MAX-PLANCK-GESELLSCHAFT

T-duality and new string geometries

Dieter Lüst, LMU and MPI München

KOUNNAS-FEST: Nicosia, September 28 - 30, 2012

Costas made several very profound and important contributions to theoretical physics !

Often I was working also on related subjects (construction of 4-dimensional strings, effective supergravity actions, supersymmetry breaking, fluxes, ..) and I enjoyed various very nice collaborations with Costas:

Nuclear Physics B365 (1991) 431–466
North-Holland

NUCLEAR
PHYSICS B

DUALITY-INVARIANT PARTITION FUNCTIONS AND AUTOMORPHIC SUPERPOTENTIALS FOR (2,2) STRING COMPACTIFICATIONS*

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We define the topological free energy for string compactifications, which is relevant for the discussion of perturbative as well as non-perturbative effects in string theory. This moduli-dependent functional, originating from the integration over massive string modes, is determined by automorphic functions of the target-space duality group. We explicitly construct these automorphic functions for symmetric orbifold and Calabi–Yau compactifications.

1. Introduction

CERN-TH.6494/92
LPTENS 92-16

Cosmological String Backgrounds from Gauged WZW Models

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Ecole Normale Supérieure, Paris, France

and

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CERN, Geneva, Switzerland

Abstract

We discuss the four-dimensional target-space interpretation of bosonic strings based on gauged WZW models, in particular of those based on the non-compact coset space $SL(2,\mathbf{R}) \times SO(1,1)^2/SO(1,1)$. We show that these theories lead, apart from the recently broadly discussed black-hole type of backgrounds, to cosmological string backgrounds, such as an expanding Universe. Which of the two cases is realized depends on the sign of the level of the corresponding Kac-Moody algebra. We discuss various aspects of these new cosmological string backgrounds.

CERN-TH.6975/93
HUB-IEP-93/3
LPTENS 93/31
hep-th/9308124

A Large Class of New Gravitational and Axionic Backgrounds for Four-dimensional Superstrings

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and

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ABSTRACT

A large class of new 4-D superstring vacua with non-trivial/singular geometries, spacetime supersymmetry and other background fields (axion, dilaton) are found. Killing symmetries are generic and are associated with non-trivial dilaton and antisymmetric tensor fields. Duality symmetries preserving N=2 superconformal invariance are employed to generate a large class of explicit metrics for non-compact 4-D Calabi-Yau manifolds with Killing symmetries. We comment on some of our solutions which have interesting singularity properties and cosmological interpretation.

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CPTH-RR060.0707

AdS_4 flux vacua in type II superstrings and their domain-wall solutions

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Abstract: We investigate the emergence of supersymmetric negative-vacuum-energy ground states in four dimensions. First, we rely on the analysis of the effective superpotential, which depends on the background fluxes of the internal manifold, or equivalently has its origin in the underlying gauged supergravity. Four-dimensional, supersymmetric anti-de Sitter vacua with all moduli stabilized appear when appropriate Ramond and Neveu–Schwarz fluxes are introduced in IIA. Geometric fluxes are not necessary. Then the whole setup is analyzed from the perspective of the sources, namely D/NS-branes or Kaluza–Klein monopoles. Orientifold planes are also required for tadpole cancellation. The solutions found in four dimensions correspond to domain walls interpolating between AdS_4 and flat spacetime. The various consistency conditions (equations of motion, Bianchi identities and tadpole cancellation conditions) are always satisfied, albeit with source terms. We also speculate on the possibility of assigning (formal) entropies to AdS_4 flux vacua via the corresponding dual brane systems.

Keywords: anti-de Sitter vacua, fluxes, branes.

T-duality has always played an important role in our common work.

- Relates different string geometries: large and small backgrounds (low and high temperature).
- Relates different string topologies: mirror symmetry.
- It is a stringy symmetry: needs momentum and winding (dual momentum) modes.
- Suggests a doubling of space coordinates: Doubled geometry.
- Relates conventional (Riemannian) geometries to new stringy geometries (non-commutative & non-associative)

T-duality: $$T: \quad R \longleftrightarrow \frac{\alpha'}{R}, \quad M \longleftrightarrow N$$

$$T: \quad p \longleftrightarrow \tilde{p}, \quad p_L \longleftrightarrow p_L, \quad p_R \longleftrightarrow -p_R.$$

- Dual space coordinates: $\tilde{X}(\tau,\sigma) = X_L - X_R$

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T-duality is part of stringy diffeomorphism group.

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Doubled field theory:

(O. Hohm, C. Hull, B. Zwiebach (2009/10))

- Manifestly O(D,D) invariant string action.
- Coordinates: use O(D,D) vector $X^M = (\tilde{X}_i, X^i)$

Outline:

II) Non-geometric flux compactifications

III) Non-commutative/non-associative geometries from non-geometric string backgrounds

D. Lüst, JEHP 1012 (2011) 063, arXiv:1010.1361; arXiv:1205.0100
R. Blumenhagen, A. Deser, D.Lüst, E. Plauschinn, F. Rennecke, J. Phys A44 (2011), 385401, arXiv:1106.0316
C. Condeescu, I. Florakis, D. Lüst, JHEP 1204 (2012) 121, arXiv:1202.6366
D. Andriot, M. Larfors, D. Lüst, P. Patalong, to appear

IV) Outlook & open problems

II) Non-geometric flux compactifications

Non-geometric backgrounds: Asymmetric orbifolds

Covariant lattices

Fermionic constructions

T-folds

......

We will encounter two different interesting situations:

- Non-geometric Q-fluxes: spaces that are locally still Riemannian manifolds but not anymore globally.

Transition functions between two coordinate patches are not only diffeomorphisms but also T-duality transformations:

$$\mathrm{Diff}(M) \quad \rightarrow \quad \mathrm{Diff}(M) \times SO(d,d)$$

Q-space will become non-commutative: $[X_i, X_j] \neq 0$

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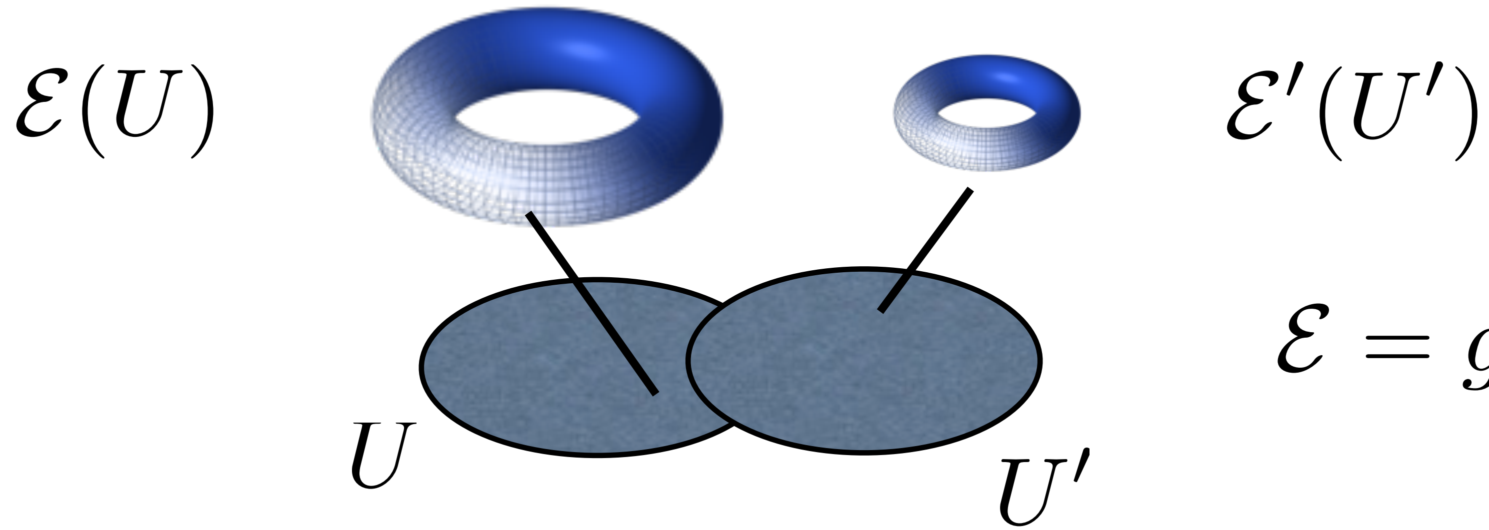
Physics is nevertheless smooth and well-defined!

Q-space

T-fold: Patching uses T-duality.

e.g. torus fibrations



$$\mathcal{E} = g + b$$

Geometric background: $\mathcal{E}' = a\mathcal{E}a^t$ in $U \cap U'$, $a \in GL(d, Z)$

Non-geometric background:

$$\mathcal{E}' = \frac{a\mathcal{E} + b}{c\mathcal{E} + d} \quad \text{in} \quad U \cap U'$$

III) Non-commutative/non-associative geometries from non-geometric string backgrounds

Now we want to derive the stringy quantum geometry of non-geometric backgrounds .

⇒ Deformed (NC/NA) string geometry with

Q- reps. R-flux as deformation parameters.

i) Elliptic monodromy: symmetric ↔ asymmetric orbifold

D. Lüst, JEHP 1012 (2011) 063, arXiv:1010.1361; arXiv:1205.0100

C. Condeescu, I. Florakis, D. Lüst, JHEP 1204 (2012) 121, arXiv:1202.6366.

ii) Parabolic monodromy: T-duality as canonical transformation

A. Andriot, M. Larfors, D. Lüst, P. Patalong, to appear; I. Bakas, D. Lüst, work in progress

iii) CFT amplitude computation

R. Blumenhagen, A. Deser, D.Lüst, E. Plauschinn, F. Rennecke, J. Phys A44 (2011), 385401, arXiv:1106.0316

i) Elliptic = finite order monodromy

ω - background, geometric space

Symmetric (freely acting orbifold): commutative

$\updownarrow$ T-duality

Q-background, non-geometric space

Asymmetric (freely acting orbifold): non-commutative

- The model is an exactly solvable CFT

- Partition function:

$$Z = \frac{1}{\eta^{12}\,\bar{\eta}^{12}}\; R \sum_{\tilde{m},n\in\mathbb{Z}} e^{-\frac{\pi R^2}{\tau_2}|\tilde{m}+\tau n|^2}\; Z_L{\textstyle\left[{h \atop g}\right]}(\tau)\,\tilde{Z}_R(\bar{\tau})\,\Gamma_{(5,5)}{\textstyle\left[{h \atop g}\right]}(\tau,\bar{\tau})$$

ii) Parabolic = infinite order monodromy

Four different 3-dimensional closed string flux backgrounds, which are related by T-duality: (Shelton, Raylor, Wecht, 2005; Dabholkar, Hull, 2005)

Chain of 3 T-dualities:

$$F^{(3)}: \qquad H \underset{T_{x_1}}{\longleftrightarrow} \omega \underset{T_{x_2}}{\longleftrightarrow} Q \underset{T_{x_3}}{\longleftrightarrow} R$$

(not isometry)

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Twisted (curved) Riemannian 3-torus

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$$[X_i, X_j] \neq 0$$

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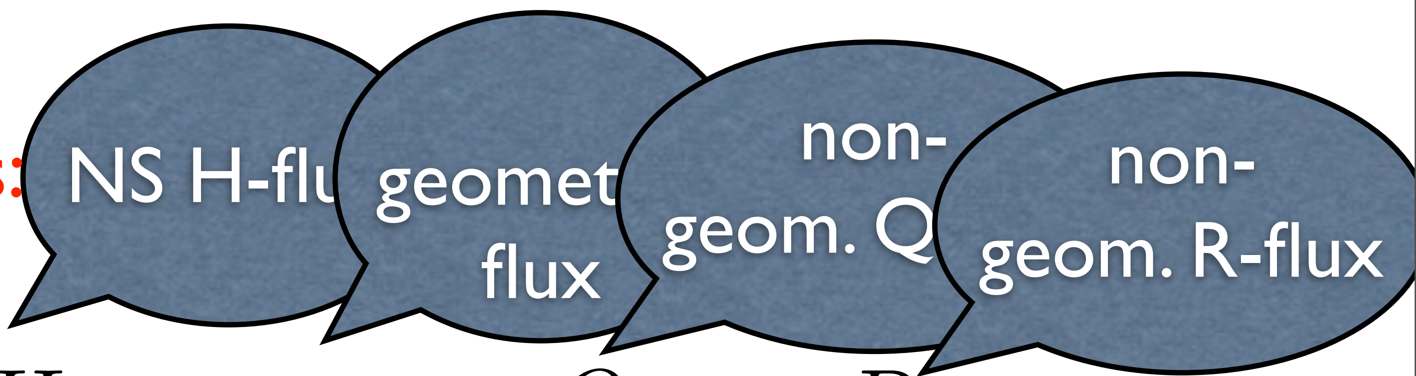

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non-comm. T-fold with Q-flux:

$$[X_i, X_j] \neq 0$$

Non-associative „Space“ with R-flux

$$[[X_i, X_j], X_k] \neq 0$$

Procedure for the quantization of these backgrounds:

1. step: Standard canonical quantization of H
and ω - backgrounds

$$[\mathcal{X}^\mu(\tau,\sigma),\mathcal{X}^\nu(\tau,\sigma')]=0$$

$$[\mathcal{P}_\mu(\tau,\sigma),\mathcal{P}_\nu(\tau,\sigma')]=0$$

$$[\mathcal{X}^\mu(\tau,\sigma),\mathcal{P}_\nu(\tau,\sigma')]=i\ \delta^\mu_\nu\ \delta(\sigma-\sigma')$$

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- Obeying the following closed string boundary (SO(2,2)-monodromy) conditions:

$$\begin{aligned}
Y^1(\tau,\sigma+2\pi) &= Y^1(\tau,\sigma)+2\pi N_Y^3 H Y^2(\tau,\sigma)\,,\\
Y^2(\tau,\sigma+2\pi) &= Y^2(\tau,\sigma)\,,\\
\tilde{Y}^1(\tau,\sigma+2\pi) &= \tilde{Y}^1(\tau,\sigma)\,,\\
\tilde{Y}^2(\tau,\sigma+2\pi) &= \tilde{Y}^2(\tau,\sigma)+HN_Y^3\tilde{Y}^1(\tau,\sigma)\,;\\
Y^3(\tau,\sigma+2\pi) &= Y^3(\tau,\sigma)+2\pi N_Y^3\,.
\end{aligned}$$

2. step: T-duality as canonical (Buscher) transformation:

(E. Alvarez, L. Alvarez-Gaume, Y. Lozano, 1994;
I. Bakas, K. Sfetsos, 1995)

$H \leftrightarrow \omega$: T-d. along $\iota = 1$

$$\begin{array}{r} \partial_\tau X^1 = \partial_\sigma Y^1 - HY^3 \partial_\sigma Y^2 \\ \partial_\sigma X^1 = \partial_\tau Y^1 - HY^3 \partial_\tau Y^2 \\ \partial_\tau X^{2,3} = \partial_\tau Y^{2,3} \\ \partial_\sigma X^{2,3} = \partial_\sigma Y^{2,3} \end{array} \quad \Big| \quad \begin{array}{c} \Longleftrightarrow \\ \text{(all orders)} \end{array} \quad \Big| \quad \begin{array}{l} \partial_\tau Y^1 = \partial_\sigma X^1 + HX^3 \partial_\tau X^2 \\ \partial_\sigma Y^1 = \partial_\tau X^1 + HX^3 \partial_\sigma X^2 \\ \partial_\tau Y^{2,3} = \partial_\tau X^{2,3} \\ \partial_\sigma Y^{2,3} = \partial_\sigma X^{2,3} \end{array}$$

$\omega \leftrightarrow Q$: T-d. along $\iota = 2$

$$\begin{array}{r} \partial_\tau Y^2 = \partial_\sigma Z^2 + HZ^3 \partial_\tau Z^1 \\ \partial_\sigma Y^2 = \partial_\tau Z^2 + HZ^3 \partial_\sigma Z^1 \\ \partial_\tau Y^{1,3} = \partial_\tau Z^{1,3} \\ \partial_\sigma Y^{1,3} = \partial_\sigma Z^{1,3} \end{array} \quad \Big| \quad \begin{array}{c} \Longleftrightarrow \\ \text{(up to } O(H^2)) \end{array} \quad \Big| \quad \begin{array}{l} \partial_\tau Z^2 = \partial_\sigma Y^2 - HY^3 \partial_\sigma Y^1 \\ \partial_\sigma Z^2 = \partial_\tau Y^2 - HY^3 \partial_\tau Y^1 \\ \partial_\tau Z^{1,3} = \partial_\tau Y^{1,3} \\ \partial_\sigma Z^{1,3} = \partial_\sigma Y^{1,3} \end{array}$$

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T-dual SO(2,2)-monodromy conditions:

$$\begin{aligned} Z^1(\tau,\sigma+2\pi) &= Z^1(\tau,\sigma) - 2\pi N_Z^3 H \tilde{Z}^2(\tau,\sigma)\,, \\ Z^2(\tau,\sigma+2\pi) &= Z^2(\tau,\sigma) + 2\pi N_Z^3 H \tilde{Z}^1(\tau,\sigma)\,, \\ \tilde{Z}^1(\tau,\sigma+2\pi) &= \tilde{Z}^1(\tau,\sigma)\,, \\ \tilde{Z}^2(\tau,\sigma+2\pi) &= \tilde{Z}^2(\tau,\sigma)\,; \\ Z^3(\tau,\sigma+2\pi) &= Z^3(\tau,\sigma) + 2\pi N_Z^3\,. \end{aligned}$$

Mix coordinates with dual coordinates.

↕

Non-geometric background.

3. step: Derive (non-canonical) quantization for Q-background:
(consistent with the non-geometrical monodromy conditions)

$$[Z^1(\tau,\sigma), Z^2(\tau,\sigma)] = -\frac{1}{2}\frac{\pi^2}{3} H \tilde{p}^3$$

T-duality does not preserve the canonical commutation relations!

Corresponding uncertainty relation:

$$(\Delta Z^1)^2 (\Delta Z^2)^2 \geq L_s^6 \, H^2 \, \langle \tilde{p}^3 \rangle^2$$

The spatial uncertainty in the X_1, X_2 - directions grows with the dual momentum in the third direction: non-local strings with winding in third direction.

4. step: T-duality in x^3-direction $\Rightarrow$ R-flux

$$\tilde{p}^3 \longrightarrow p^3$$

$\Rightarrow$ For the case of non-geometric R-fluxes one gets:

$$[Z^1(\tau,\sigma), Z^2(\tau,\sigma)] = -\frac{1}{2}\frac{\pi^2}{3} H p^3$$

Use $[p^3, X^3] = -i$

$$\Rightarrow \quad [[Z^1, Z^2], Z^3] + \text{perm.} \simeq H$$

Non-associative algebra!

This nicely agrees with the non-associative closed string structure found by Blumenhagen, Plauschinn in the SU(2) WZW model: arXiv:1010.1263

Twisted Poisson structure (same as for point particle in the field of a magnetic monopole, being related to co-cycles)

IV) Outlook & open questions

- Mixed closed string bound conditions (in analogy to mixed D-N boundary conditions for D-branes) lead to closed string non-commutativity. This is a stringy, non-local effect - Wilson loop operator.

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- Non-associative $\triangle$- product for functions:

$$f_1(y) \triangle f_2(y) \triangle \ldots \triangle f_N(y) :=$$

$$\exp\left[\sum_{m<n<r} F^{abc}\, \partial_a^{y_m}\, \partial_b^{y_n}\, \partial_c^{y_r}\right]\, f_1(y_1)\, f_2(y_2) \ldots f_N(y_N)\Big|_{y_1=\ldots=y_N=y}$$

(see also: K. Savvidy (2002))

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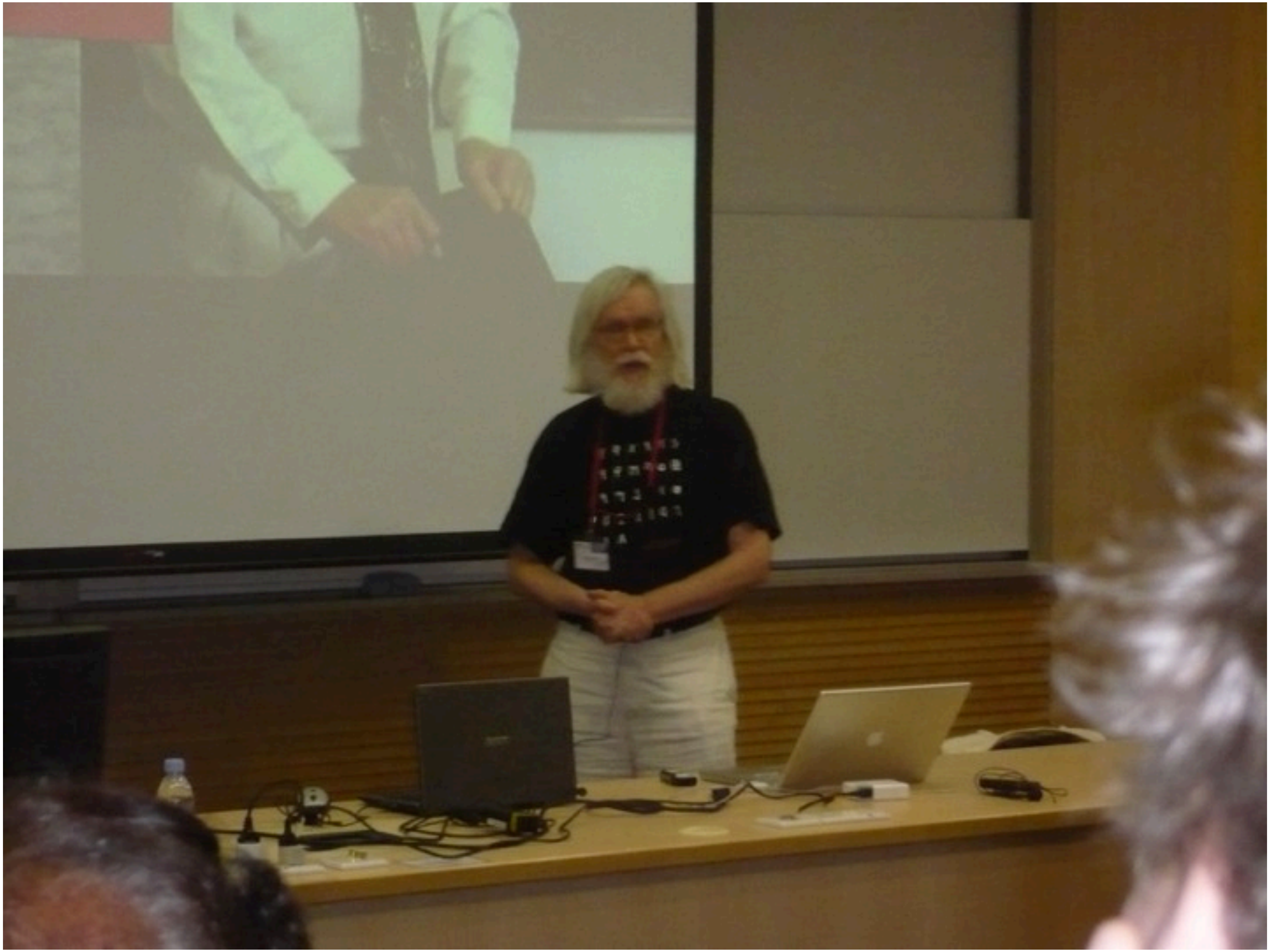
- Is there are non-commutative (non-associative) theory of gravity? (Non-commutative geometry & gravity: P.Aschieri, M. Dimitrijevic, F. Meyer, J. Wess (2005 L.Alvarez-Gaume, F. Meyer, M. Vazquez-Mozo (2006))

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Happy Birthday and all the best wishes to you and your family, Costa!