e^{3\phi} & 0 \\ 0 & 0 & 0 & e^{\phi} \end{array}\right)$$

block diagonal transformation depending only on 5D scalars

$$G(\lambda) = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & E^{-1} \end{array}\right)$$

$E\left(\lambda\right) \equiv \left(a^{-1/2}\right)_a{}^J = E_a{}^J$ coset representative for G5/H5

Symplectic representative in the axion basis

$$\mathbf{L}(a^I,\phi,E\,(\lambda))=\left(\begin{array}{cc|cc} e^{-3\phi} & 0 & 0 & 0 \\ a^I e^{-3\phi} & e^{-\phi}{E_a}^I & 0 & 0 \\ \hline -\frac{1}{6}de^{-3\phi} & -\frac{1}{2}d_K {E_a}^K e^{-\phi} & e^{3\phi} & -a^K (E^{-1})^a{}_K e^{\phi} \\ \frac{1}{2}d_I e^{-3\phi} & d_{IK}{E_a}^K e^{-\phi} & 0 & e^{\phi}(E^{-1})^a{}_I \end{array}\right) = \sqrt{2}\begin{pmatrix} \mathrm{Re}\, f & -\mathrm{Im}\, f \\ \mathrm{Re}\, h & -\mathrm{Im}\, h \end{pmatrix}$$

In this bases L is lower triangular, Im f =0
(different from the usual basis of N=2:) $\quad (f,h) = (L^\Lambda, \bar{D}_{\bar{a}}\bar{L}^\Lambda; M_\Lambda, \bar{D}_{\bar{a}}\bar{M}_\Lambda)$

Dependence on d_{IJK} only in lower left block

Can use it to compute vielbeins, connections on coset spaces

Some Properties

$$\mathcal{M} = \begin{pmatrix} \mathbb{1} & -\mathrm{Re}\mathcal{N} \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} \mathrm{Im}\mathcal{N} & 0 \\ 0 & (\mathrm{Im}\mathcal{N})^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ -\mathrm{Re}\mathcal{N} & \mathbb{1} \end{pmatrix} \equiv \mathcal{R}^T \mathcal{M}_D \mathcal{R}$$

$$\mathcal{M} \quad = -(\mathbf{L}\,\mathbf{L}^T)^{-1}$$

$$Z_A = < Q, V_A > = Q^T \Omega V_A = f^\Lambda{}_A q_\Lambda - h_{\Lambda\ A} p^\Lambda$$

$$V_{BH} = -\frac{1}{2} Q^t \mathcal{M}(\mathcal{N}) Q = < Q, V_A > < Q, \bar{V}^A >$$

$$Z_A = \mathbf{L}^{-1} Q$$

Example: N=8

Jordan Triple Sistem is the euclidean Jordan algebra $J_3^{\mathbb{O}_s}$

Scalar manifold $\frac{G_5}{H_5} = \frac{E_{6(6)}}{USp(8)}, \ \dim_{\mathbb{R}} = 42$ $\qquad (a^{-1})_I{}^J$

d_{IJK} invariant tensor of 27 fundamental irrep of $E_{6(6)}$

$Sp(56,\mathbb{R})$ matrix $\mathbf{L}(a^I, \phi, E(\lambda))$ is coset representative of

$$\frac{G_4}{H_4} = \frac{E_{7(7)}}{SU(8)}, \ \dim_{\mathbb{R}} = 70$$

$$SU(8) \supset USp(8);$$

$$\mathbf{70} = \underset{\lambda^x}{\mathbf{42}} + \underset{a^I}{\mathbf{27}} + \underset{\phi}{\mathbf{1}}.$$

$\mathbf{28} \quad Z_{AB} \longrightarrow Z_A \quad \mathbf{27} + \mathbf{1}$

Application to BH: STU model

- Simple example of cubic special geometry, with prepotential F=STU
- 3 complex scalar S, T, U each parametrizing SU(1,1)/U(1)
- Can be viewed as a truncation of N=8
- It yields a non trivial test of the use of the axion frame

First order black hole attractor flows

A.CERESOLE
G. DALL'AGATA
2009

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}\left(c^4 \frac{dr^2}{\sinh^4(cr)} + \frac{c^2}{\sinh^2(cr)} d\Omega^2_{S^2}\right)$$

- Defining a real $W(\phi, \bar{\phi})$, extremal black holes are described by

$$\begin{cases} U' & = & -e^U W \\ \phi'^i & = & -2e^U g^{i\bar{\jmath}} \partial_{\bar{\jmath}} W \end{cases}$$

$$V_{BH}(\phi, q, p) = W^2 + 4g^{i\bar{\jmath}} \partial_i W \partial_{\bar{\jmath}} W$$

- BPS BH's are a special case with W = |Z|
- But **other possible solutions are the non-BPS BH's !**
- $\partial_i W(\phi, \bar{\phi}) = 0$ **gives non-BPS critical points!**

$W(\phi, \bar{\phi})$ "fake" superpotential $e^U W|_\infty \sim M_{ADM}$

CERESOLE, DALL'AGATA, SF, YERANYAN 2009

W was found for STU model in generic charge configuration using duality invariance:

BELLUCCI, SF, MARRANI, YERANYAN 2008

1. Take W for STU model in S=T=U limit
2. Compute it in simple charge configuration and then boost it to generic charges by a duality transformation

$$W^2 = \frac{i_1 + i_2}{4} + \frac{3}{8}\left[\left(4\, i_3 \sqrt{-I_4} - (i_1 + i_2)\, I_4 + \left(i_1 - \frac{i_2}{3}\right)^3\right)^{1/3} + \right.$$
$$\left. + \left(-4\, i_3 \sqrt{-I_4} - (i_1 + i_2)\, I_4 + \left(i_1 - \frac{i_2}{3}\right)^3\right)^{1/3}\right].$$

$\Longrightarrow$ non polynomial expression, but at non-BPS attractor point:

$$i_2 = 3 i_1 = \frac{3}{4}\sqrt{-I_4}\,, i_3 = 0 \implies S_{BH} = W^2 = \sqrt{-|I_4|}$$

- Give W in terms of a complete set of duality invariants for N=2
- respect the Sp(2n+2,R) structure

CERCHIAI MARRANI SF ZUMINO 2009

$$i_1 = Z\overline{Z}$$

$$i_2 = g^{i\bar{\jmath}} Z_i \overline{Z}_{\bar{\jmath}} \qquad (Z_i = D_i Z \;,\; \overline{Z}_{\bar{\imath}} = \overline{D}_{\bar{\imath}}\, \overline{Z}),$$

$$i_3 = \frac{1}{6}\left[Z N_3(\overline{Z}) + \overline{Z N}_3(Z_i)\right], \qquad i_4 = \frac{i}{6}\left[Z N_3(\overline{Z}) - \overline{Z N}_3(Z)\right],$$

$$i_5 = g^{i\bar{\imath}} C_{ijk} C_{\bar{\imath}\bar{\jmath}\bar{k}} \overline{Z}^j \overline{Z}^k Z^{\bar{\jmath}} Z^{\bar{k}},$$

cubic norms:

$$N_3(\overline{Z}) = C_{ijk} \overline{Z}^i \, \overline{Z}^j \, \overline{Z}^k, \qquad \overline{N}_3(Z) = C_{\bar{\imath}\bar{\jmath}\bar{k}} Z^{\bar{\imath}} \, Z^{\bar{\jmath}} \, Z^{\bar{k}}.$$

Ansatz:

$$W(\phi, \bar{\phi}) = W(i_1, i_2, i_3, i_4, i_5)$$

CERESOLE, DALL'AGATA, SF, YERANYAN 2009

$$I_4 = (i_1 - i_2)^2 + 4 i_4 - i_5 \qquad \partial_i I_4 = 0$$

T^3, ST^2, STU

BOSSARD, MICHEL, PIOLINE ARXIV:0908.1742

compute W by "reduction over time"

W^2 given implicitly as a "non standard diagonalization problem": solution of a sextic polynomial in whose coefficients are SU(8) invariants

CERESOLE, GNECCHI, SF, MARRANI 2012

CHECK: using the axion basis for the central charges, after a unitary rotation, find the fake superpotential for p0 q0 charge configuration of T^3 model

Summary

d-geometry is relevant for extended (even N) supergravities

- Interesting to use universal parametrization of scalar manifold reflecting 5D origin (axion frame)
- Coset representative has lower triangular form, f section is real
- Application to black hole flows, explain stratification of charge orbits, computation of fake superpotential
- Hope: useful to understand the structure of quantum corrections in extended SG, in particular N=8

RENATA KALLOSH


Happy
Birthday