

Gravitational waves from inflation: the good, the bad & the ugly

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STANFORD UNIVERSITY



DEPARTMENT OF PHYSICS



Direct evidence of cosmic inflation
Mon, 2014-03-17

The detection of gravitational waves by the BICEP2 experiment at the South Pole supports the cosmic inflation theory of how the universe came to be. The discovery, made in part by

17 March 2014 Last updated at 14:46

Cosmic inflation: 'Spectacular' discovery hailed

Researchers with an experiment based at the South Pole have discovered the long-sought "smoking gun" for inflation.



Telescope captures view of gravitational waves



CERN COURIER

Apr 30, 2014

BICEP2 finds evidence of cosmic inflation



HARVARD-SMITHSONIAN
CENTER FOR ASTROPHYSICS

First Direct Evidence of Cosmic Inflation
Monday, March 17, 2014 - 10:45am

BICEP2 COLLABORATORS CONFIRM COSMIC INFLATION

NSF-funded BICEP2 collaborators confirm cosmic inflation, a cataclysmic event that followed a fraction of a second after the Big Bang.

Gravitational Waves from Big Bang Detected

SCIENTIFIC AMERICAN

Big Bang breakthrough announced; gravitational waves detected

By Elizabeth Landau, CNN

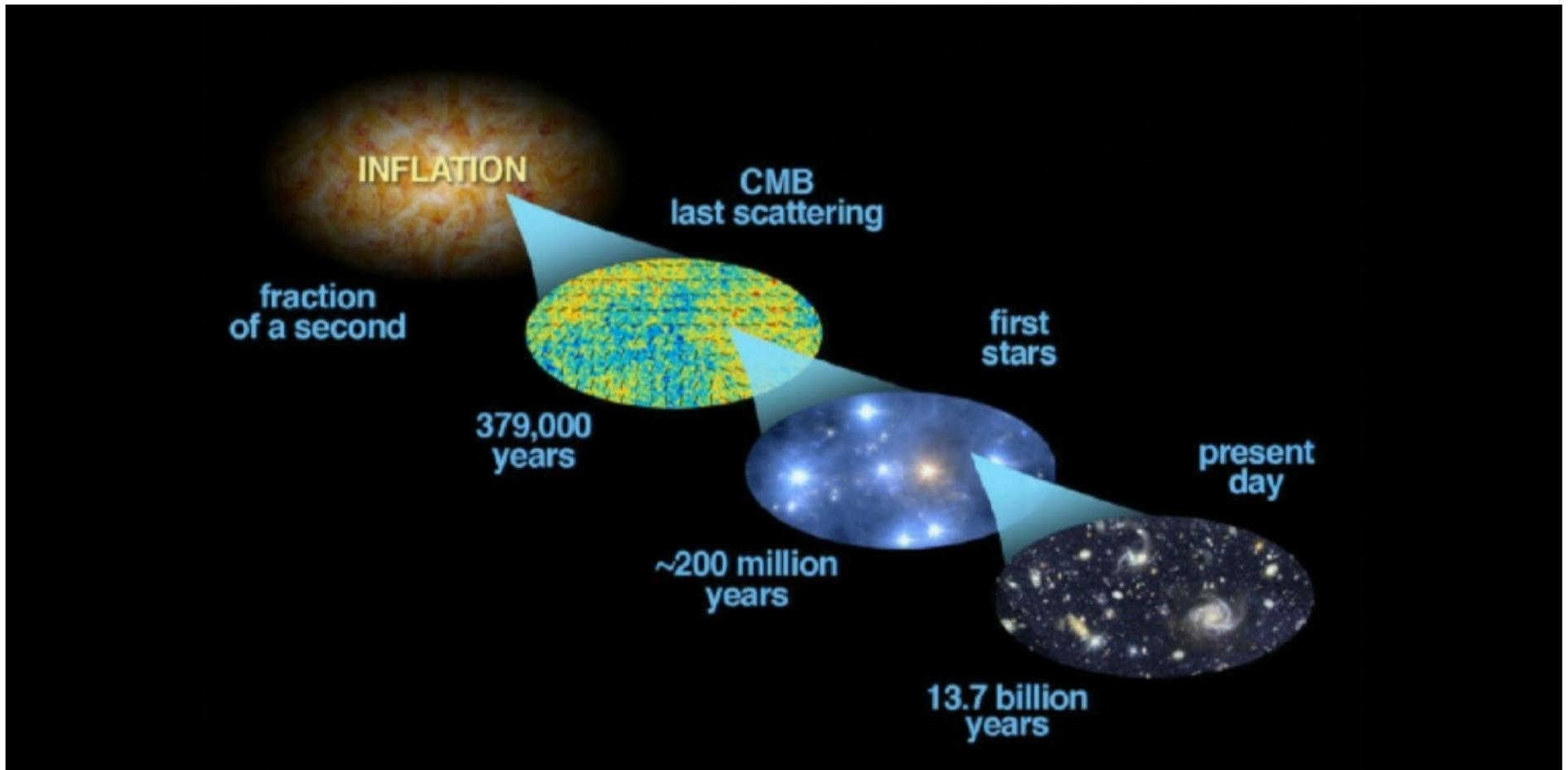
physicsworld.com

BICEP2 finds first direct evidence of cosmic inflation

03/17/2014
BICEP2 Discovers First Direct Evidence of Inflation and Primordial Gravitational Waves

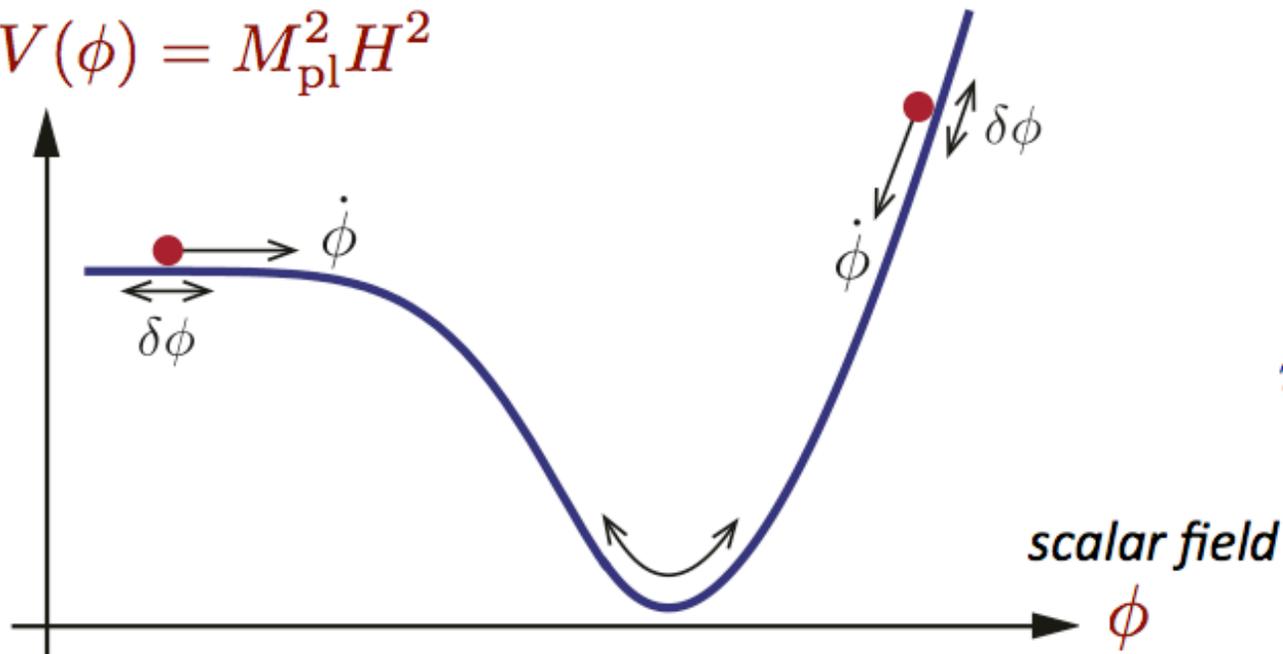
Caltech

Why was the BICEP2 claim so very exciting ...



We have a nearly complete picture of the growth of **large-scale structure** through **gravitational instability** in a sea of **dark matter**, starting with **scalar density perturbations** which we have detected imprinted on the **cosmic microwave background** ... if these were created by '**inflation**' then seeing the associated **tensor perturbations** would *prove* that inflation actually occurred!

$$V(\phi) = M_{\text{pl}}^2 H^2$$



Toy model of
slow-roll inflation:

$$\epsilon = \frac{M_{\text{pl}}^2}{2} \left(\frac{V'}{V} \right)^2 \ll 1$$

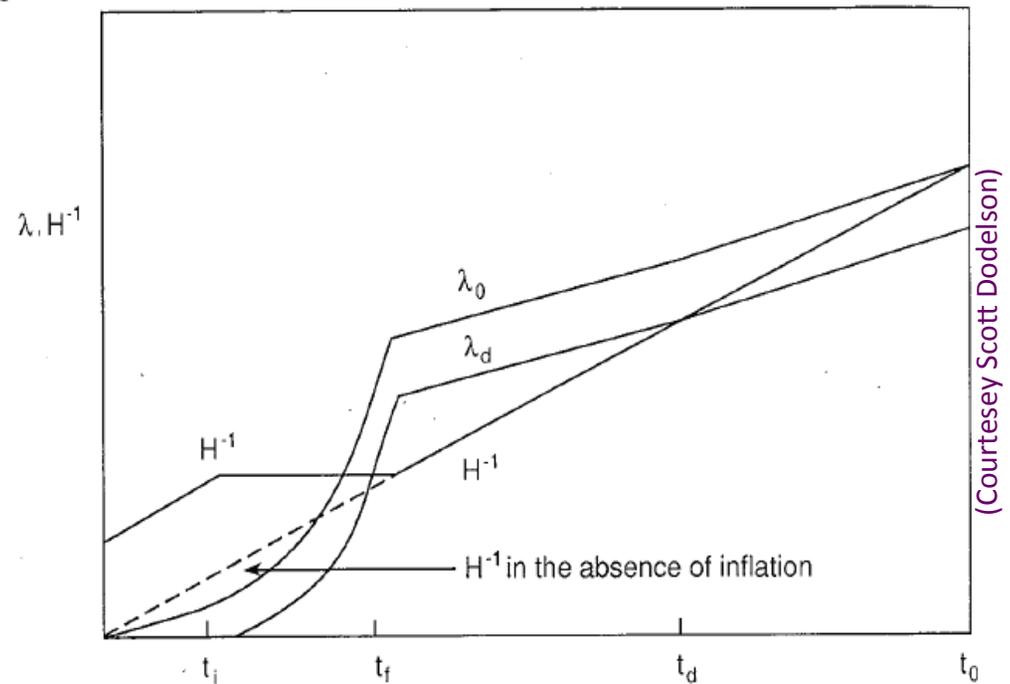
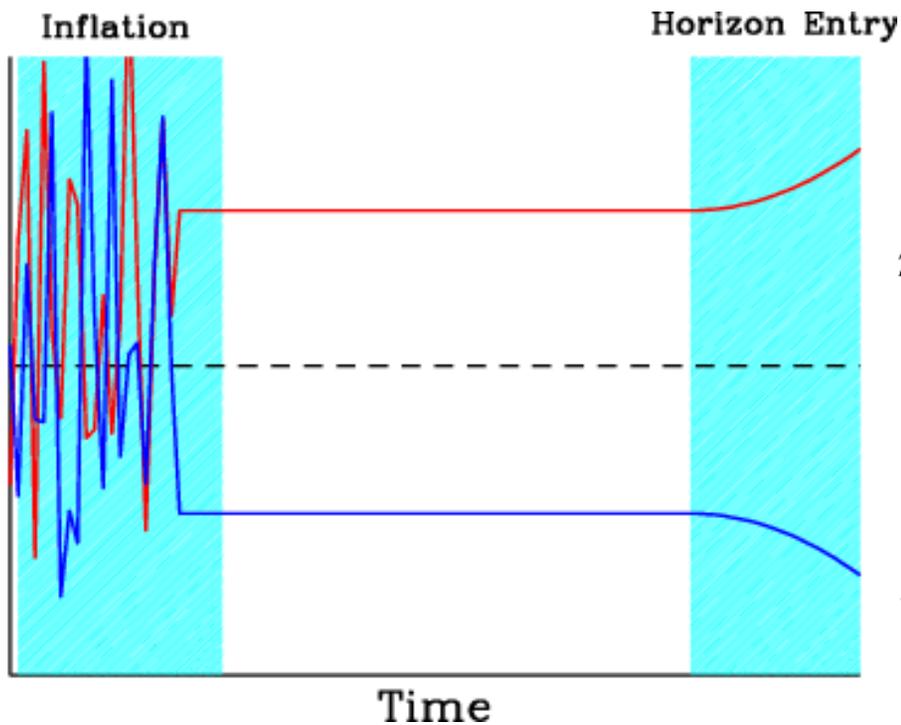
$$\eta = M_{\text{pl}}^2 \frac{V''}{V} \ll 1$$

The slow evolution of a scalar field down a nearly flat part of its potential during which its vacuum energy is nearly constant so:

$$a \propto e^{H_{\text{infl}} t}, \quad \text{with} \quad H_{\text{infl}} = \sqrt{\frac{8\pi G_{\text{N}}}{3} V_0}$$

If the number of e-folds $N(\phi) = \int_{\phi_{\text{ini}}}^{\phi_{\text{end}}} \frac{H_{\text{infl}}}{\dot{\phi}} d\phi$ exceeds $\sim 50-60$, the region within the present Hubble radius would have been causally connected at the inflationary epoch, thus solving the ‘horizon problem’ (+ ‘monopole problem’)

- Quantum mechanical fluctuations: $\langle \Psi(\mathbf{k}) \Psi(\mathbf{k}') \rangle = (2\pi)^3 \delta^3(\mathbf{k}-\mathbf{k}') P_\Psi(\mathbf{k})$
- Inflation *stretches* wavelength beyond horizon: $\Psi(\mathbf{k}, t)$ becomes constant (until horizon reentry after inflation ends – *first out, last in*)
- Infinite number of independent perturbations with independent amplitudes, but ... **inflation synchronizes all modes!**



During inflation, Ψ fluctuates quantum mechanically around a smooth background ... its *mean* value is zero, but its variance is:

$$\begin{aligned} \langle \Psi^2(\vec{x}) \rangle &= \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} e^{i\vec{k}'\cdot\vec{x}} \langle \tilde{\Psi}(\vec{k}) \tilde{\Psi}(\vec{k}') \rangle \\ &= \int \frac{dk}{k} \frac{k^3 P_{\Psi}(k)}{2\pi^2} \end{aligned}$$

... so get equal contributions on all scales if $P_{\Psi} \propto k^{-4+n}$ with $n = 1$ (“scale-invariant” spectrum)

In the toy model of inflation the slope of the potential (which provides the ‘arrow of time’) induces a ‘tilt’ to the spectrum of scalar density perturbations which have amplitude:

$$\Delta_s^2 \equiv \left(\frac{H^2}{2\pi\dot{\psi}} \right)^2$$

Inflation also generates a spectrum of tensor perturbations (gravitational waves) with amplitude:

$$\Delta_t^2 \equiv \frac{2}{\pi^2} \frac{H^2}{M_{\text{Pl}}^2}$$

The ratio of tensor to scalar perturbations is therefore: (characteristic of the inflationary potential)

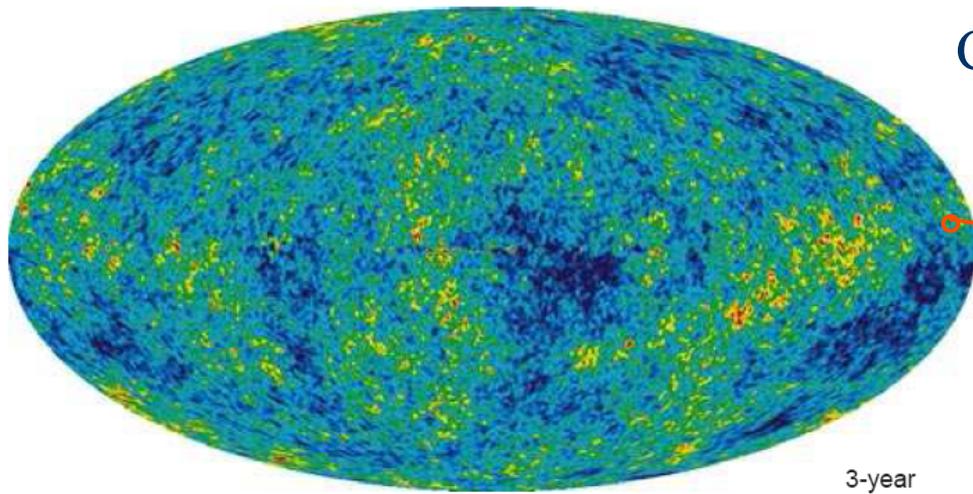
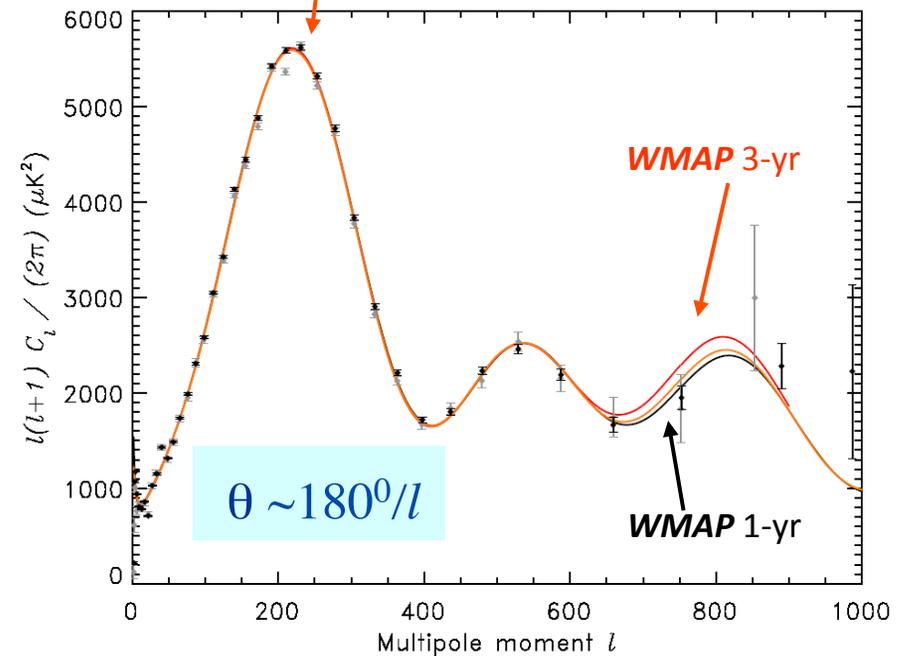
$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} = \frac{8}{M_{\text{Pl}}^2} \left(\frac{\dot{\psi}}{H} \right)^2$$

Coherent oscillations in photon-baryon plasma ... excited by density perturbations on *super*-horizon scales

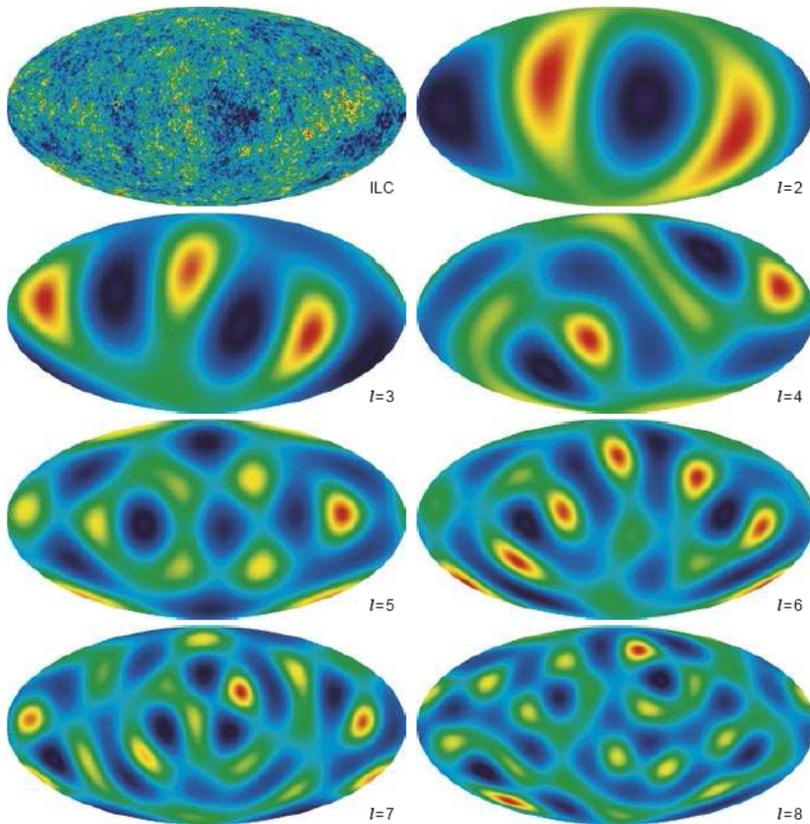
(Hubble radius at t_{rec})

$$\Delta T(\mathbf{n}) = \sum a_{lm} Y_{lm}(\mathbf{n})$$

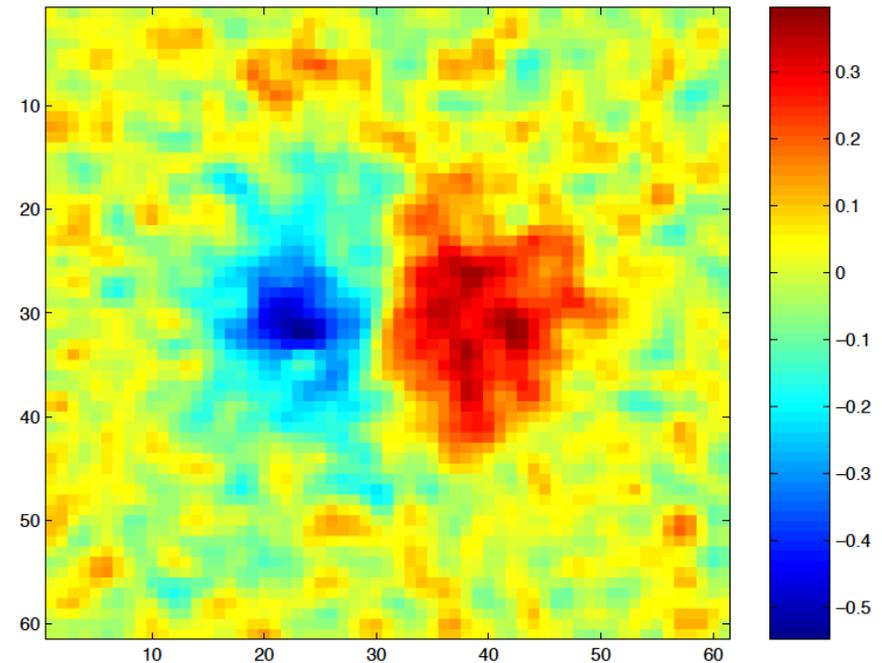
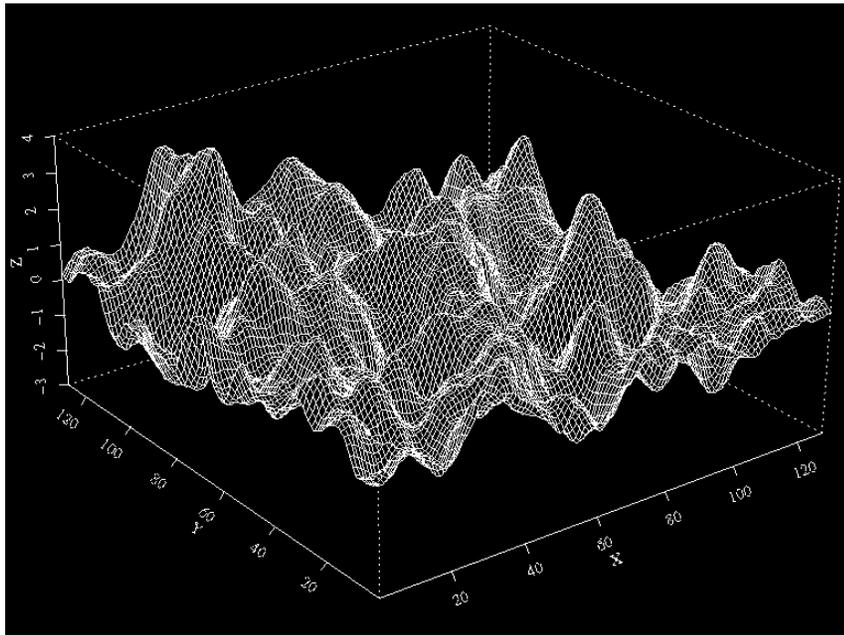
$$C_l \equiv \frac{1}{2l+1} \sum |a_{lm}|^2$$



-200 T(μK) +200



$O(10^7)$ pixels can be reduced to $O(10^3)$ multipoles *only* by assuming that it is a random Gaussian density field!

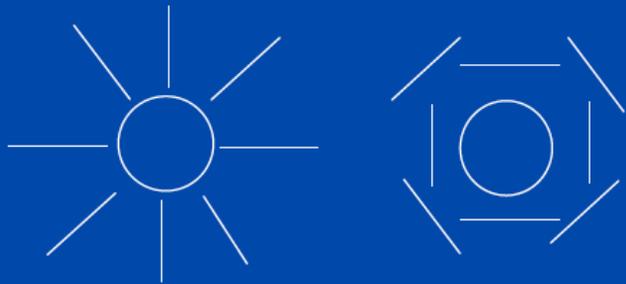


... and $O(10^3)$ multipoles can be fitted by a power-law spectrum characterized by 2 parameters (amplitude & slope) *only* by assuming it is close to a scale-invariant spectrum

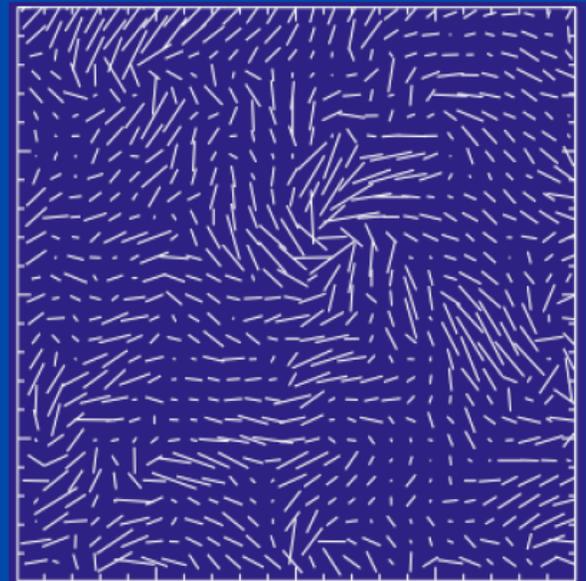
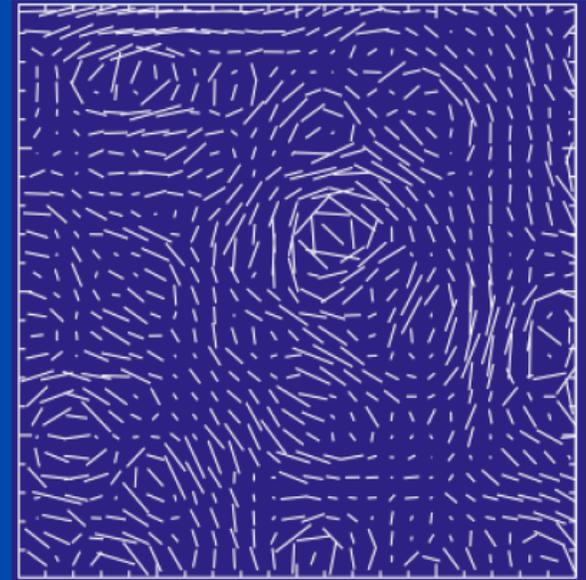
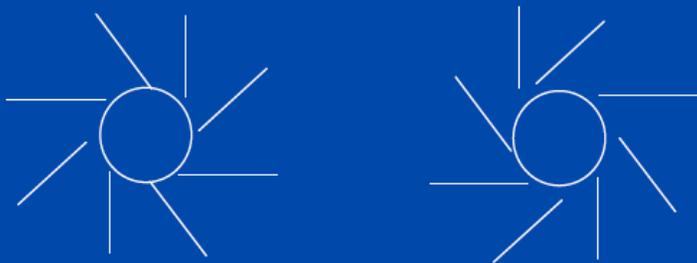
Gaussianity and scale –invariance are characteristic of the quantum fluctuations of a free massless scalar field in a \sim De Sitter background ... so we implicitly *assume* that slow-roll inflation is the origin of CMB temperature fluctuations

E and B modes polarization (similar to gradient/curl decomposition of vector field)

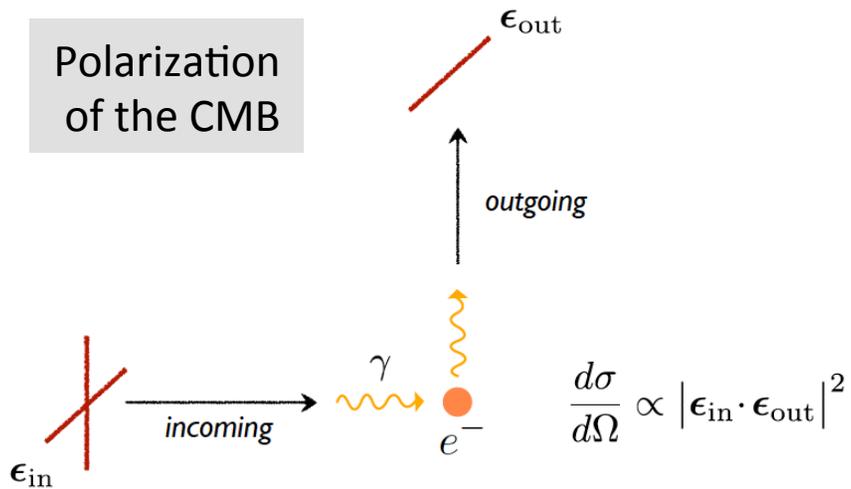
E polarization
from scalar, vector and tensor modes



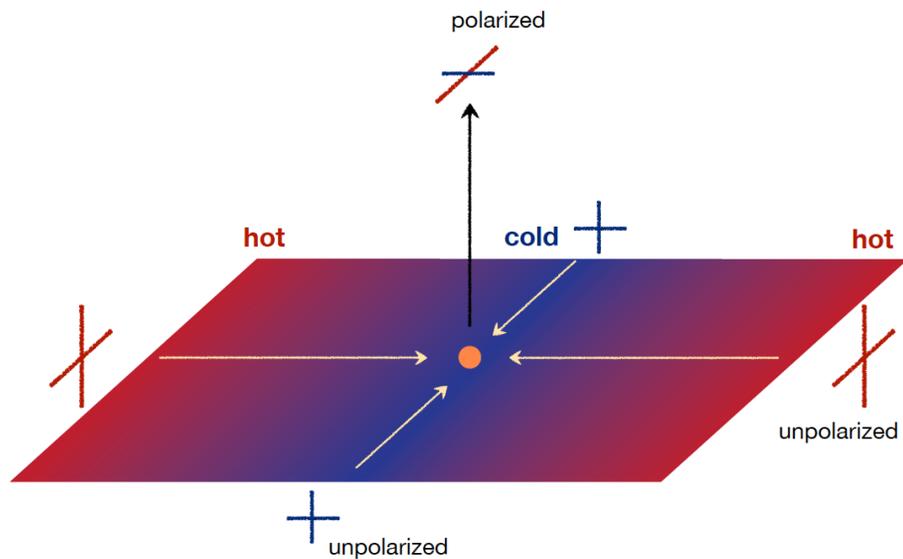
B polarization only from (vector)
tensor modes (and gravitational lensing of E polarization)



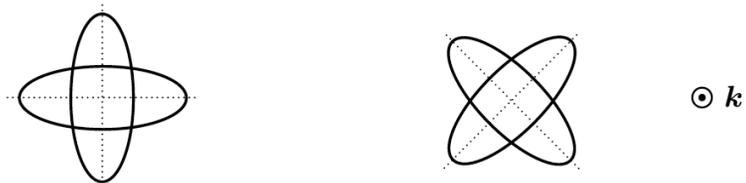
Polarization of the CMB



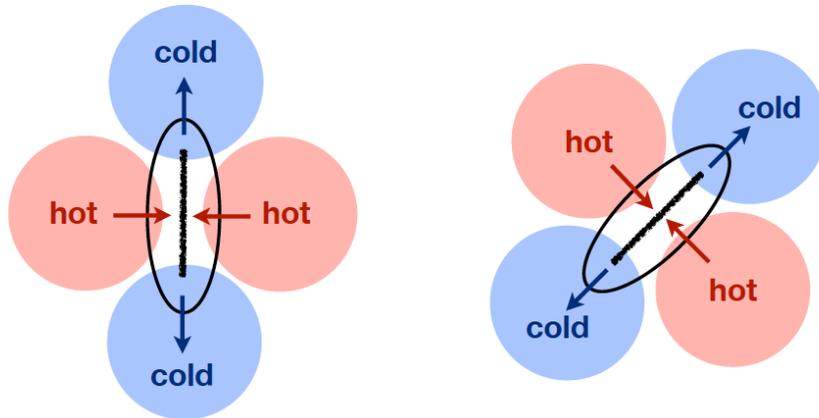
Quadrupolar temperature anisotropy leads to linear polarization:



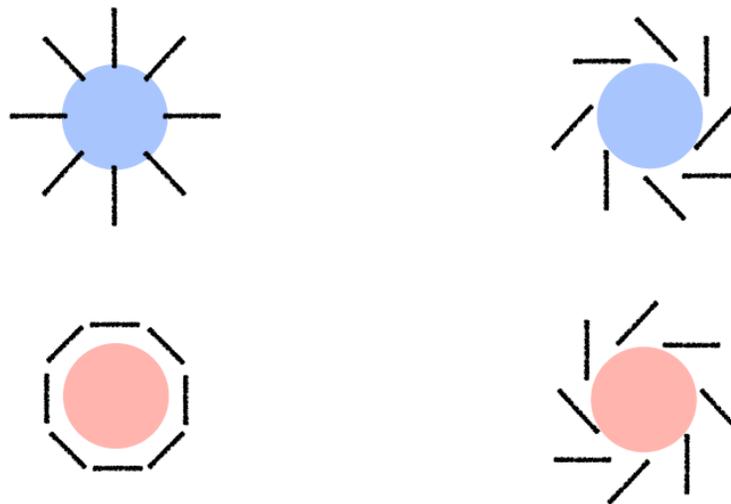
Recall the two polarization modes of a gravitational wave:



The anisotropic stretching of space induces a temperature quadrupole and scattering produces two types of polarization



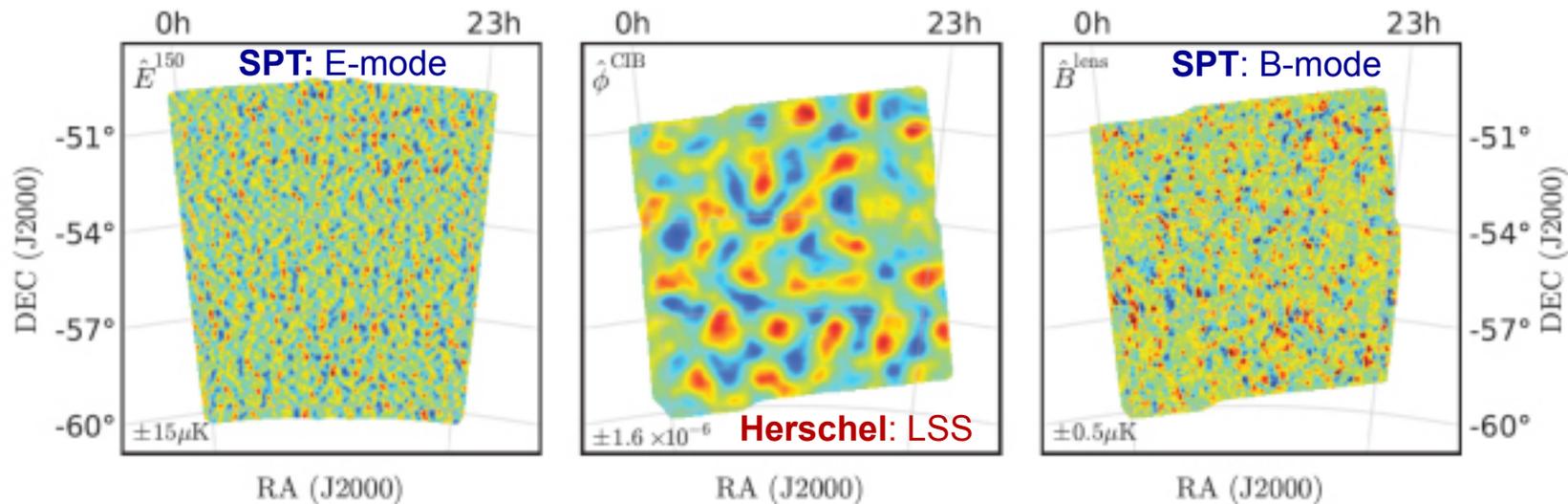
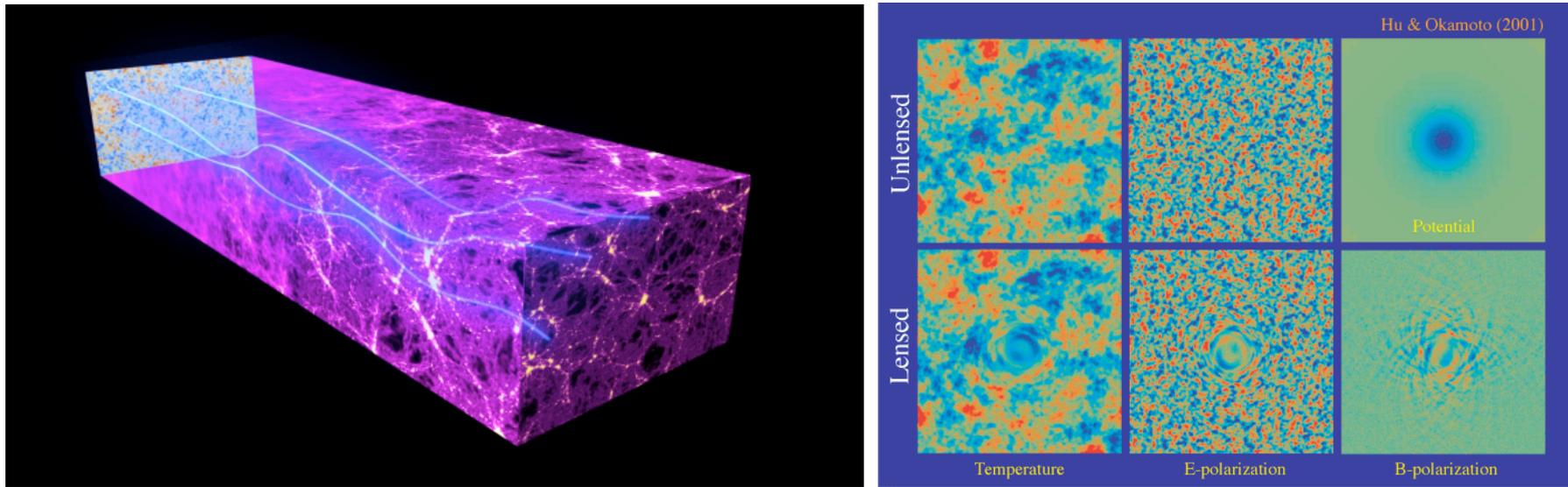
Summing over many waves, we get the following polarization patterns around hot and cold spots:



E-mode
(grad)

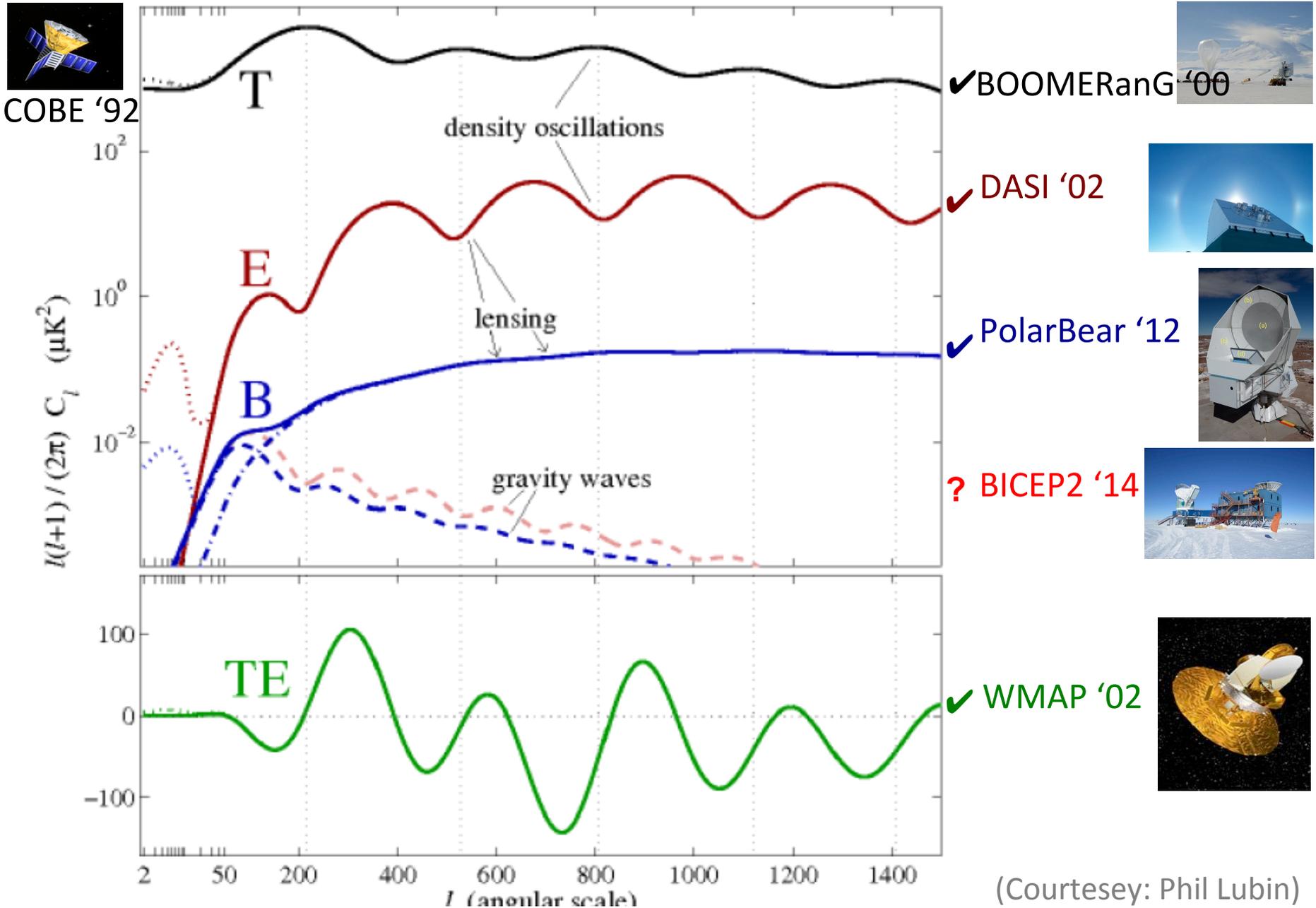
B-mode
(curl)

E mode \rightarrow B mode through gravitational lensing of the CMB



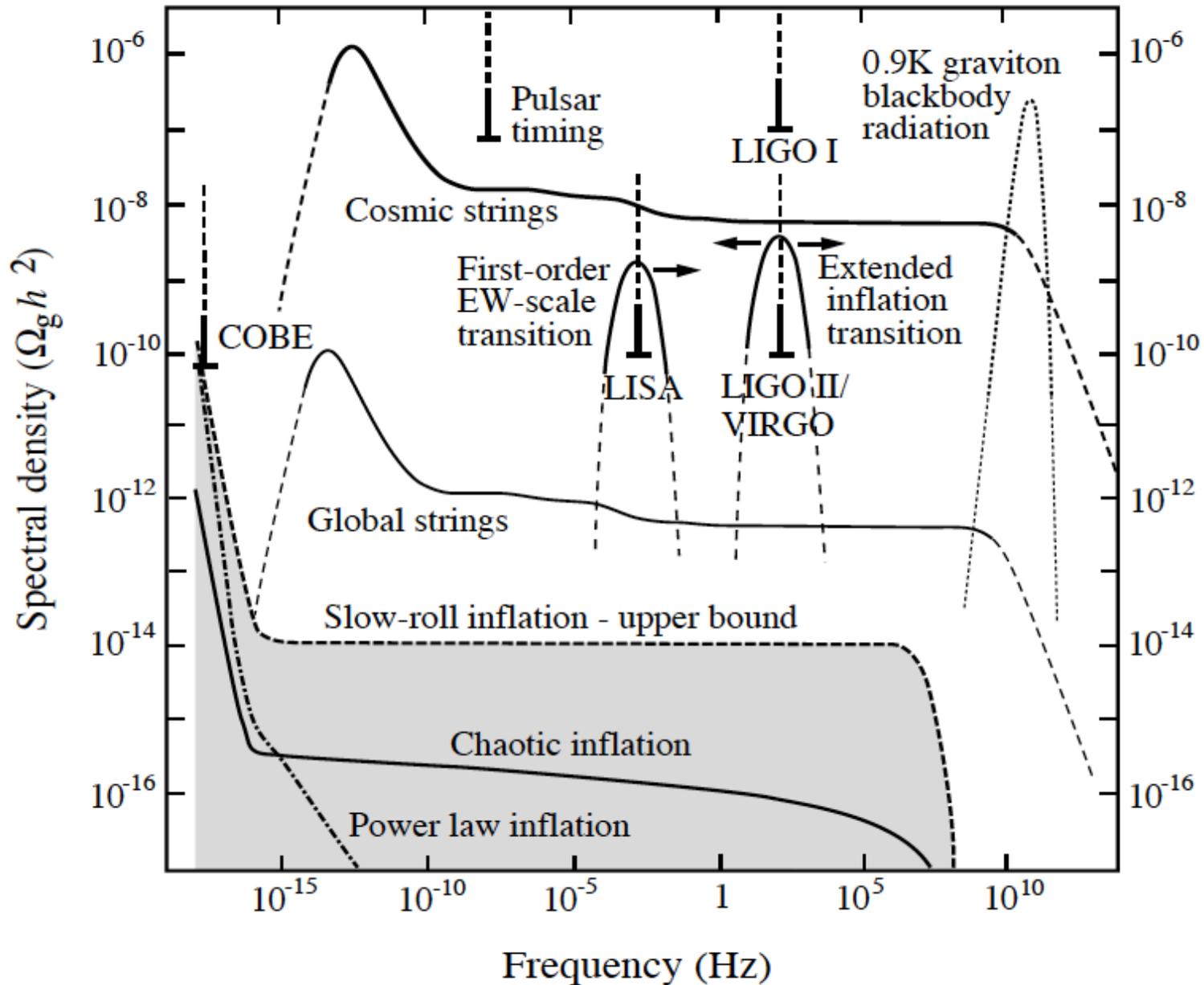
Depicts: E-modes and B-modes in the CMB polarisation (left and right panels, respectively) and the gravitational potential of the large-scale distribution of matter that is lensing the CMB (central panel)
 Copyright: Image from D. Hanson, et al., 2013, Physical Review Letters

Inflationary predictions for (adiabatic) CMB fluctuations

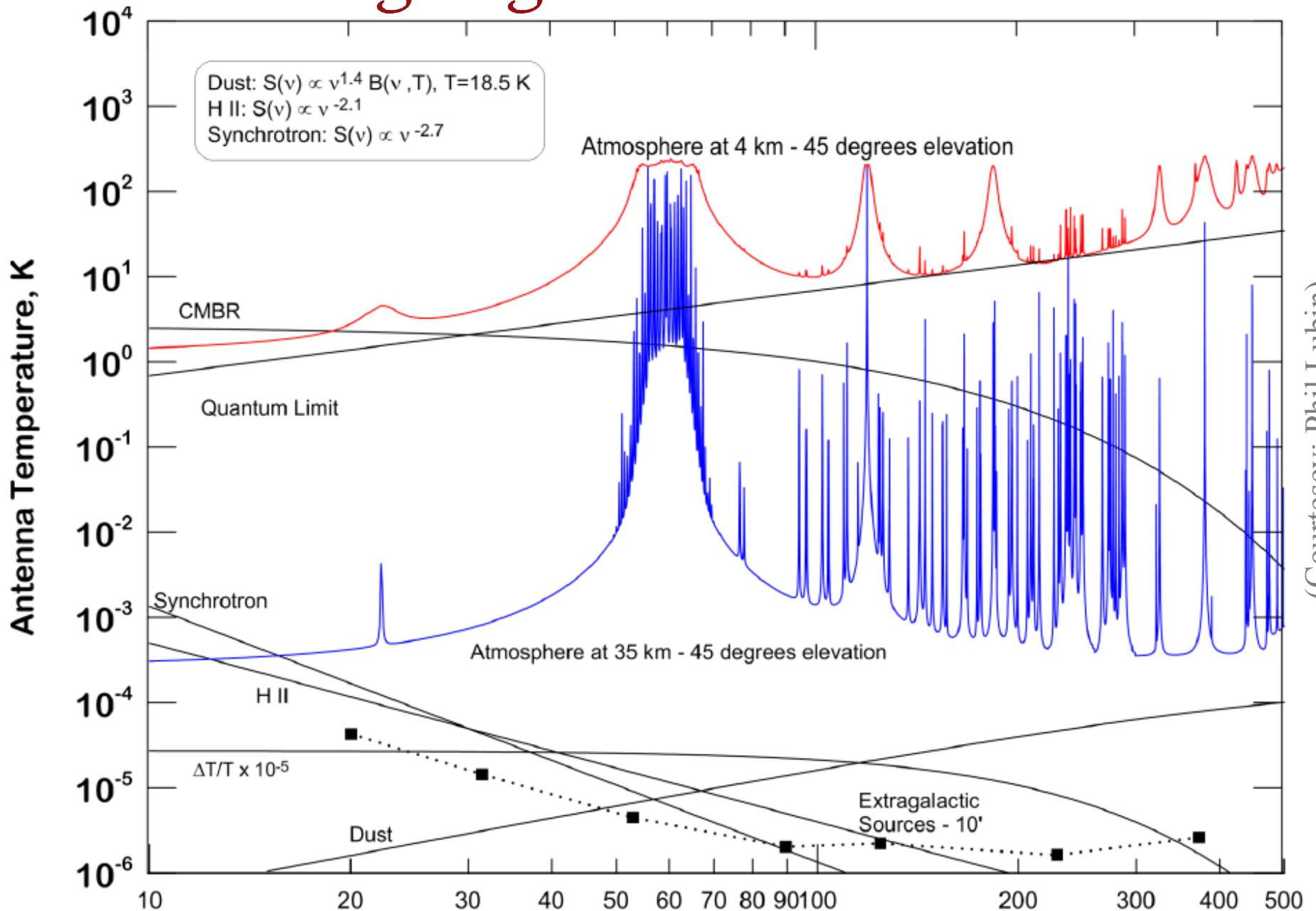


(Courtesy: Phil Lubin)

... well below the sensitivity of gravitational wave detectors



The challenge of ground-based CMB observations

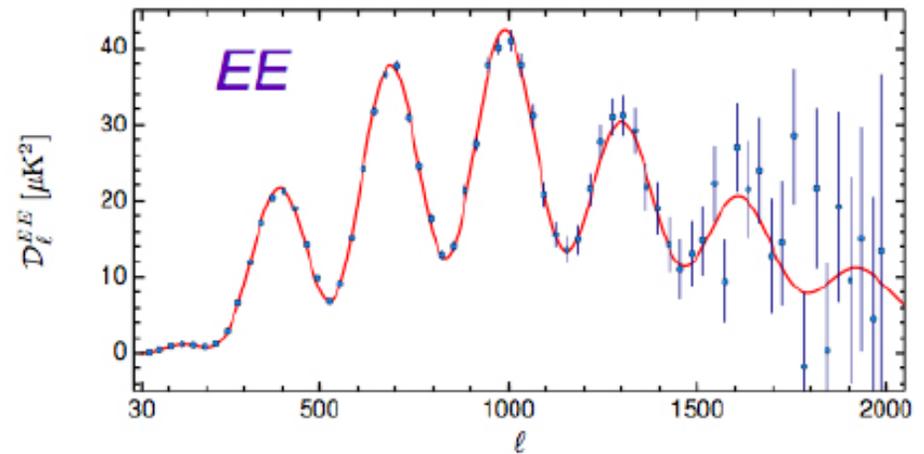
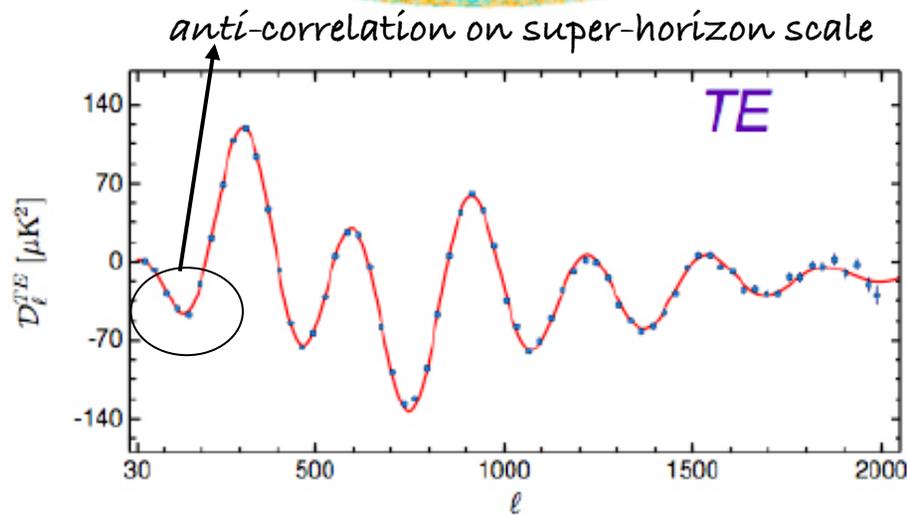
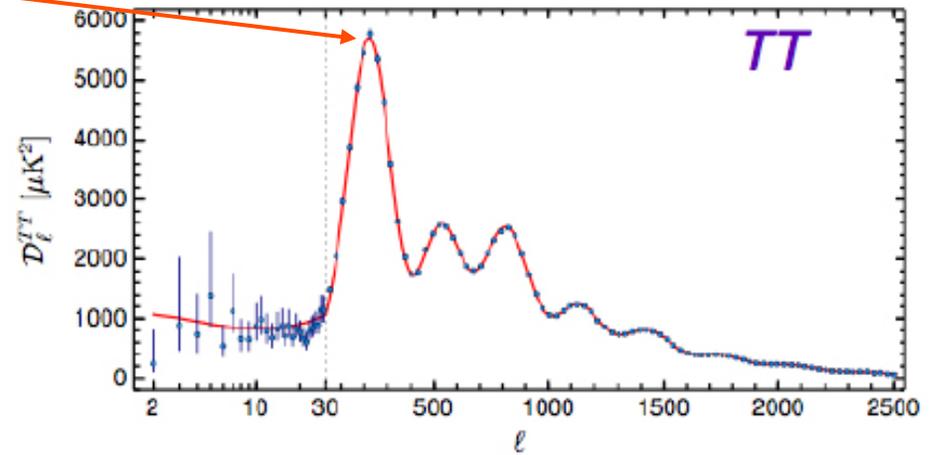
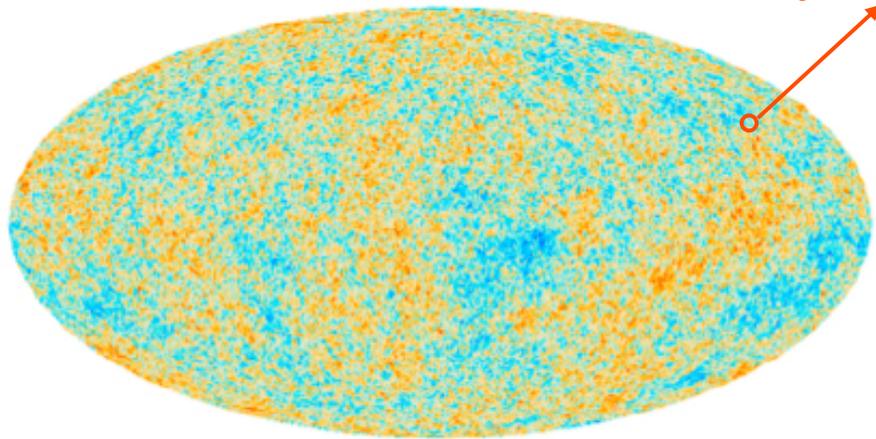


Planck data release II – 1st December 2014

$$\Delta T(\mathbf{n}) = \sum a_{lm} Y_{lm}(\mathbf{n})$$

$$C_l \equiv \frac{1}{2l+1} \sum |a_{lm}|^2$$

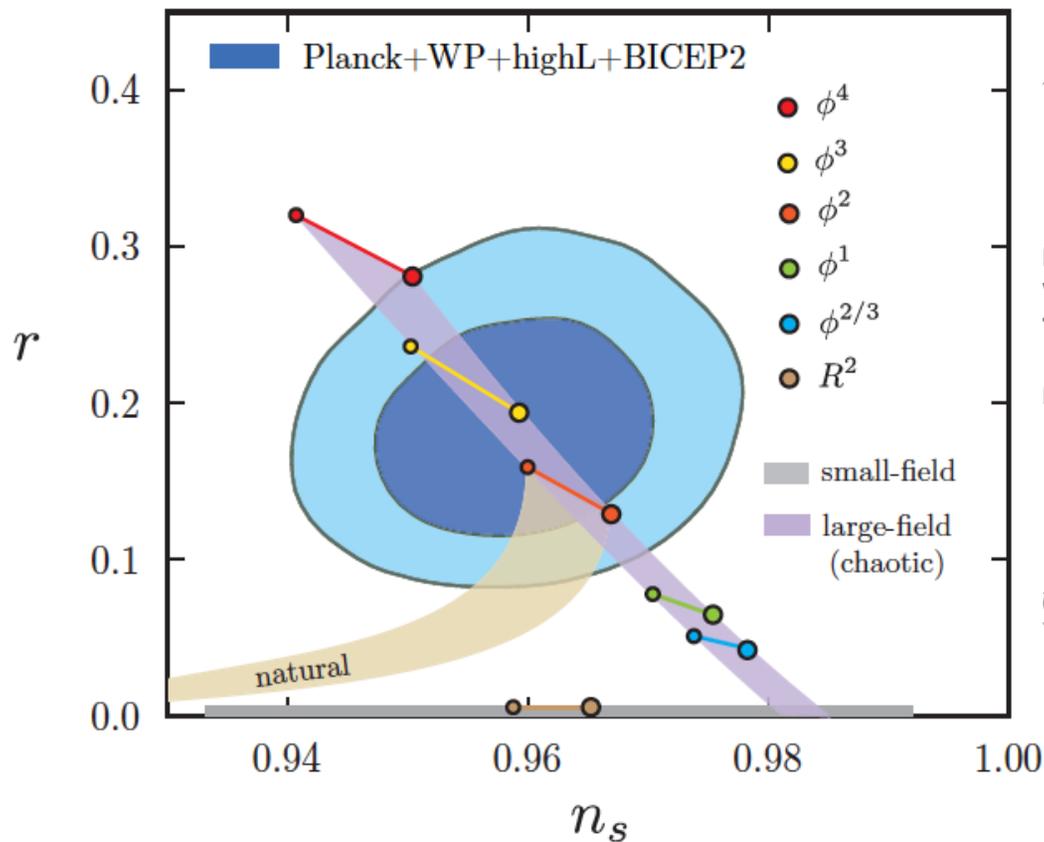
spatial scale of today's universe at (re)combination



Coherent oscillations in a photon+baryon plasma excited by primordial scalar density perturbations on *super-horizon* length scales

So had the *BICEP2* claim that $r \approx 0.2$ been correct
 ... then we would have learnt that:

- The energy scale of inflation is: $V^{1/4} \approx 2.1 \times 10^{16} \text{ GeV} (r/0.2)^{1/4} \sim M_{\text{GUT}}$
- The field excursion was super-Planckian: $\Delta\phi \approx 4 M_{\text{Pl}} (r/0.2)^{1/2}$



- The vacuum energy was cancelled to 1 part in 10^{112} after inflation!

The BICEP2 Telescope

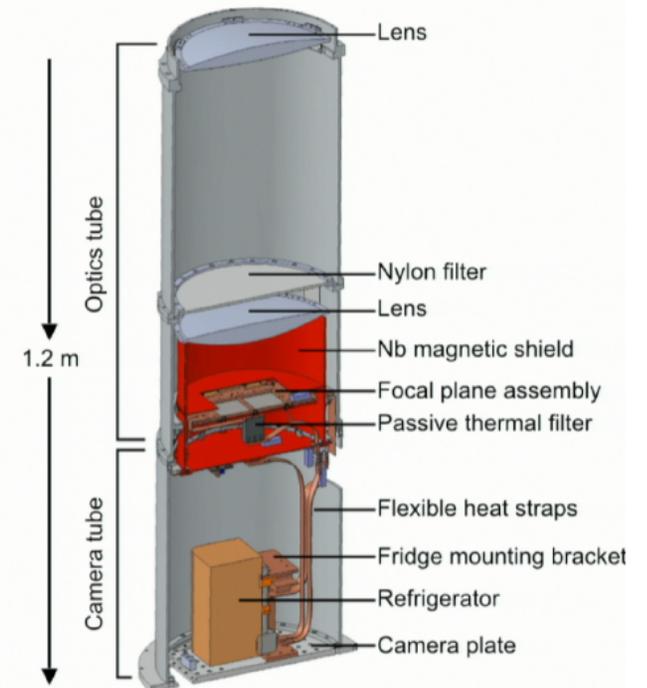


Telescope as compact as possible while still having the angular resolution to observe degree-scale features.

On-axis, refractive optics allow the entire telescope to rotate around boresight for polarization modulation.

Liquid helium cools the optical elements to 4.2 K.

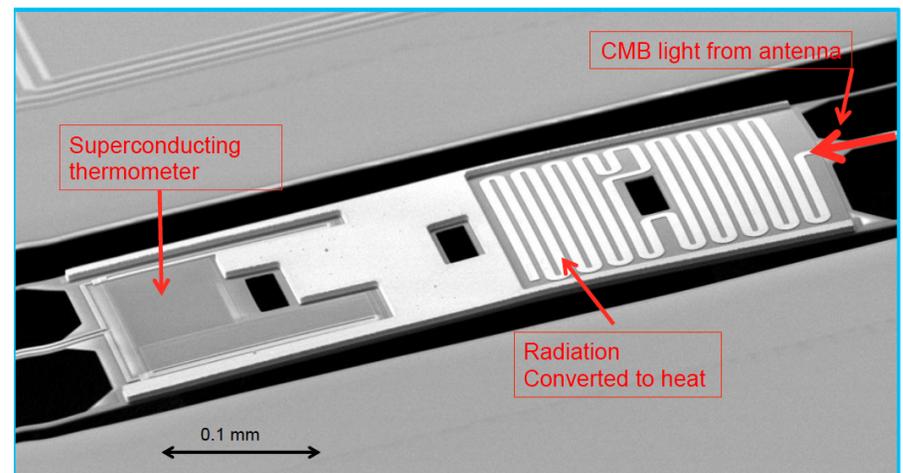
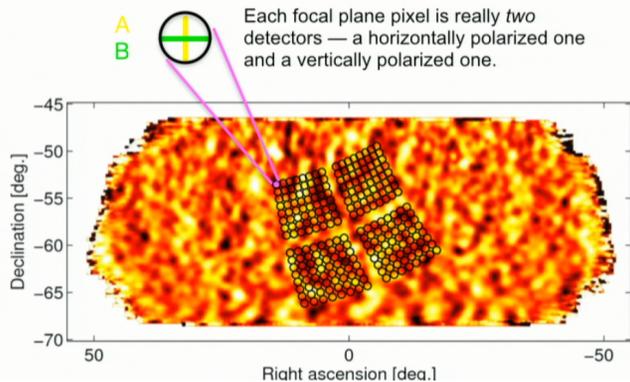
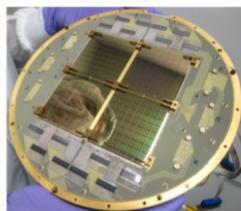
A 3-stage helium sorption refrigerator further cools the detectors to 0.27 K.



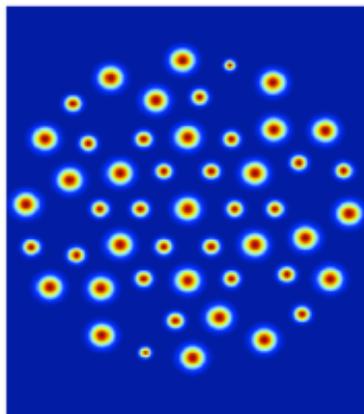
Scan the telescope back and forth on the sky.

Measure CMB T by summing the signal from orthogonally polarized detector pairs.

Measure CMB polarization by differencing the signal.



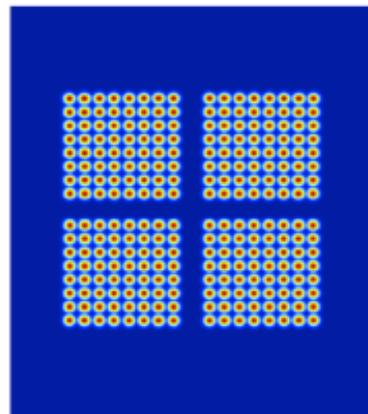
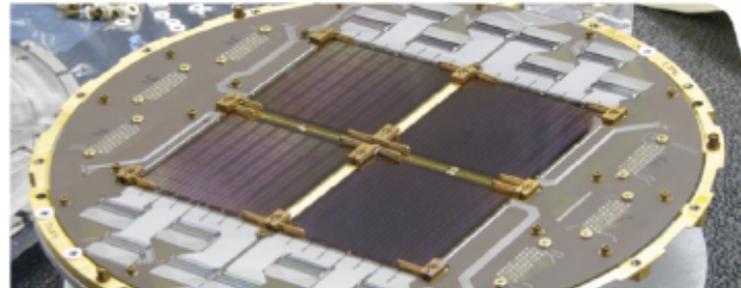
BICEP1 (2006-2008)



-5 0 5
Longitude (degrees)

98 NTDs (95/150 GHz)

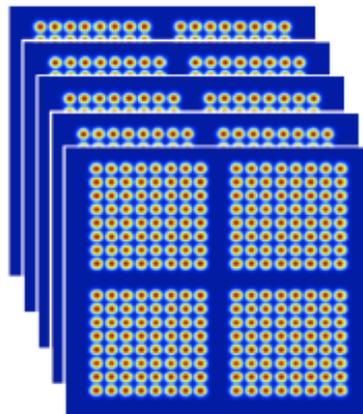
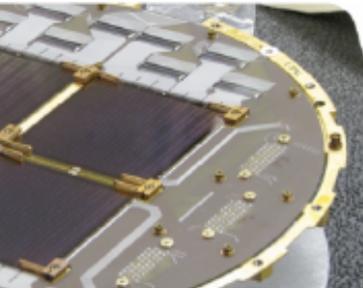
BICEP2 (2010-2012)



-5 0 5
Longitude (degrees)

512 TESs (150 GHz)

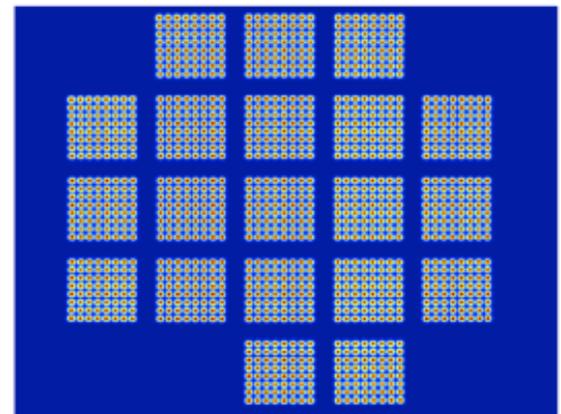
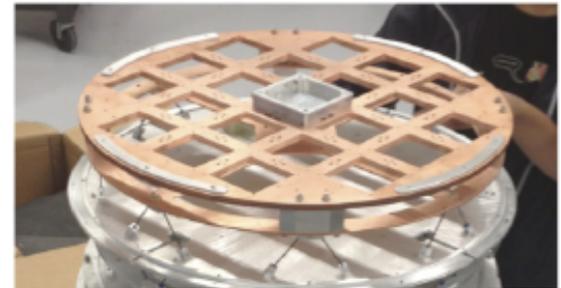
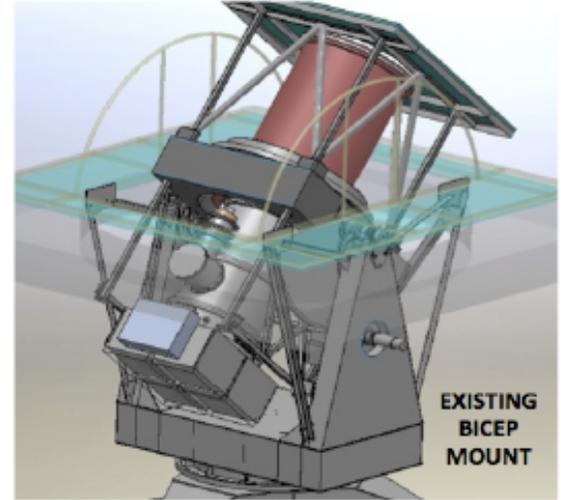
Keck Array (2011-2016)



-5 0 5
Longitude (degrees)

2560 TESs (150 GHz)

BICEP3 (2015-2016)



-10 -5 0 5 10
Longitude (degrees)

2560 TESs (95 GHz)

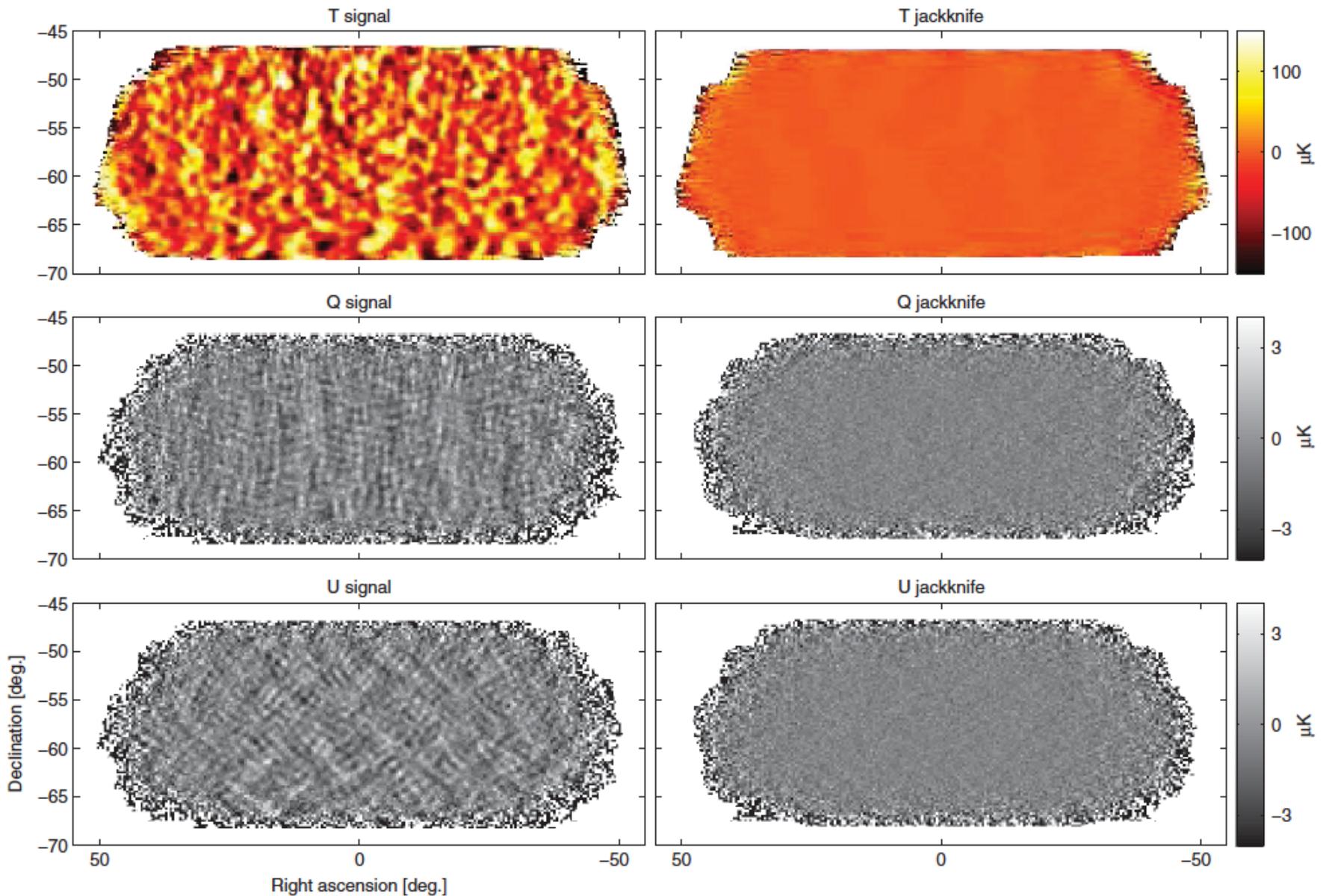


FIG. 1 (color). BICEP2 T , Q , U maps. The left column shows the basic signal maps with 0.25° pixelization as output by the reduction pipeline. The right column shows difference (jackknife) maps made with the first and second halves of the data set. No additional filtering other than that imposed by the instrument beam (FWHM 0.5°) has been done. Note that the structure seen in the Q and U signal maps is as expected for an E -mode dominated sky.

BICEP2 claimed to have detected the B-mode signal from inflation!

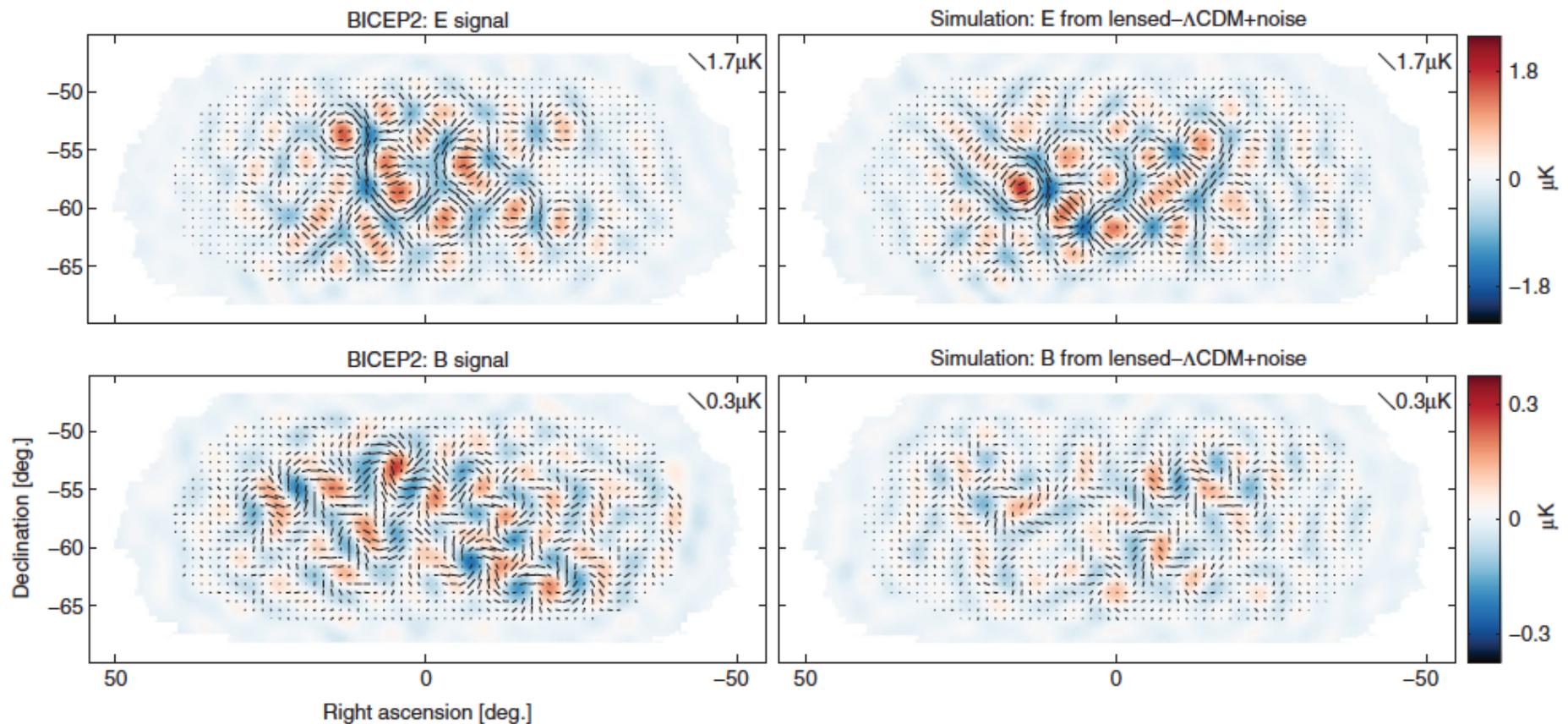
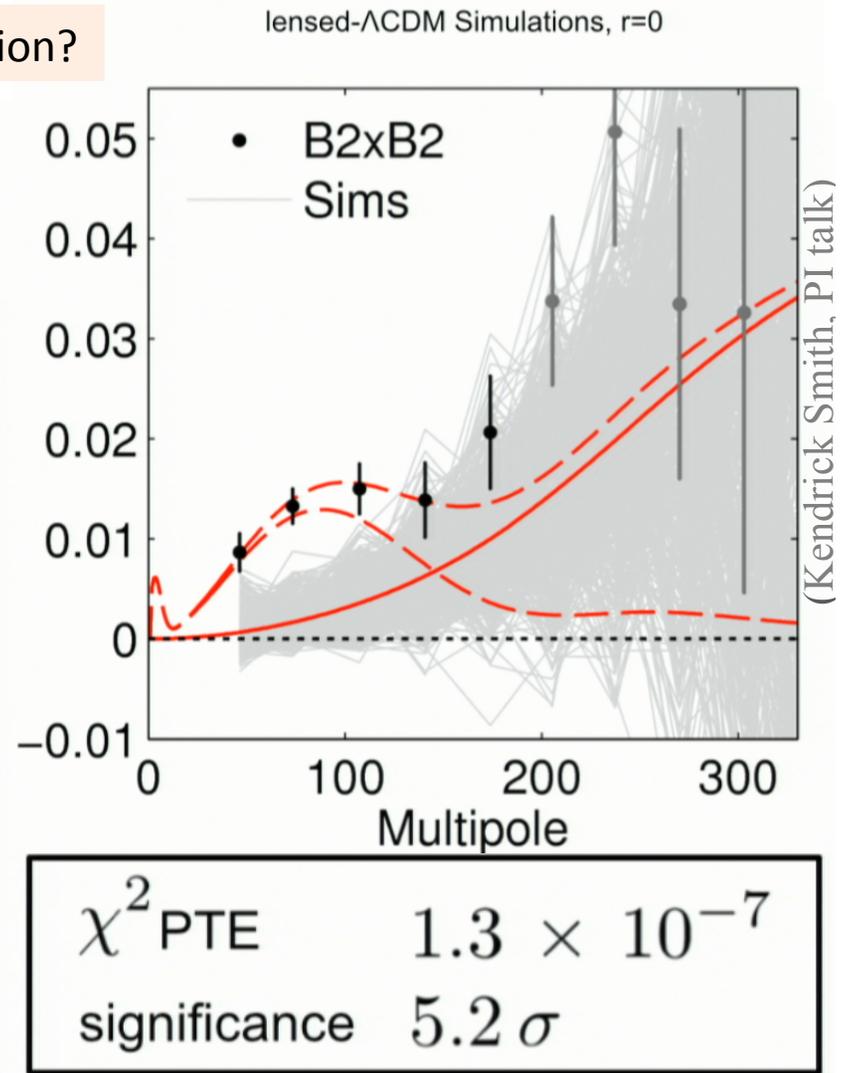
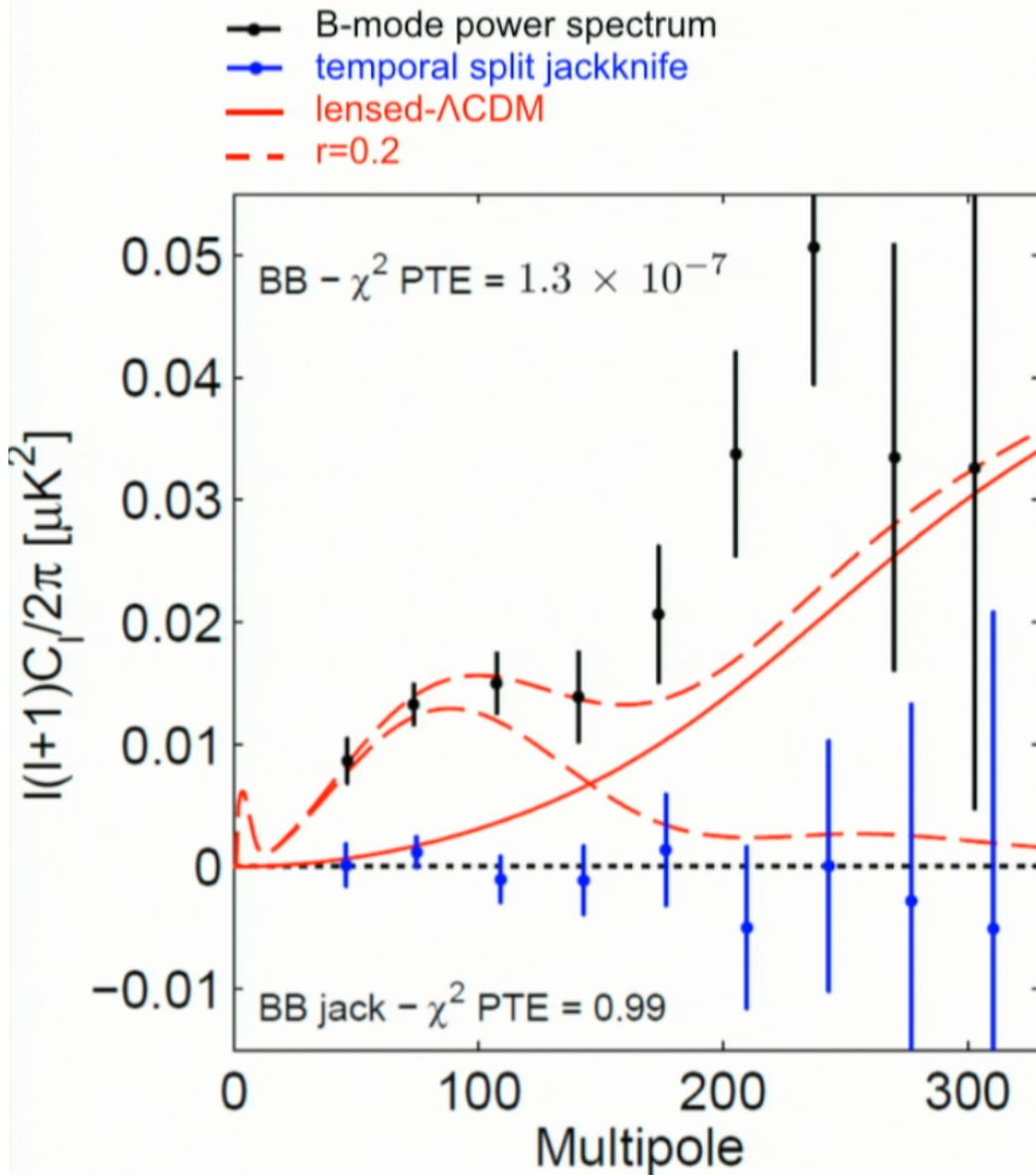


FIG. 3 (color). Left: BICEP2 apodized E -mode and B -mode maps filtered to $50 < \ell < 120$. Right: The equivalent maps for the first of the lensed- Λ CDM + noise simulations. The color scale displays the E -mode scalar and B -mode pseudoscalar patterns while the lines display the equivalent magnitude and orientation of linear polarization. Note that excess B mode is detected over lensing+noise with high signal-to-noise ratio in the map ($s/n > 2$ per map mode at $\ell \approx 70$). (Also note that the E -mode and B -mode maps use different color and length scales.)

What is the actual significance of the B -mode detection?

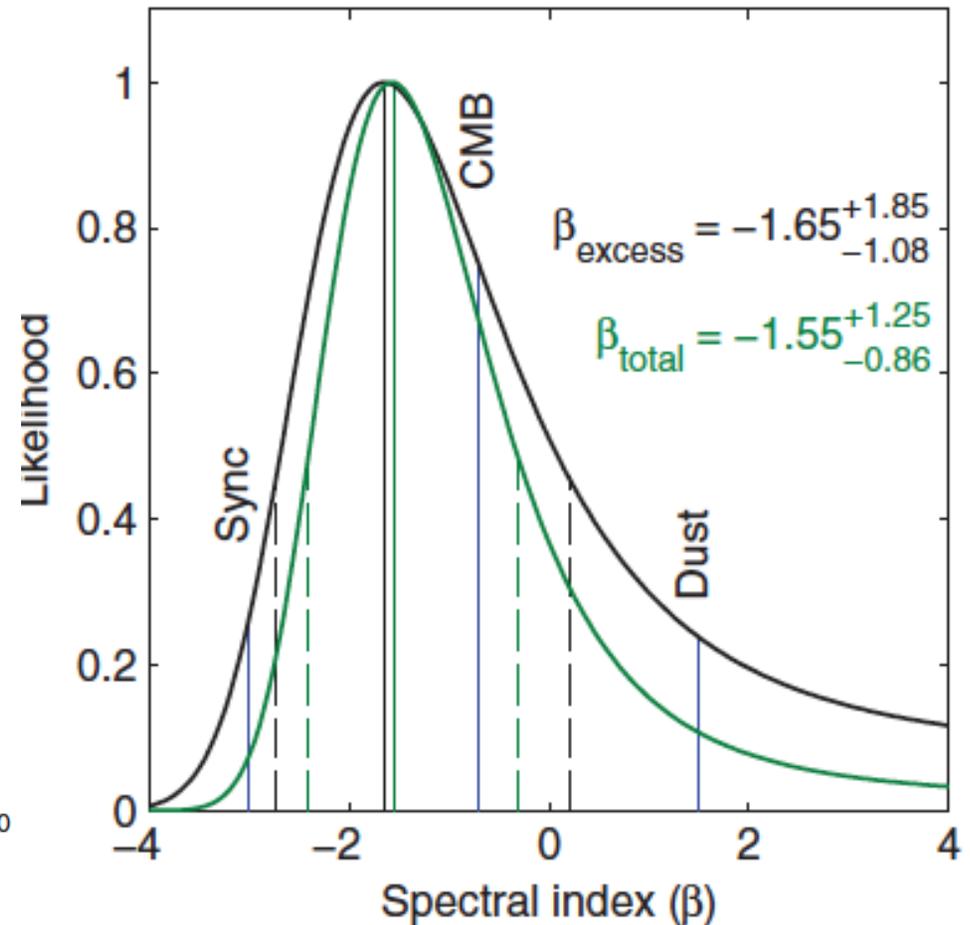
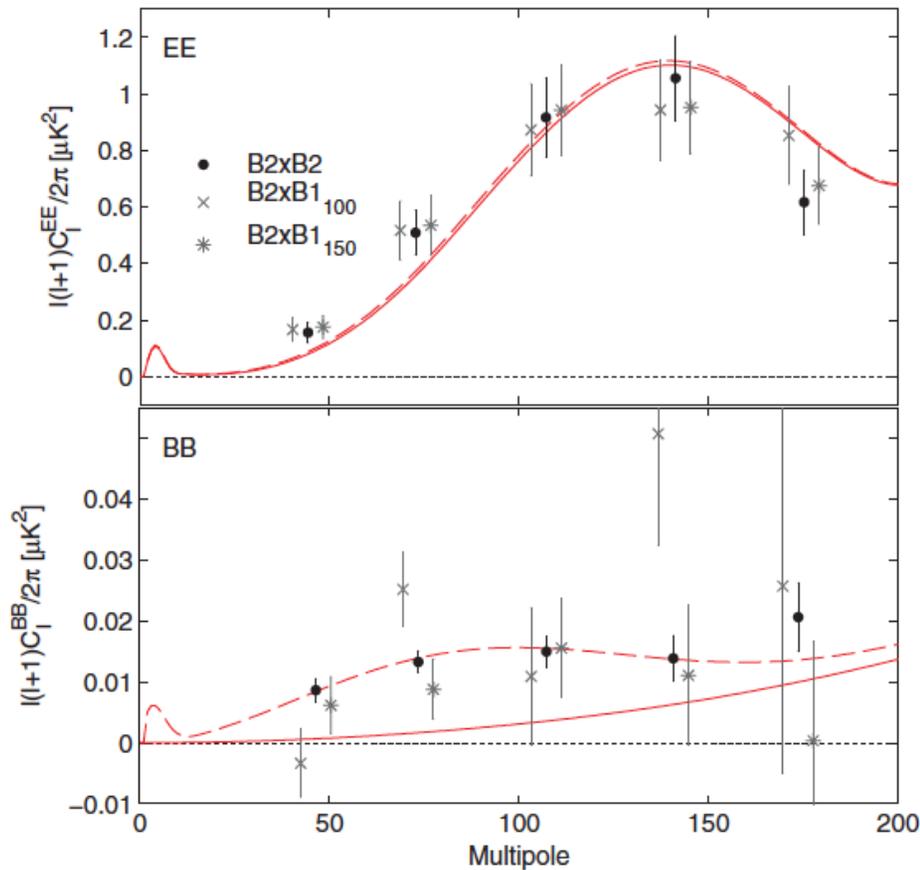


Ade *et al*, PRL 112:241101,2014

This is just the chance probability of the observed B -mode signal to arise as a fluctuation of the lensed E -mode signal ... it is *not* a ' $>5\sigma$ detection' of a CMB signal

“We can use the BICEP2 auto and BICEP2xBICEP1₁₀₀ spectra to constrain the frequency dependence of the nominal signal, If the signal at 150 GHz were due to synchrotron we would expect the frequency cross spectrum to be much larger in amplitude than the BICEP2 auto spectrum. Conversely if the 150 GHz power were due to polarized dust emission we would not expect to see a significant correlation with the 100 GHz sky pattern.”

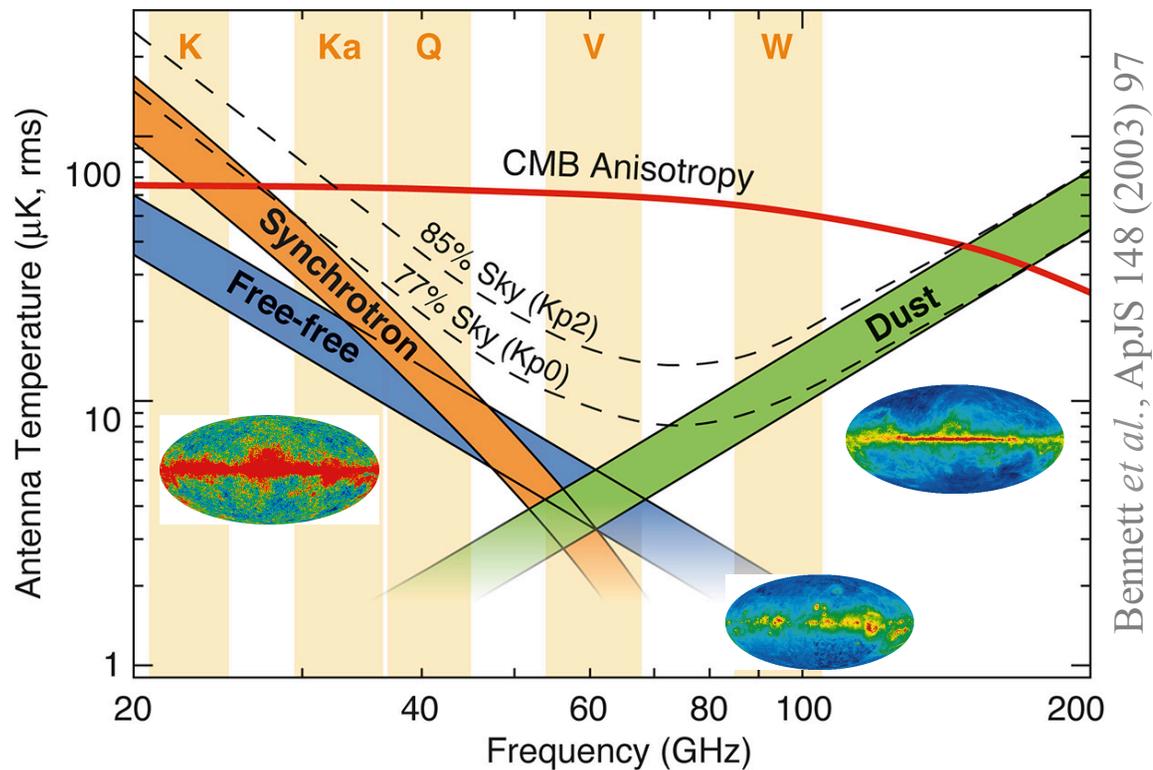
Ade *et al*, PRL 112:241101,2014



... so the significance with which the observed signal was likely to be **CMB** ($\beta \sim -0.7$) rather than either **synchrotron** ($\beta \sim -3$) or **dust** ($\beta \sim 1.5$) emission was *only* $1.6 - 1.7\sigma$

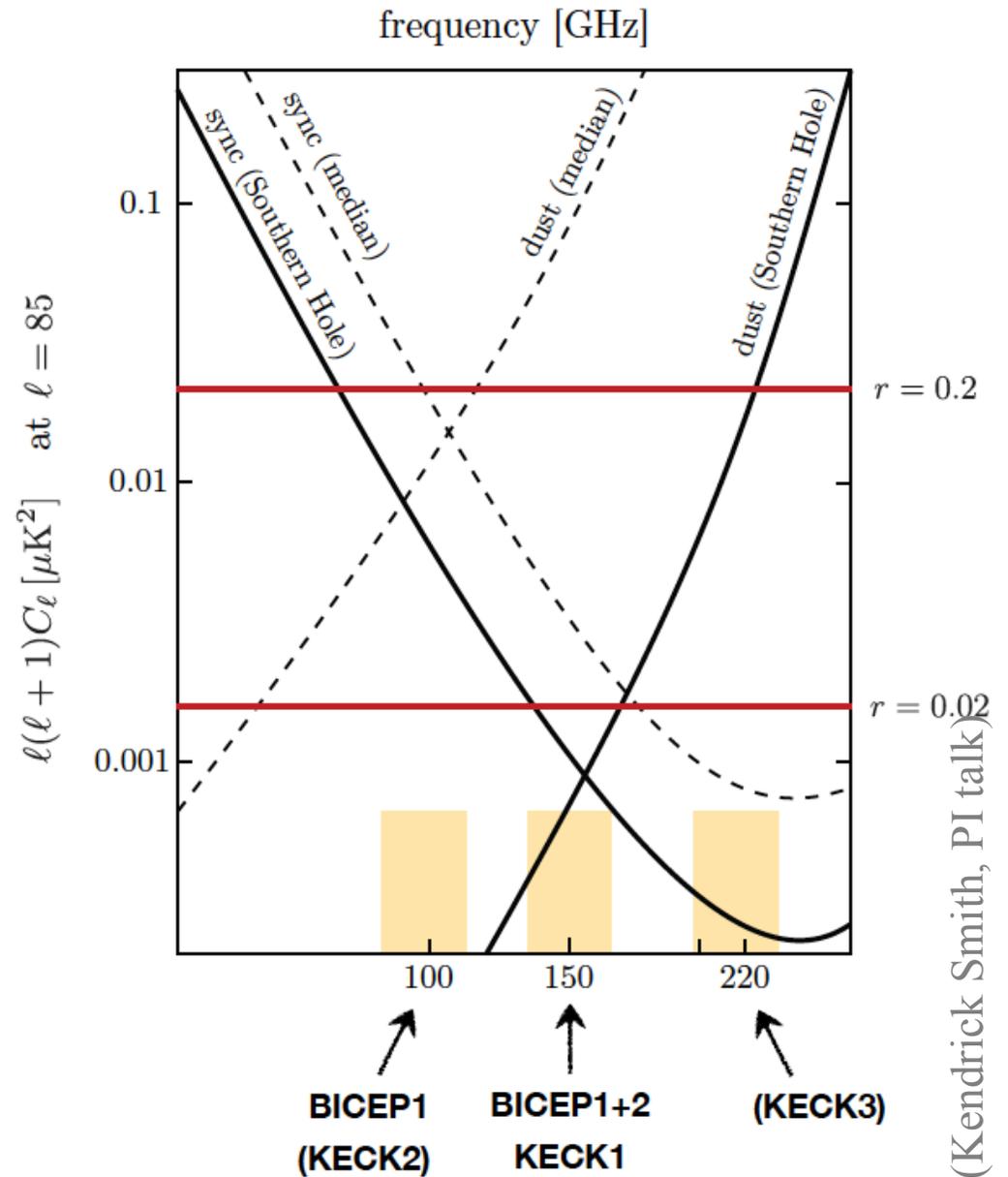
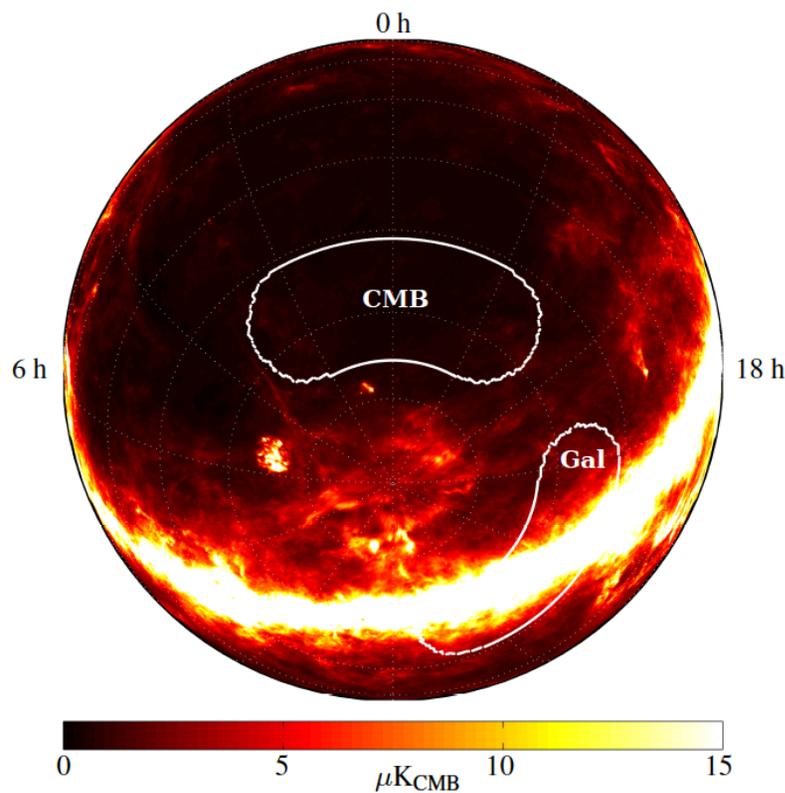
At CMB frequencies the most important sources of foregrounds are:

- Synchrotron radiation by cosmic ray electrons in the (ordered + turbulent) Galactic magnetic field (strongly polarised)
- Free-free emission from ionised hydrogen (unpolarised)
- Thermal dust emission (weakly polarised) + ‘spinning dust’ (unpolarised) + ?



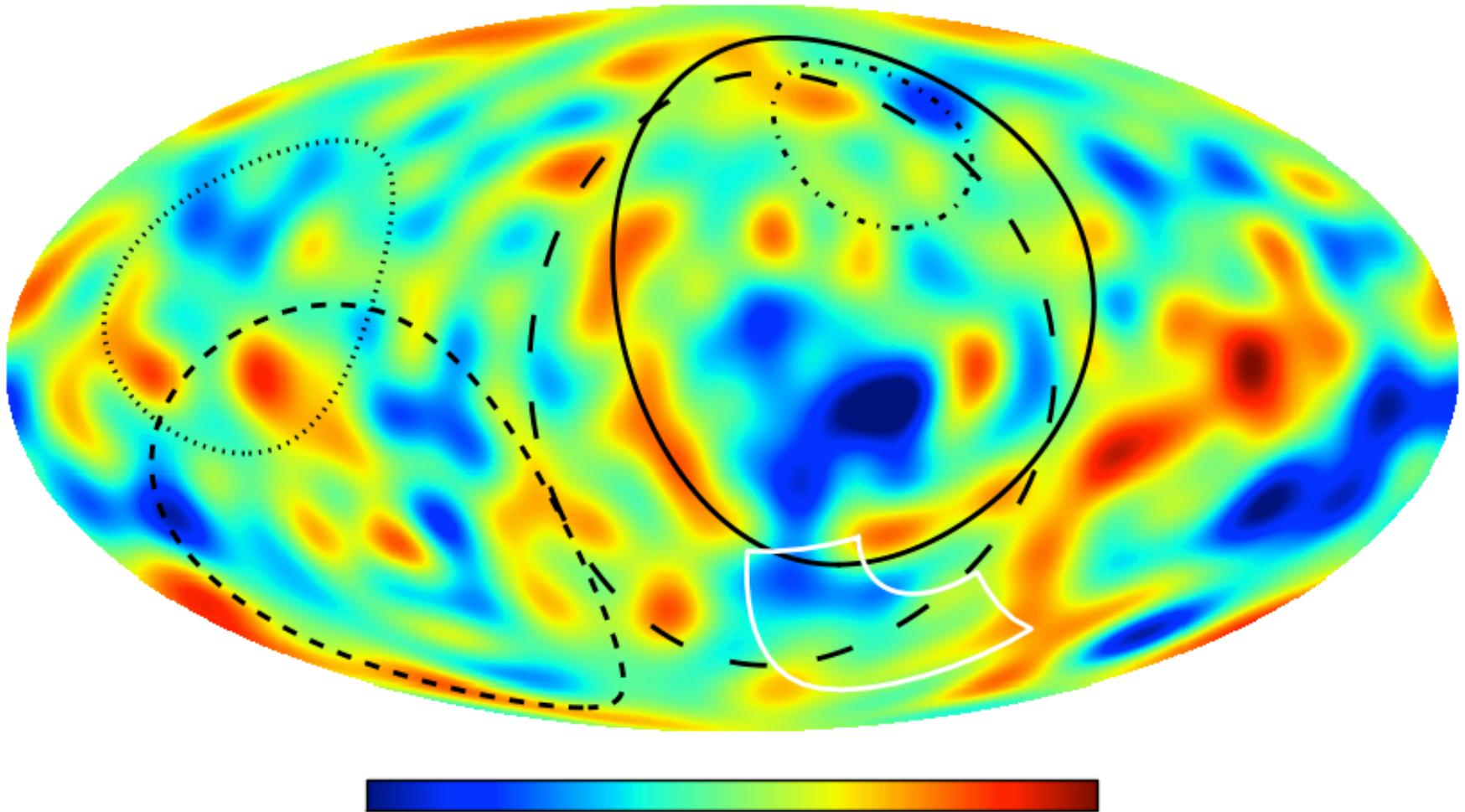
To subtract out the foregrounds, observe at multiple frequencies and isolate the CMB by its blackbody spectrum ... and/or look at high galactic latitude away from Milky Way

BICEP2 observed a small patch of high-latitude sky chosen to minimise these foregrounds ... but the levels were estimates (*not* observations)



(Kendrick Smith, PI talk)

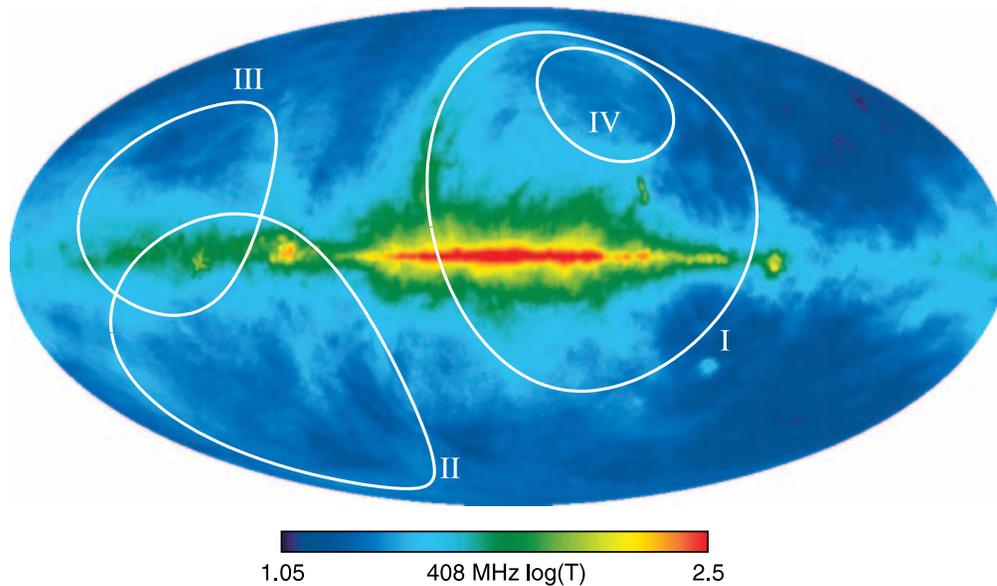
This particular patch of sky was chosen to be observed because:
“... such ultra clean regions are very special – at least an order of magnitude cleaner than the average $b > 50^\circ$ level” *Ade et al, PRL 112:241101,2014*



However it is in fact crossed by a galactic ‘radio loop’!

What are the 'radio loops'?

Haslam *et al*, A&AS 47:1,1982

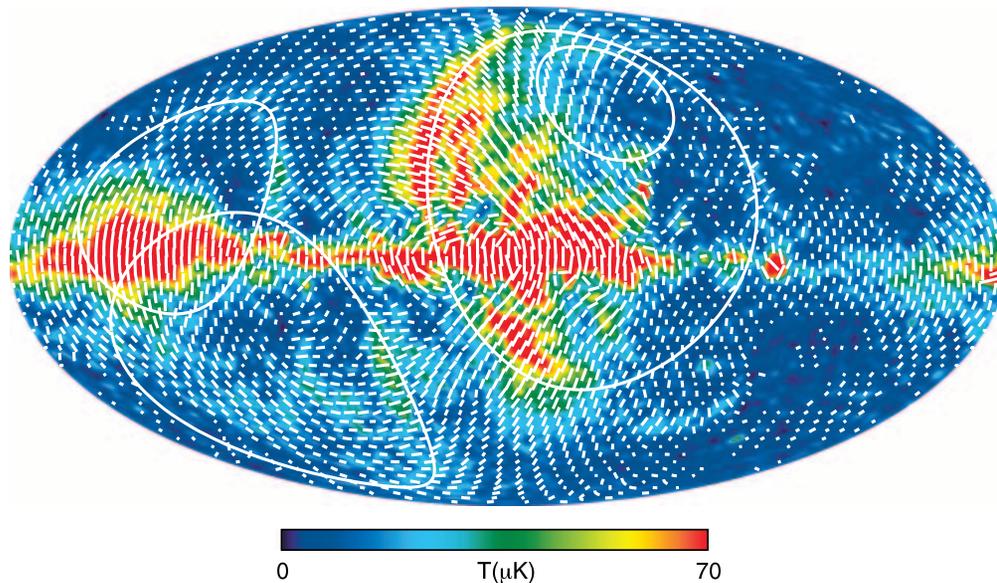


✧ Probably the radiative shells of very old supernova remnants

✧ Can only see 4-5 of these in the 408 MHz radio sky

Berkhuijsen *et al*, A&A 14:252,1971

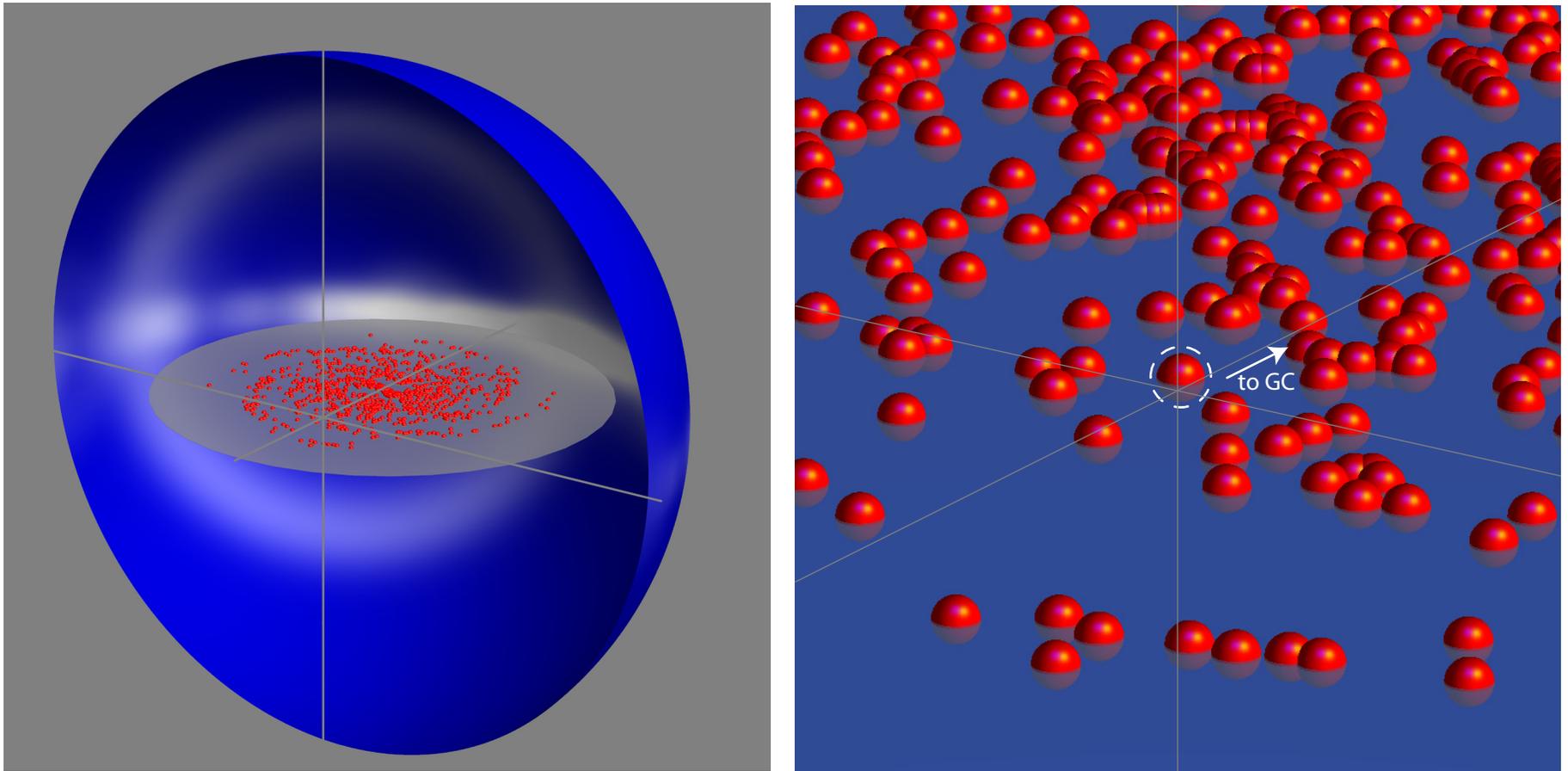
Page *et al*, ApJS 170:335,2007



✧ However there must be *several thousand* loops in the Galaxy which cannot be resolved against the galactic radio background ... indeed they probably constitute most of the 'diffuse' background

Sarkar, MNRAS 199:97,1982

Simulating the galactic distribution of old SNRs



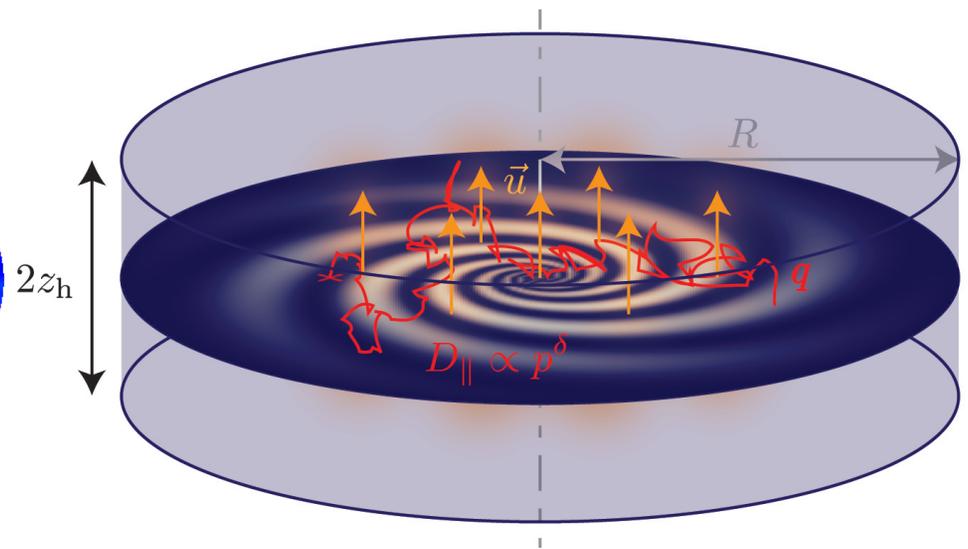
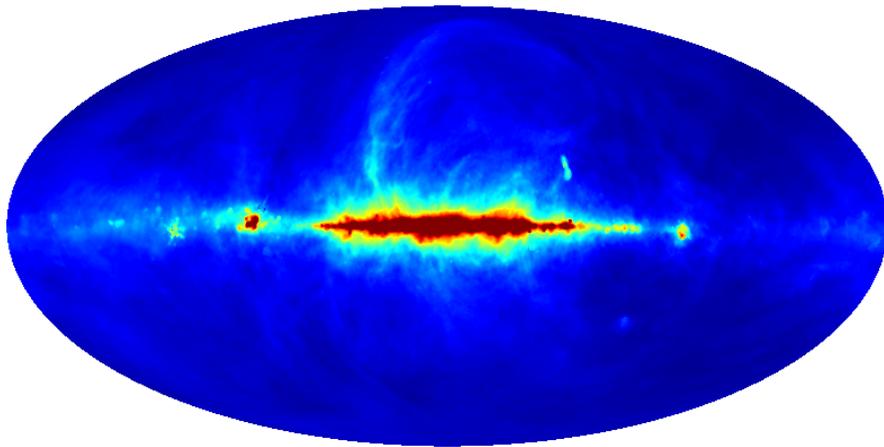
With ~ 3 SN/century, there must be *several thousand* old SNRs in the radiative phase of evolution ... their shells will compress the interstellar magnetic field – and the *coupled* cosmic ray electrons – to high values, significantly boosting the synchrotron emissivity

The galactic radio background

Synchrotron radiation by **relativistic cosmic ray electrons** spiralling in the **galactic magnetic field** (regular spiral + turbulent component):

$$P(\mathbf{r}; \nu) = \int dE n_e(\mathbf{r}; E) \frac{\sqrt{3} e^3 B_{\perp}(\mathbf{r})}{8\pi^2 \epsilon_0 c m_e} F\left(\frac{\nu}{\nu_c}\right)$$

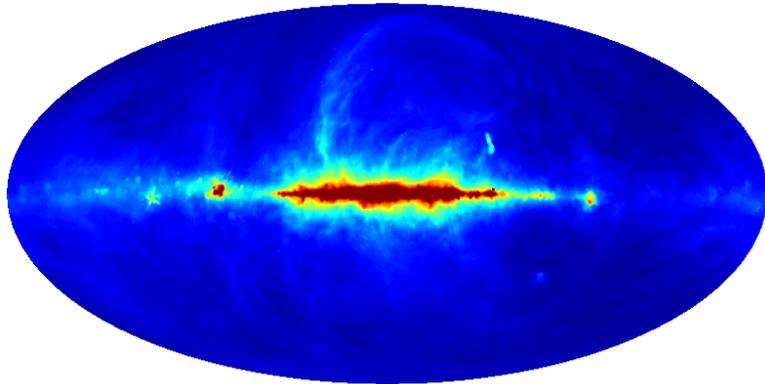
where $\nu_c = \frac{3}{2} \left(\frac{E}{m_e}\right)^2 \frac{B_{\perp}(\mathbf{r})}{B(\mathbf{r})}$, $F(x) = x \int_x^{\infty} dx' K_{5/3}(x')$



Can model using GALPROP code which solves for the diffusion of cosmic rays in the Galaxy (assumed to be a cylindrical slab + extended 'halo')
+ add emissivity on small-scales from MHD turbulence (with Kolmogorov spectrum)

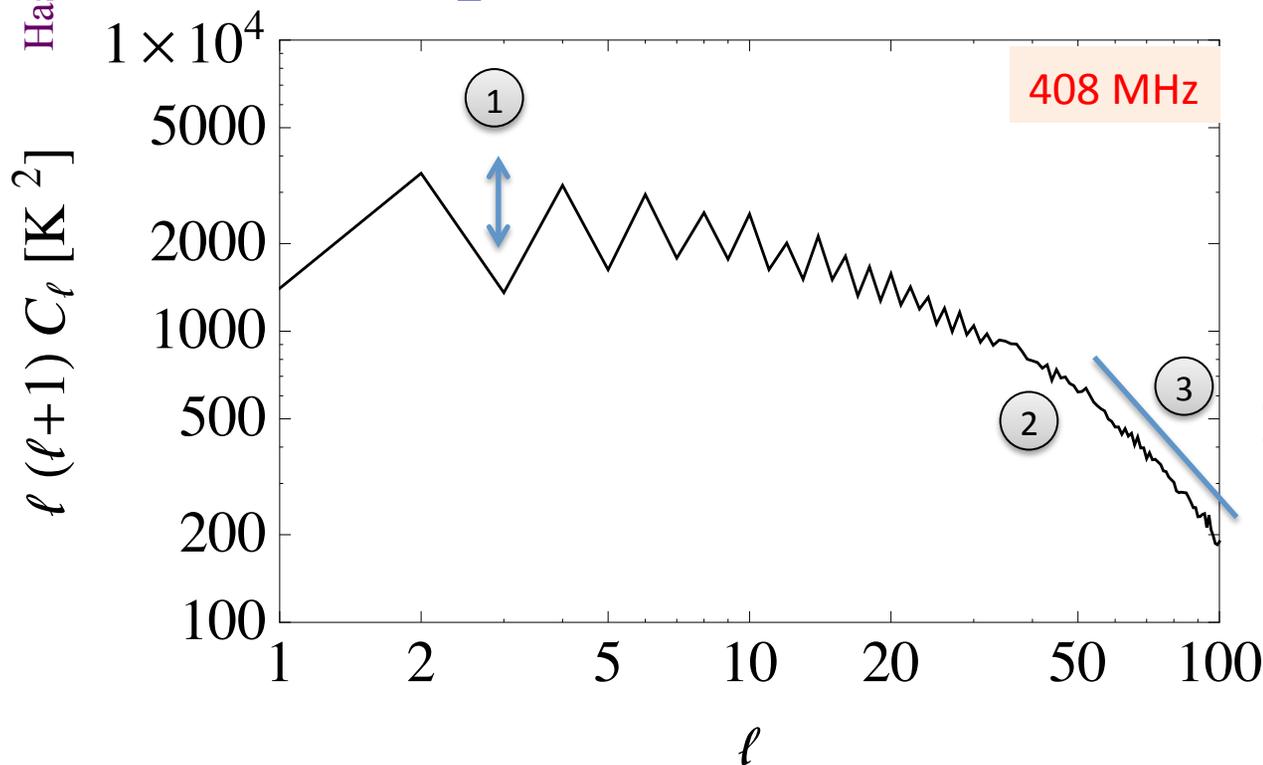
Angular power spectrum of radio emission

Haslam *et al.*, A&AS 47:1, 1982



$$J(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

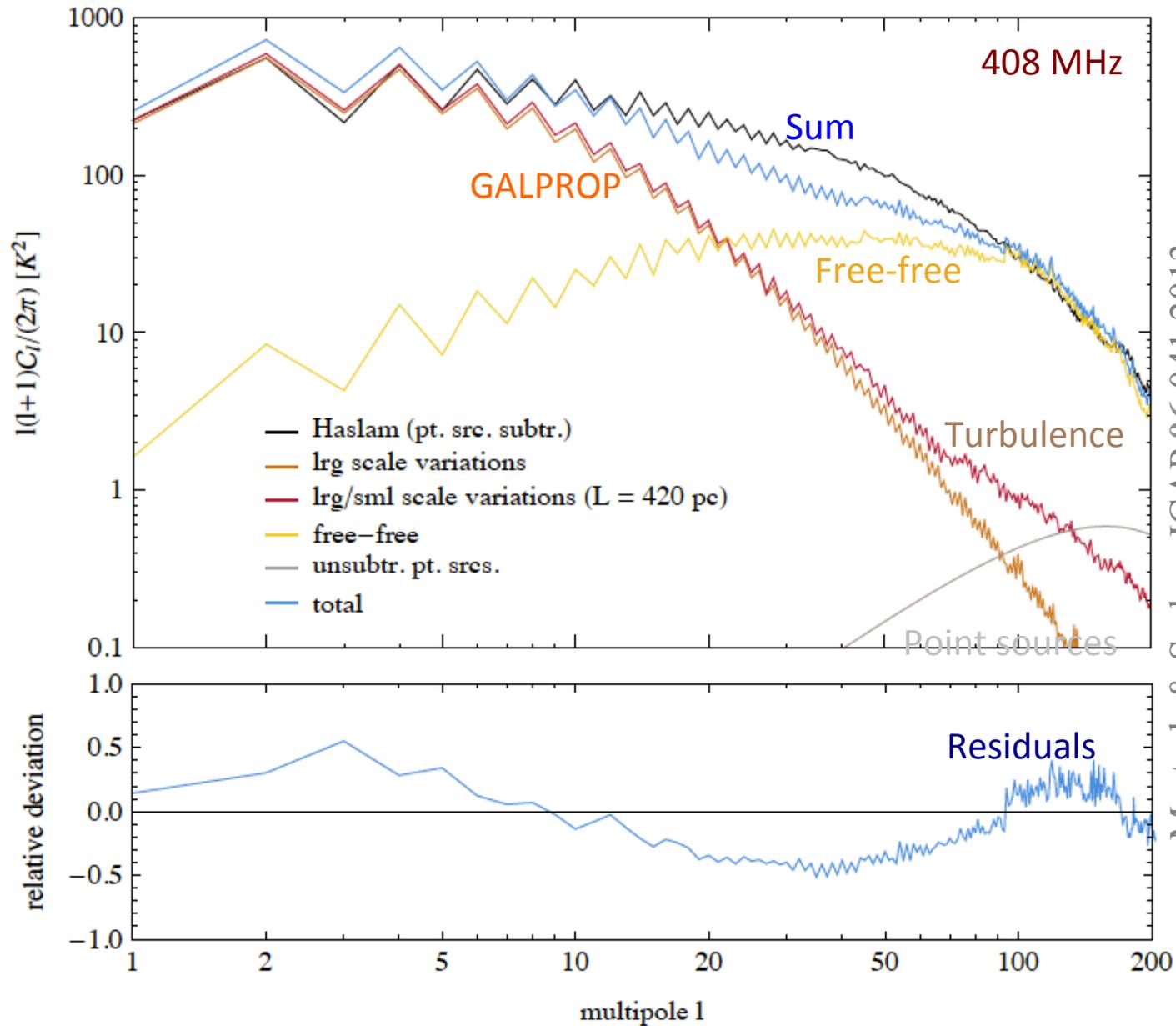
$$C(\ell) \equiv \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$



1. Even/odd structure reflects planar symmetry of Galaxy

2. At higher multipoles $C_\ell \propto \ell^{-m}$, $m \sim 11/3$ (Kolmogorov turbulence)

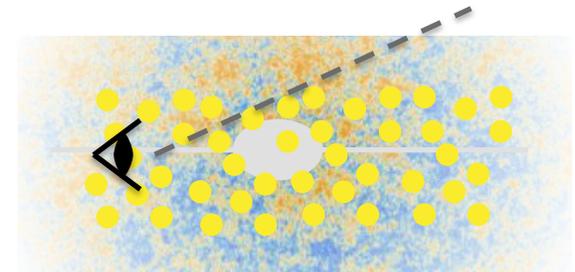
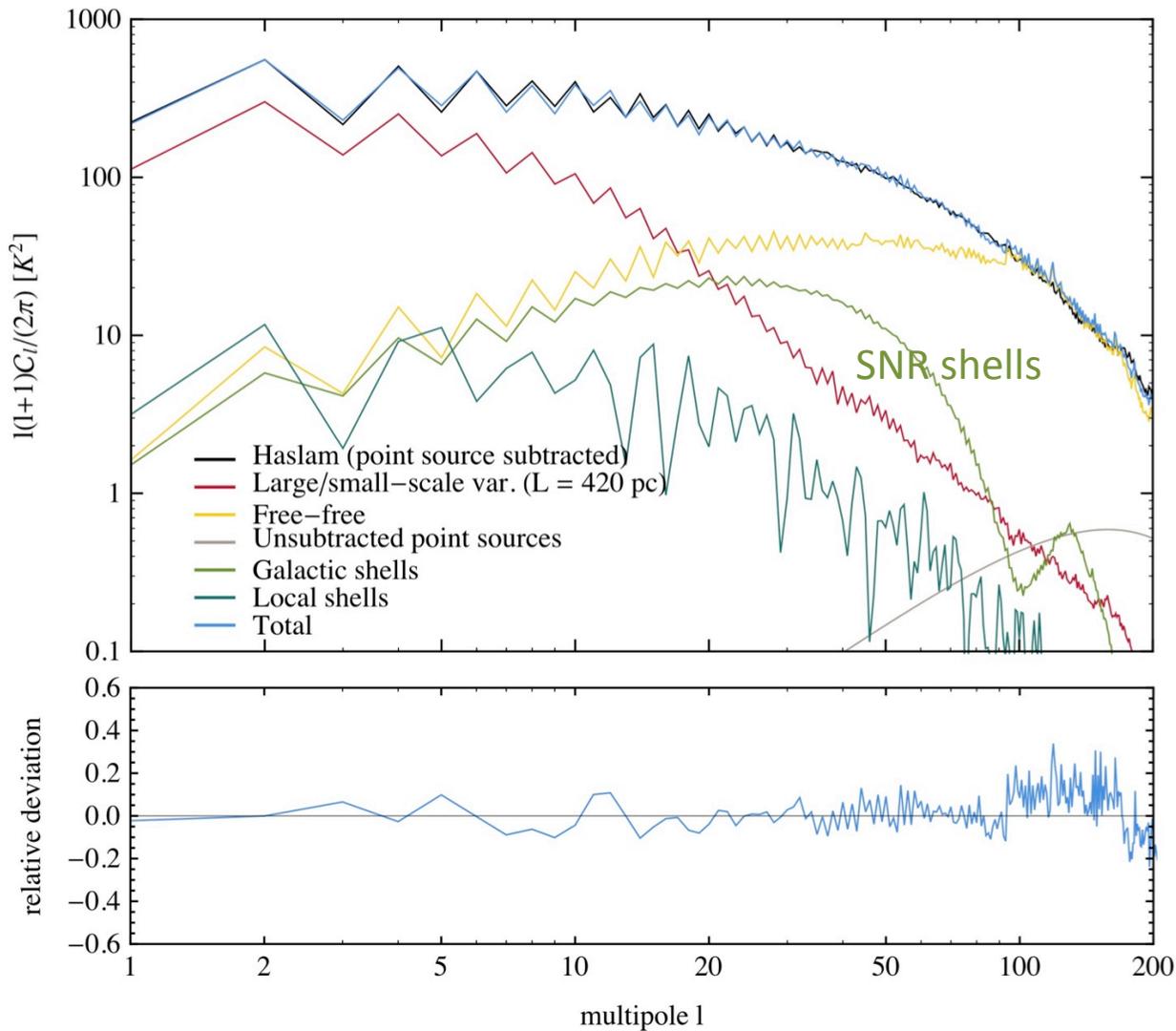
The uniform galaxy model (+ small-scale turbulence) does *not* provide a match to the angular power spectrum of the radio background



Mertsch & Sarkar, JCAP 06:041, 2013

... but adding a population of old SNRs does!

Mertsch & Sarkar, JCAP 06:041,2013



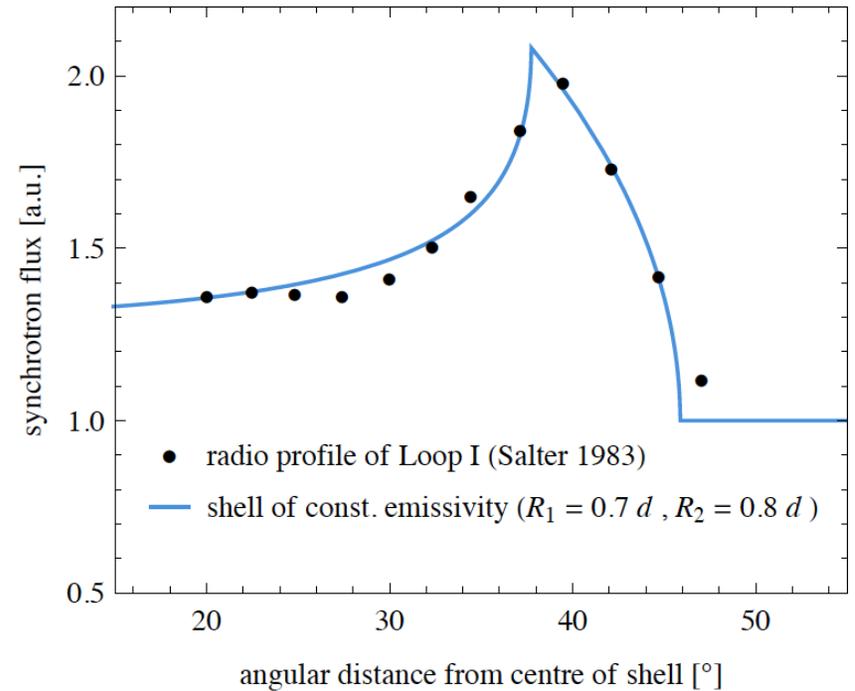
- Several thousand shells of old SNRs in Galaxy
- We know 4 local shells (Loop I-IV) but others are modeled in MC approach
- They contribute in *just* the required multipole range

Angular Power Spectrum of a SNR shell

... after projection along line-of-sight, the shell of homogeneous emissivity has angular profile $g(r)$

$$C_l^j = \frac{1}{2l+1} \sum_{m=-l}^l |a_{lm}^j|^2$$

$$= \frac{1}{4\pi} \left(f_j(\nu) \int_{-1}^1 dz' P_l(z') g_j(z') \right)^2$$

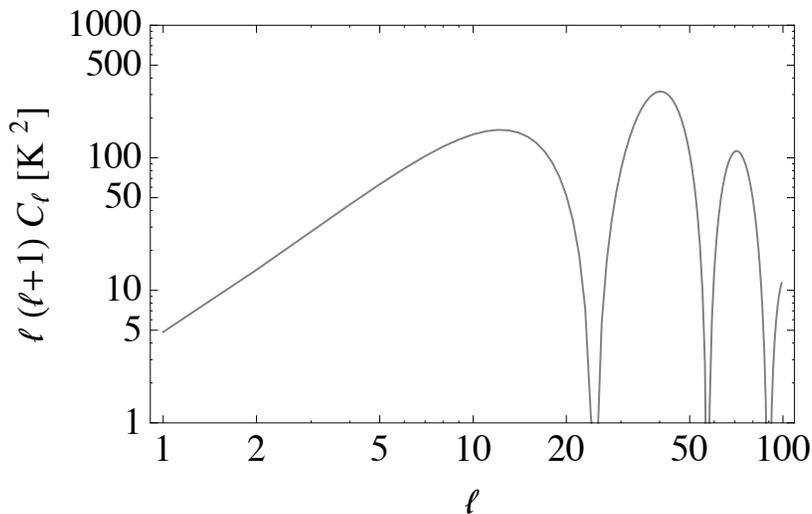


Mertsch & Sarkar, JCAP 06:041,2013

Angular power spectrum for shell i :

$$C_i(\ell) \propto \left(P_\ell \left(\cos \frac{R_i}{d_i} \right) \right)^2$$

... thickness of shell determines cut-off



Modelling an ensemble of shells

Assumption: flux from one shell factorises into angular part and frequency part: $J_{\text{shell } i}(\nu, \ell, b) = \varepsilon_i(\nu)g_i(\ell, b)$



Frequency part: $\varepsilon_i(\nu)$

Magnetic field gets compressed in SNR shell

Electrons get betatron accelerated

Emissivity increased with respect to ISM

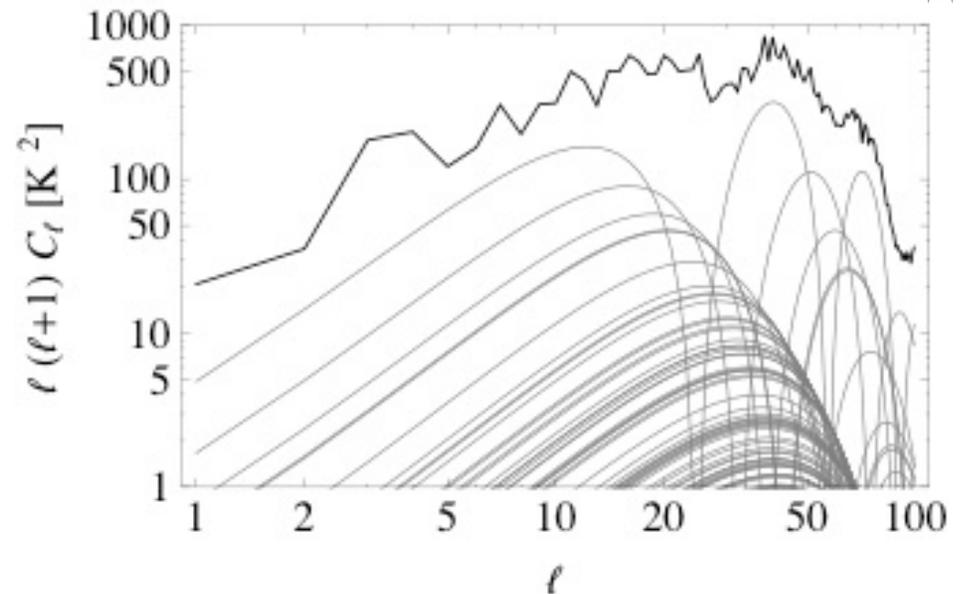
Angular part: $g_i(\cos \psi)$

Assume constant emissivity in shell:

$$a_{lm}^i \sim \varepsilon_i(\nu) \int_{-1}^1 dz' P_l(z') g_i(z')$$

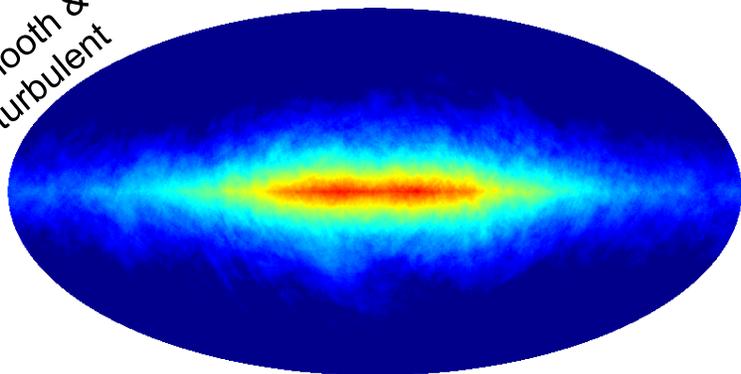
Add up contribution from all shells:

$$a_{lm}^{\text{total}} = \sum_i a_{lm}^i$$

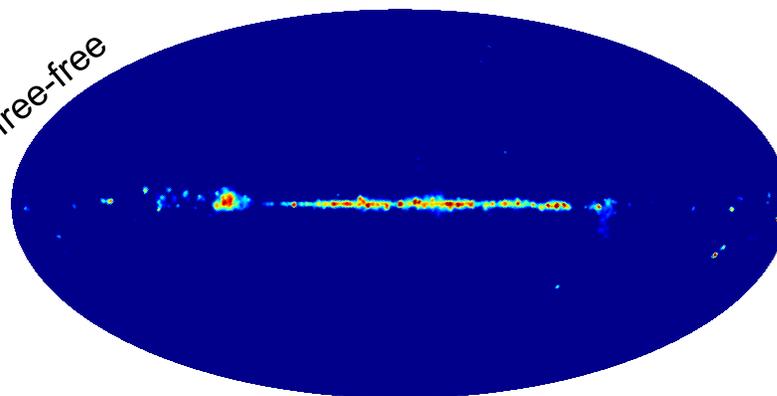


This model has structure at high latitude (like the *real*/radio sky)

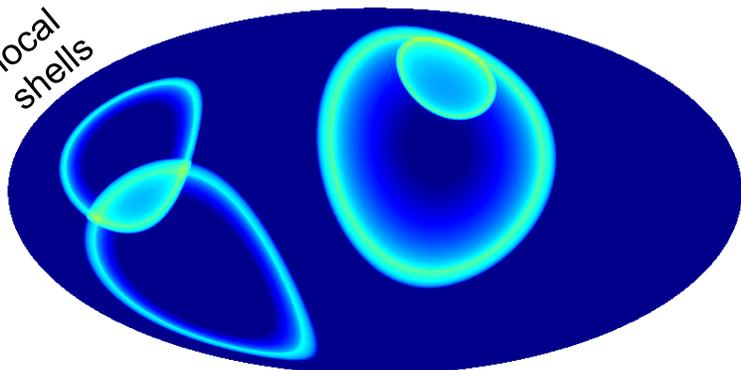
smooth &
turbulent



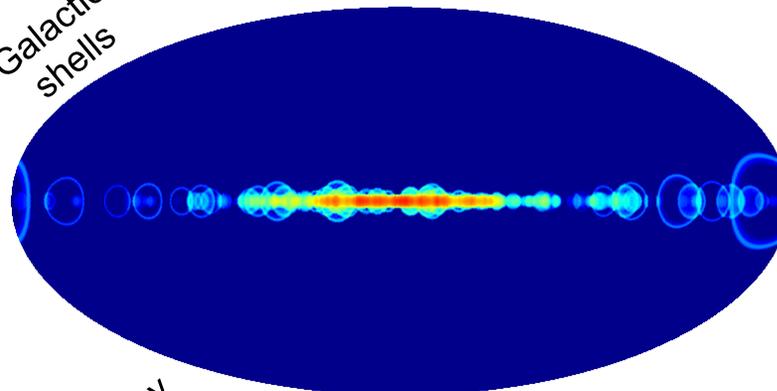
free-free



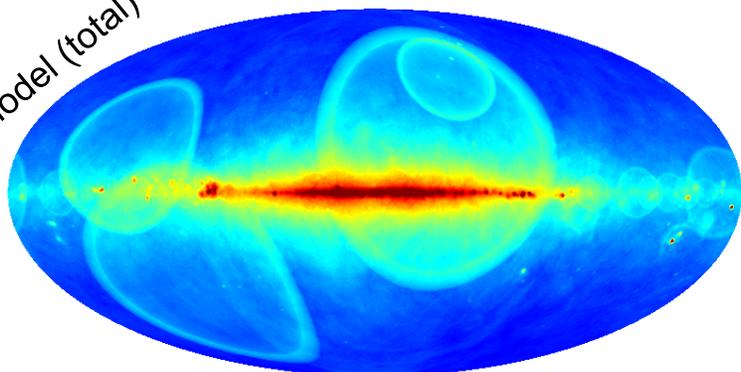
local
shells



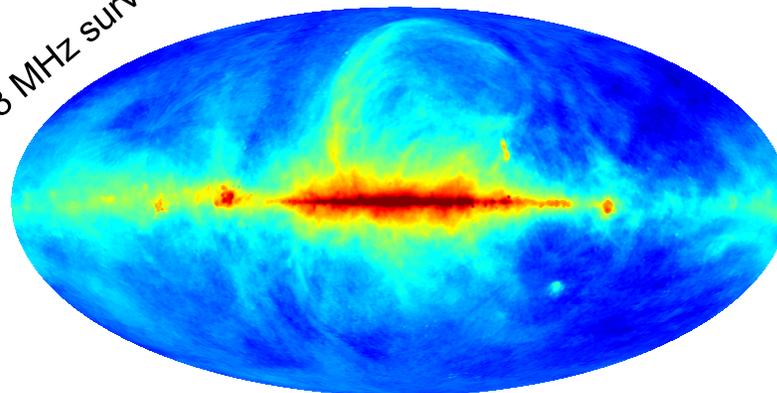
Galactic
shells



Model (total)



408 MHz survey

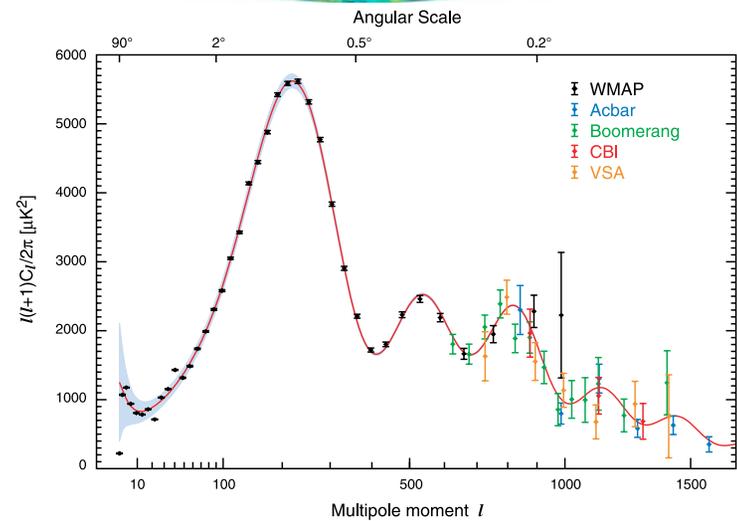
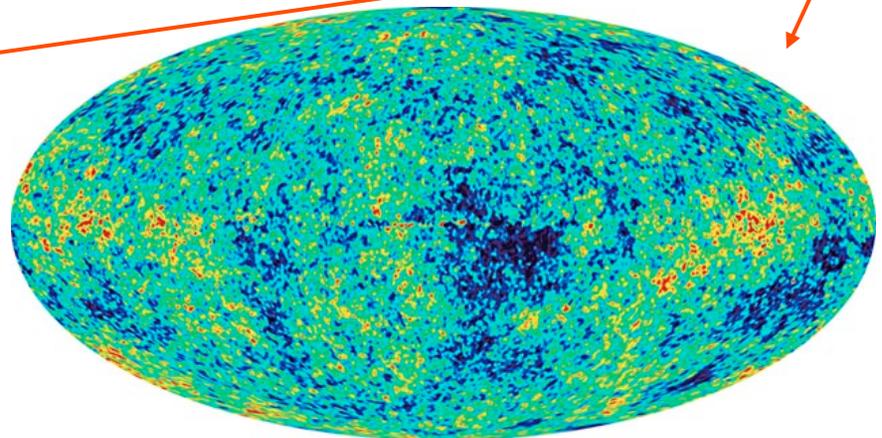
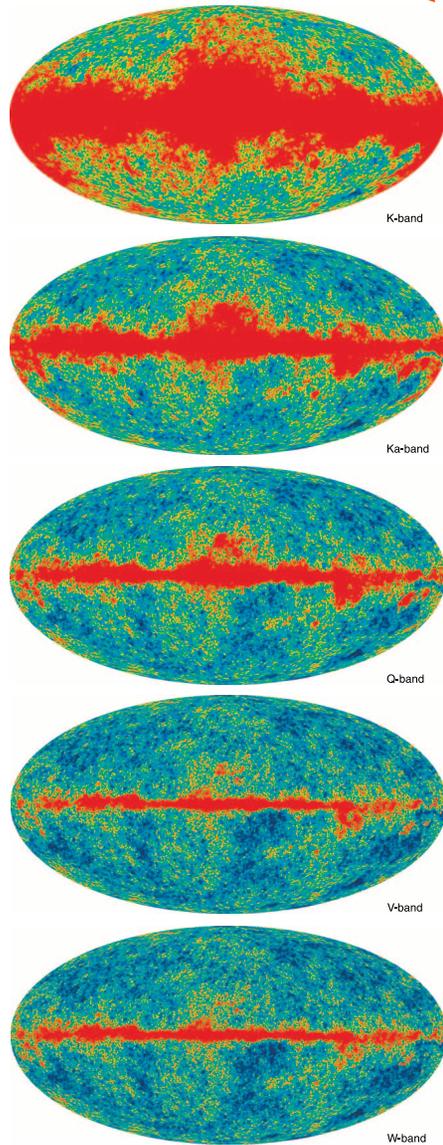


1.0 2.5 Log (K)

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CMB foreground removal: How do we get from this to this?

Hinshaw *et al.*, ApJS 170:288, 2007



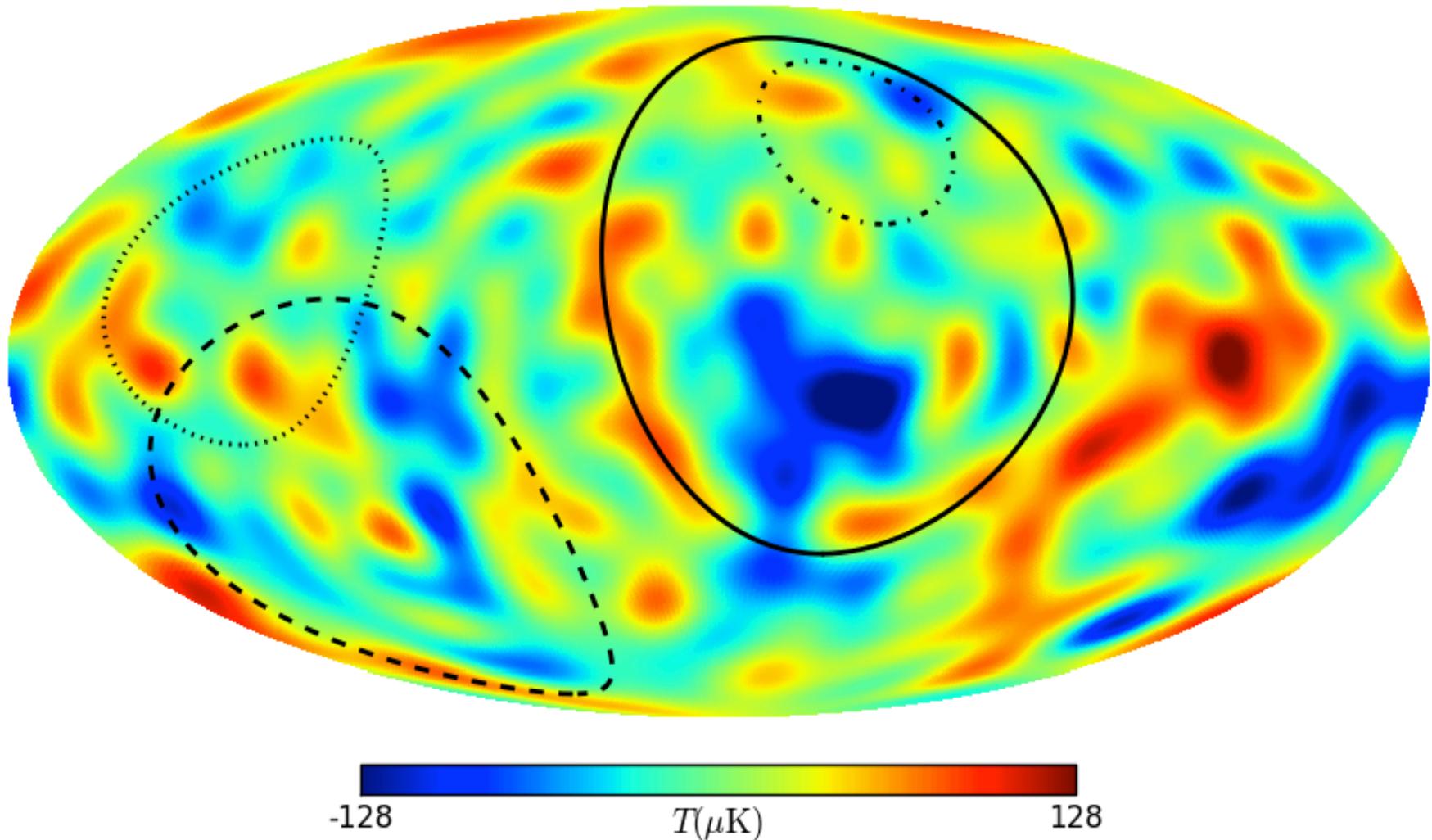
Answer: ILC ~ Internal Linear Combination (SMICA for Planck)

$$T_{\text{ILC}} = \sum_i \zeta_i T_i = \sum_i (T_{\text{CMB}} + S_i T_{\text{foreground}})$$

... and minimise the variance σ_{ILC}^2

But this technique may fail *locally* in regions where there is *both* synchrotron and dust emission - e.g. in old supernova remnant shells (nearby ... so at high latitude)

Anomalies in WMAP-9 Internal Linear Combination map ($\ell \leq 20$)



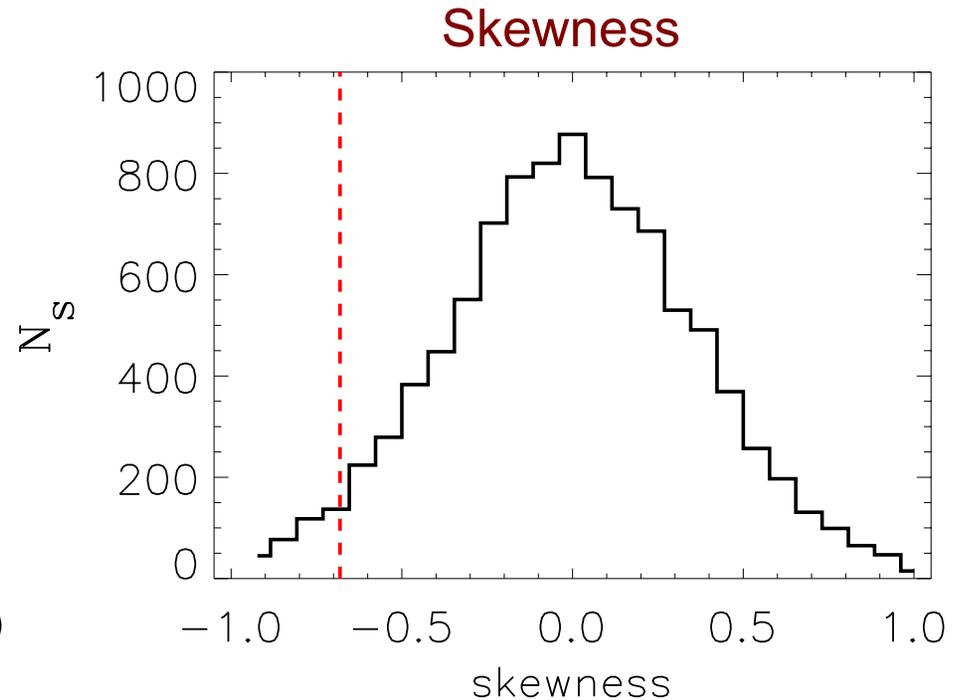
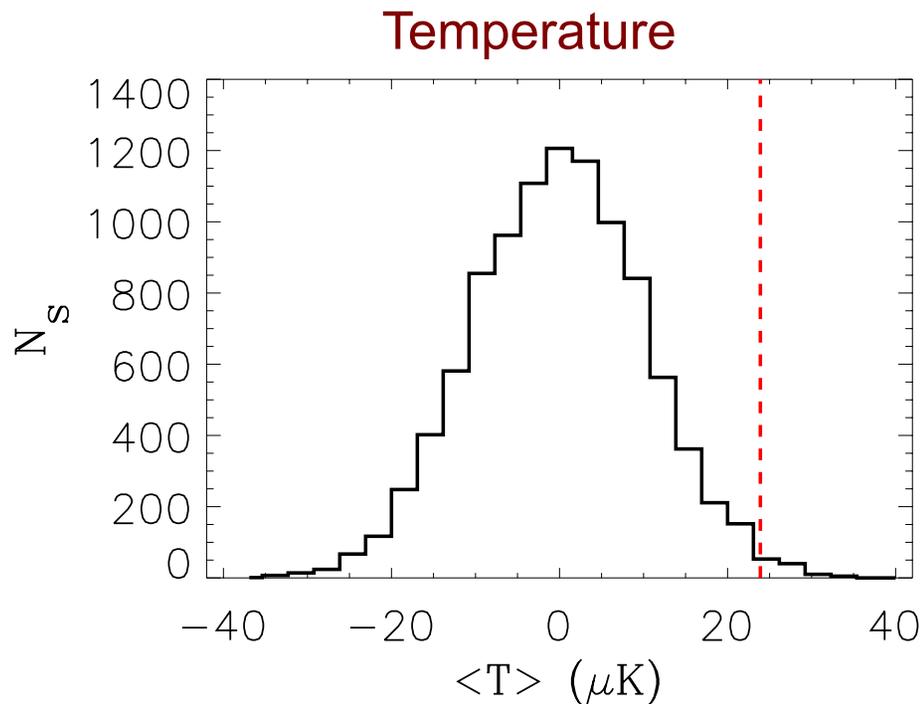
Bennett et al, ApJS 208:20, 2013

Are the radio loops visible (even in microwaves)?

Anomalies in WMAP-9 Internal Linear Combination map ($\ell \leq 20$)

There is a 22 *mK* excess temperature in ring around Loop I

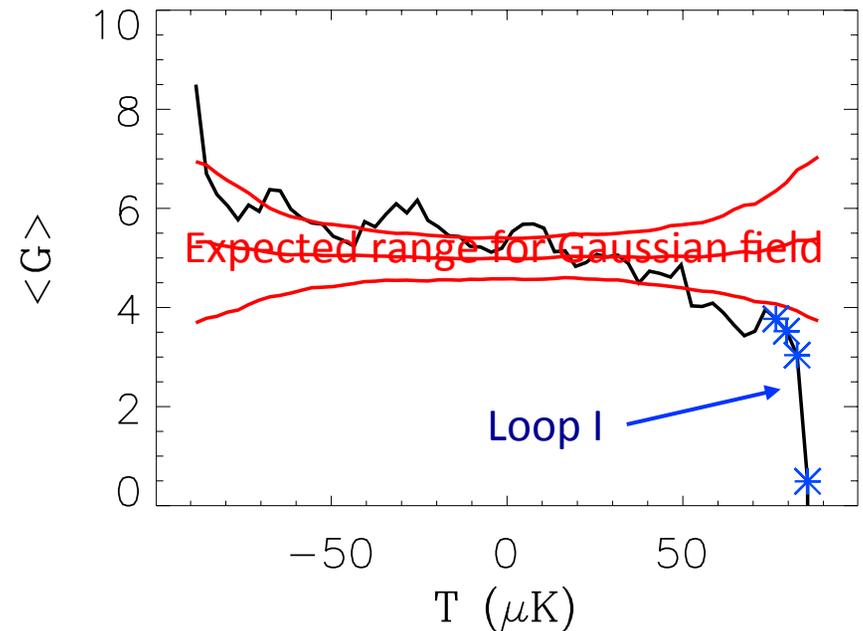
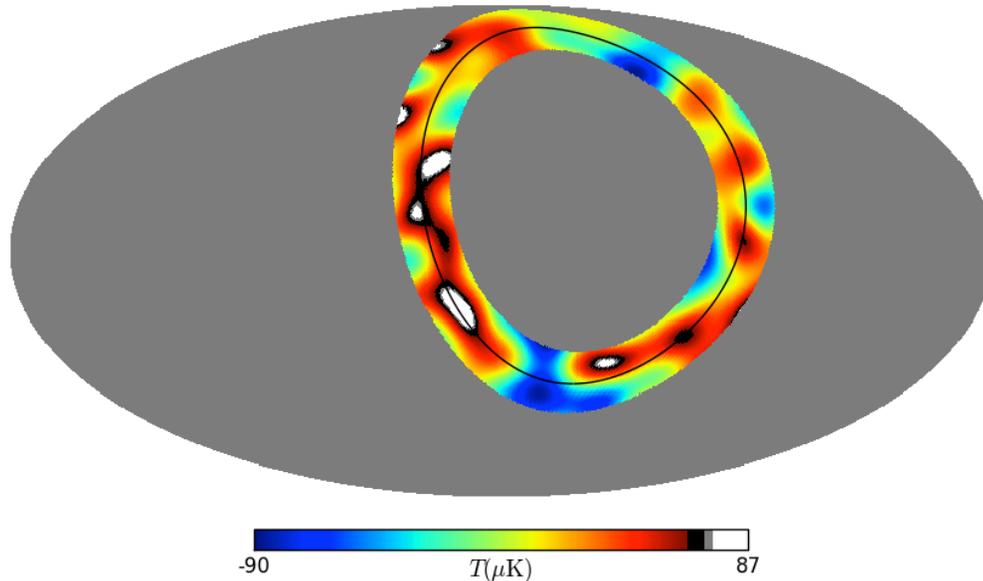
(NB: This is $\sim 1/4$ of the total TT signal in the ‘cleaned’ CMB map)



Compare with MC \Rightarrow p-values of $\mathcal{O}(10^{-2})$

Anomalies in WMAP-9 Internal Linear Combination map ($\ell \leq 20$)

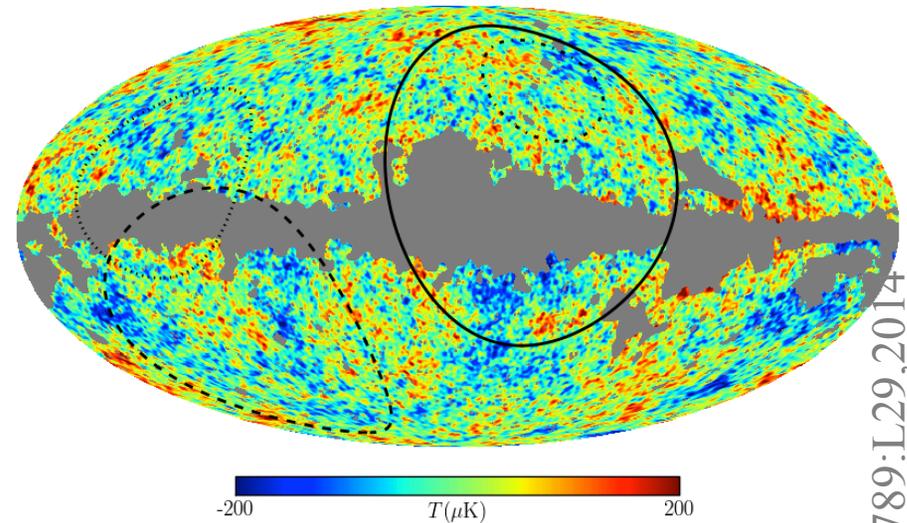
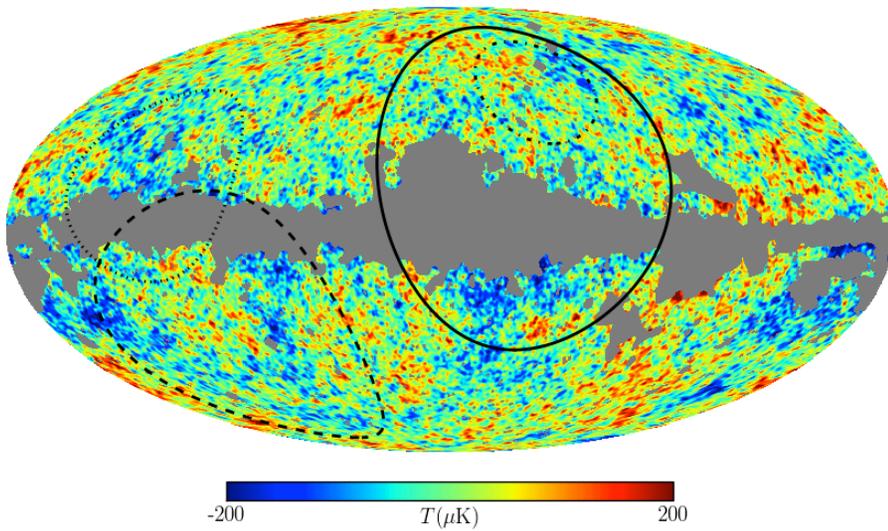
Cluster analysis (Naselsky & Novikov, ApJ 444:1,1995): Compute for each pixel the angular distance G from Loop I along great circles crossing both the pixel and the loop center and compare with random realisation of best-fit Λ CDM model



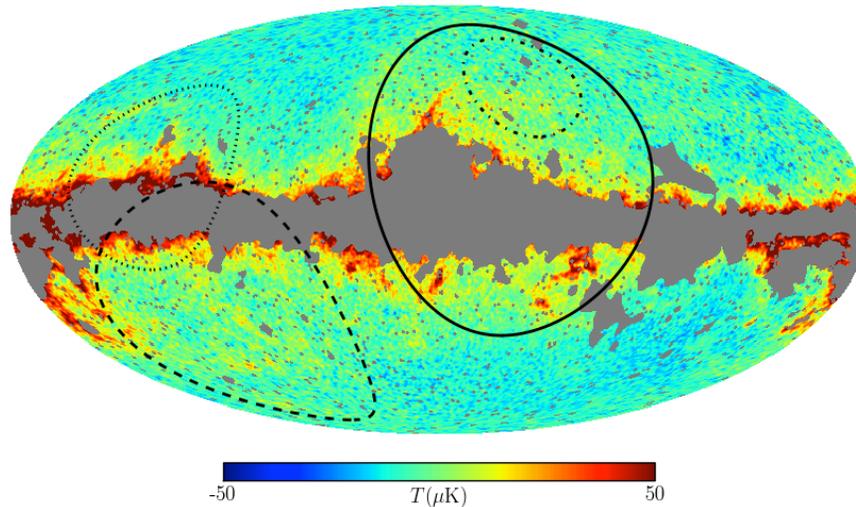
From 100,000 MC runs: probability for *smaller* $\langle G \rangle$ in last 4 bins $\sim 10^{-4}$

ILC coefficients from Loop I region

ILC coefficients from rest of sky



Difference $\text{ILC}_{\text{rest}} - \text{ILC}_{\text{Loop I}}$



There *is* an imprint of the radio loops in the ‘internal linear combination’ (also SMICA) maps of the CMB which have *supposedly* been cleaned of all foreground emissions!

What do we know about the Loop I anomaly?

- Spatially correlates with Loop I
- *Unlikely* to be synchrotron (checked with our synchrotron model)
- Frequency dependence:

Simple toy model: $\xi(\hat{\mathbf{n}}) = \tau(\hat{\mathbf{n}})T_s \Theta(\nu_{\min} \leq \nu_j \leq \nu_{\max})$

with $\tau(\hat{\mathbf{n}}) \sim 10^{-6}$ and $T_s \sim 20$ K

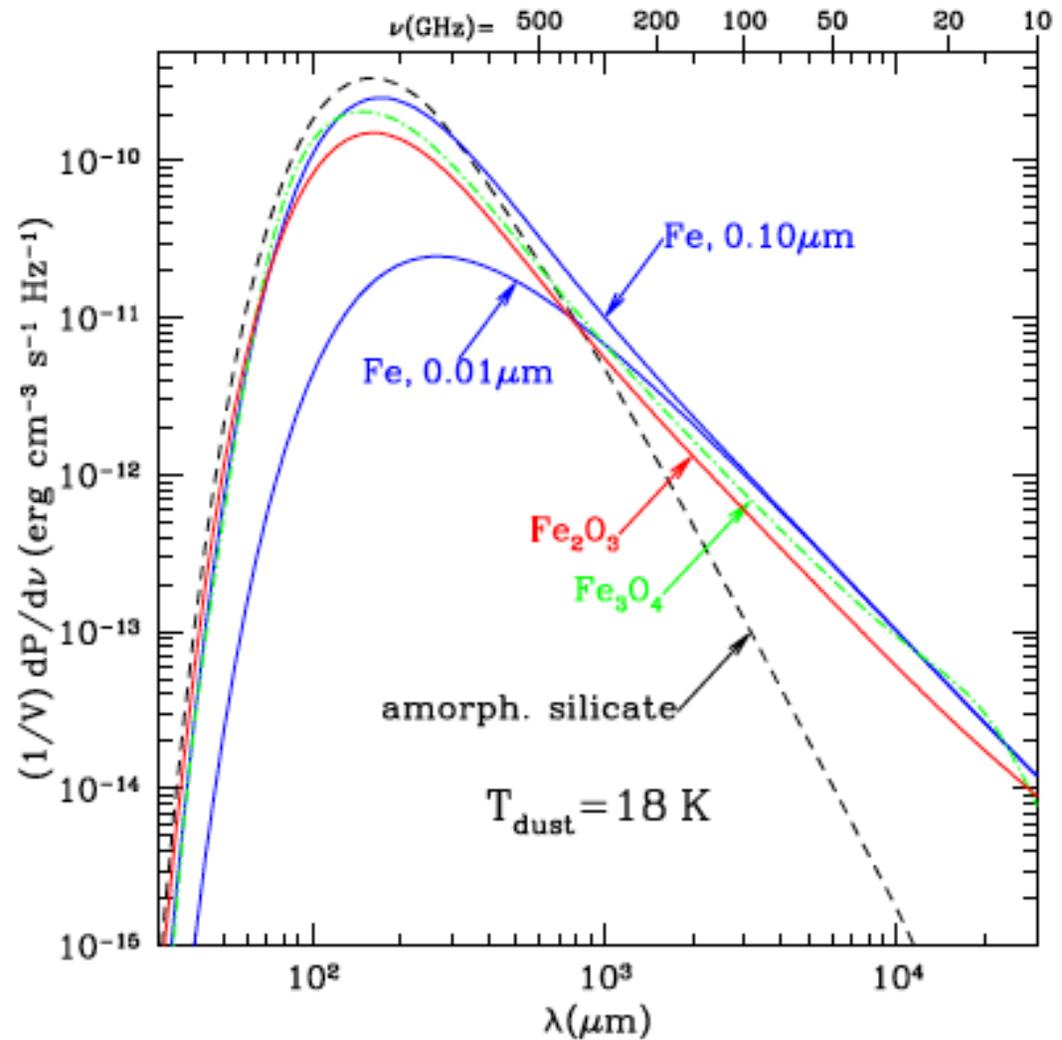
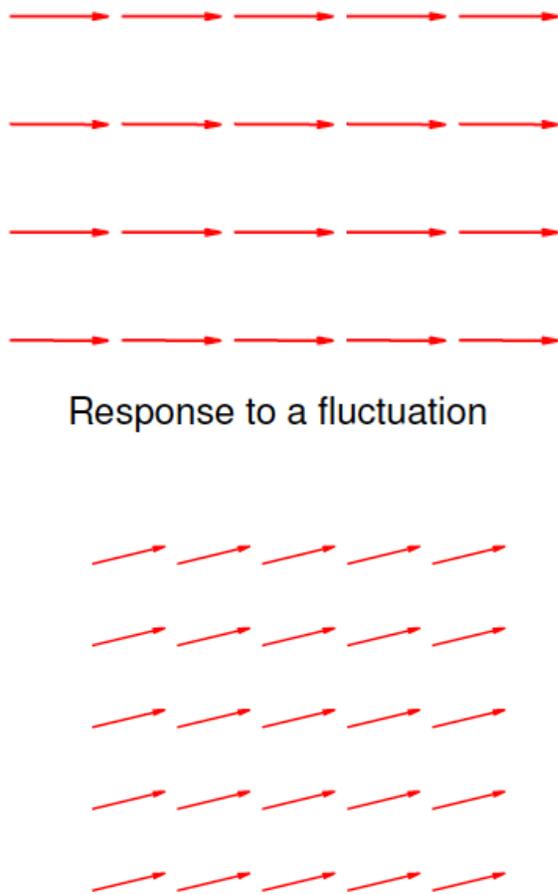
If $\tau(\hat{\mathbf{n}})$ depends only weakly on ν , can estimate frequency dependence from

$$\sum_j W_j \tau(\hat{\mathbf{n}}) T_s \propto \sum_j W_j$$

... Can also use polarised V- and W-bands to get handle on spectral index

Could it be *magnetic* dipole radiation from dust (with ferrimagnetic inclusions)?

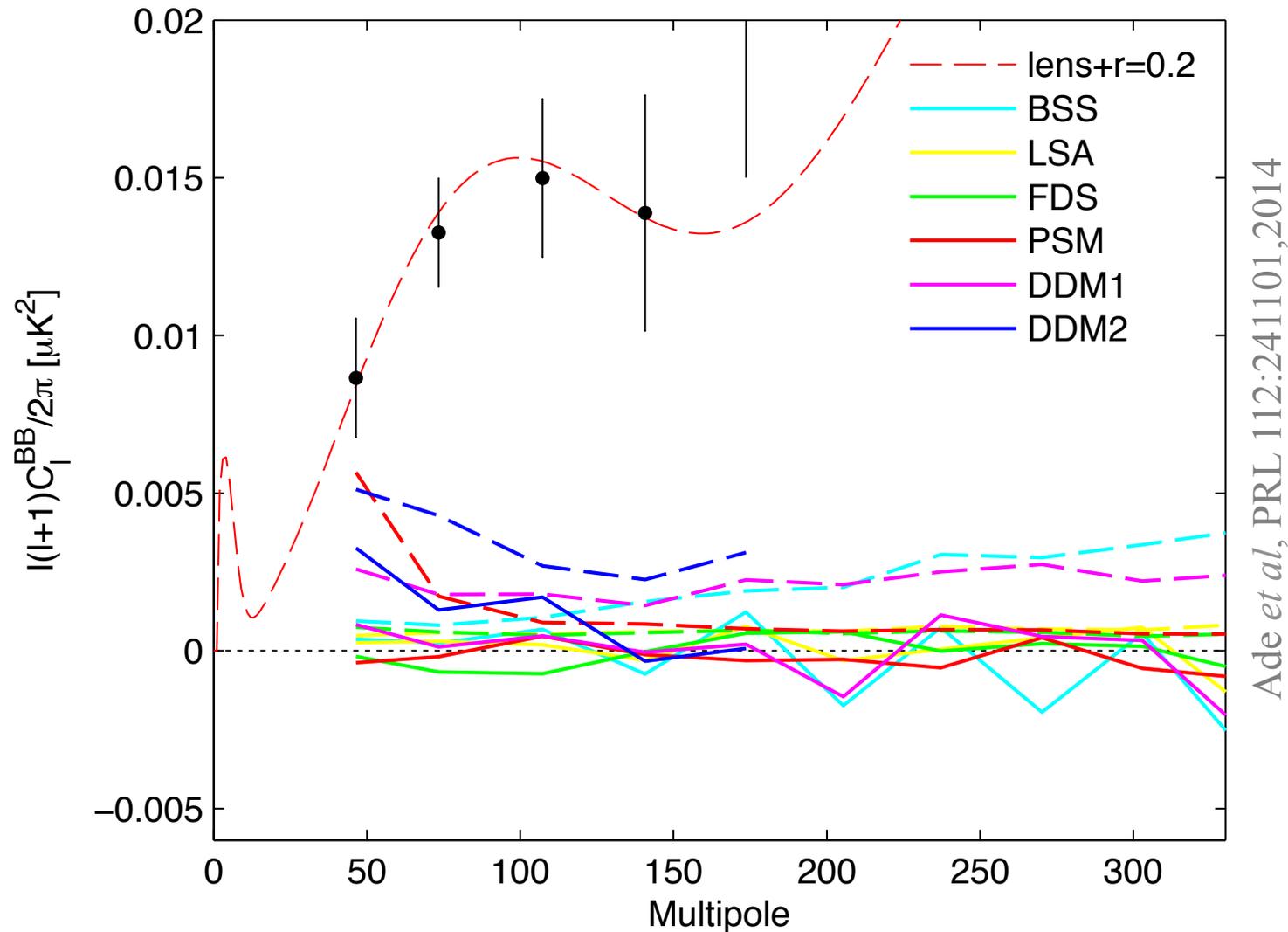
Could it be *magnetic* dipole radiation from dust in the loops (with iron or ferrimagnetic inclusions)?



Draine & Hensley, ApJ 765:139, 2013

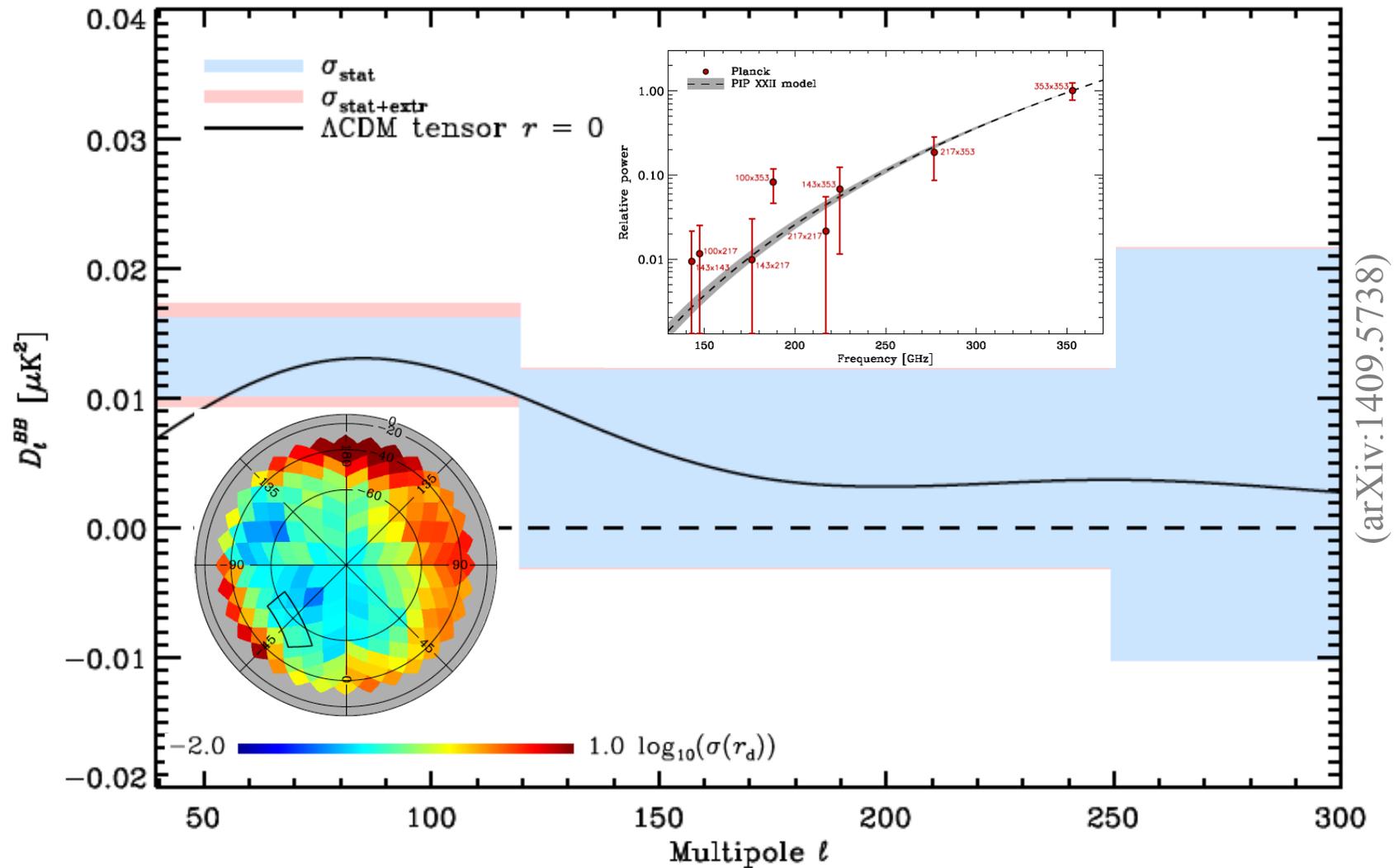
This is also suggested by *Planck* from the observed decrease of the polarization fraction of dust emission between 353 & 70 GHz [arXiv:1405.0874v1 – but withdrawn in v2!]

BICEP2 signal was said *not* to correlate with 'known foregrounds'



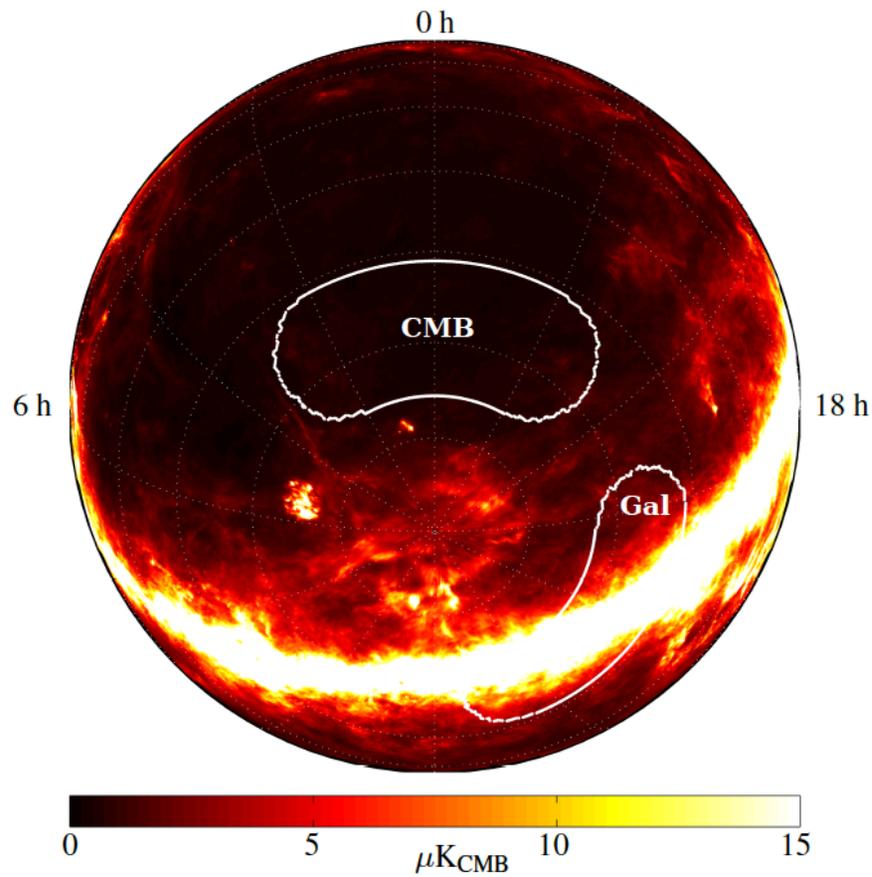
However the new foreground we have identified is *not* included in any of the models...

The 353 GHz polarised dust emission map from *Planck* shows high latitude emission from dust with a high polarisation fraction of $\sim 20\%$ - extrapolated to 150 GHz, this is comparable to the BICEP2 'signal'!

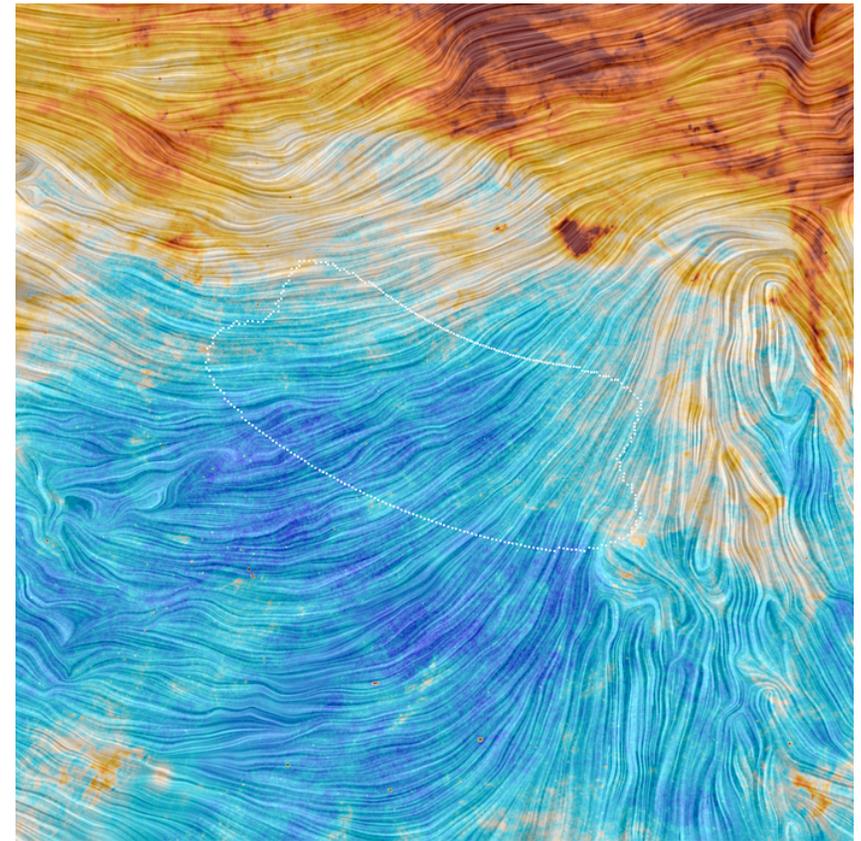


(arXiv:1409.5738)

We had to wait for cross-correlation between *BICEP2* and *Planck* to settle the issue ...



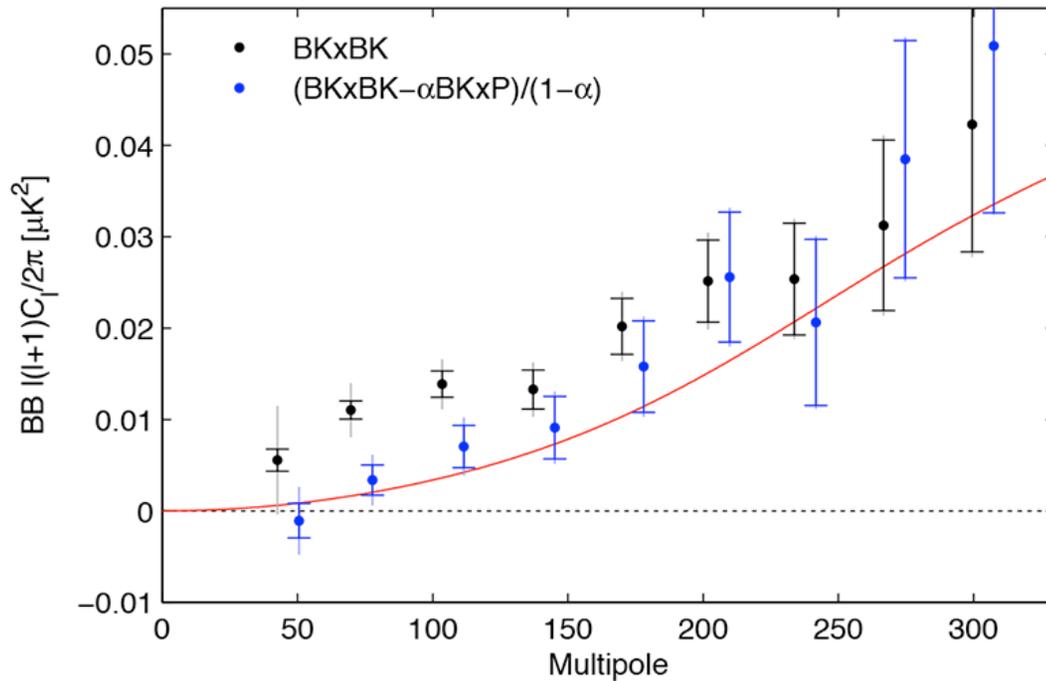
[arXiv:1403.3985]



Planck view of BICEP2 field
Released 30th January 2015

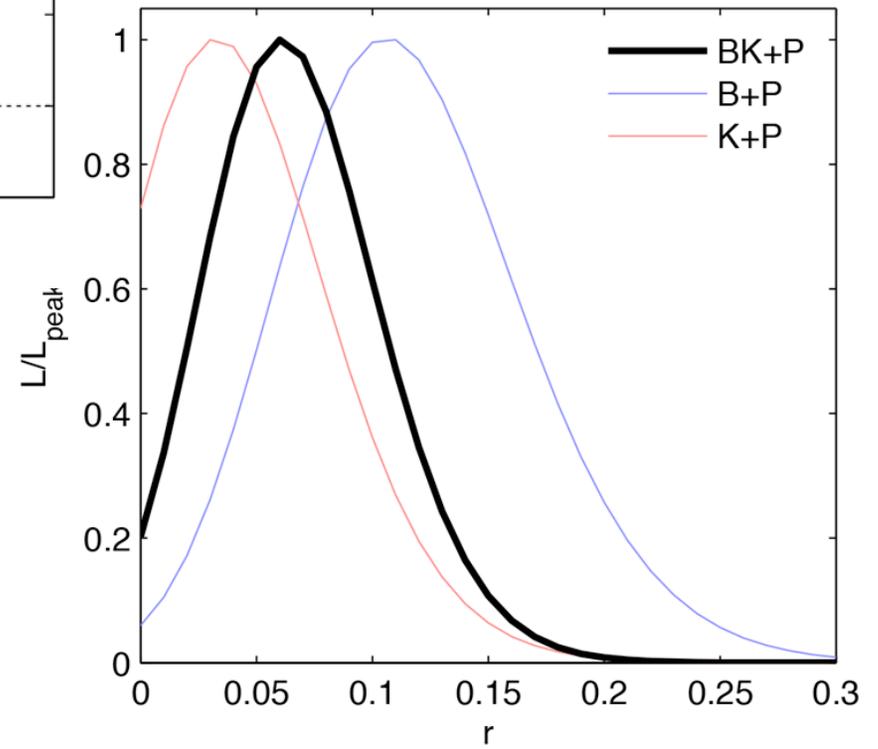
“The *BICEP 2* field is centered on Galactic coordinates $(l, b) = (316^{\circ}, -59^{\circ})$ and was originally selected on the basis of exceptionally low contrast in the FDS dust maps (Finkbeiner et al. 1999). ~~It must be emphasized that these ultra clean regions are very special—at least an order of magnitude cleaner than the average $b > 50^{\circ}$ level.~~”

The *Planck* 353 GHz dust map does correlate with *BICEP2* (and *Keck*) ...

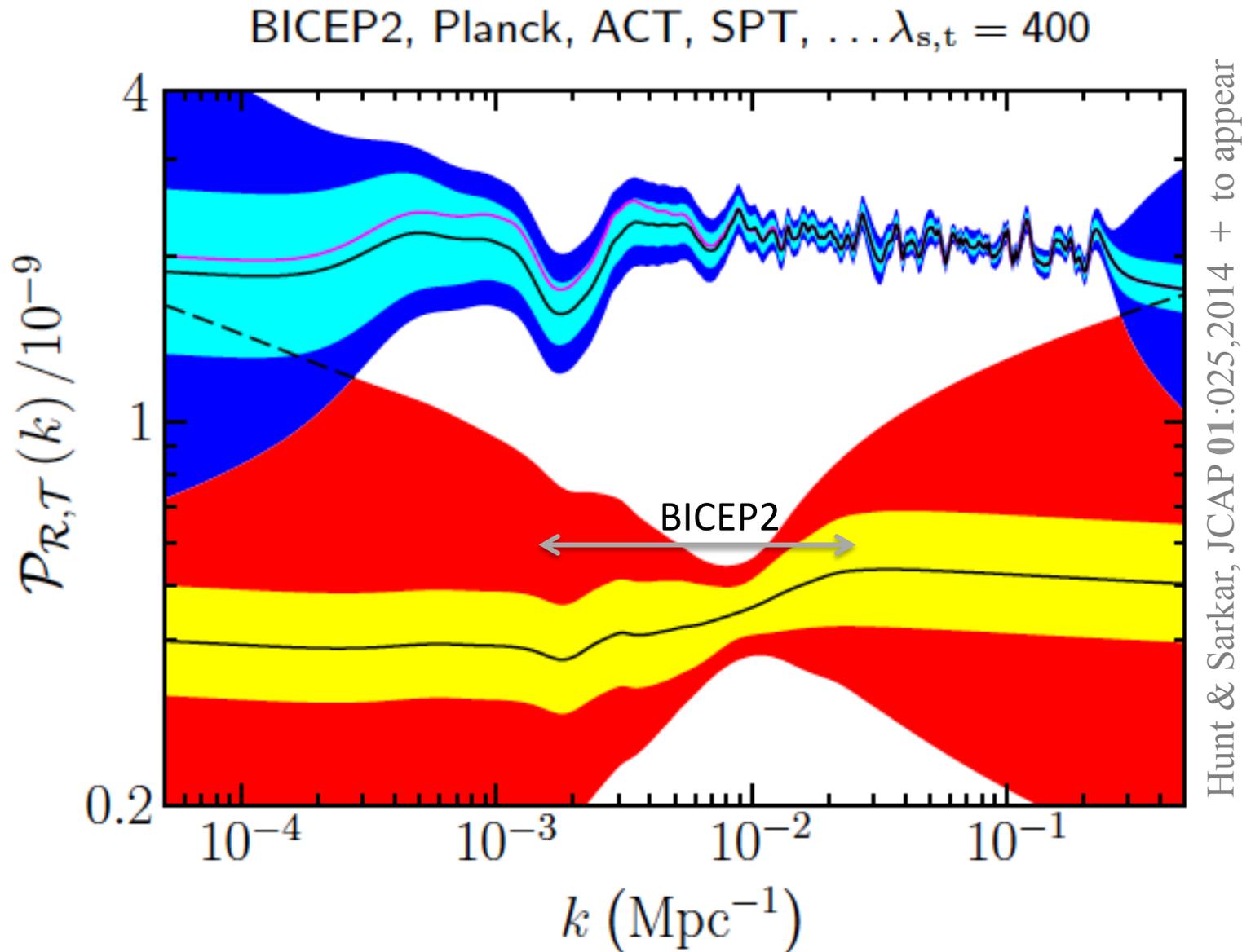


The *B*-mode polarisation detected by *BICEP2* in the ‘Southern Hole’ was in fact galactic foreground emission

“Inflationary gravitational waves remain elusive ...”
 $r < 0.11$

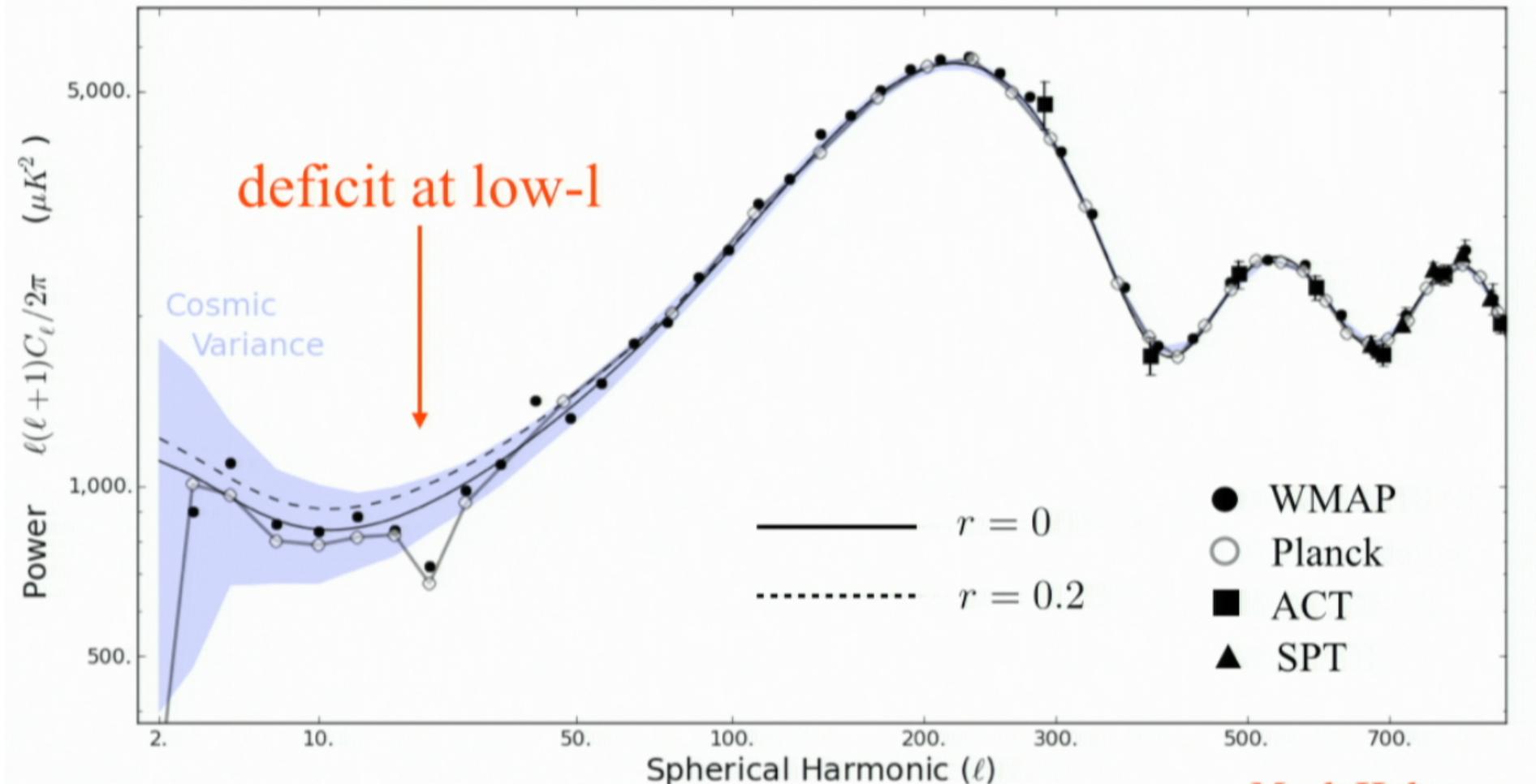


Deconvolution of CMB data shows *deviations* from a power-law spectrum



Moreover it is hard to reconcile the BICEP2 claim with TT data ... because the spectral slope of the ‘tensor signal’ is of *opposite* sign to the slow-roll expectation ($\underline{n}_T = -r/8$)

Given that the power on large scales is already *low*, adding a gravitational wave component exacerbates the problem (and requires a *positive* tilt in its spectrum to match the data)



This is the *opposite* of what is expected in single-field slow-roll inflation

Summary

Inflation driven by the slow roll of a scalar field is a convenient paradigm which enables us to engage with CMB observations ... but it is very challenging to realise in a physical field-theoretical framework without rather unnatural *fine-tuning* of parameters

Lacking a fundamental understanding of how vacuum energy couples to gravity, inflation must in any case be considered a 'toy model'

... unless of course we detect the predicted gravitational waves!

However this will be hard unless we learn how to model the Galactic foreground emission *far more accurately* than we can at present

Meanwhile there is an indication that the primordial spectrum of fluctuations cuts off on the scale of the *present* Hubble radius H_0^{-1} ?!