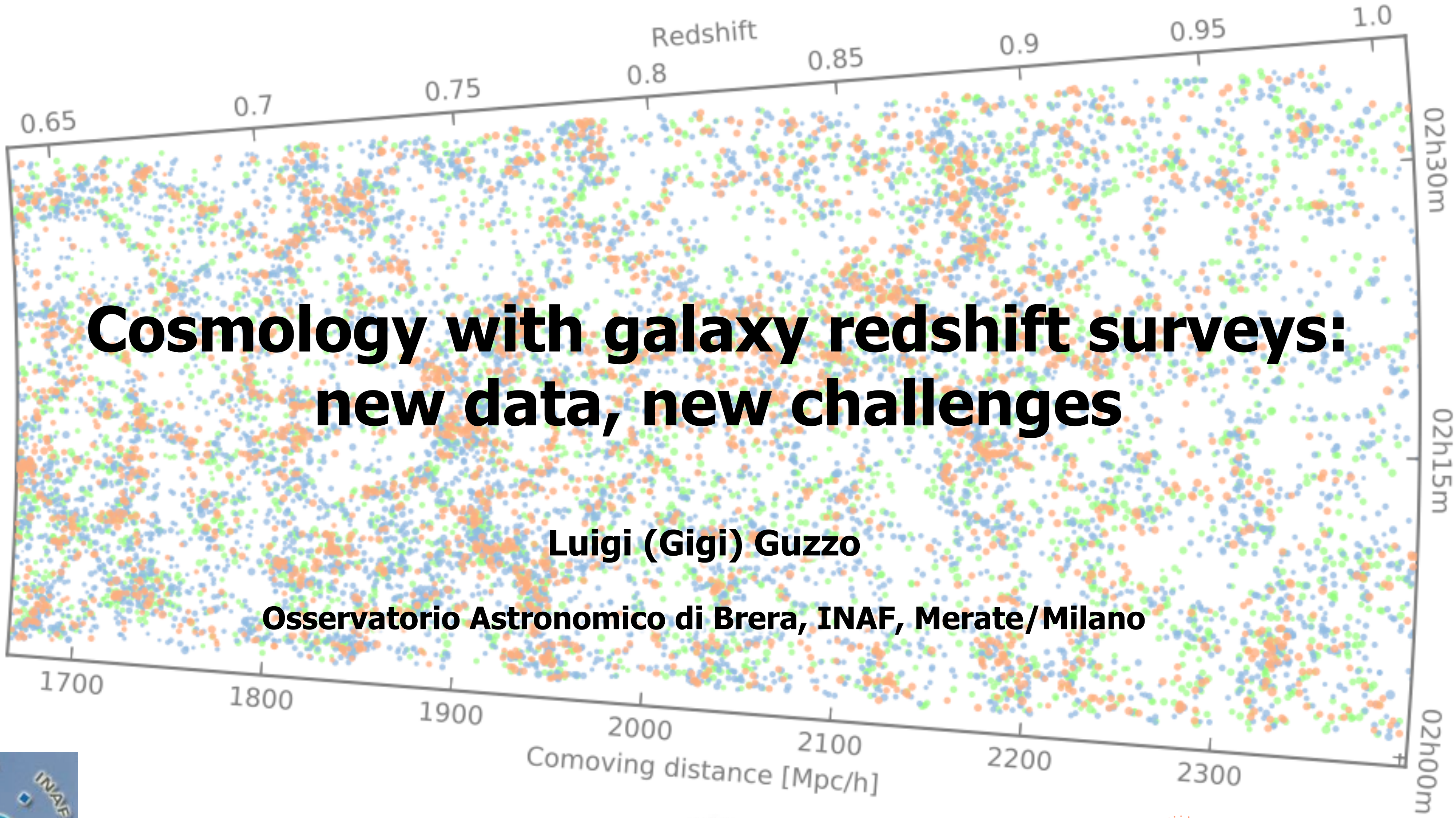
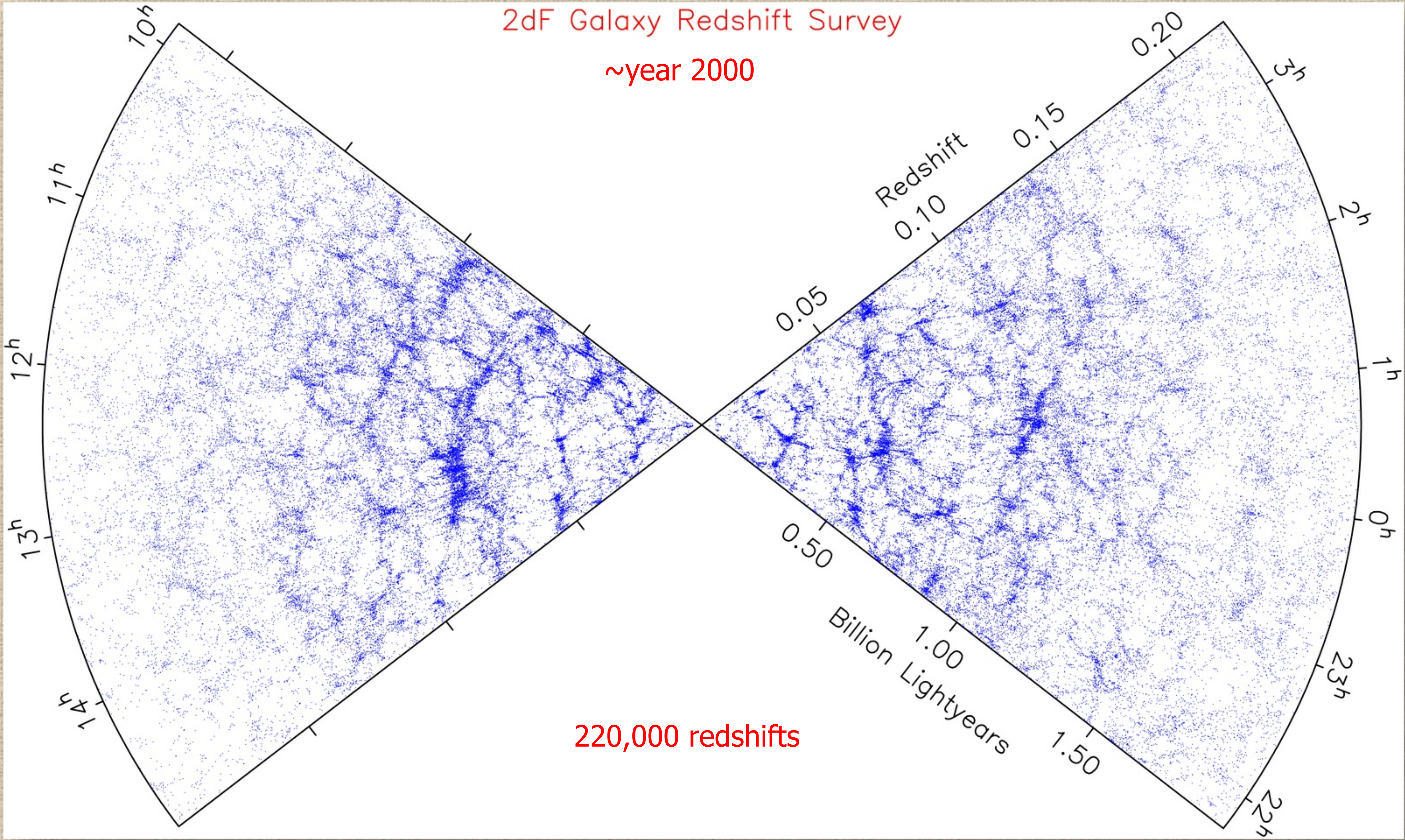



# Cosmology with galaxy redshift surveys: new data, new challenges

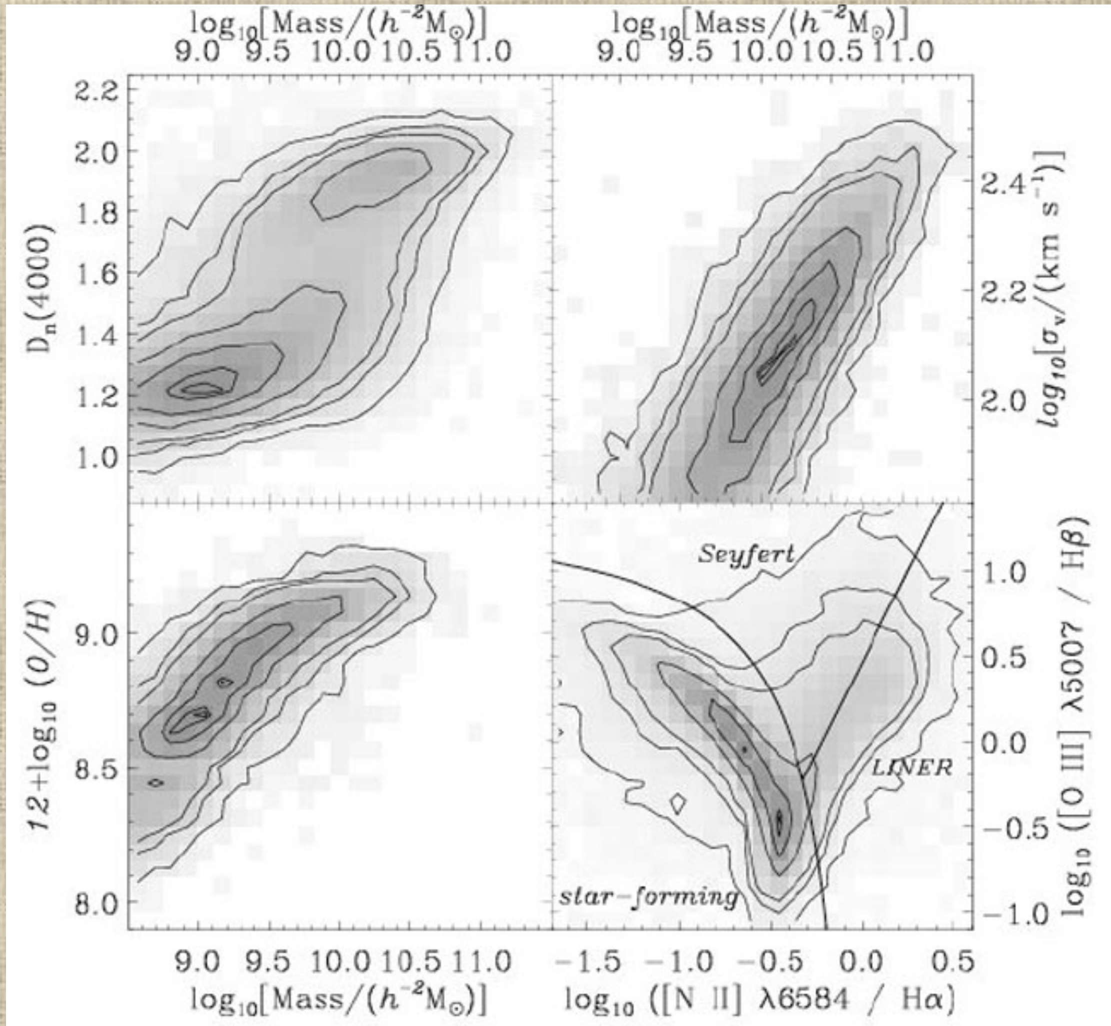
**Luigi (Gigi) Guzzo**

**Osservatorio Astronomico di Brera, INAF, Merate/Milano**

# Galaxy redshift surveys: a major pillar of the cosmological model…

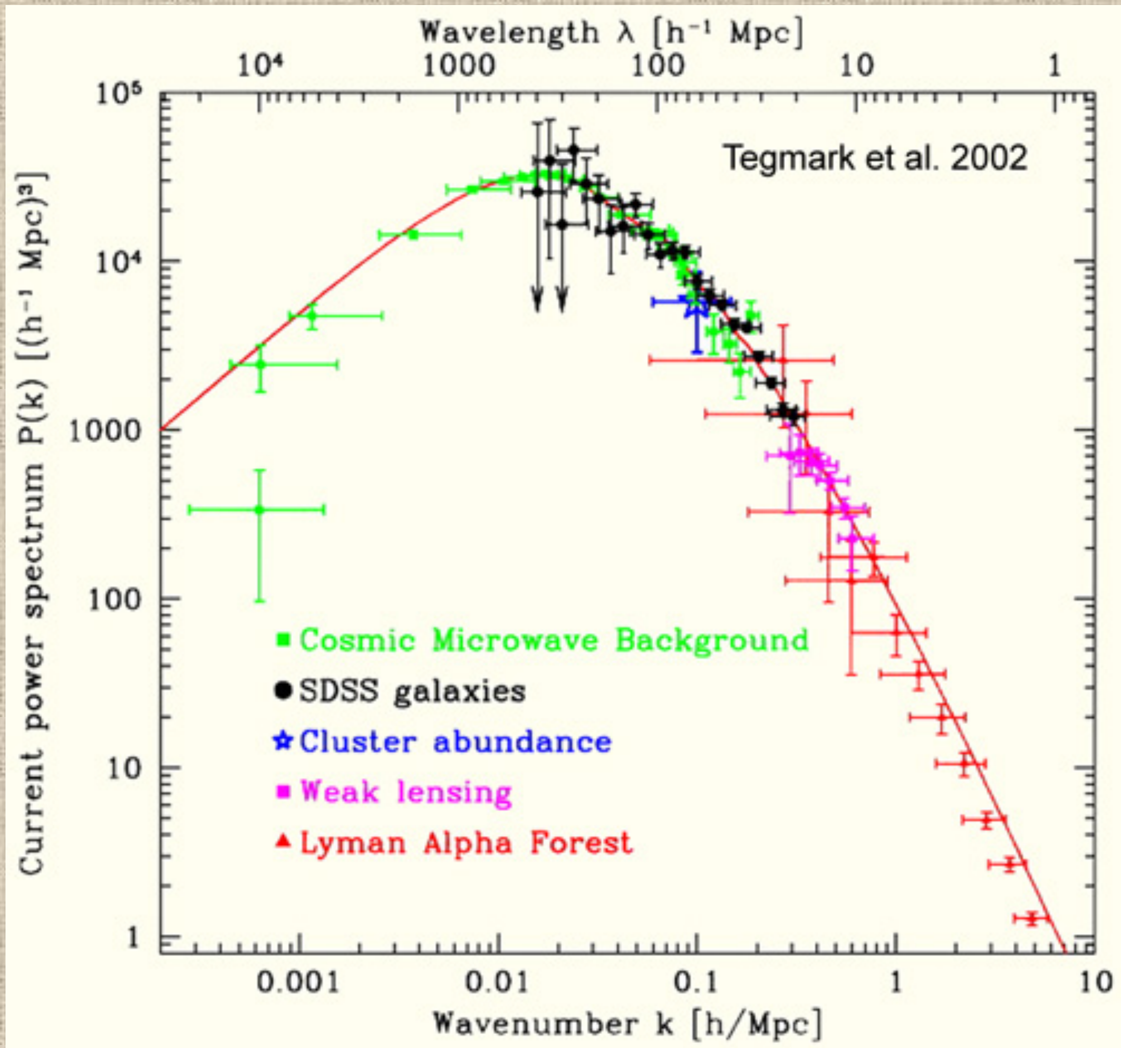


# …but also of our understanding of how galaxies form and evolve…

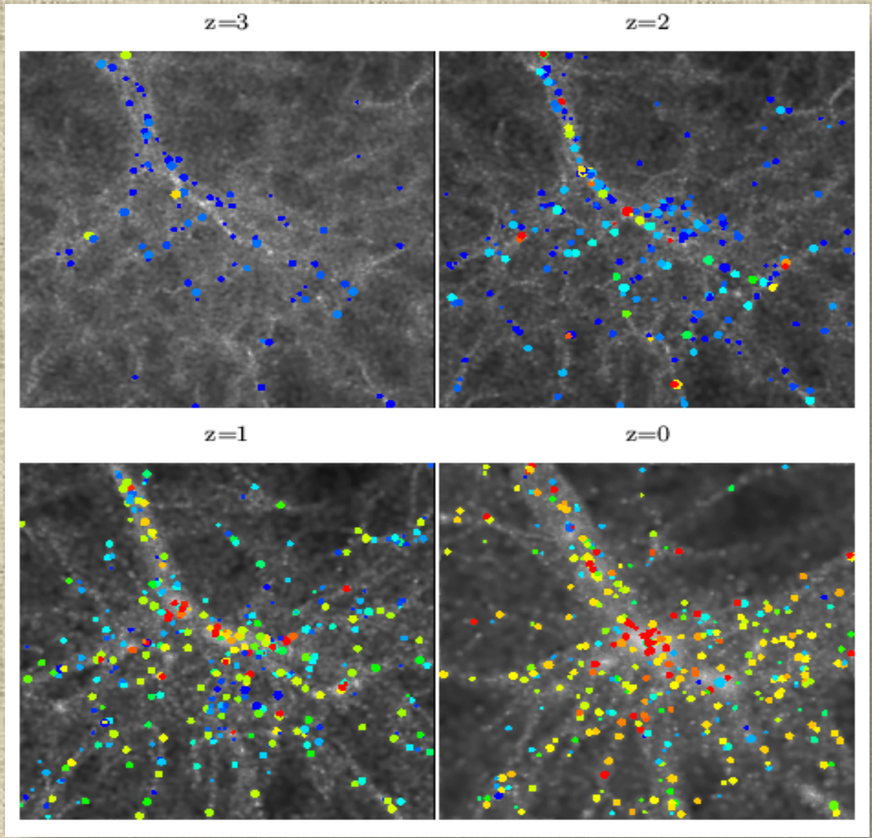



- SDSS: statistical distribution of galaxy properties for $\sim10^6$ galaxies

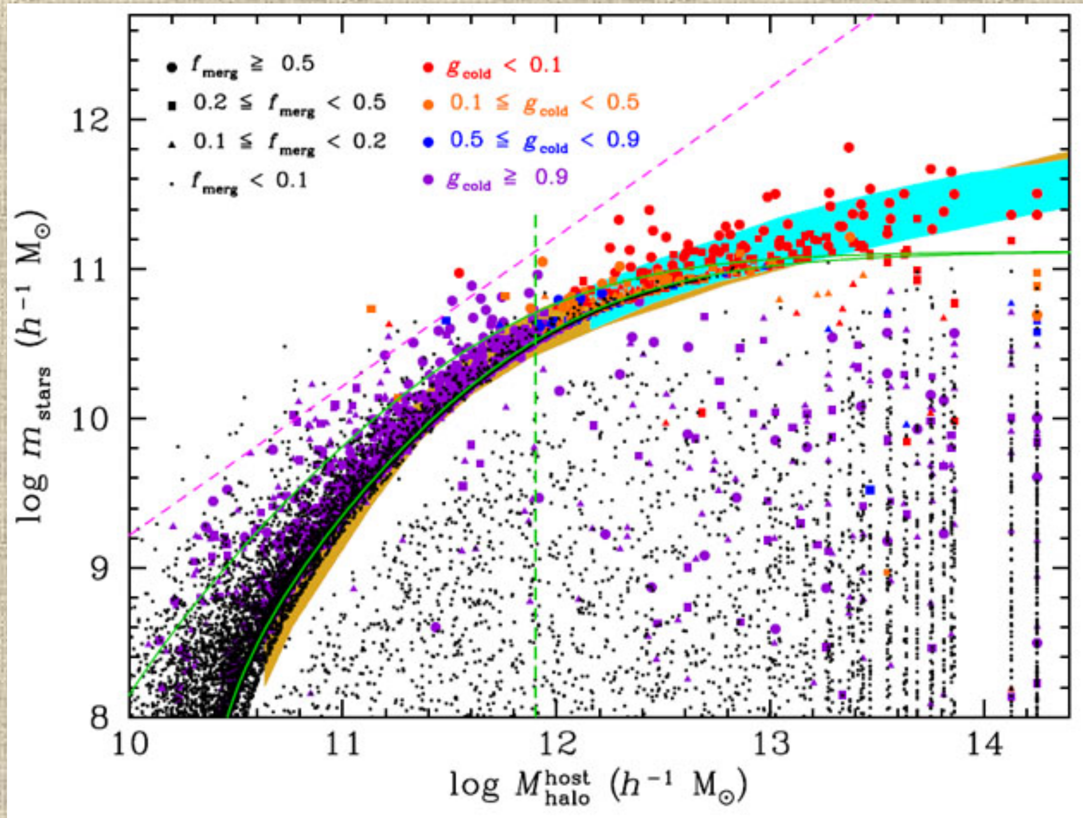
# The clustering power spectrum: a probe of the underlying cosmology



# We need to understand galaxies, to do cosmology…

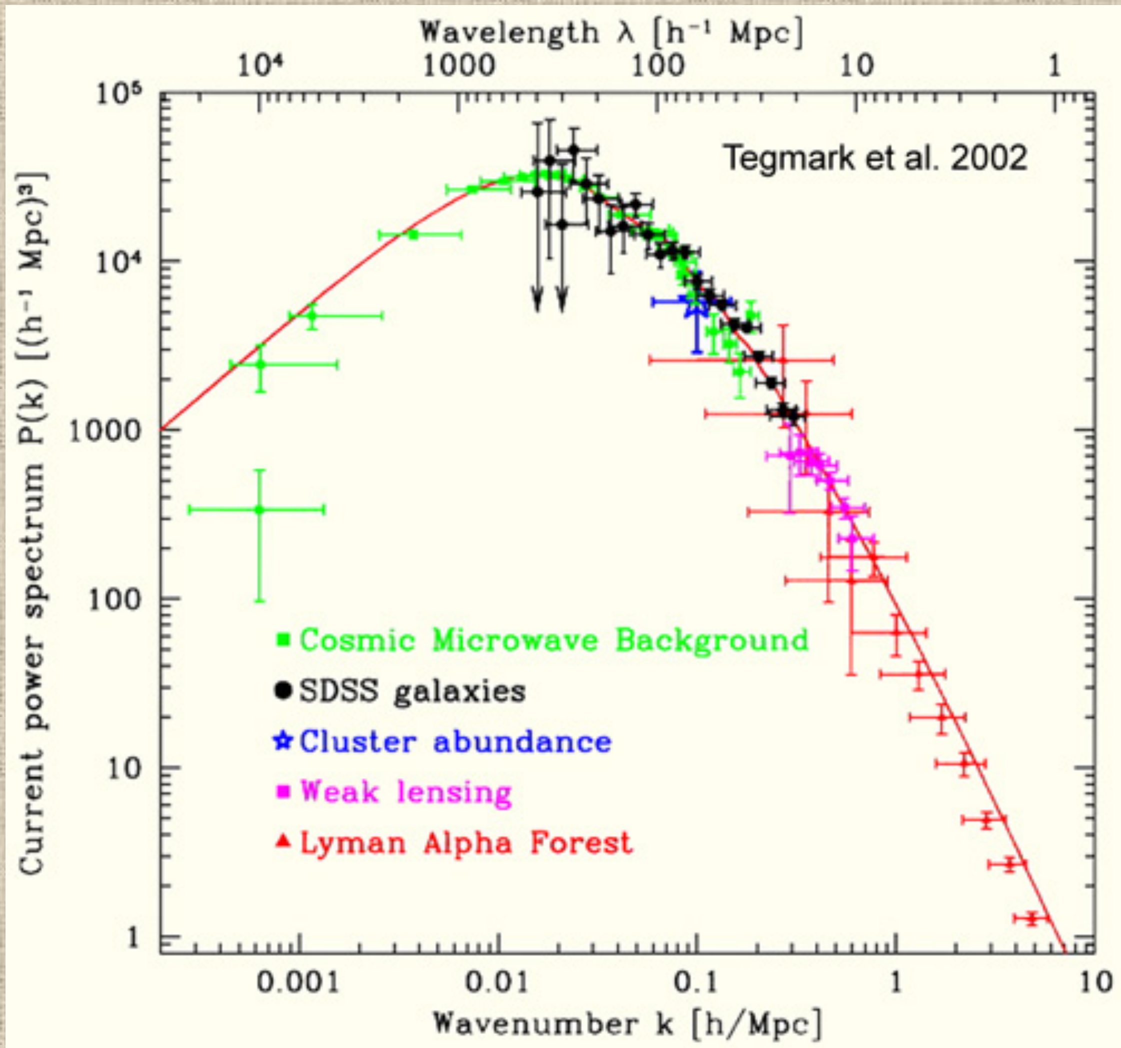



Kauffman & Diaferio 1998

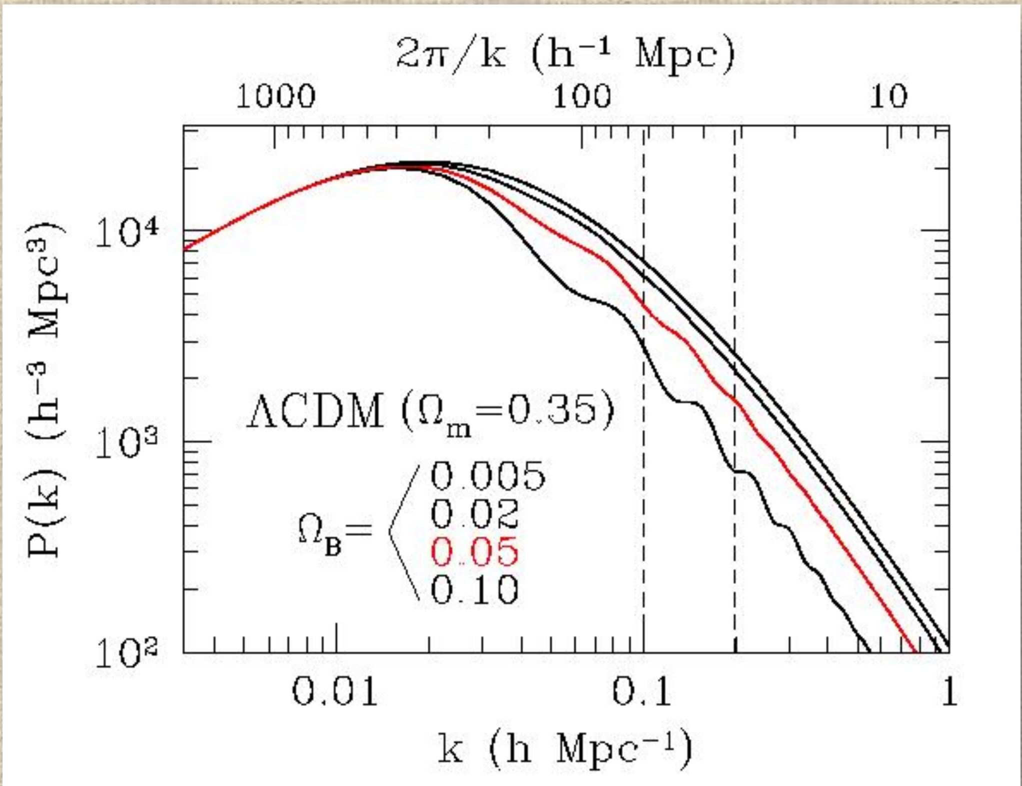


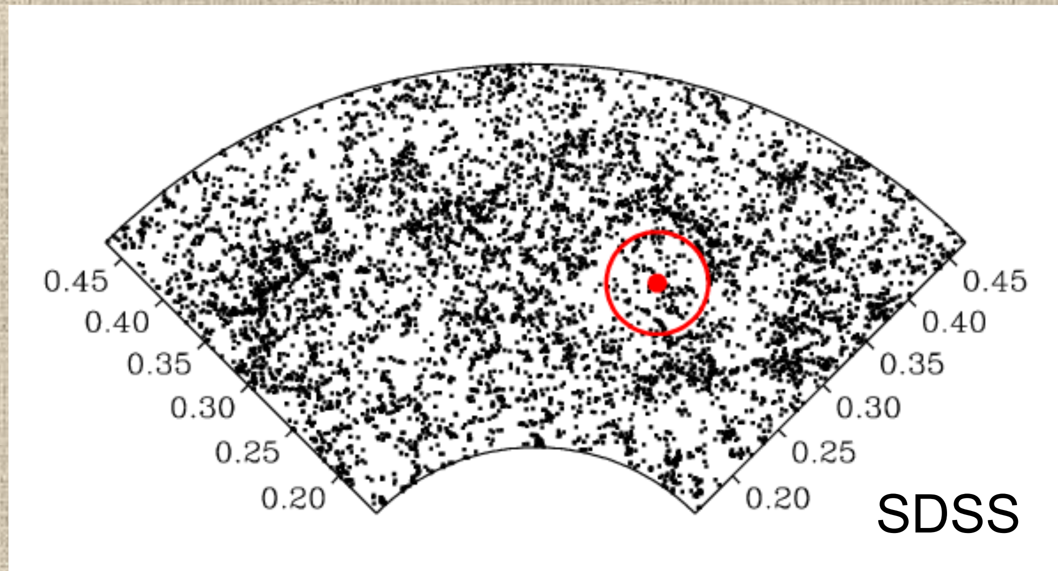

Cattaneo et al. 2011 – halo mass vs stellar mass;
(toy model on high-resolution simulation DM halos)

# The clustering power spectrum: a probe of the underlying cosmology



# Baryonic Acoustic Oscillations: a standard ruler to measure H(z)

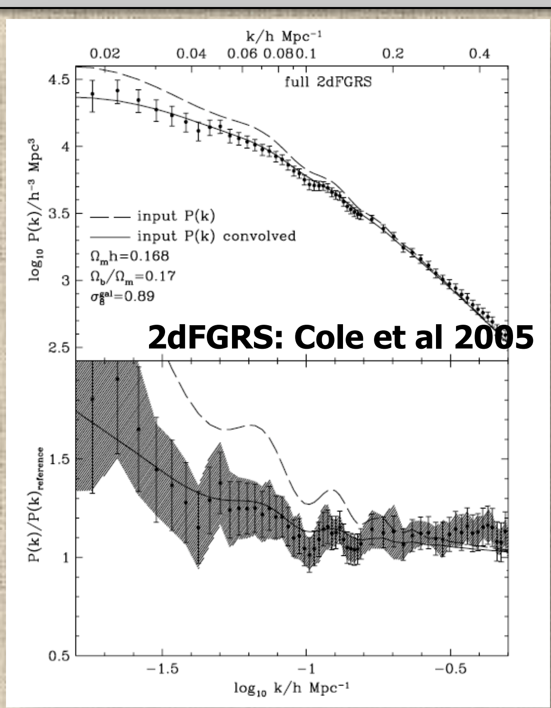


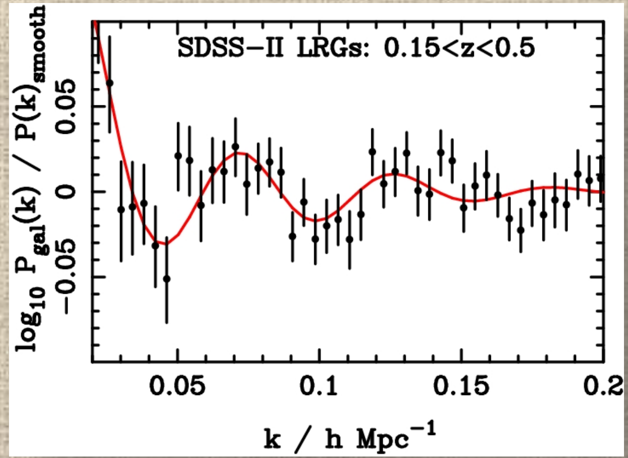

D. Eisenstein 2007

# BAO detection in galaxy redshift surveys

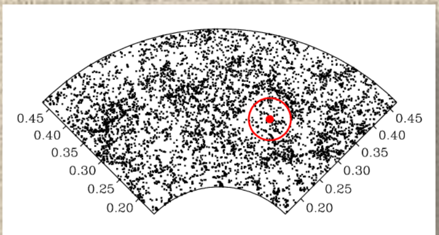
**Fourier Space (wiggles):**



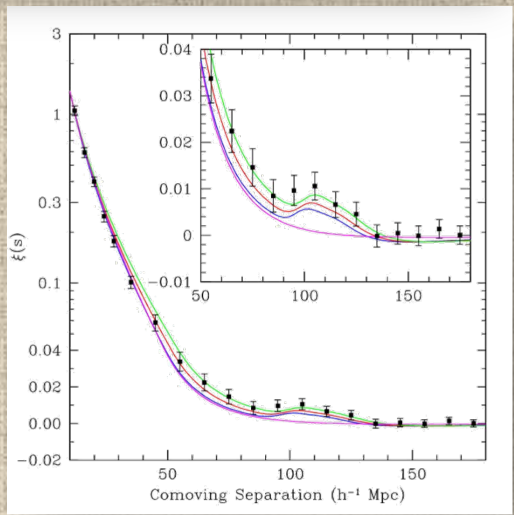

**2dFGRS: Cole et al 2005**



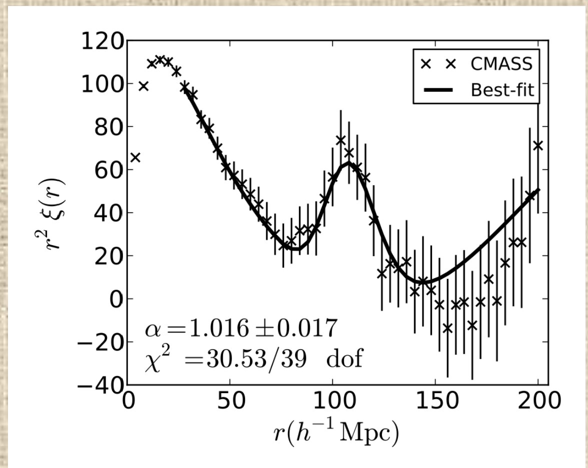

**Percival et al 2010**



**Configuration Space (BAO peak):**



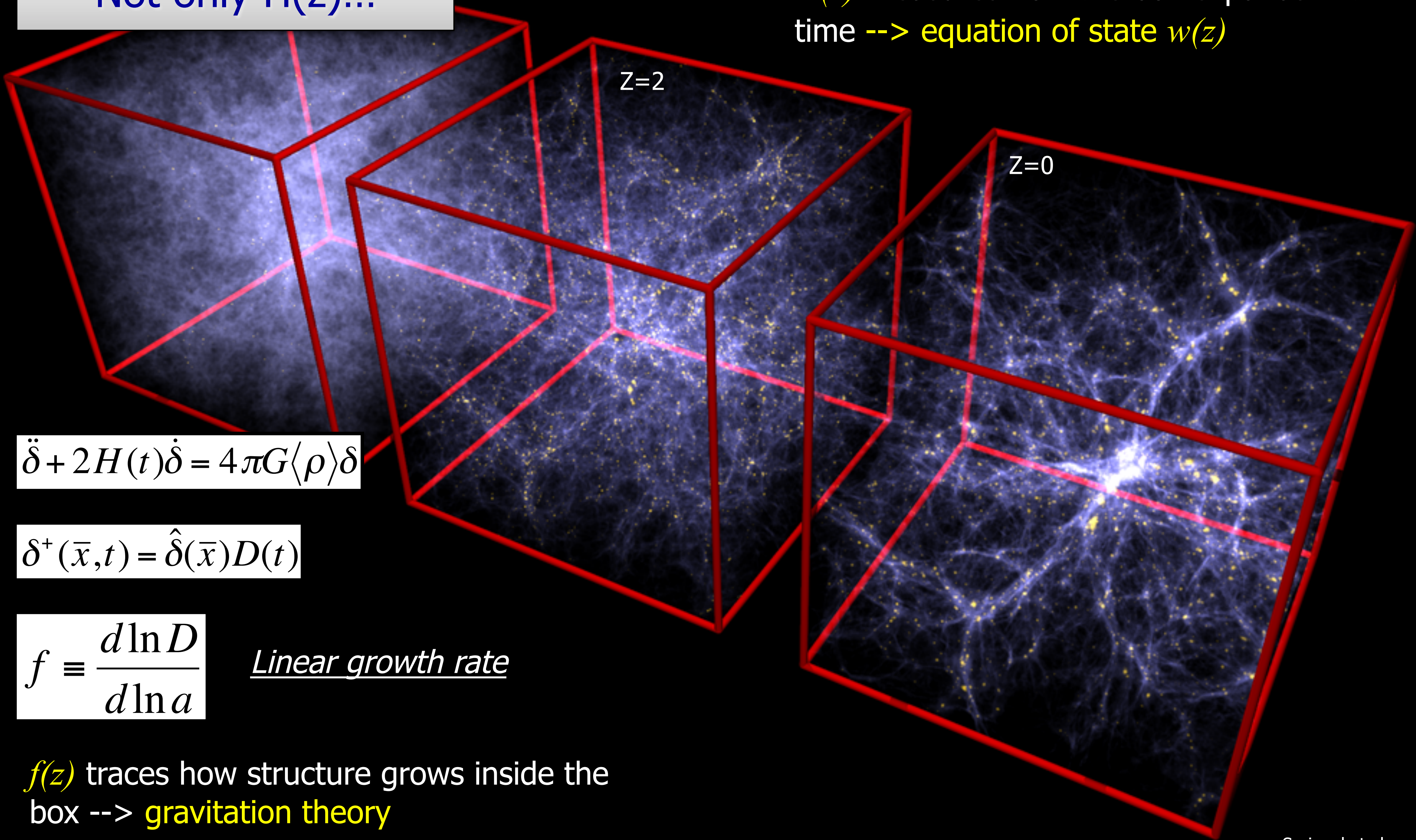

**SDSS: Eisenstein et al 2005**




**BOSS: Anderson et al. 2012, 2013**

# Not only H(z)…

*H(z)* measures how the box expands with time --> equation of state *w(z)*




Springel et al.

$$\ddot{\delta} + 2H(t)\dot{\delta} = 4\pi G\langle\rho\rangle\delta$$

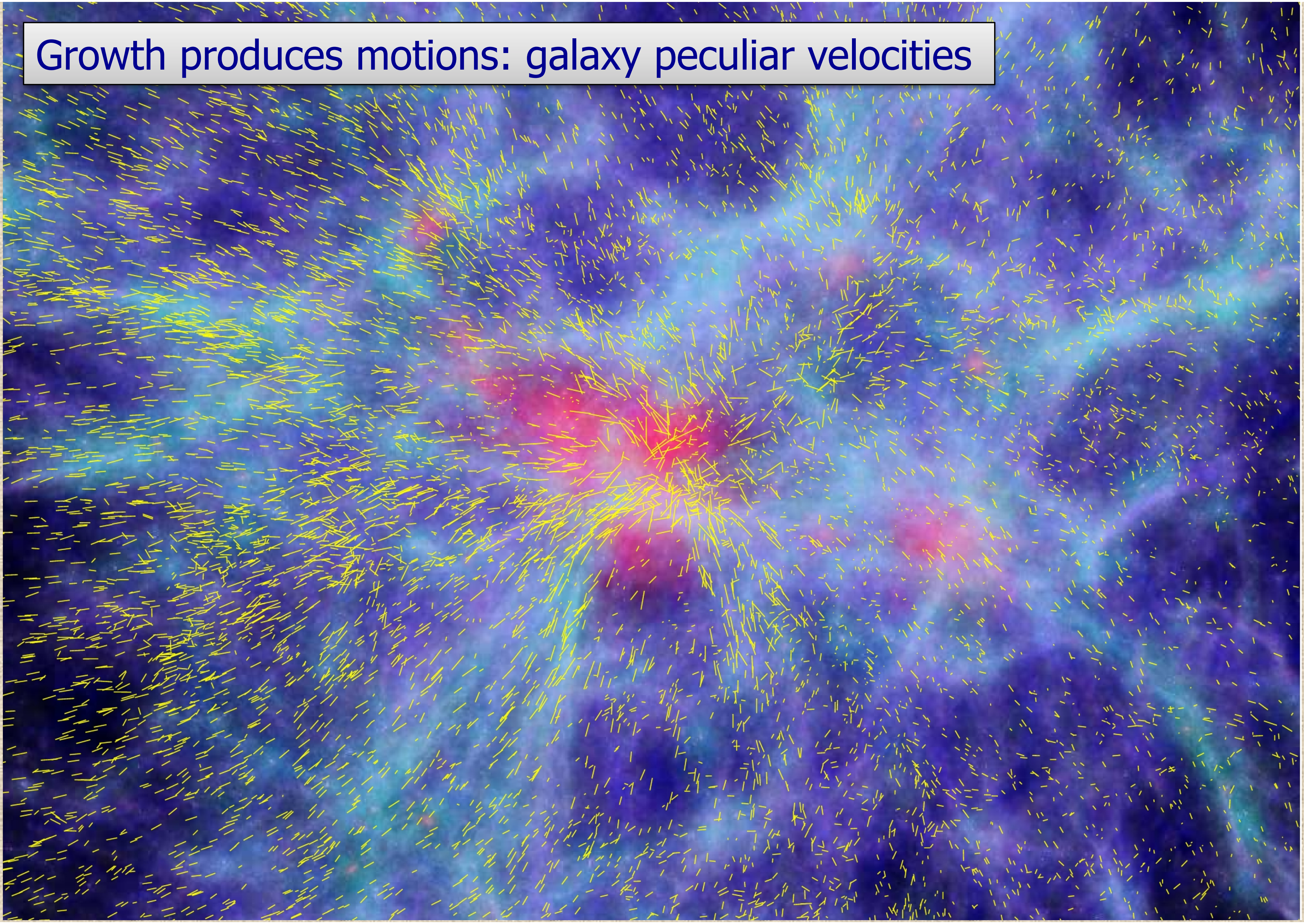
$$\delta^{+}(\bar{x},t) = \hat{\delta}(\bar{x})D(t)$$

$$f \equiv \frac{d\ln D}{d\ln a}$$

*Linear growth rate*

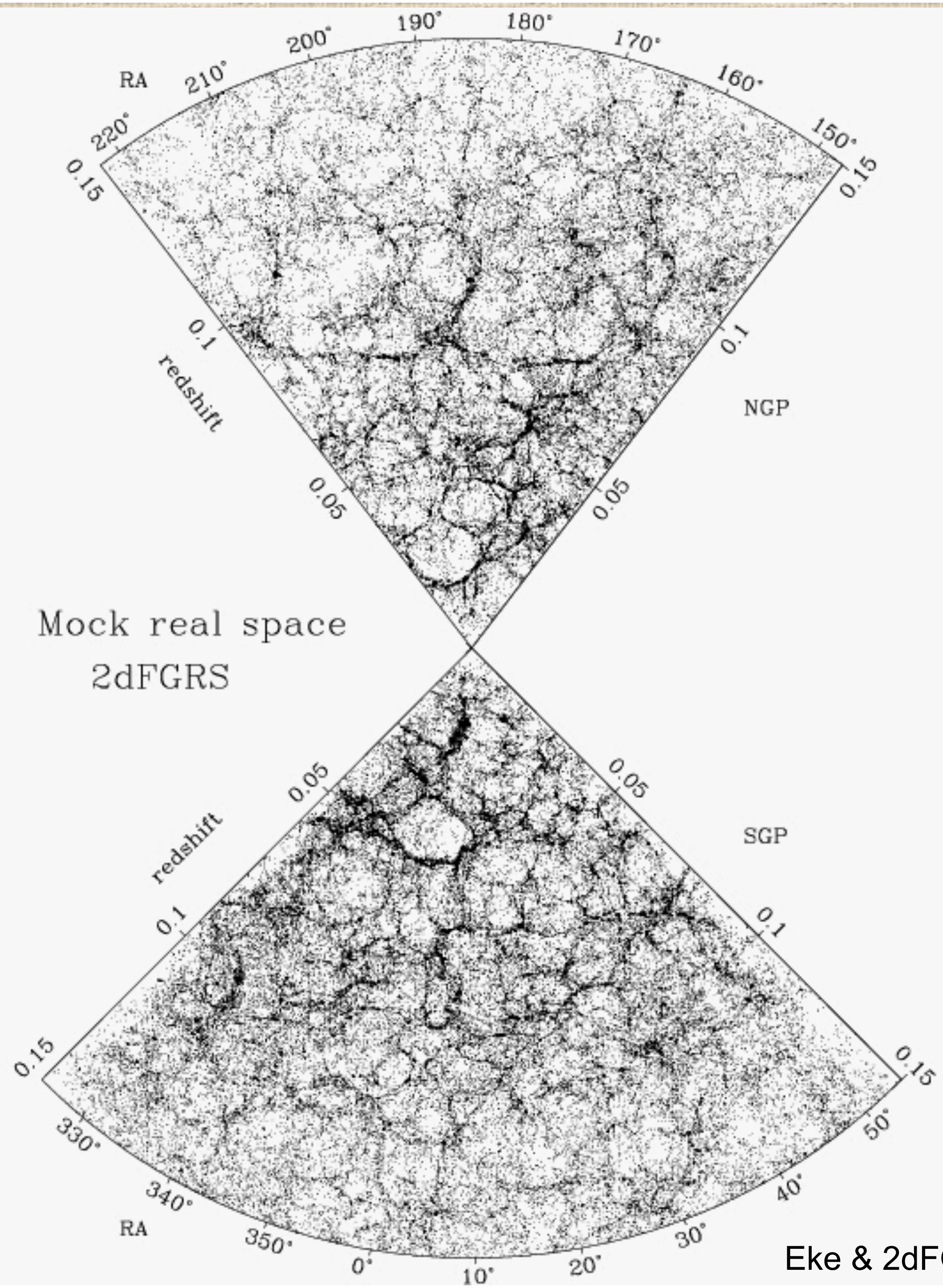
*f(z)* traces how structure grows inside the box --> gravitation theory

# Growth produces motions: galaxy peculiar velocities



Peculiar velocities manifest themselves in galaxy redshift surveys as *redshift-space distortions* (Kaiser 1987)

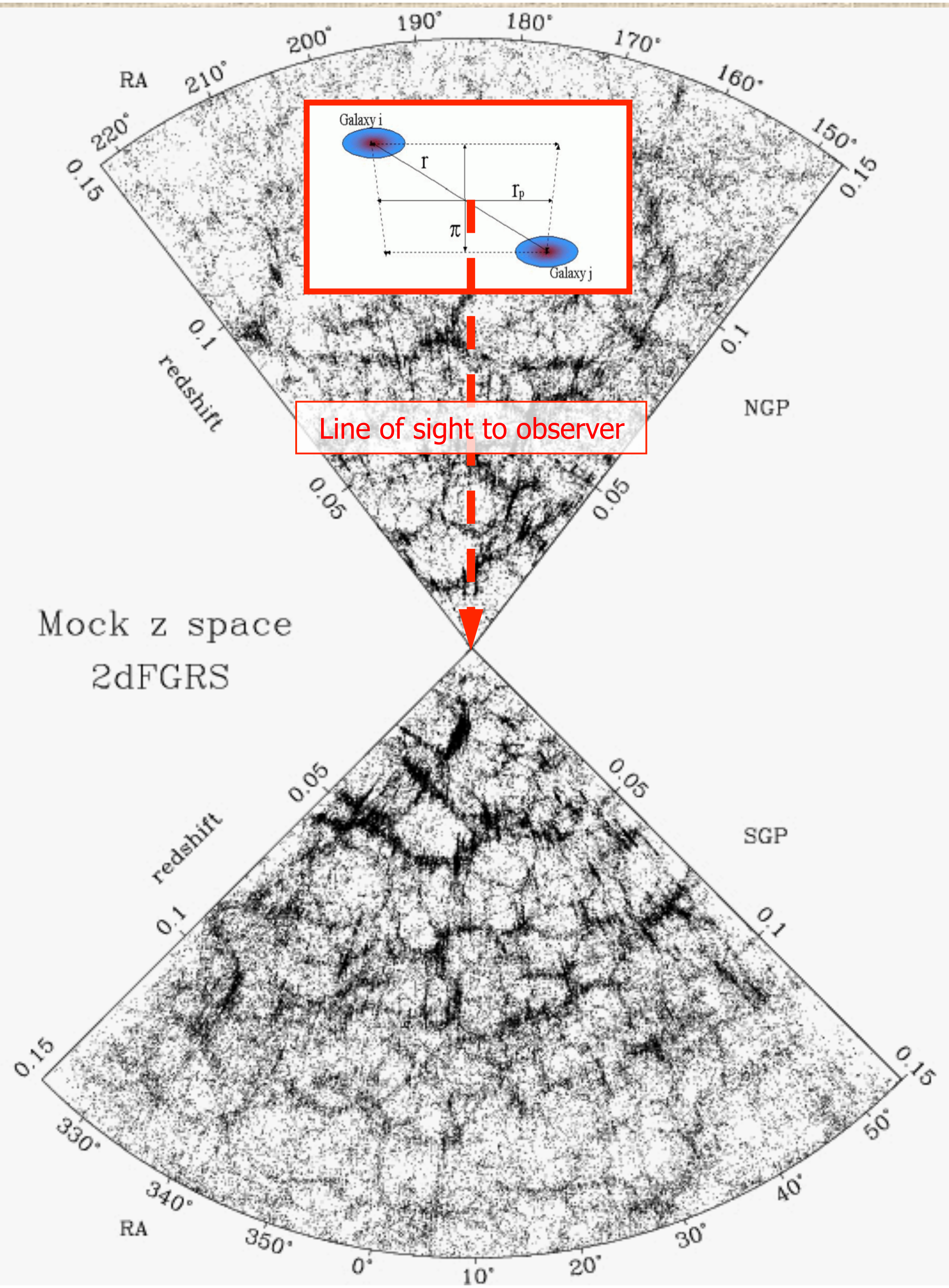
**real space**




Eke & 2dFGRS 2003

Peculiar velocities manifest themselves in galaxy redshift surveys as *redshift-space distortions* (Kaiser 1987)

**redshift space**

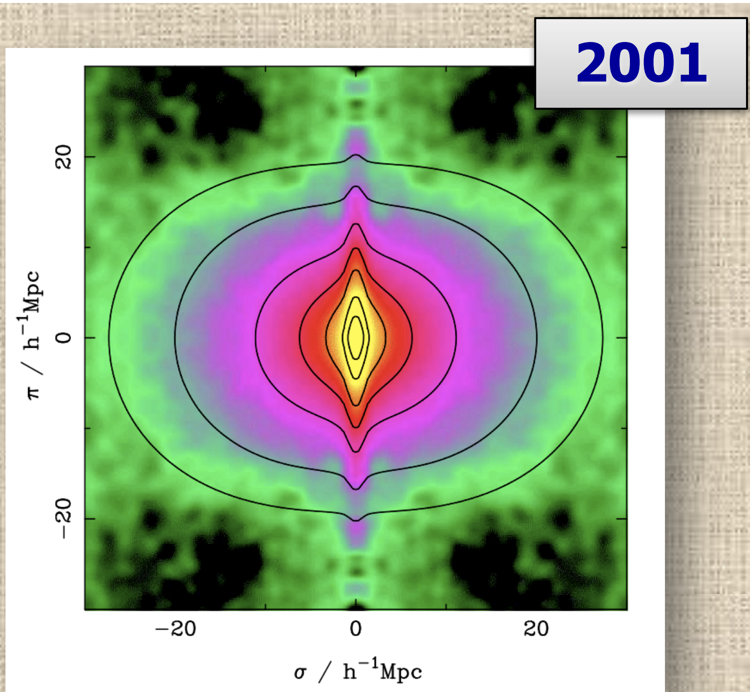


# Redshift-Space Distortions: an old way to look at a new thing…

**Nature 410, 169 (2001)**

## A measurement of the cosmological mass density from clustering in the 2dF Galaxy Redshift Survey

John A. Peacock[1], Shaun Cole[2], Peder Norberg[2], Carlton M. Baugh[2], Joss Bland-Hawthorn[3], Terry Bridges[3], Russell D. Cannon[3], Matthew Colless[4], Chris Collins[5], Warrick Couch[6], Gavin Dalton[7], Kathryn Deeley[6], Roberto De Propris[6], Simon P. Driver[8], George Efstathiou[9], Richard S. Ellis[9,10], Carlos S. Frenk[2], Karl Glazebrook[11], Carole Jackson[4], Ofer Lahav[9], Ian Lewis[3], Stuart Lumsden[12], Steve Maddox[13], Will J. Percival[1], Bruce A. Peterson[4], Ian Price[4], Will Sutherland[1,7] & Keith Taylor[3,10]



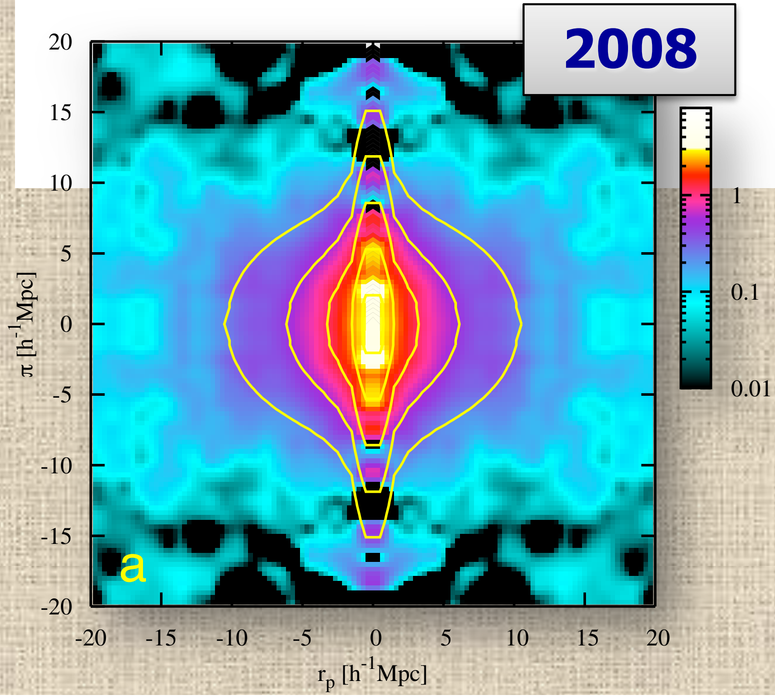

Vol 451|31 January 2008|doi:10.1038/nature06555 nature

**Nature 451, 541 (2008)**

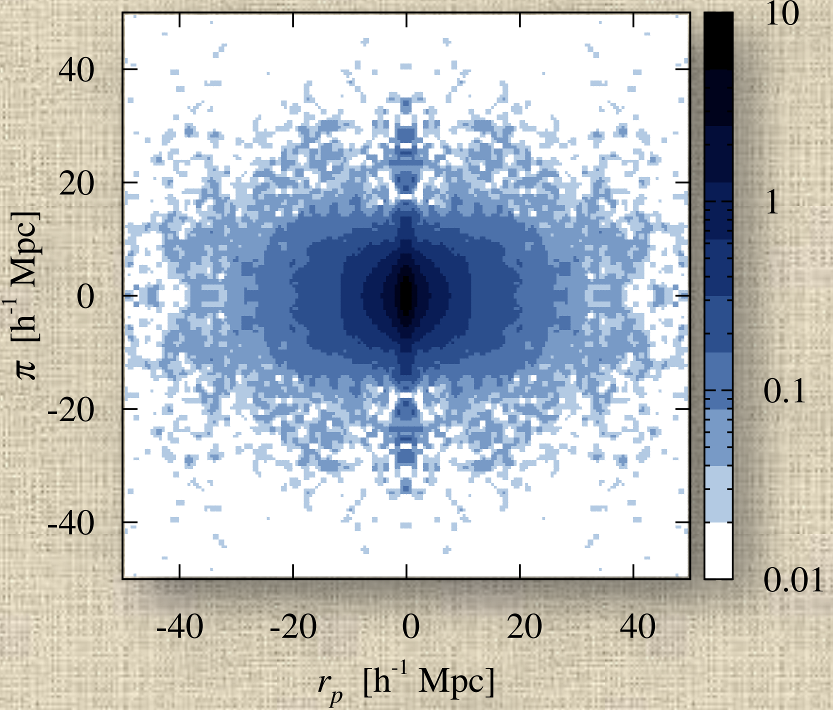
LETTERS

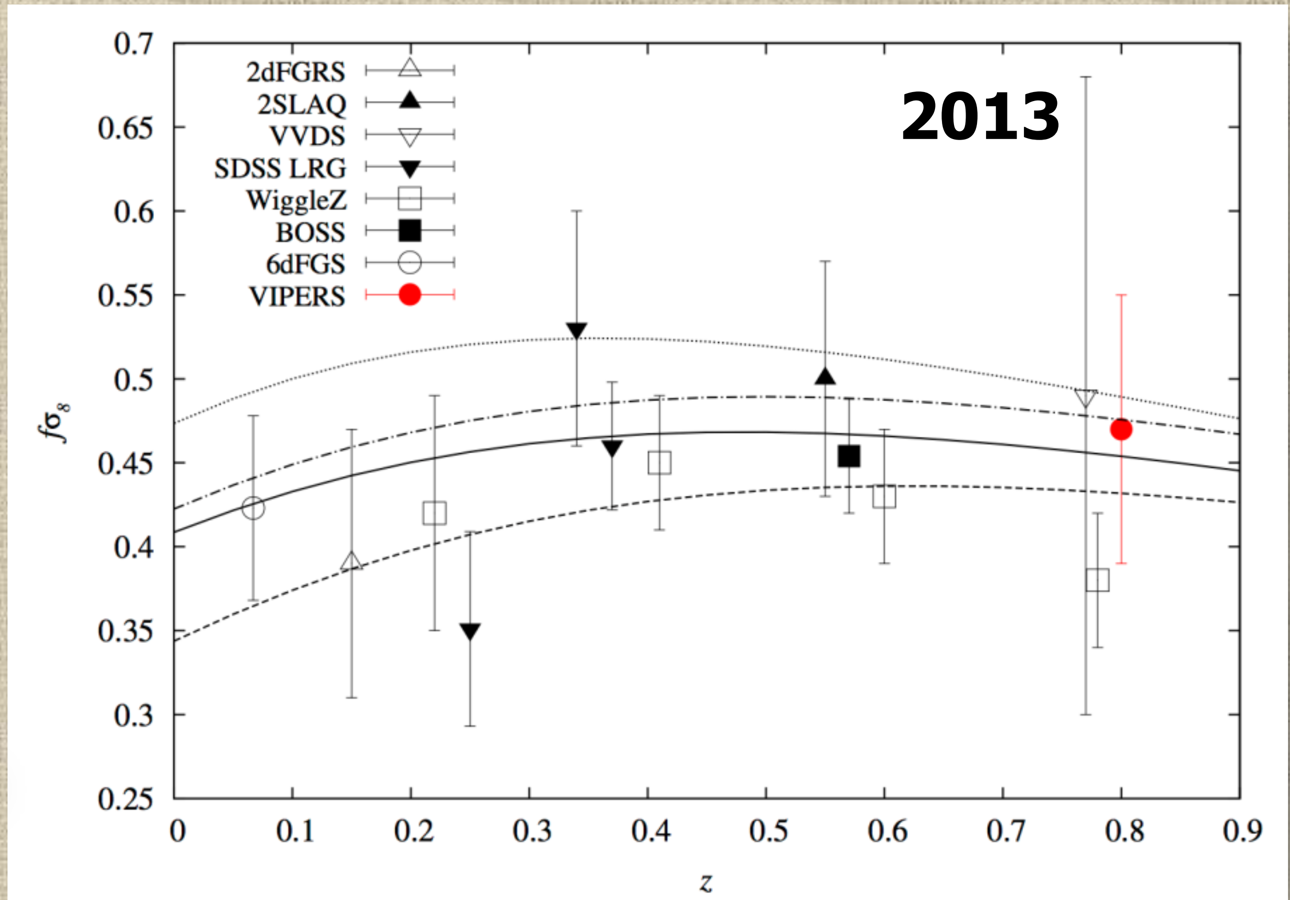
## A test of the nature of cosmic acceleration using galaxy redshift distortions

L. Guzzo[1,2,3,4], M. Pierleoni[3], B. Meneux[5], E. Branchini[6], O. Le Fèvre[7], C. Marinoni[8], B. Garilli[5], J. Blaizot[3], G. De Lucia[3], A. Pollo[7,9], H. J. McCracken[10,11], D. Bottini[5], V. Le Brun[7], D. Maccagni[5], J. P. Picat[12], R. Scaramella[13,14], M. Scodeggio[5], L. Tresse[7], G. Vettolani[13], A. Zanichelli[13], C. Adami[7], S. Arnouts[7], S. Bardelli[15], M. Bolzonella[15], A. Bongiorno[16], A. Cappi[15], S. Charlot[10], P. Ciliegi[15], T. Contini[12], O. Cucciati[1,17], S. de la Torre[7], K. Dolag[3], S. Foucaud[18], P. Franzetti[5], I. Gavignaud[19], O. Ilbert[20], A. Iovino[1], F. Lamareille[15], B. Marano[16], A. Mazure[7], P. Memeo[5], R. Merighi[15], L. Moscardini[16,21], S. Paltani[22,23], R. Pellò[12], E. Perez-Montero[12], L. Pozzetti[15], M. Radovich[24], D. Vergani[5], G. Zamorani[15] & E. Zucca[15]



# Redshift-space distortions as a dark energy test




BOSS: $f\sigma_8(z=0.57) = 0.447 \pm 0.028$ Samushia et al. 2014

VIPERS: $f\sigma_8(z=0.8) = 0.47 \pm 0.08$ De la Torre, LG et al. 2013

## Galaxy clustering: a primary probe to answer the high-level questions…

- Nature of Dark Matter ?
- Nature of Dark Energy ?
- Behaviour of gravity at the largest scales (did Einstein have final word)?
- Physics of the initial conditions (inflation) ?

**Implications for physics**

→ the Standard Model of cosmology ($\Lambda$CDM)

→ the Standard Model of particle physics

# …if a galaxy redshift survey is properly designed

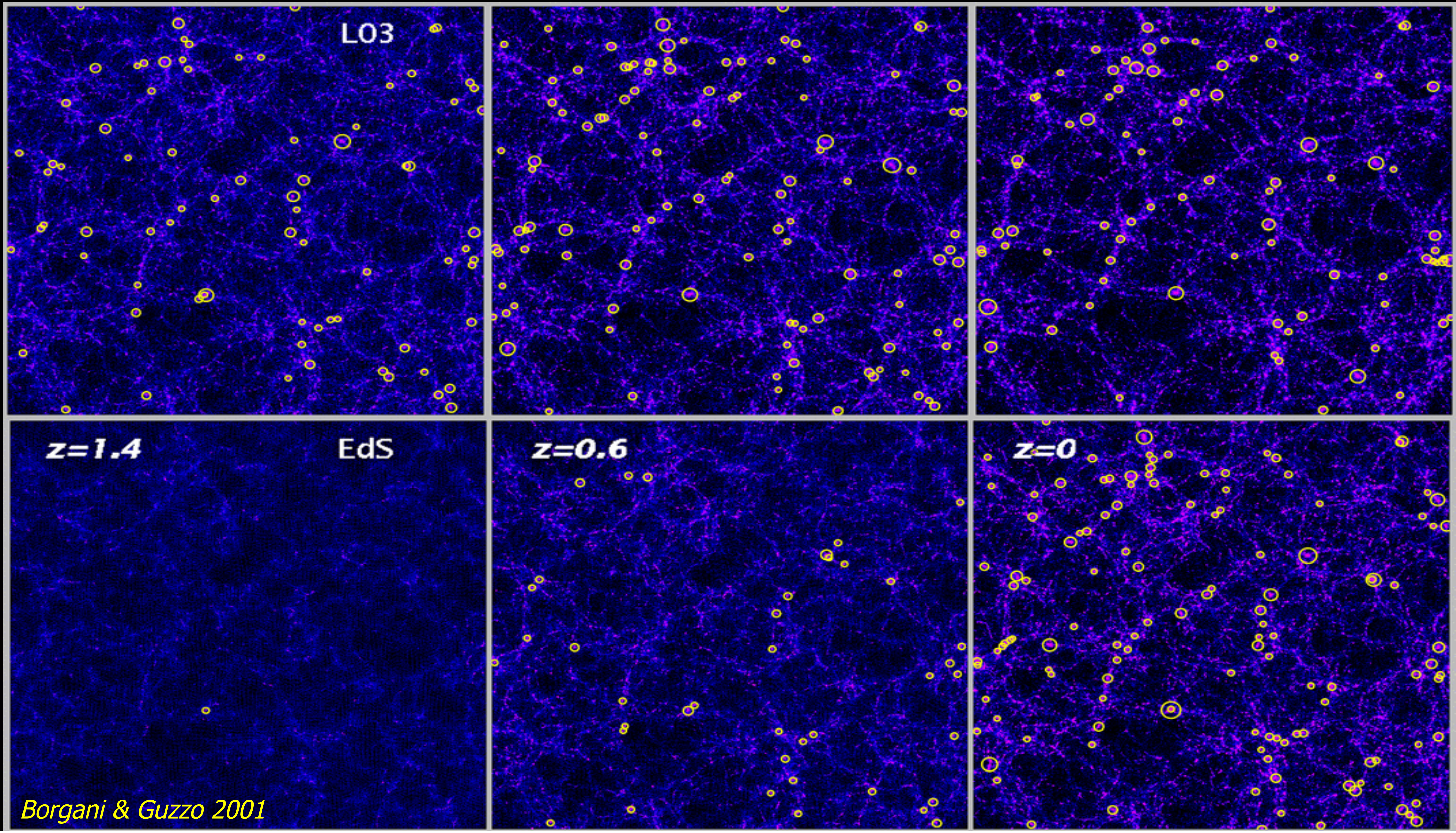
**STATISTICAL ERRORS** (not an issue nowadays?):

- Sample bigger volumes to push down sample variance, but being sufficiently dense to stay away from shot noise regime on the scales of interest
- Use multiple populations? (seemed more promising) → survey design

**SYSTEMATIC ERRORS**:

- How do my galaxy tracers sample dark-matter distribution? DM-baryon connection (**bias**) → survey design (type of tracers, …)
- Minimize impact of **non-linear clustering** → survey design (largest possible volume)
- Accuracy of **modelling** (e.g. RSD), to match requirements of precision cosmology → technical advances, but also survey design (some tracers may be less affected than others)
- Use multiple populations, as a cross-check of systematic effects → survey design

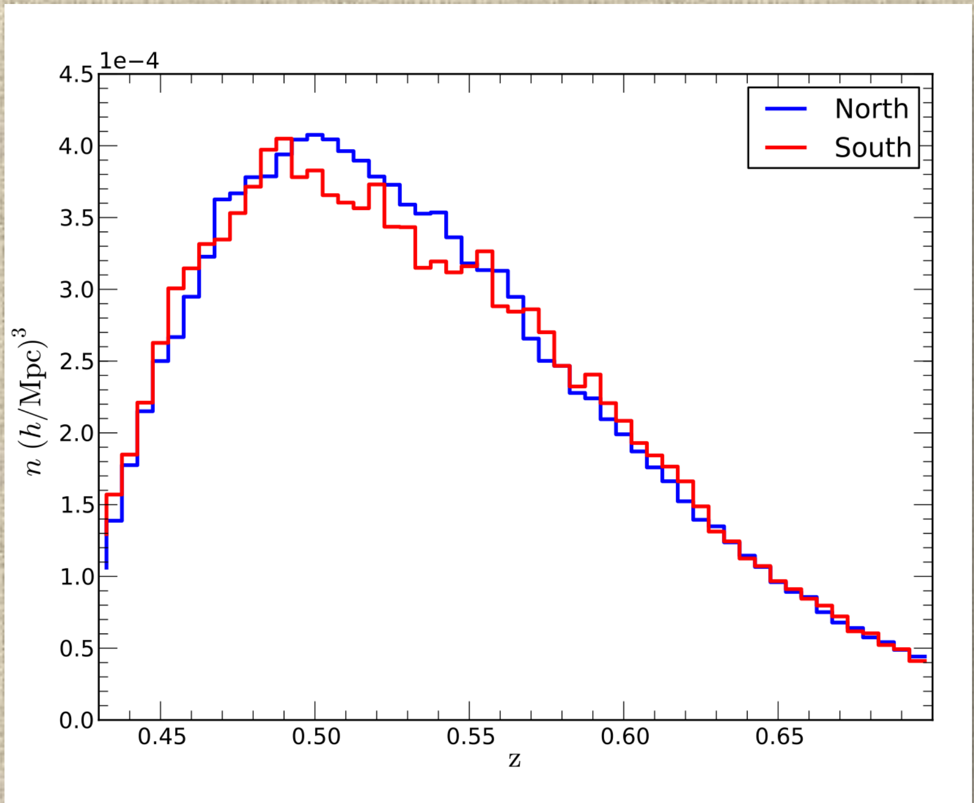
# SEE THE WHOLE MOVIE, NOT JUST THE FINAL PICTURE…

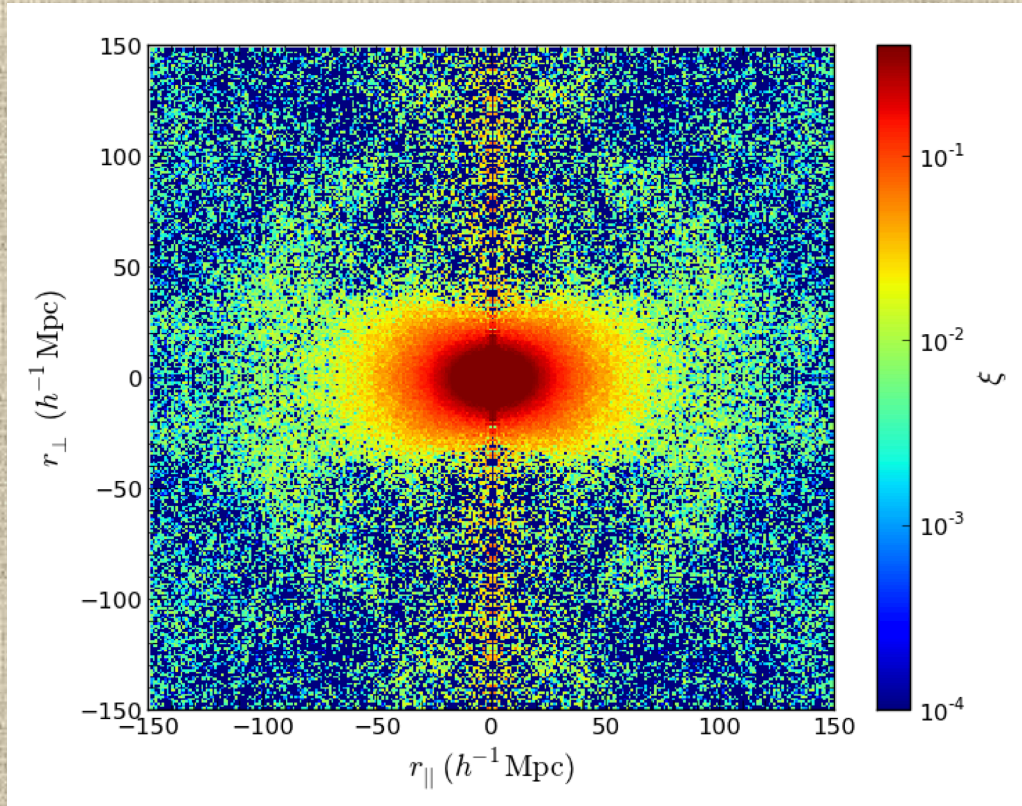


# Push deeper using a sparse "special" galaxy population…

E.g. SDSS-LRG, and **BOSS** (see also **Wigglez** – Blake et al.):

- BOSS: "CMASS" LRG-like col-col selection, "loosely selecting constant mass galaxies", **z<0.7**
- **Area=8500 deg$^2$ , Volume~6 $h^{-3}$ Gpc, Ngal = 690,000 → <n>~$10^{-4}$ $h^3$ Mpc$^{-3}$**
- Optimized for BAO measurement, excellent (a posteriori) for Redshift Space Distortions
- See e.g. Samushia et al. (2014) and references therein



…or push to higher redshift, but aiming at a volume **and density** comparable to 2dFGRS and SDSS, with similarly broad selection function

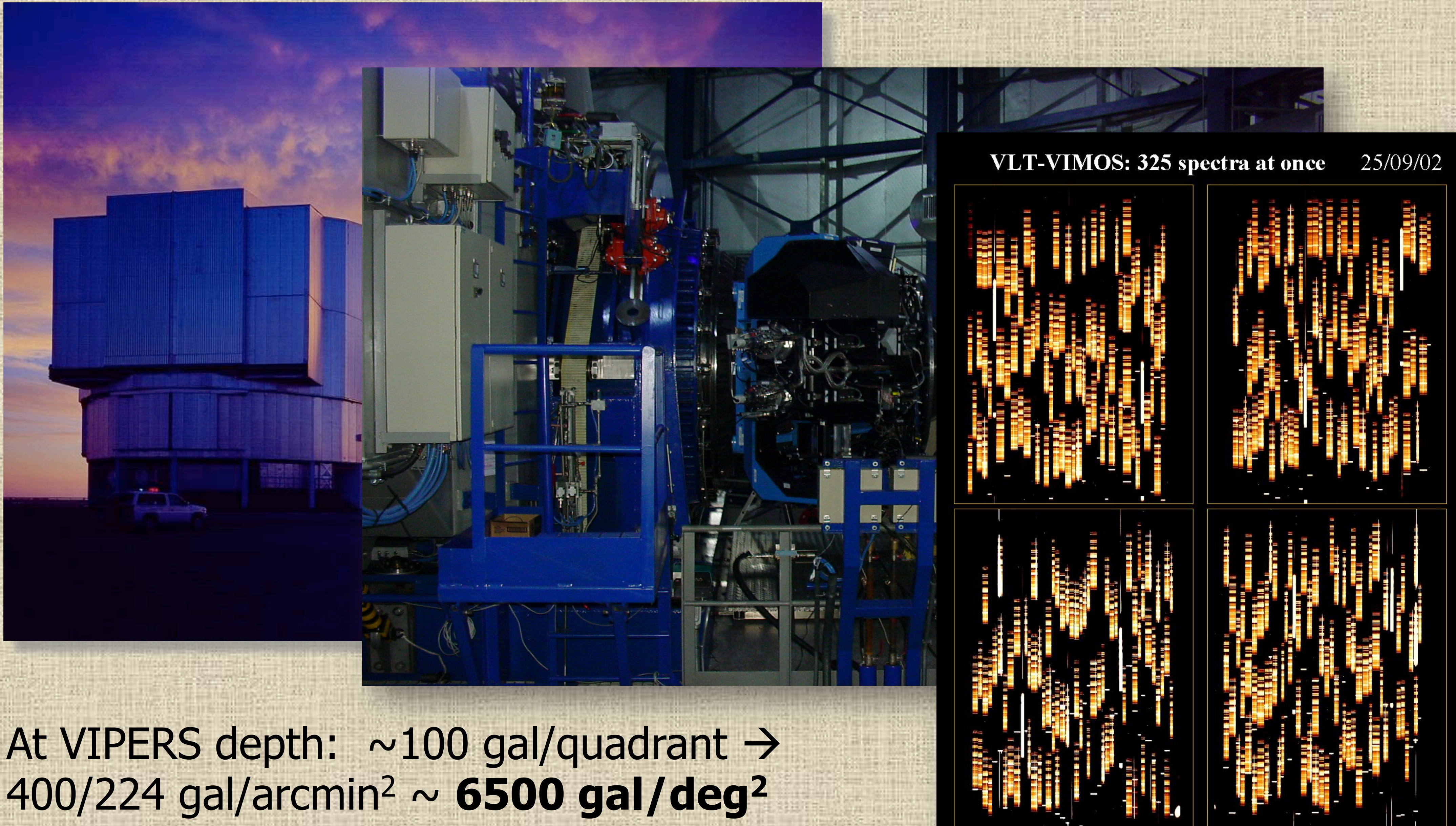
VIPERS

VIMOS PUBLIC EXTRAGALACTIC REDSHIFT SURVEY

# VIPERS headline science goals

- **Galaxy clustering at z~1 with comparable precision to z~0:**
  - Evolution of $\xi(r)$ and P(k) ($\Omega_m$, $\Omega_b$ at z~1)
  - Dependence on galaxy properties
  - Galaxy-DM relations (HOD modeling)
- **Growth rate from redshift-space distortions at z~1**
- **Evolution and non-linearity of galaxy biasing**
- **Evolution of galaxy colors and environmental effects**
- **Bright/massive/rare galaxies at z~1 and evolution of the galaxy luminosity and stellar mass functions**
- Combined clustering / weak-lensing analysis (photo-z calibr., CFHTLenS match)
- Multi-wavelength studies (SWIRE, XMM-XXL, UDS, VIDEO,…)

# VIMOS @ VLT fills unique niche in density-area space




At VIPERS depth: ~100 gal/quadrant →
400/224 gal/arcmin$^2$ ~ **6500 gal/deg$^2$**

# VIPERS strategy

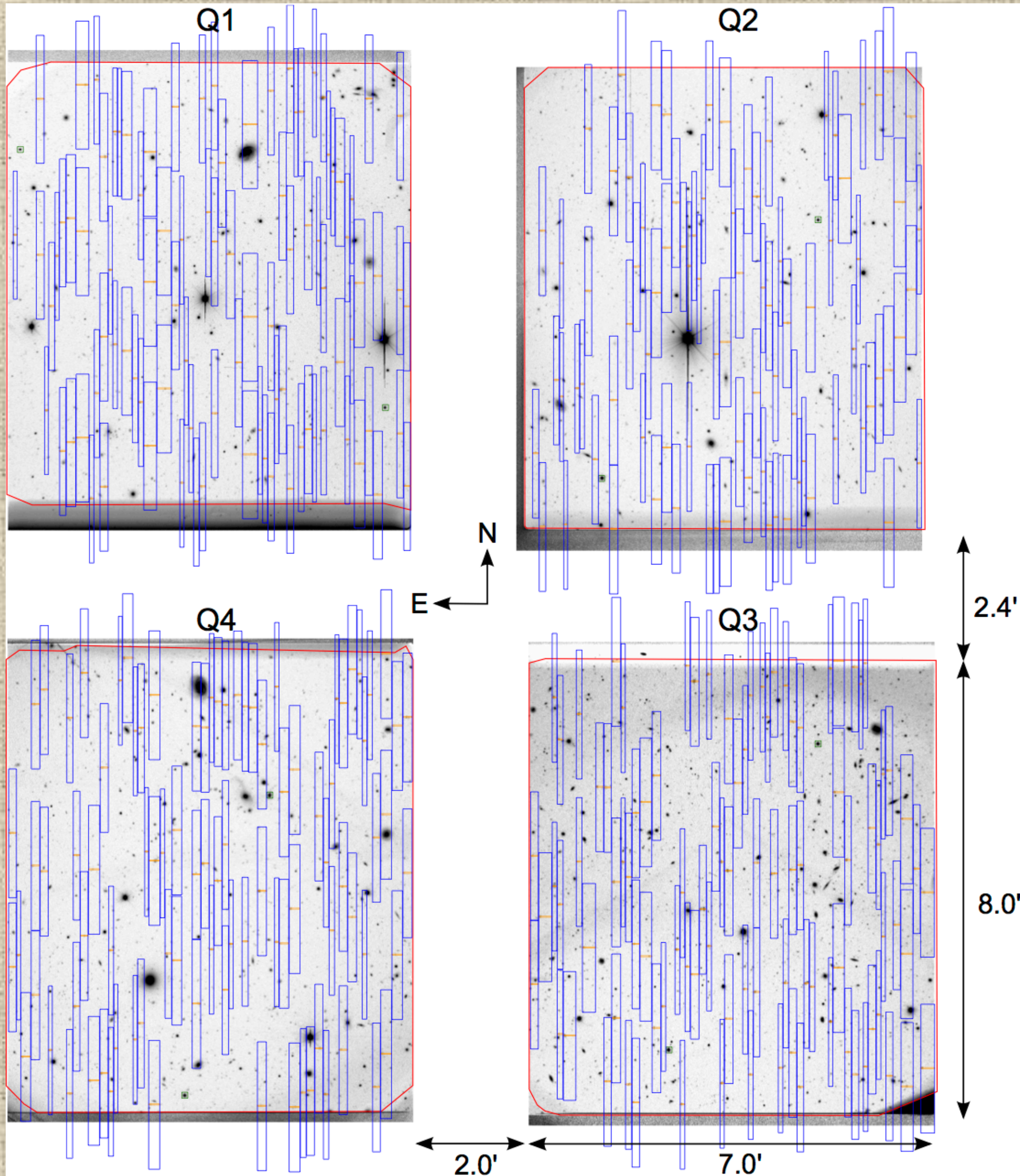
- **Want volume and density comparable to a survey like 2dFGRS, but at z=[0.5-1]:** cosmology driven, but with broader legacy return
- **Means Vol~5 x $10^7$ $h^{-3}$ $Mpc^3$, ~100,000 redshifts, close to full sampling**
- Implies **$I_{AB}$<22.5, ~24 $deg^2$**
- Improve sampling within redshift range of interest through **z>0.5 robust color-color pre-selection** (+star-galaxy separation), with also better match to VIMOS multiplexing: **>40% sampling**
- **CFHTLS Wide** (W1 and W4 fields, ~16 + 8 $deg^2$) provides accurate multi-band photometry to support this
- VIMOS LR Red grism, **45 min exposure**
- 288 pointings, **440.5 VLT hours (~55 night-equivalent)**

# VIPERS Team

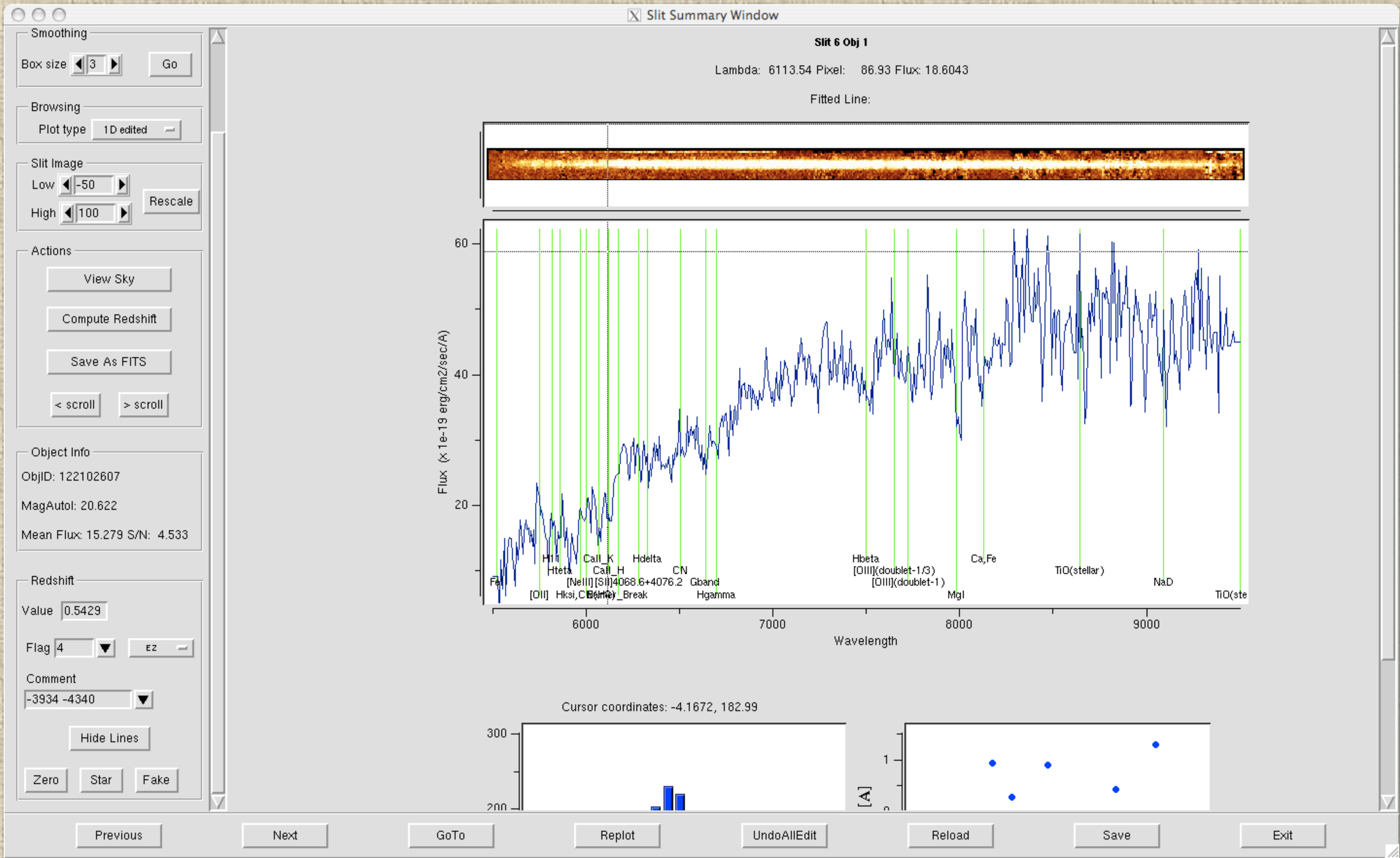
(see http://vipers.inaf.it)

# VIPERS single-shot footprint on the sky

- **On average, 360 spectra observed per VIMOS pointing, given VIPERS target sample surface density and clustering**
- **VIPERS strategy yields mean spatial density $\langle n \rangle \sim 10^{-2}\ h^3\ Mpc^{-3}$ within the range of interest**

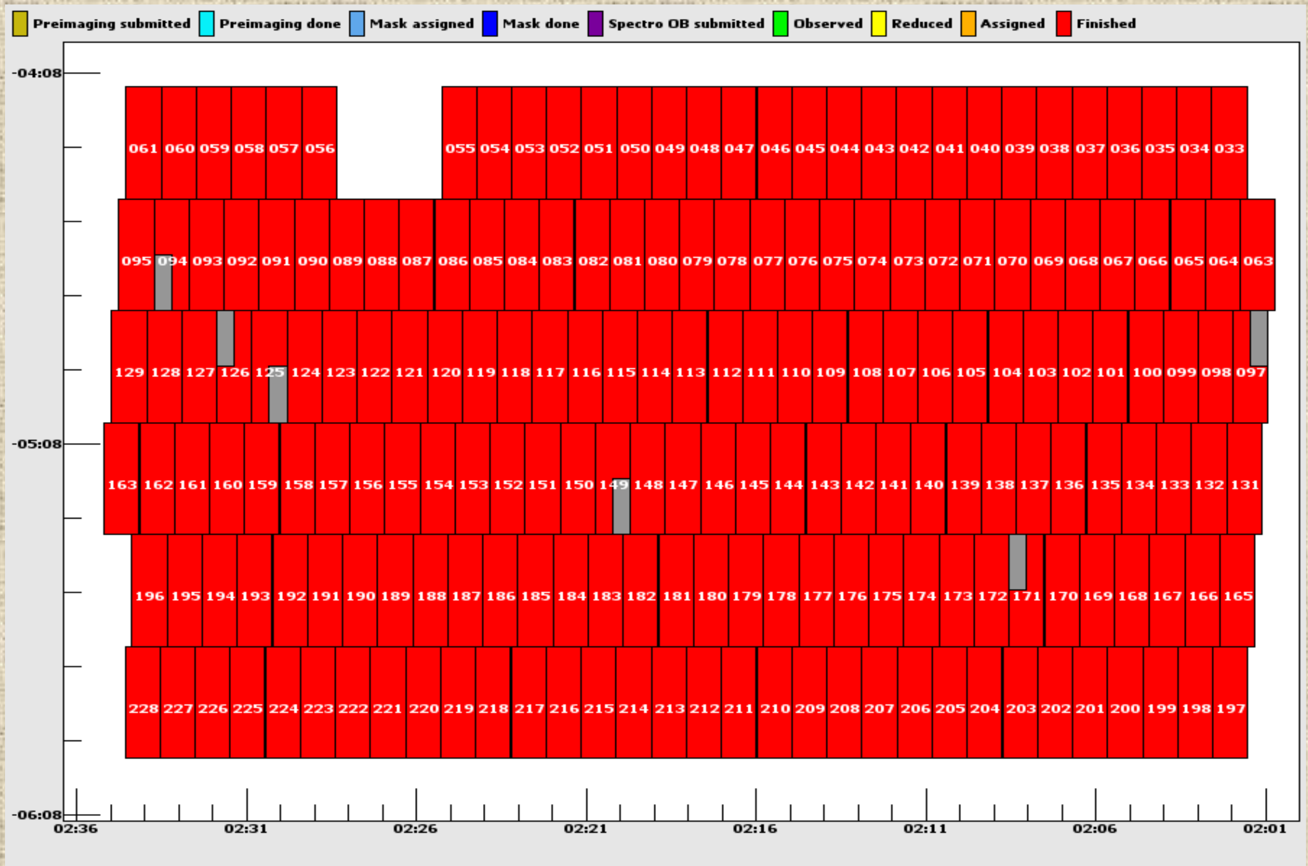


1. Automatic spectral extraction/calibration + redshift measurement: ***EasyLife*** pipeline running at INAF- IASF Milano (Garilli et al. 2012, PASP, 124)
2. Redshift review and validation: ***VIPGI*** (Scodeggio et al. 2005, PASP, 117) ***&*** ***EZ*** (Garilli et al. 2010, PASP, 122)

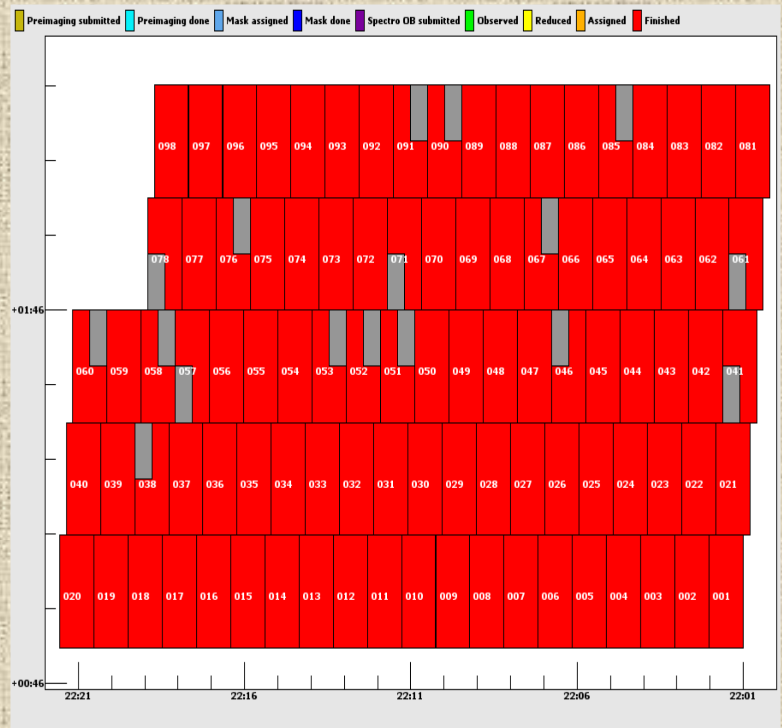


# Sky coverage today: VIPERS is finished!

## W1




## W4



# VIPERS Status

- **Survey completed in January 2015; all data now reduced and validated: internal final (V6.0) catalogue available to team:**

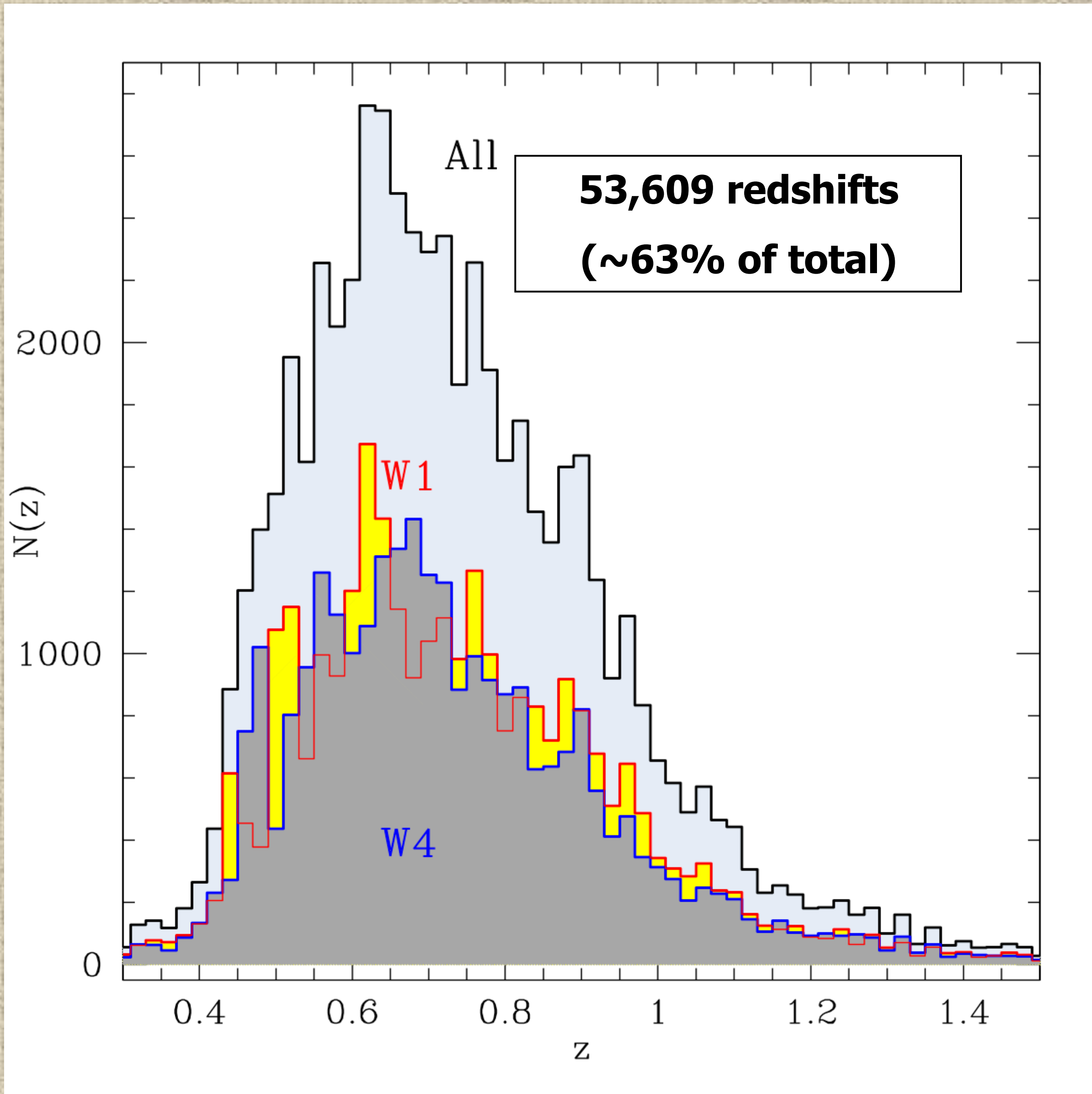
**SURVEY STATUS AS OF 14/05/2015**

| EFFECTIVE TARGETS | MEASURED REDSHIFTS | STELLAR CONTAMINATION | COVERED AREA |
|---|---|---|---|
| 93252 | 88901 | 2265 (2.5 %) | 100.0 % |

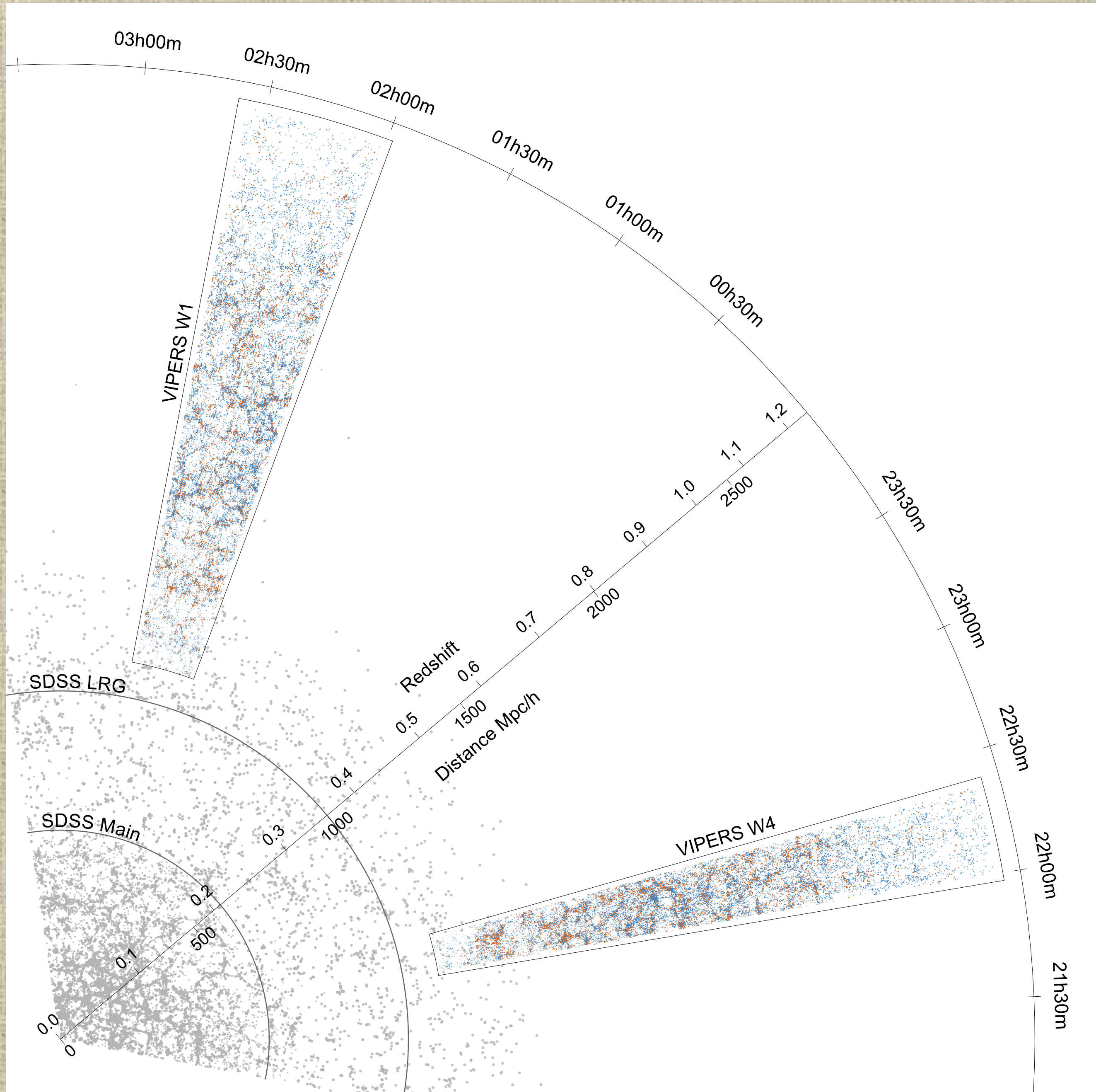
**EFFECTIVE TARGETS (ET) are all the primary targeted objects with the exclusion of the ones flagged as -10 (undetected). MEASURED REDSHIFTS (MR) are the fraction of ET for which a redshift has been measured. STELLAR CONTAMINATION are the MR objects which have been identified as stars.**

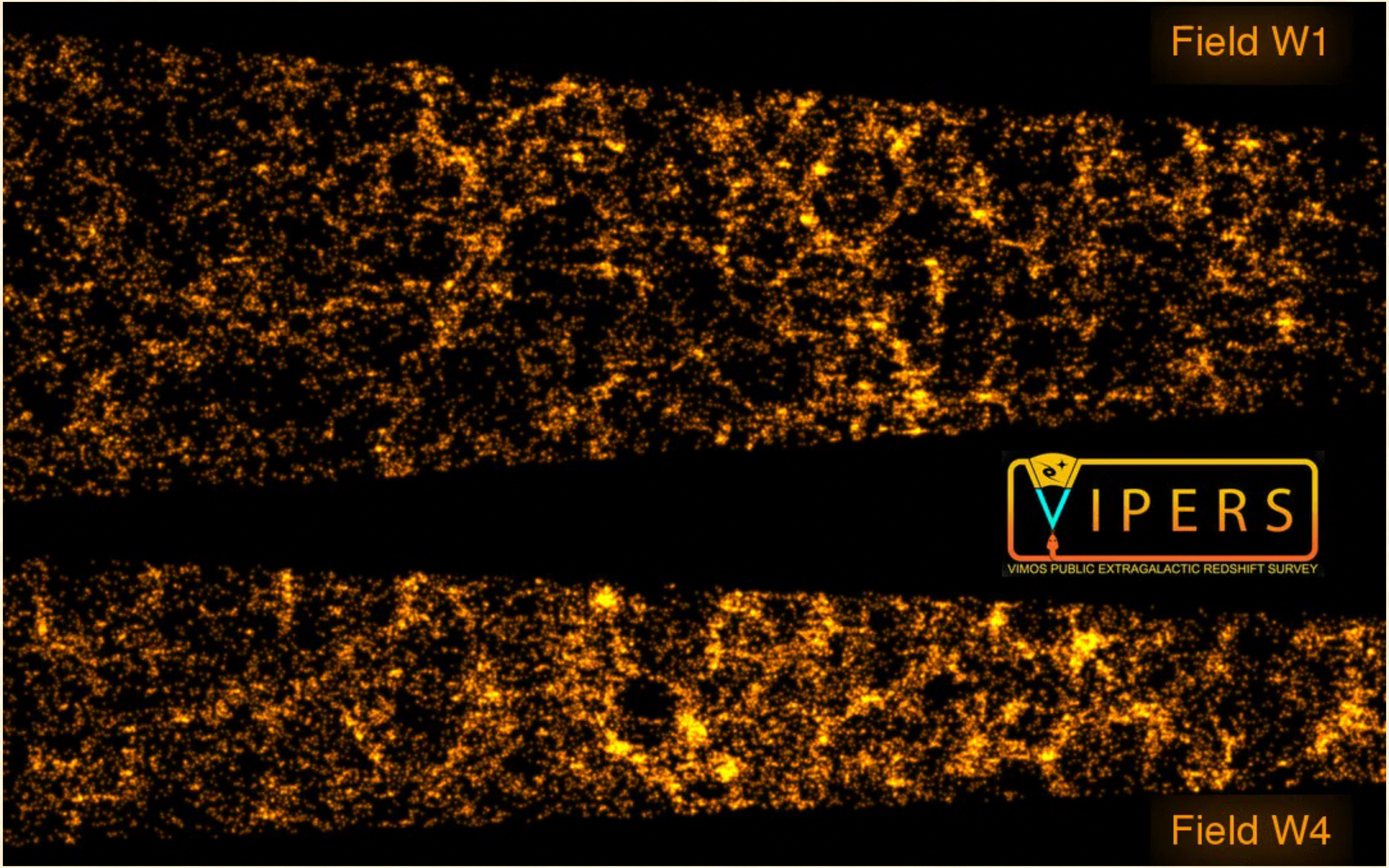
- **Summer 2016: public release of full data set**

# PDR-1 redshift distribution

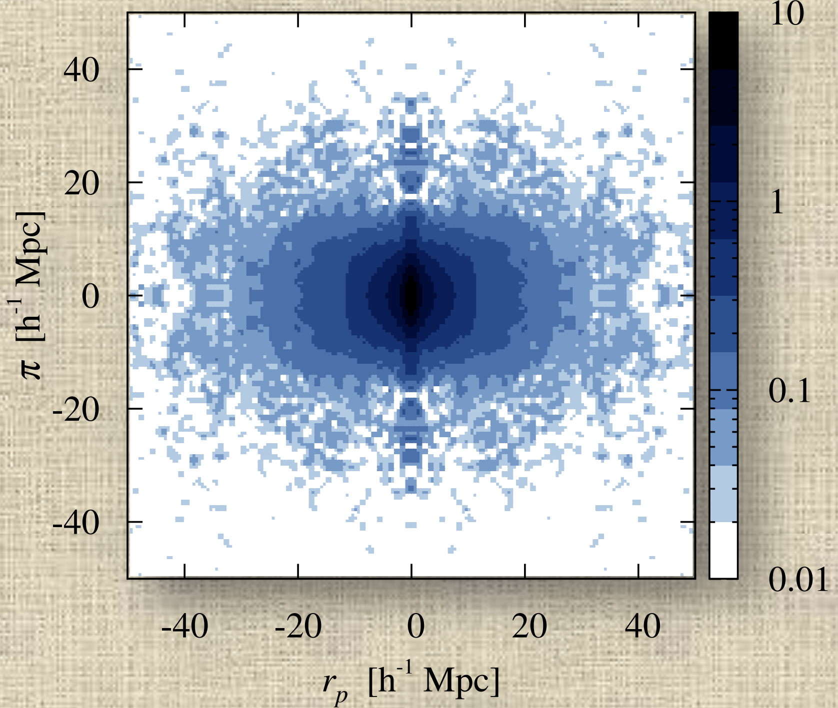


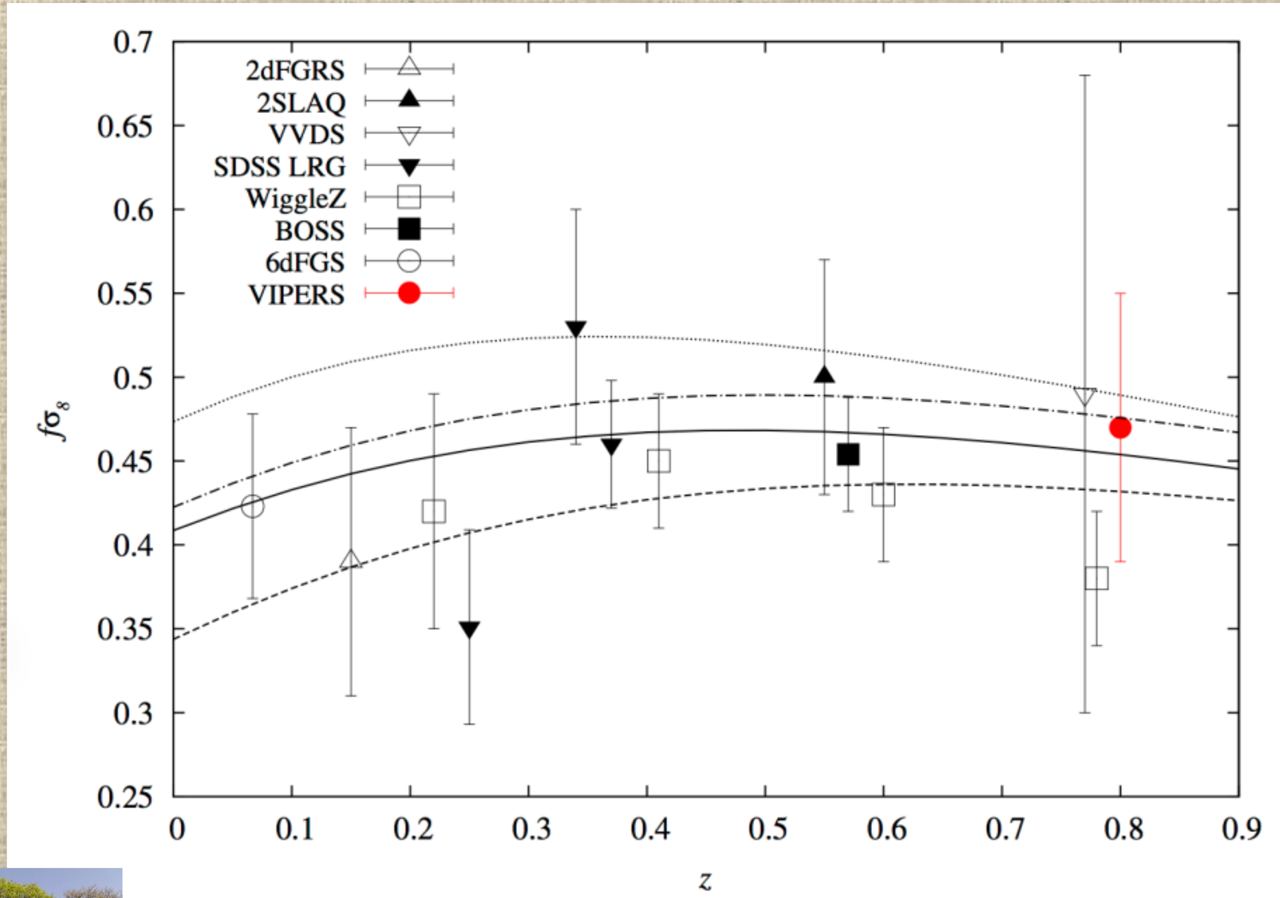

**(Guzzo et al. 2014)**


03h00m
02h30m
02h00m
01h30m
01h00m
00h30m
23h30m
23h00m
22h30m
22h00m
21h30m
VIPERS W1
VIPERS W4
SDSS LRG
SDSS Main
Redshift
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
Distance Mpc/h
0
500
1000
1500
2000
2500

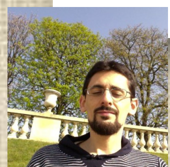

Field W1
VIPERS
VIMOS PUBLIC EXTRAGALACTIC REDSHIFT SURVEY
Field W4

# Redshift-space clustering and growth rate of structure from the PDR-1



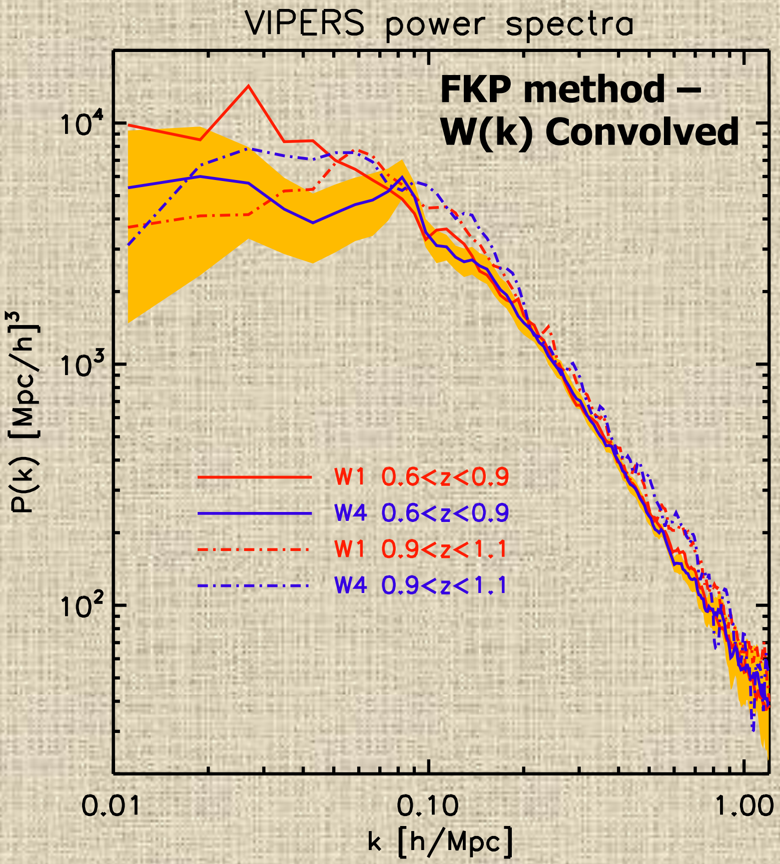



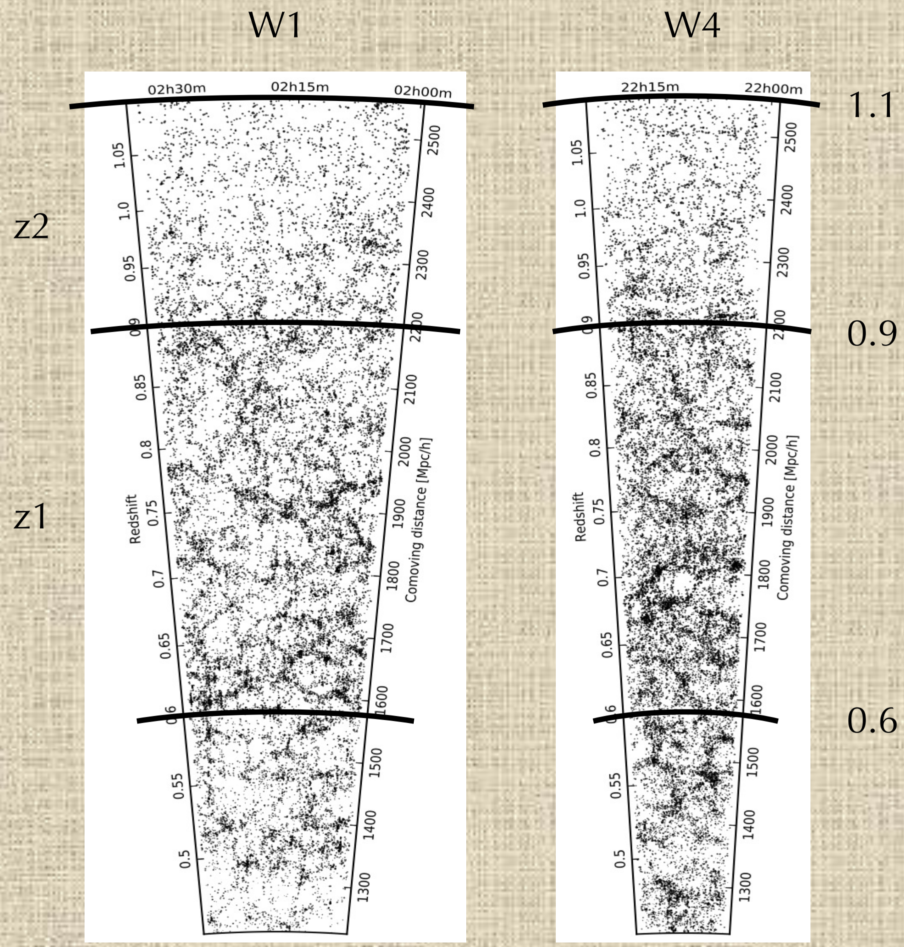

VIPERS: $f\sigma_8(z=0.8) = 0.47 \pm 0.08$



De la Torre et al. 2013

# The power spectrum of the galaxy distribution at z=0.5-1.1 from VIPERS (S. Rota PhD work)



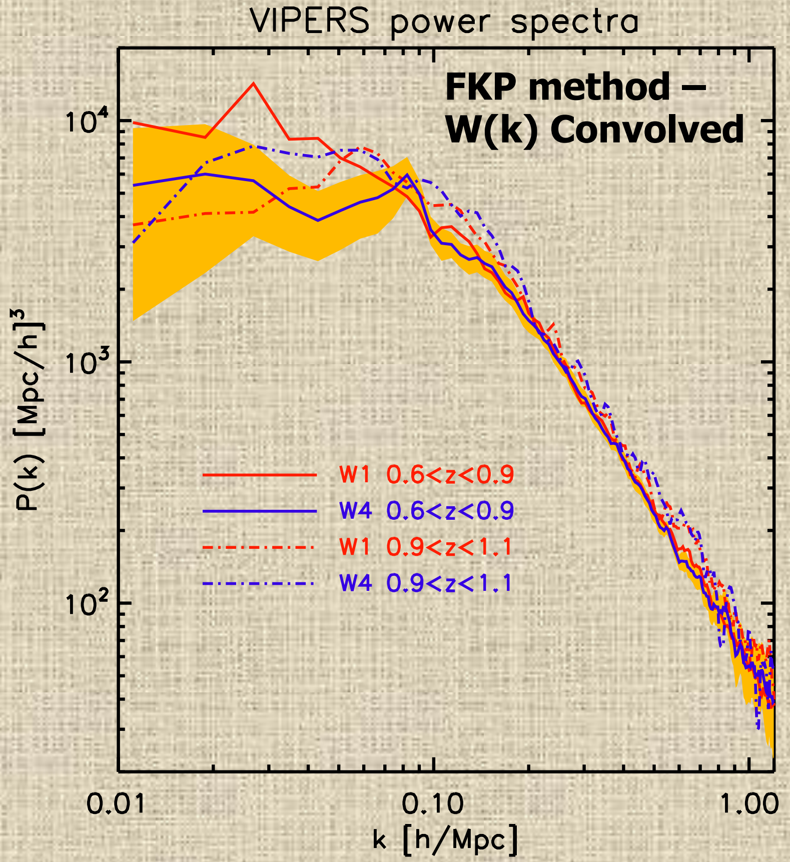



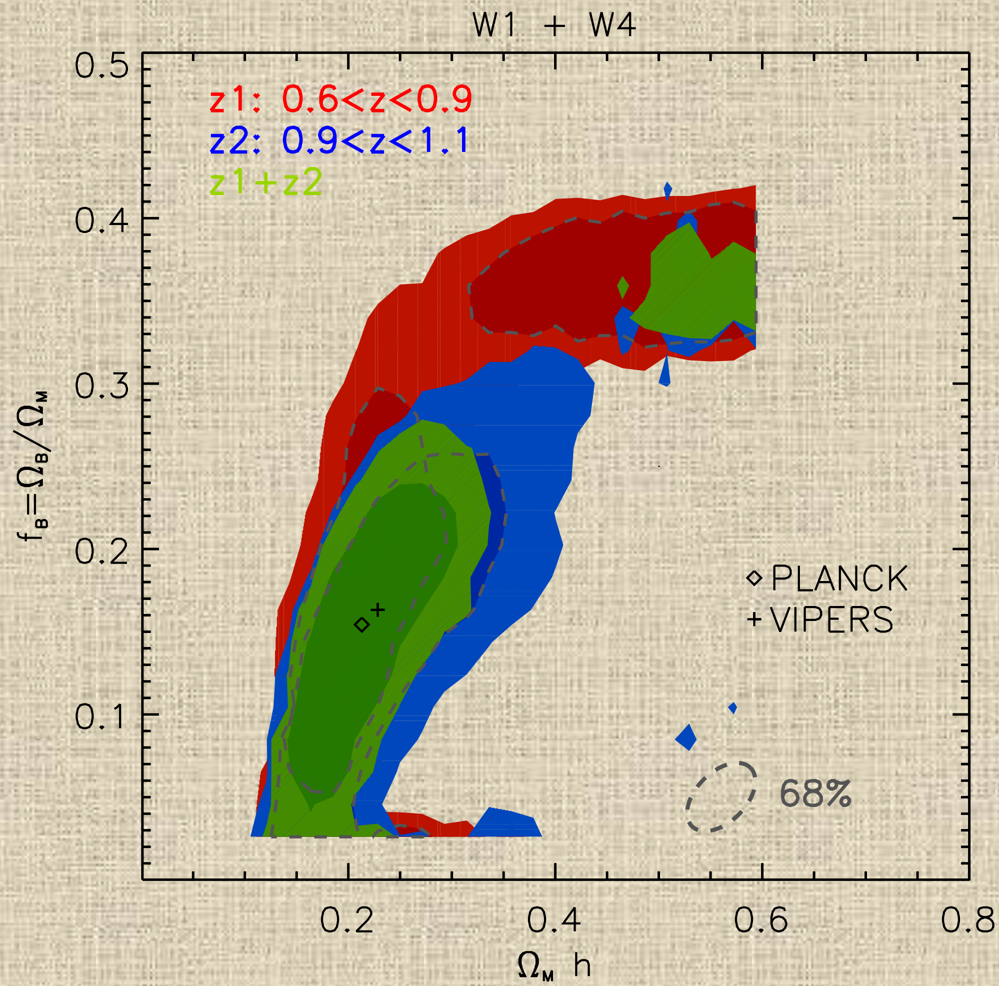

- Very careful treatment of window function

**(Rota, Bel, Granett, LG & VIPERS Team, to be submitted)**

- 4 independent estimates: 2 z bins in 2 independent fields (W1 and W4)

# The power spectrum of the galaxy distribution at z=0.5-1.1 from VIPERS (S. Rota PhD work)



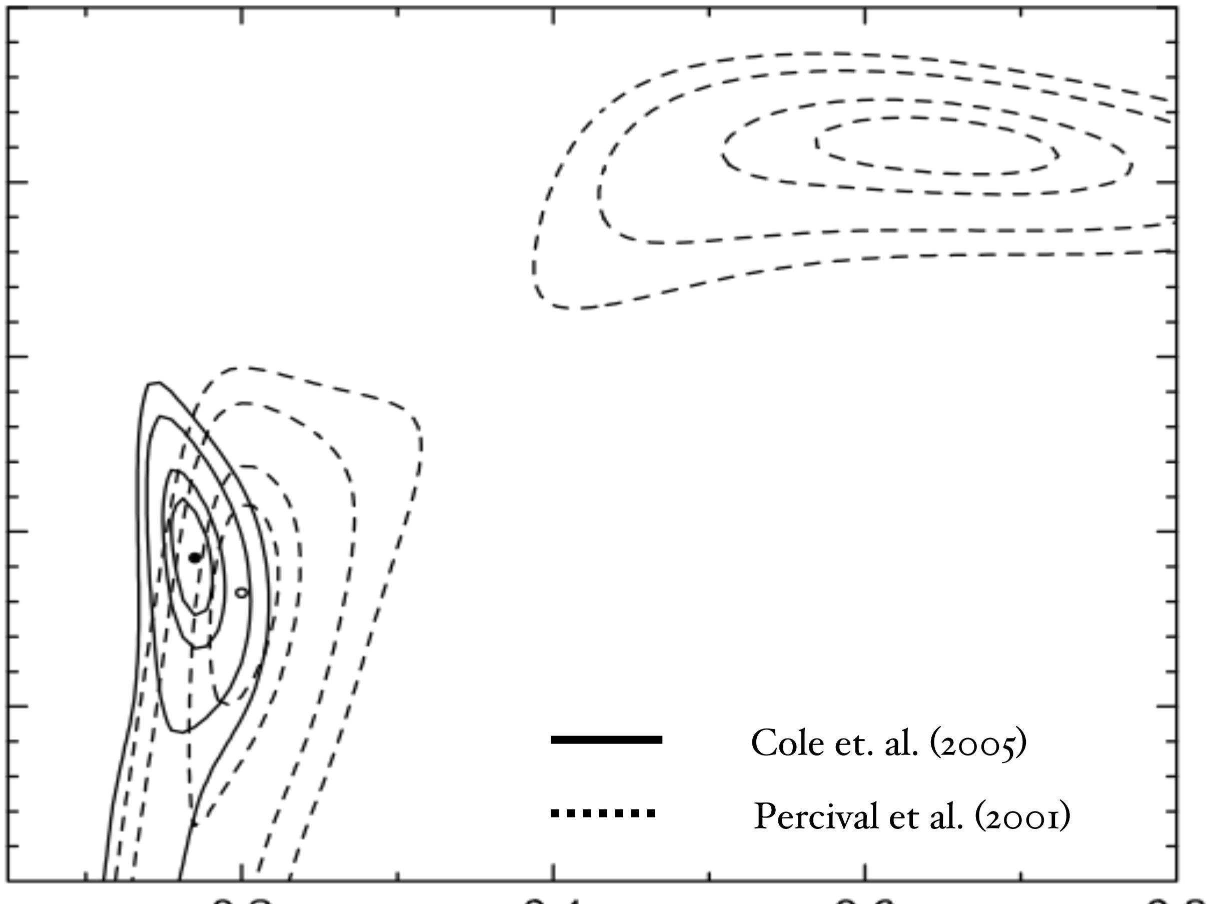




- Very careful treatment of window function

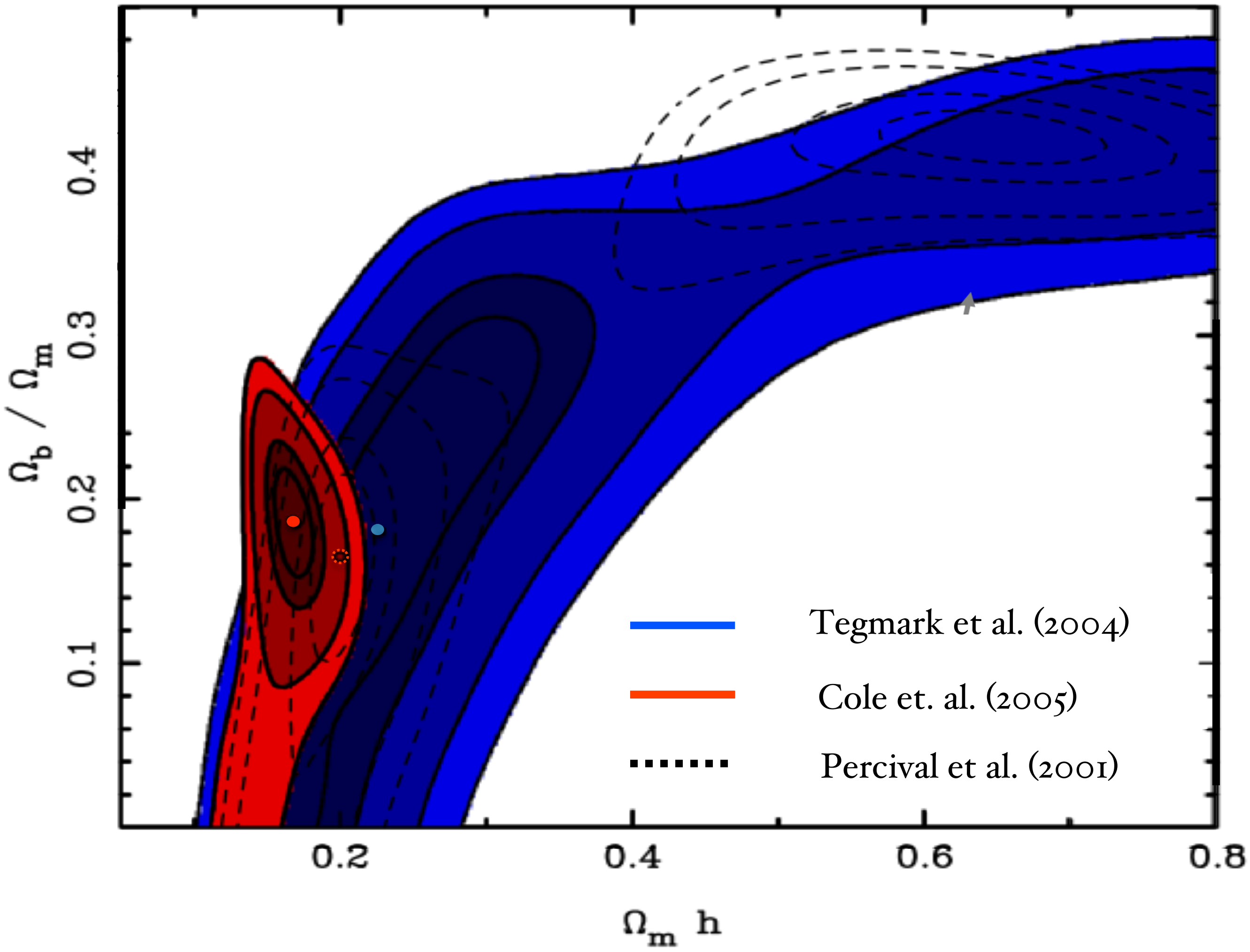
**(Rota, Bel, Granett, LG & VIPERS Team, to be submitted)**

- 4 independent estimates: 2 z bins in 2 independent fields (W1 and W4)

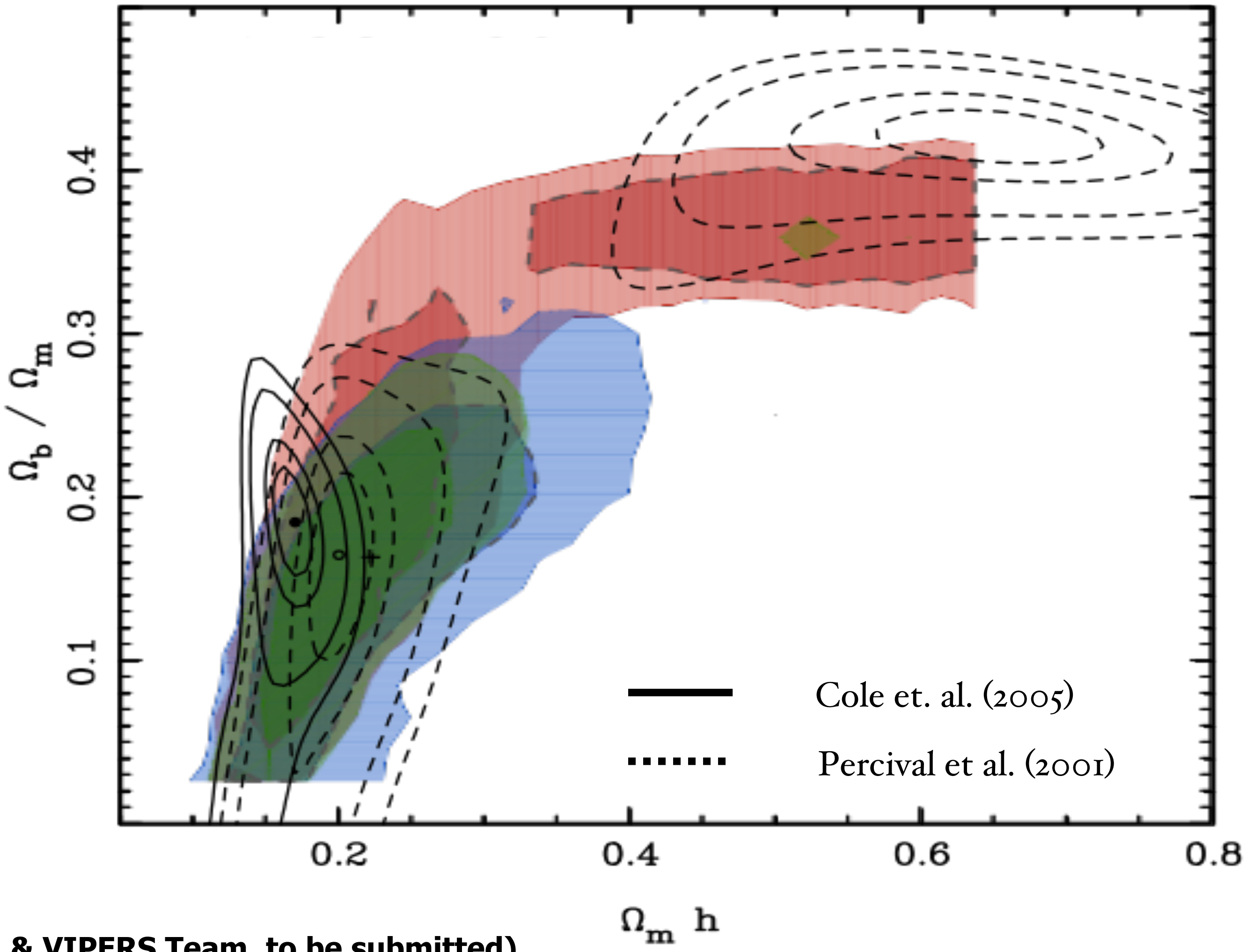
# Comparison to z~0, 2dFGRS



# Comparison to z~0, 2dFGRS vs SDSS

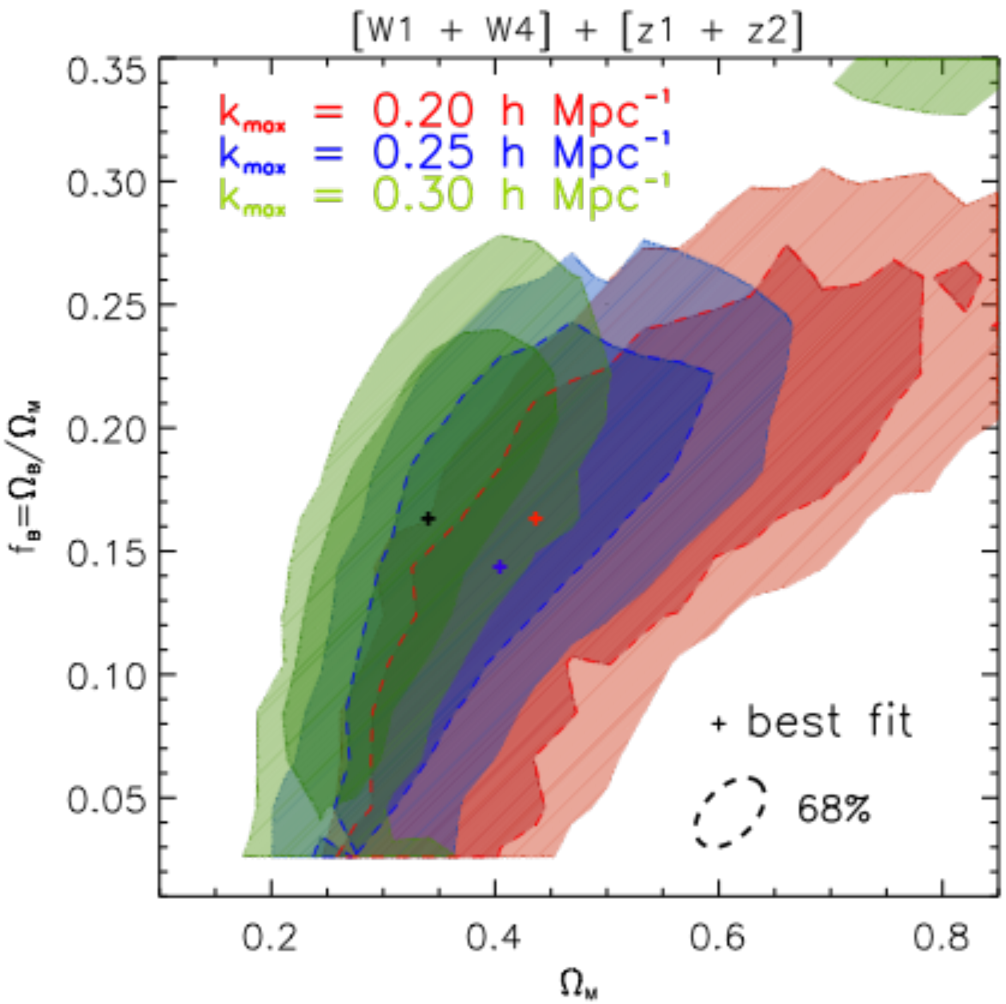


# Comparison to z~0, VIPERS vs 2dFGRS




**(Rota, Bel, Granett, LG & VIPERS Team, to be submitted)**

# Relevance of systematic effects: dependence on $k_{max}$ in the fit

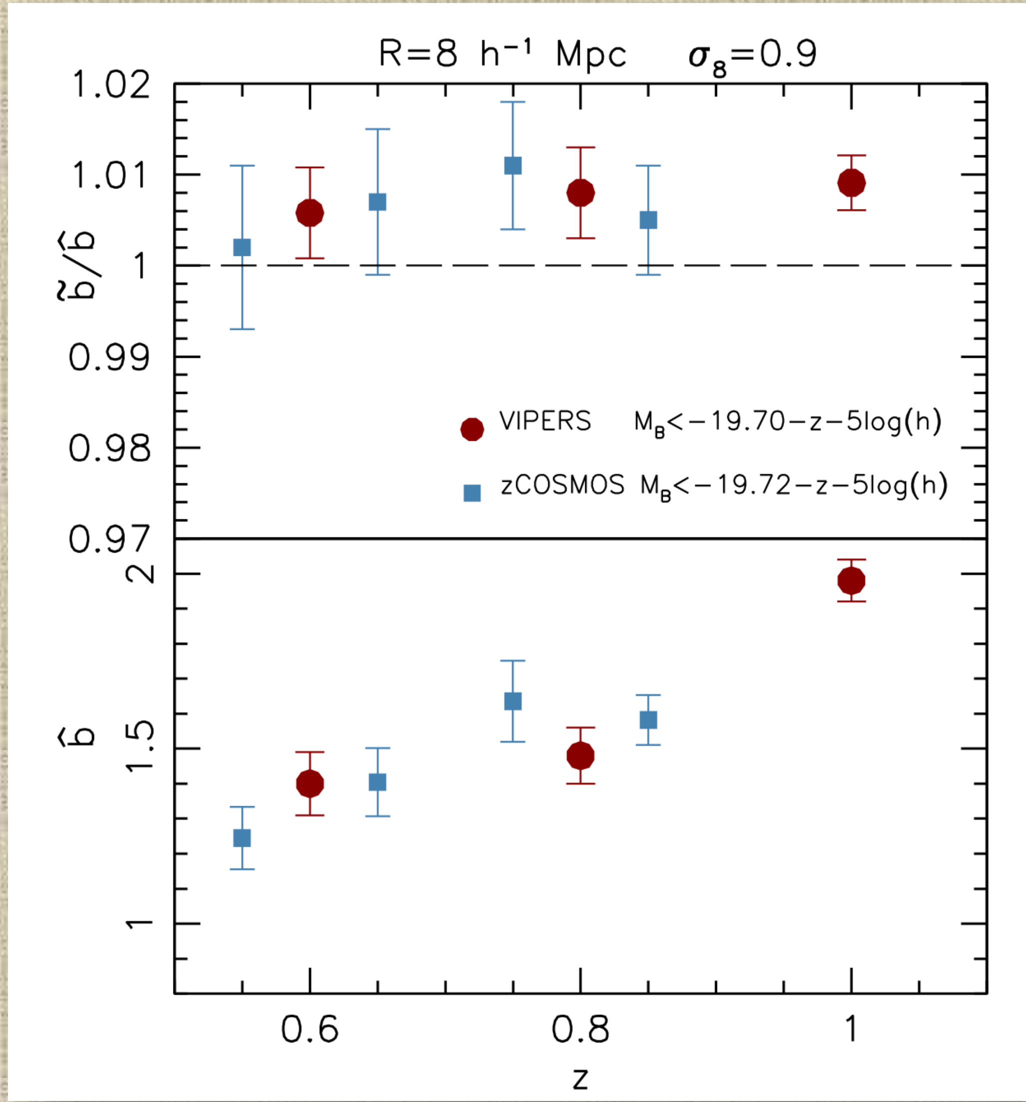



(Higher-z → less non-linearity → push to higher $k_{max}$)

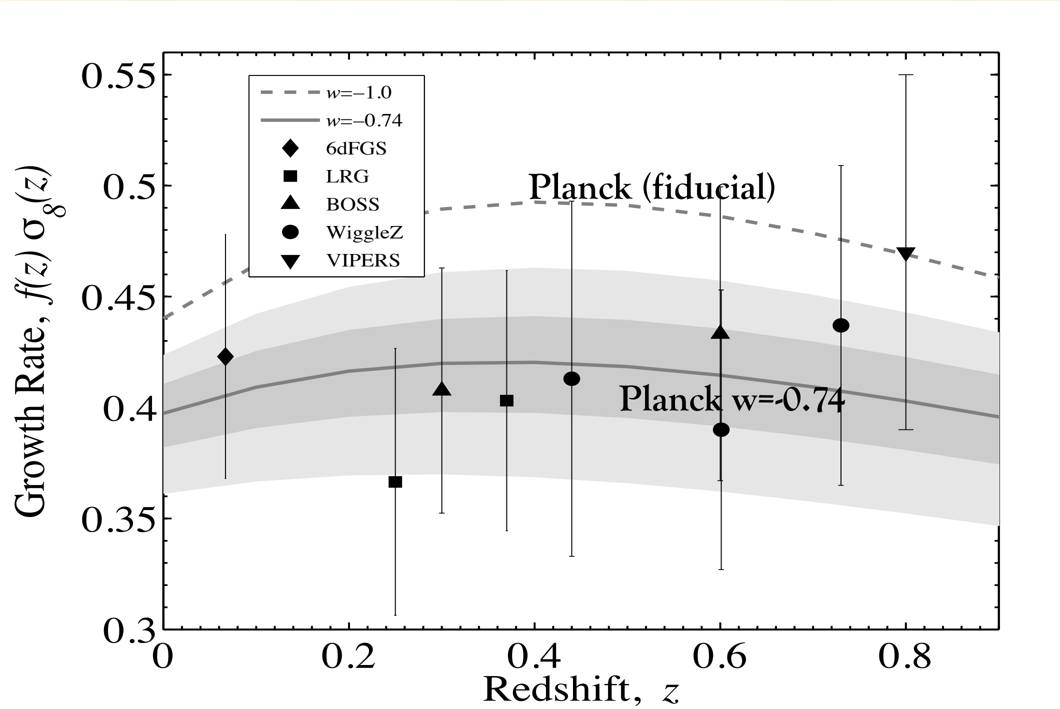
# Non-linearity of galaxy bias and its evolution

Using Sigad, Branchini & Dekel (2000) inversion technique

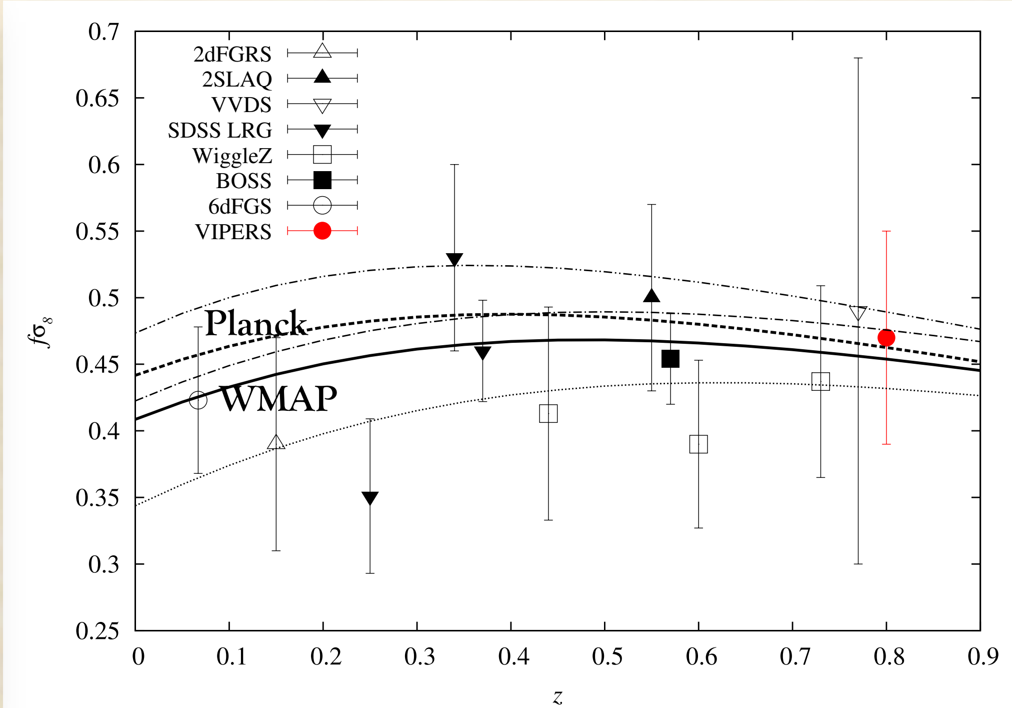
**(Di Porto, Branchini & VIPERS Team 2014)**



# Is there a real tension of current constraints on $f\sigma_8$ with GR+Planck predictions?




(Macaulay et al. 2013)

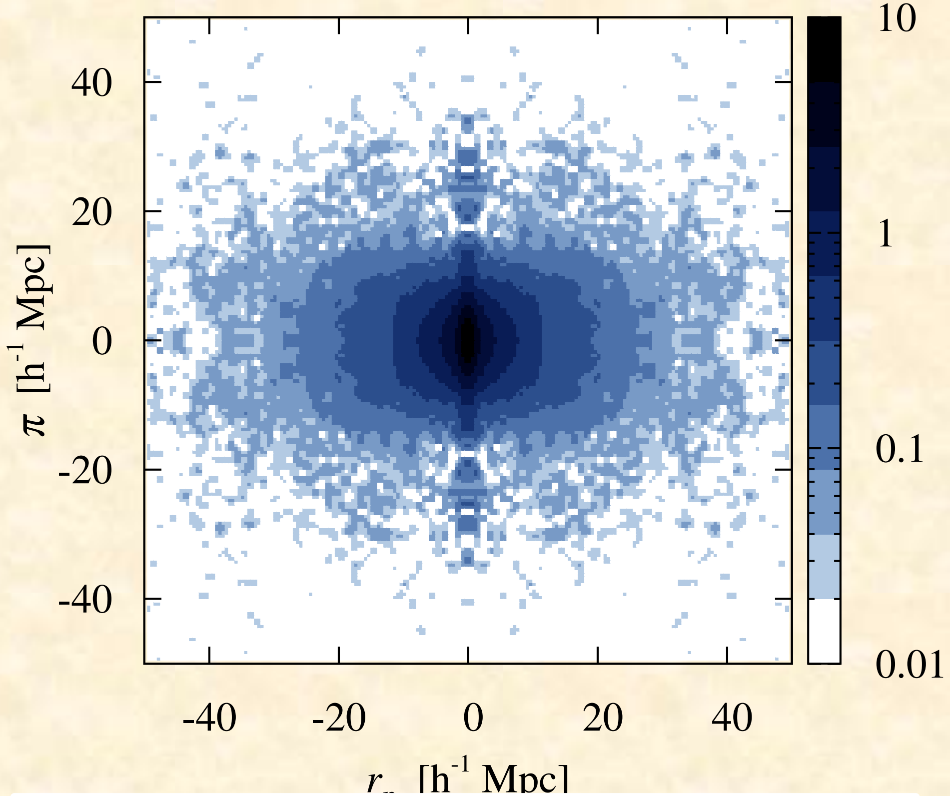



(de la Torre & VIPERS 2013)

(see also Salvatelli et al. 2014)

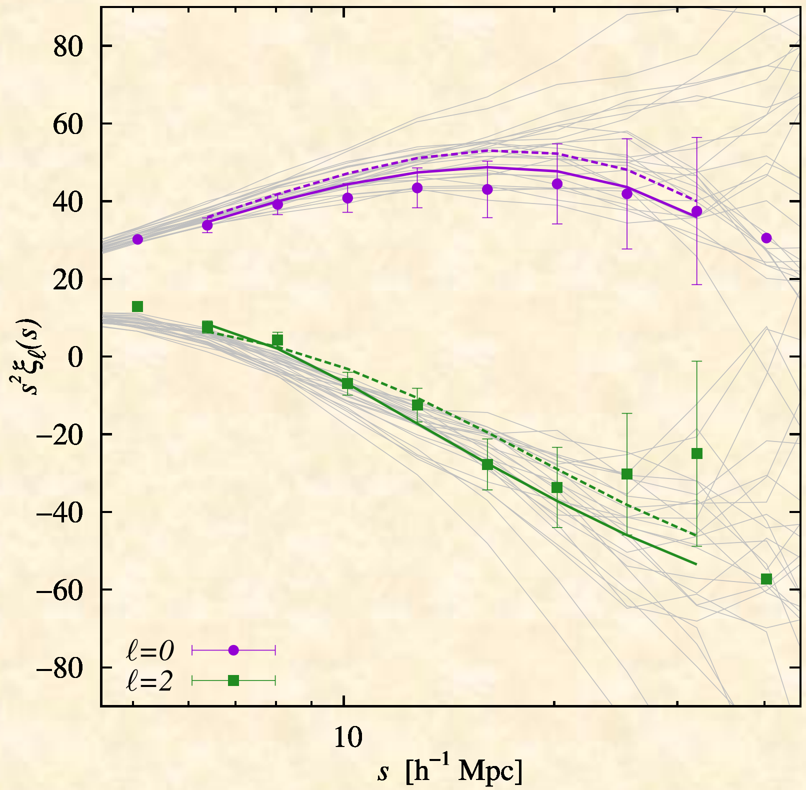
# Measuring RSD: how this is done in detail

A. **Fit the full 2D correlation function**, expressed as combination of spherical armonics (moments)




**Pros:** highly non-linear scales where FoG dominates more cleanly removed
**Cons:** lots of d.o.f. → covariance matrix estimation more difficult

B. **Fit single multipoles**

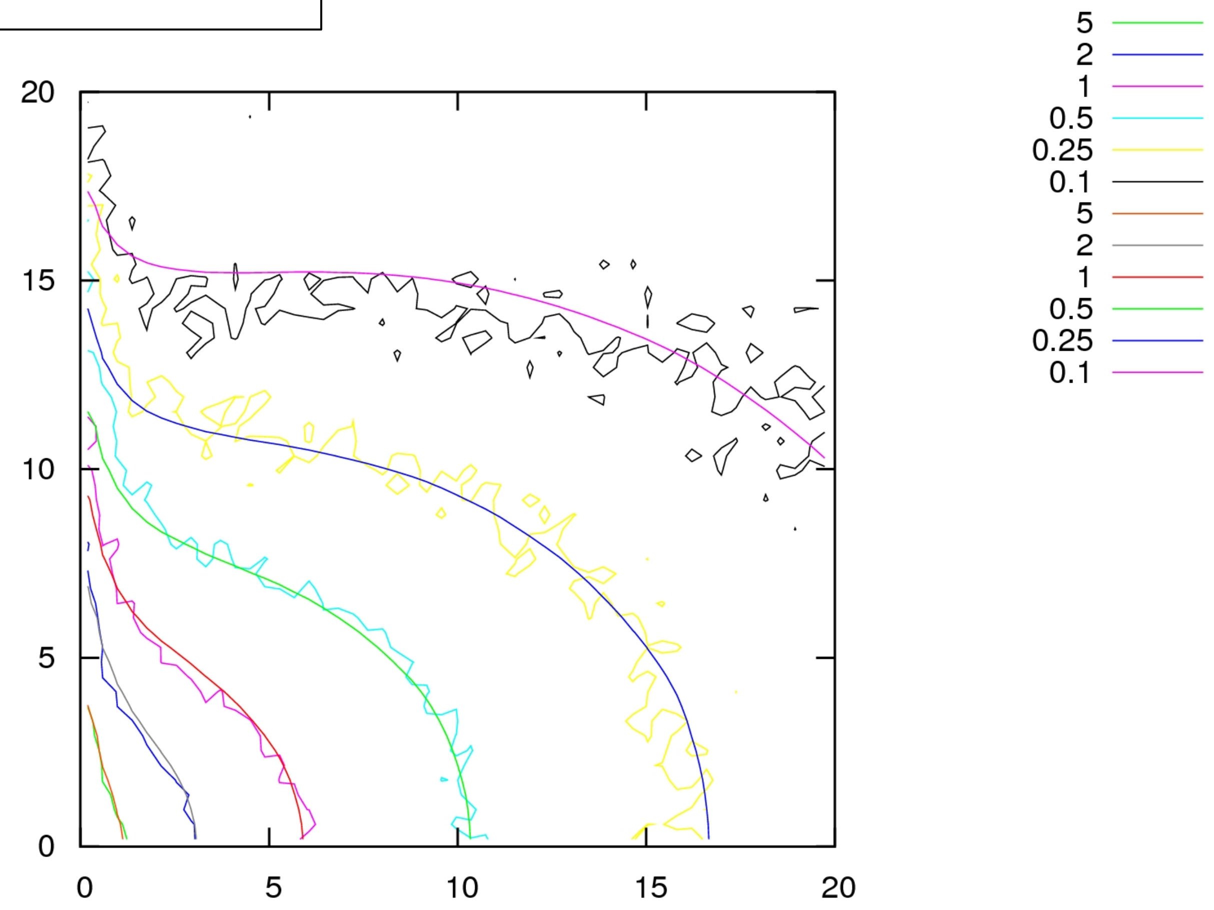



**Pros:** compress the information → easier to estimate covariance matrix
**Cons:** uncertainties in modelling small-scale non-linearity (FoG) affect all scales

# Kaiser/Hamilton linear redshift-distortion model + correction

$$P(k_{||}, k_{\perp}) = P(k)\left(1+\beta\mu^2\right)^2 D(k\mu\sigma_p).$$

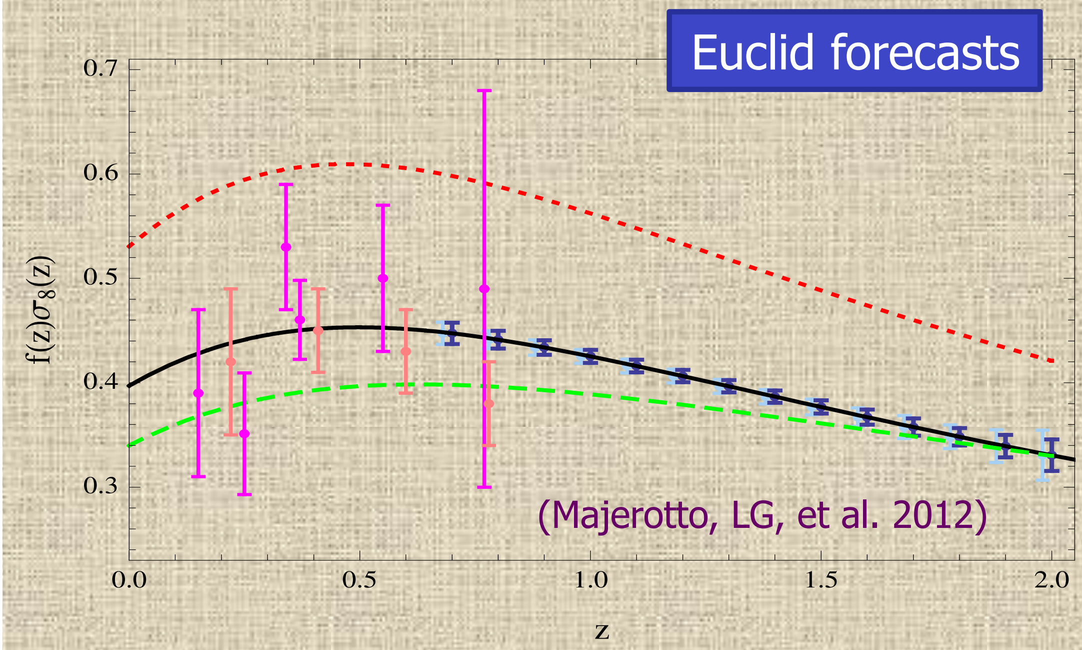
$$D(k\mu\sigma_p) = \frac{1}{1+(k\mu\sigma_p)^2/2}$$



# Systematic effects on Redshift-Space Distortions…

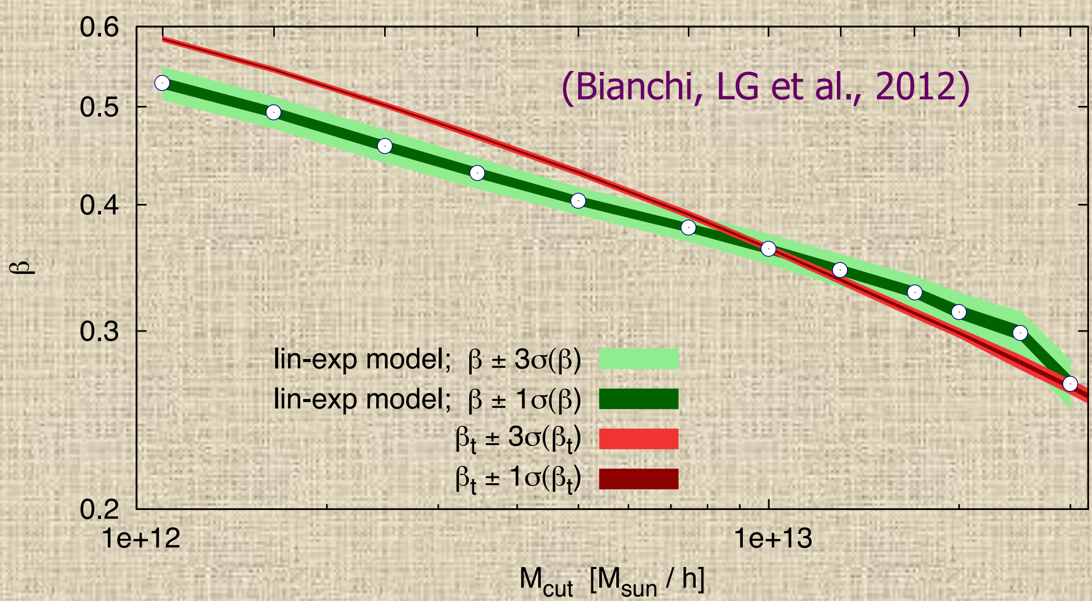
Need to improve modelling to enter *"precision RSD era"*

→ e.g. EUCLID: 1-3% precision on $f\sigma_8$




→ A lot of modelling work ongoing (Scoccimarro, Taruya+, Kwan+, Reid+, Samushia+, Seljak+, Bianchi+, Kopp+, …

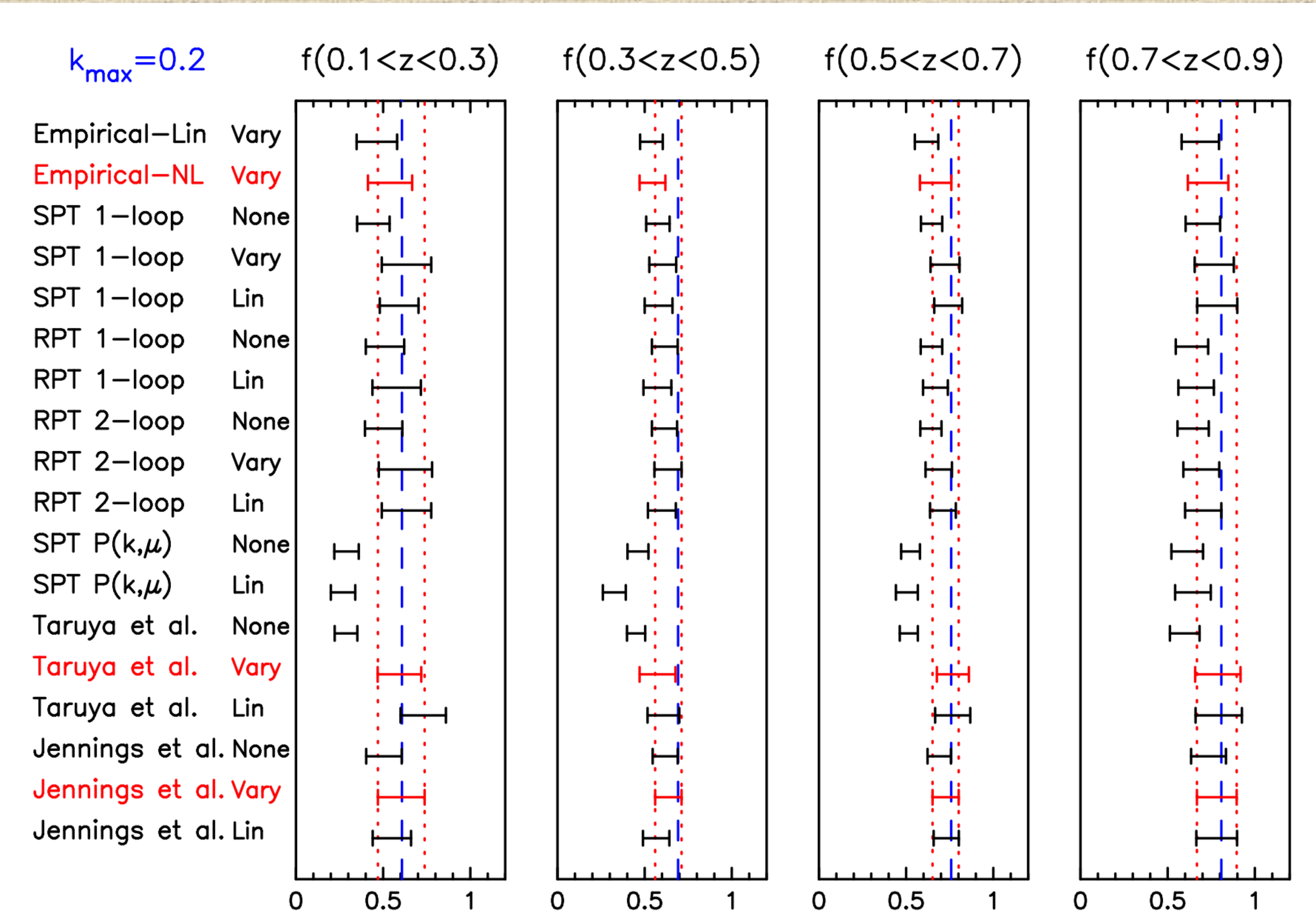
→ "Standard" RSD dispersion model: up to 10% systematic error




(also Okumura & Jing, 2011)

# Reducing systematics: better RSD models?

Blake et al. (2011) → Test of various models on WiggleZ data



# Better RSD models: understand pairwise f(v)

**D. Bianchi (now @ICG Portsmouth) PhD work – Bianchi, Chiesa & LG, 2014, MNRAS 446, 75**

- Goal: reduce degrees of freedom on description of the pairwise velocity PDF in the context of the ***streaming model***

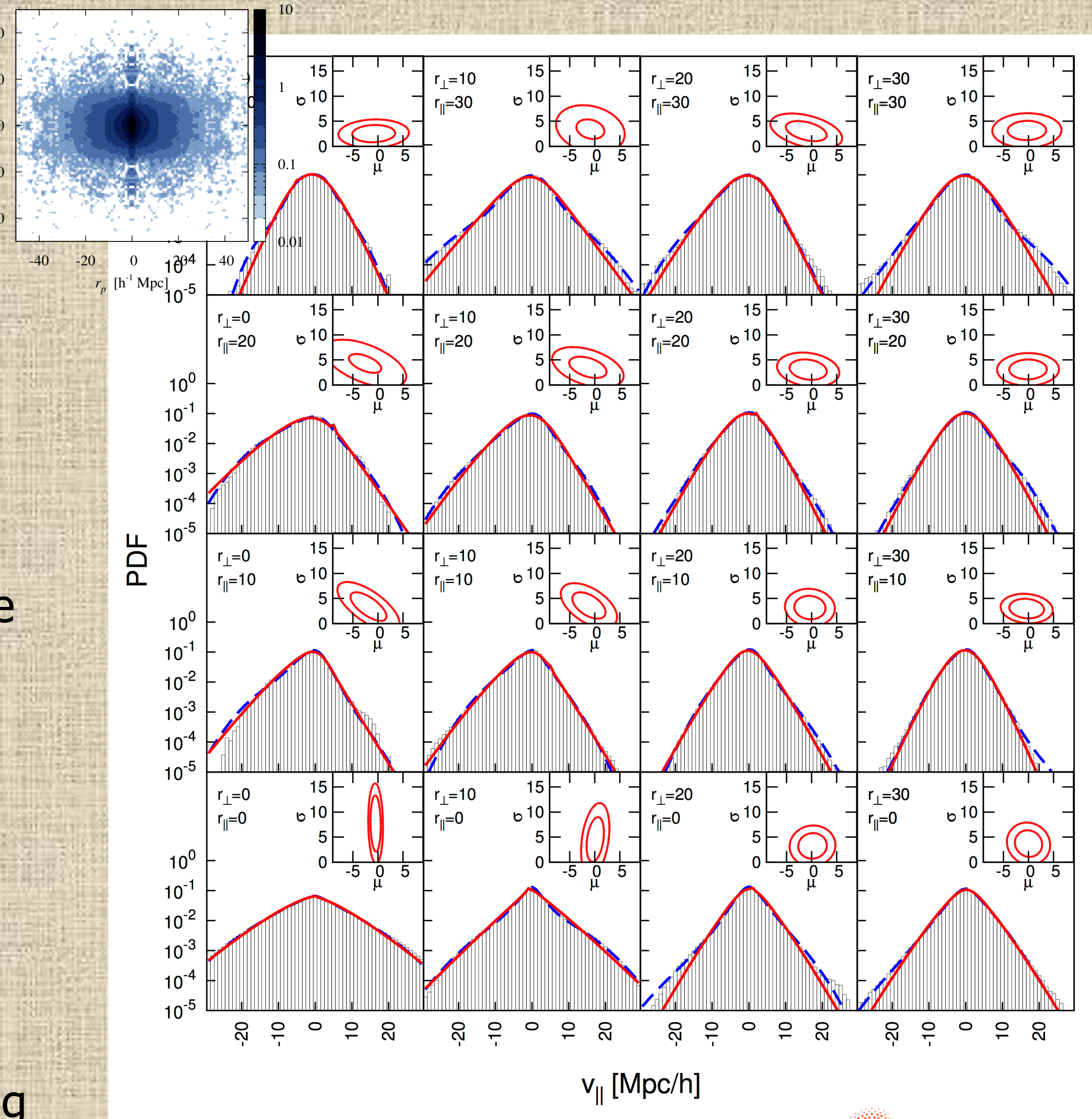
$$1 + \xi_S(s_\perp, s_\parallel) = \int dr_\parallel \; [1 + \xi_R(r)] \; \mathcal{P}(r_\parallel - s_\parallel | \mathbf{r})$$

- PDF described as weighted sum of Gaussians, whose mean and dispersion are described in turn by bivariate Gaussian

$$\mathcal{P}(v_\parallel) = \int d\mu d\sigma \; \mathcal{P}_L(v_\parallel | \mu, \sigma) \; \mathcal{F}(\mu, \sigma)$$

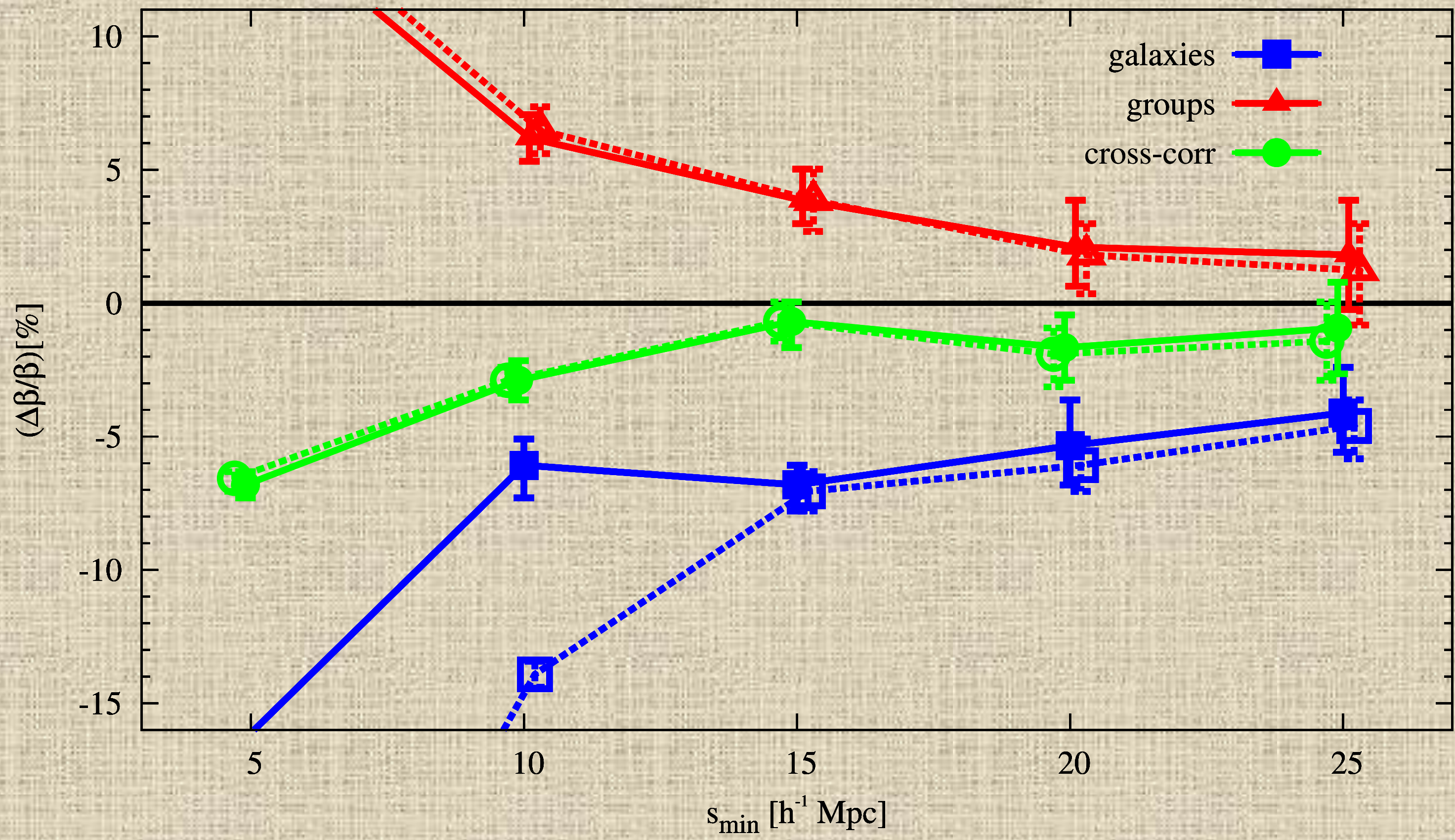
- Works extremely well: naturally provides exponential/Gaussian/skewed PDFs, depending on separation

- Uhlemann et al. (2015): development using Edgeworth expansion



# Improving RSD measurements: better tracers of LSS and v

**F. Mohammad** PhD project: **RSD from the group-galaxy cross-correlation** (Mohammad, et al., submitted), plus define **customized multipole expansion** ("truncated multipoles") to reduce weight of nonlinear scales



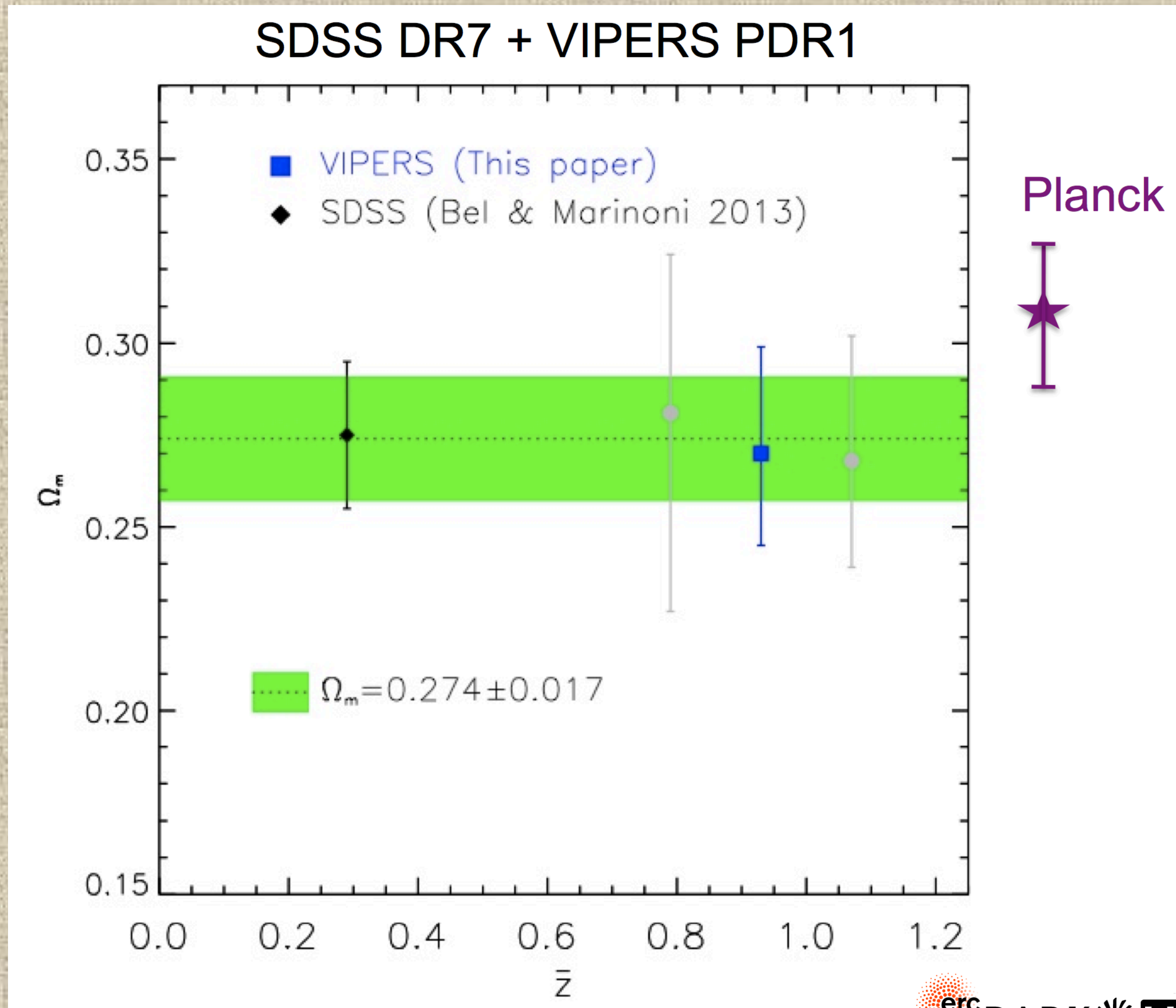
# (3) "Optimized" statistics: the "clustering ratio" from counts in cells (Bel et al.), an implicit probe of P(k) shape

The clustering ratio: $$\eta_R(r) \equiv \frac{\xi_R(r)}{\sigma_R^2}$$

where:

- R=smoothing radius of galaxy field
- r=nR (n=3,4,5) i.e. correlated on larger scales
- Ratio has favourable propertites wrt to quasi-linear/mildly nonlinear effects on the P(k): most of these factor out
- Essentially a ratio of power in two different k bands

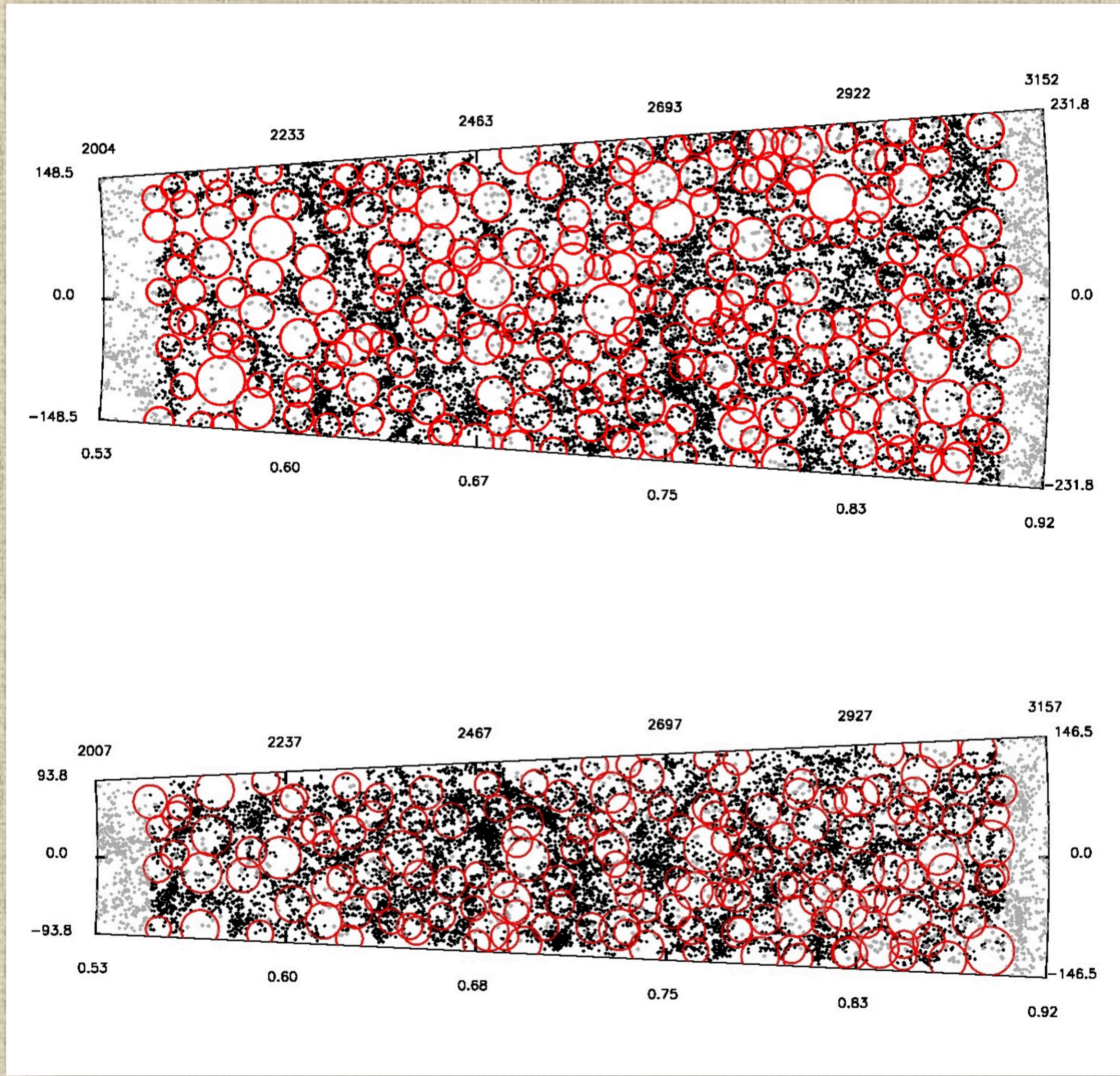
→ **Reduce the effect on P(k) shape of the "Big Three", i.e. nonlinearity, bias and RSD**




Bel et al. 2014, A&A, 563, 37

# Identify new cosmological probes: cosmic voids at z~1

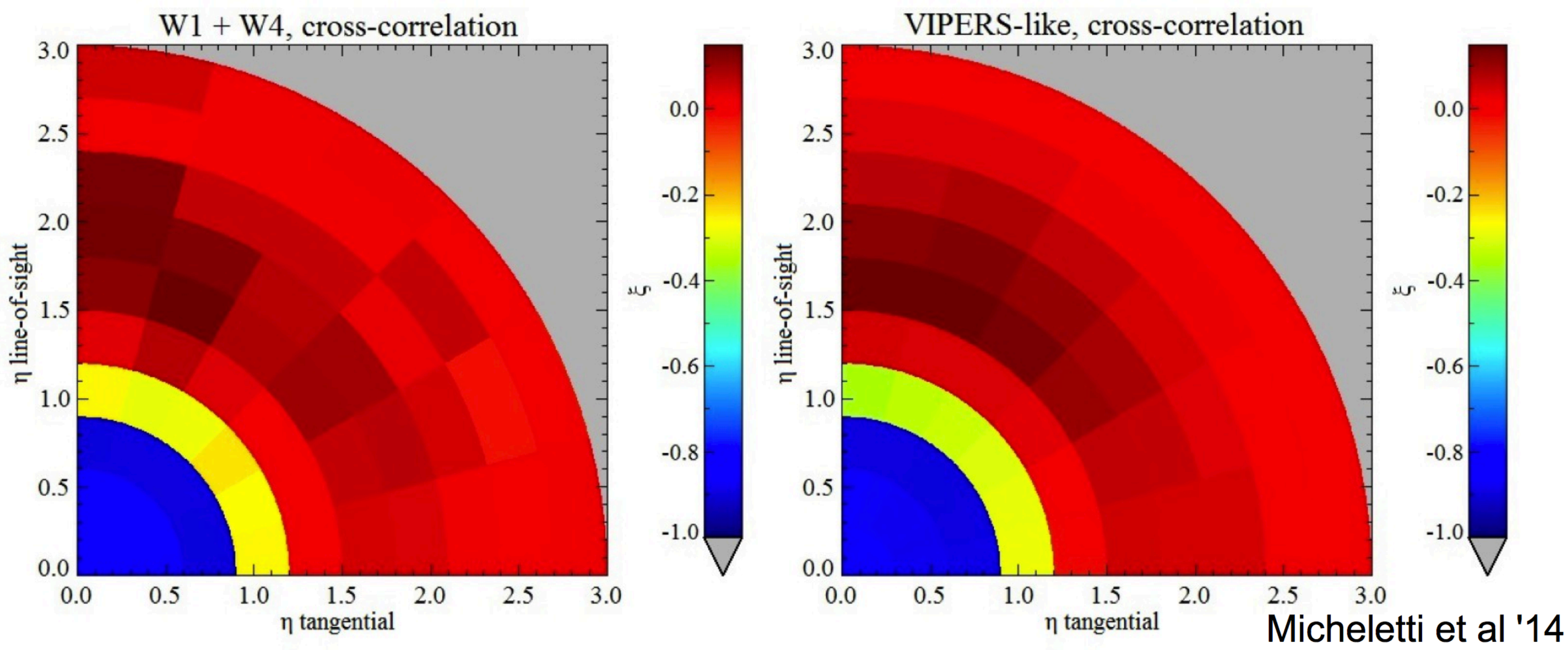
**Micheletti, Iovino, Hawken, Granett & VIPERS team, 2014**




erc DARK LIGHT

# Identify new cosmological probes: cosmic voids at z~1

## The void-galaxy cross correlation function

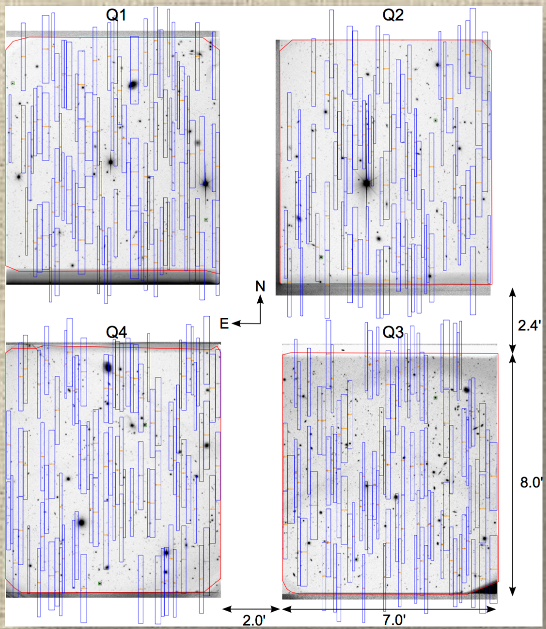


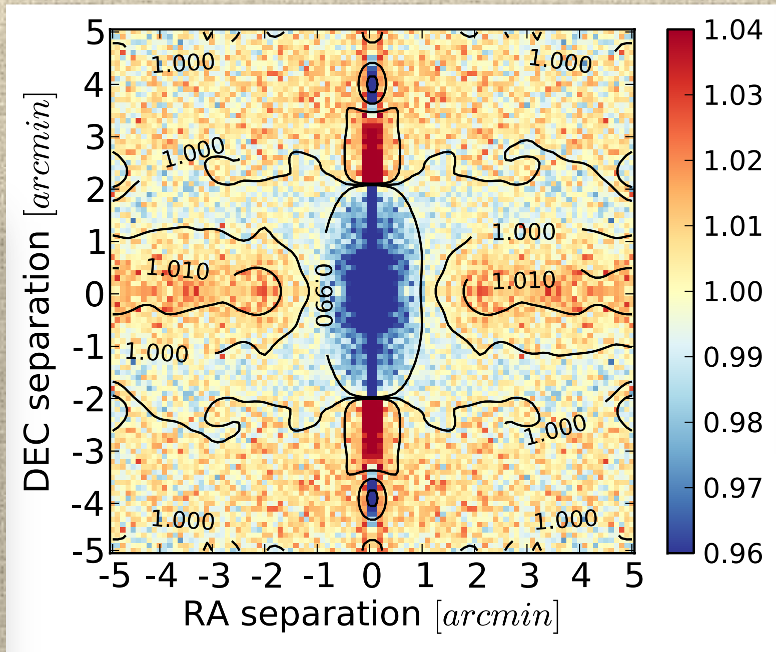

**Modelling the cross-correlation function: A. Hawken et al., in preparation**

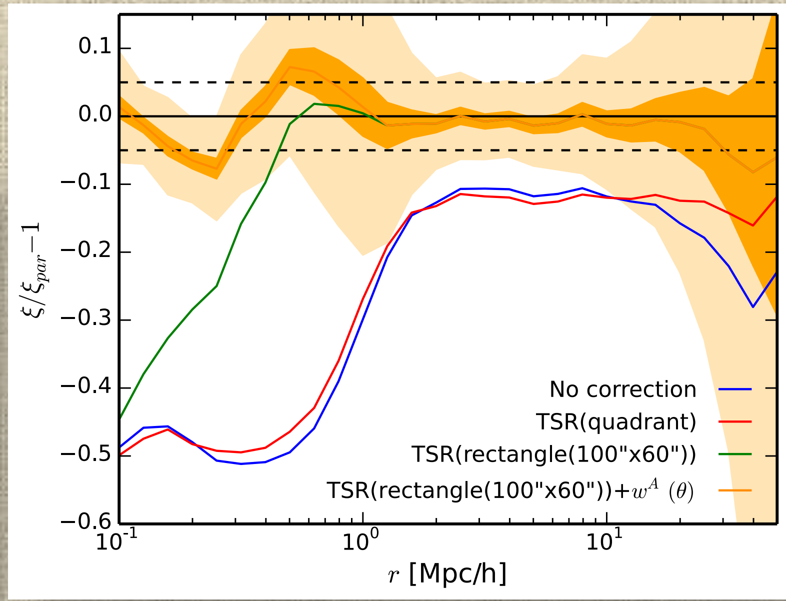
→ **How precise and accurate can this method be?**
→ **Needs highly-samples surveys like GAMA and VIPERS**

# Minimize observational effects (not obvious at 1% level!)

**E.g. detailed correction of masking effects in the VIPERS data on the estimate of two-point correlations (A. Pezzotta PhD work)**



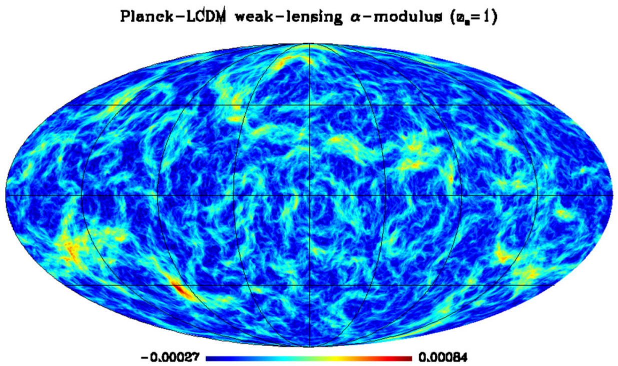



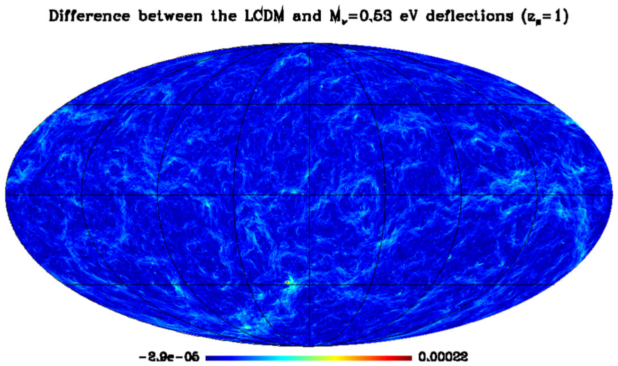

**→ This will be very relevant for Euclid slitless spectroscopic mode**

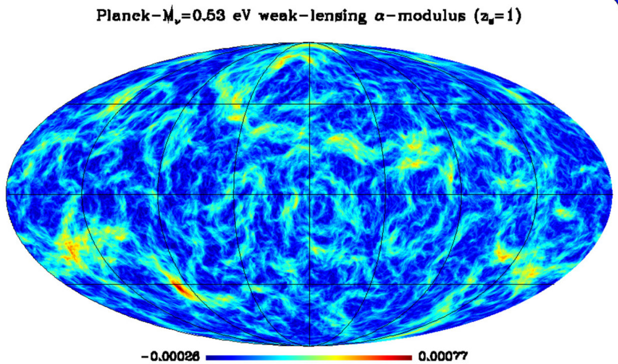
# Account for all existing components: neutrinos!

**Carbone et al., DEMNUni simulations**




**Deflection angle maps for $z_s$=1 (Carbone et al. in prep)**



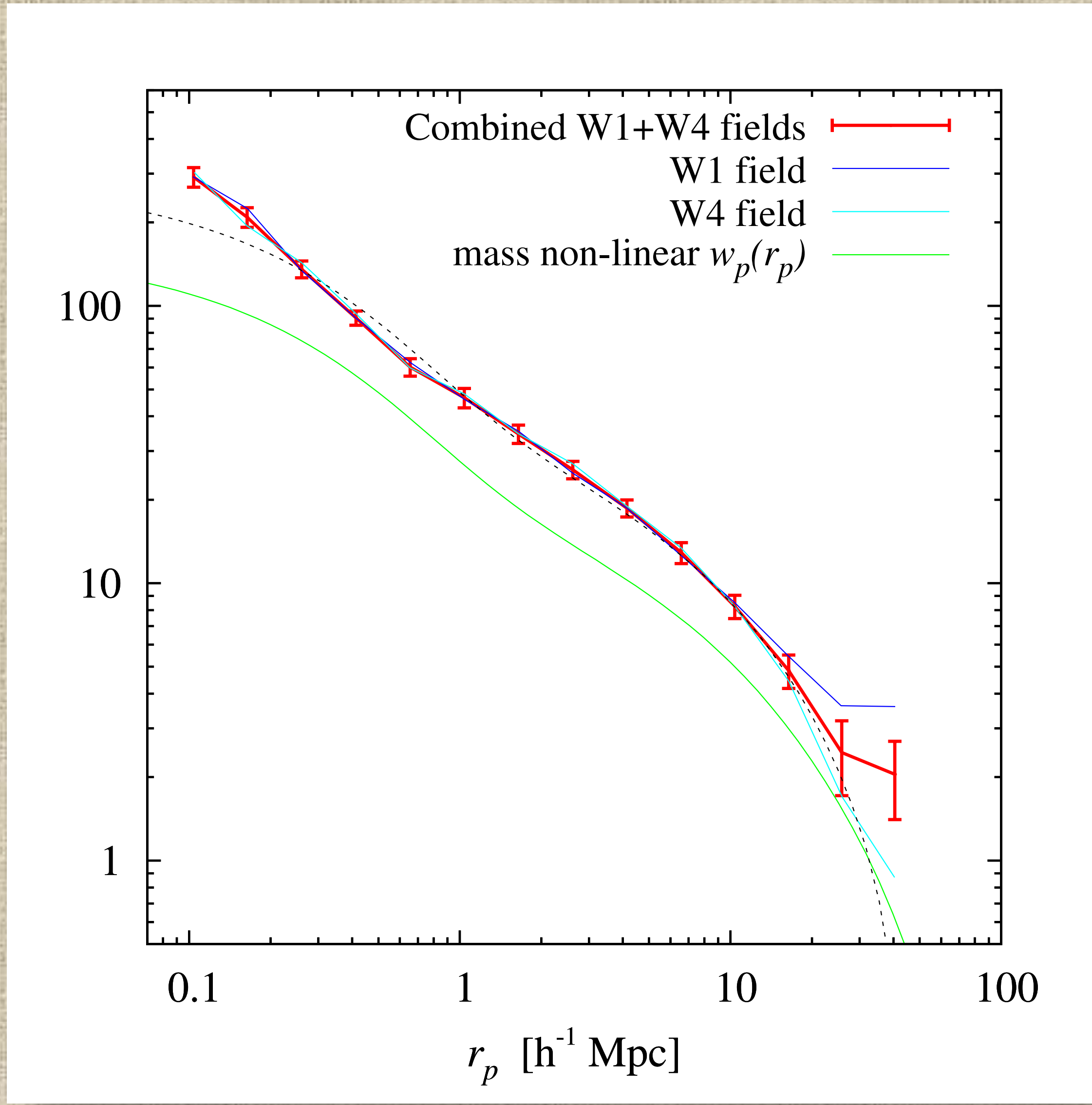




**Ray-tracing across the matter distribution of the DEMNUni simulations: L=2 Gpc/h, $N_{part}$=2x(2048)$^3$ (including massive neutrino particles)**

# Improve understanding relation between DM and baryons

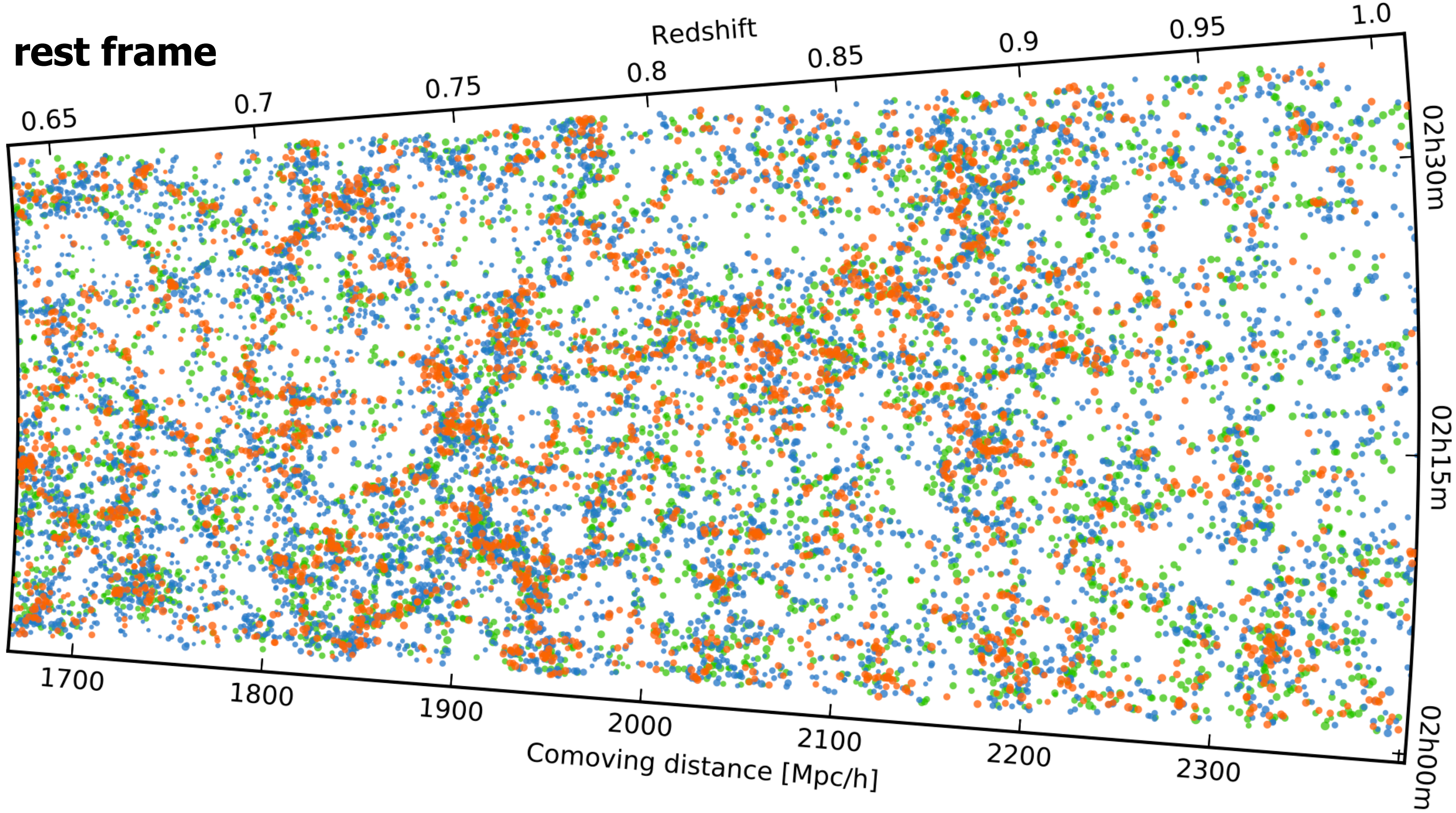
- **Halo Occupation Distribution modelling of VIPERS correlation function**

(De la Torre & VIPERS team 2015, in preparation)



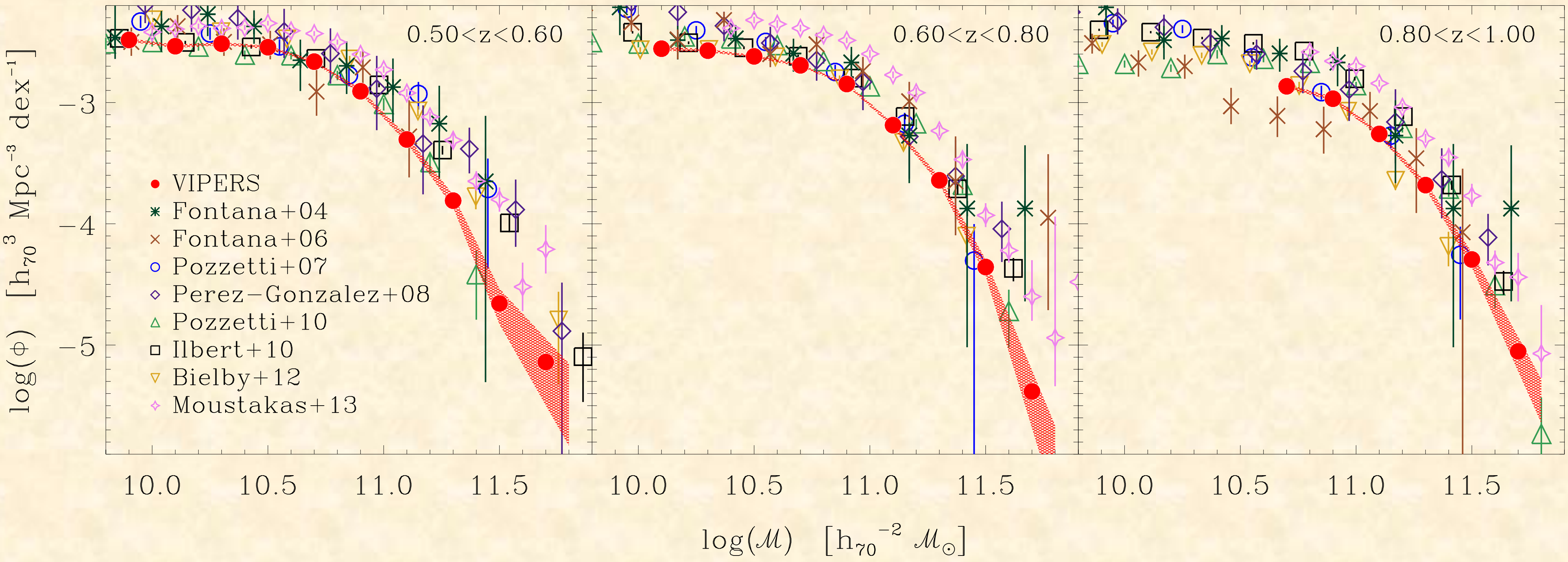
# VIPERS provides detailed structure AND galaxy properties

**(U-B) rest frame**




Color-density relation: Cucciati et al., in prep.

# Galaxy Stellar Mass Function




**MOST PRECISE MEASUREMENT EVER OF THE NUMBER DENSITY OF MASSIVE GALAXIES AT Z~1**

I. Davidzon, Bolzonella et al. 2013, A&A, 558, 23
II. Fritz et al. (CM diagram + LF), 2014, A&A, 563, 92

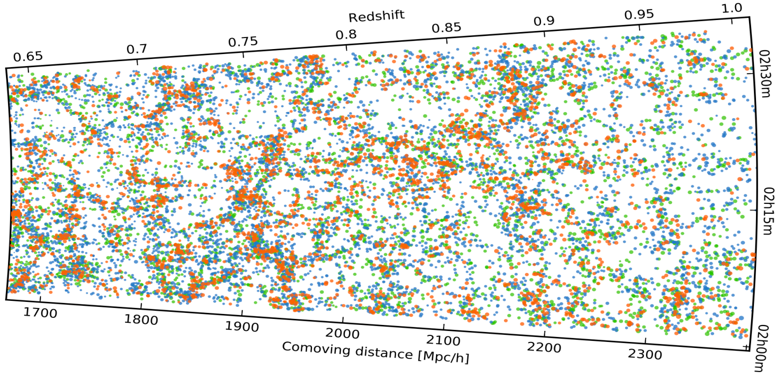
# Combining imaging and spectroscopy: the importance of photometry

$$ds^2 = -a^2(\tau)\left[(1+2\Psi)\,d\tau^2 - (1-2\Phi)\,d\vec{x}^2\right]$$

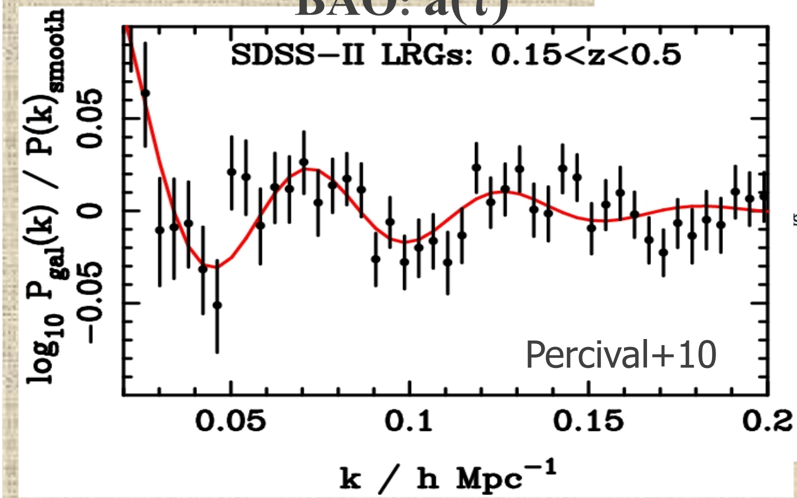
$\Phi$ : governs motion of matter

$\Psi$ : governs motion of light

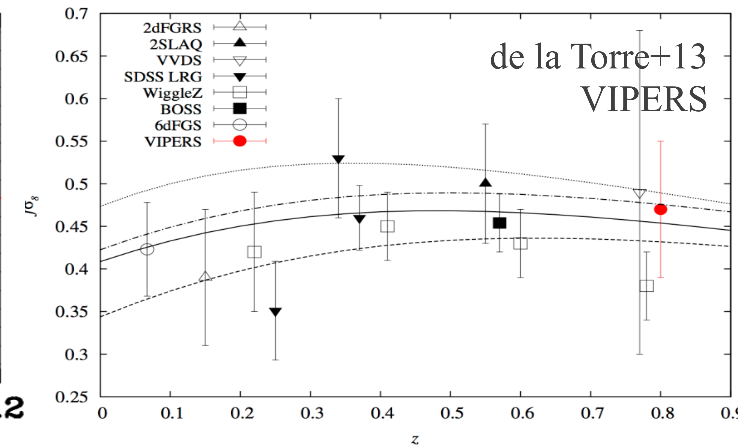
$\Phi = \Psi$ for GR



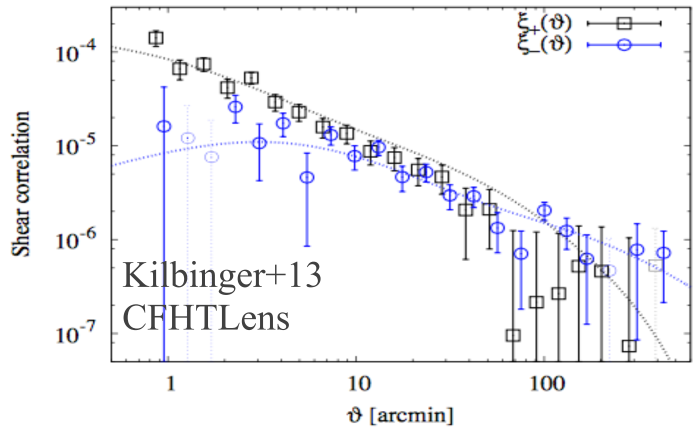

BAO: $a(\tau)$




RSD: $\Phi$

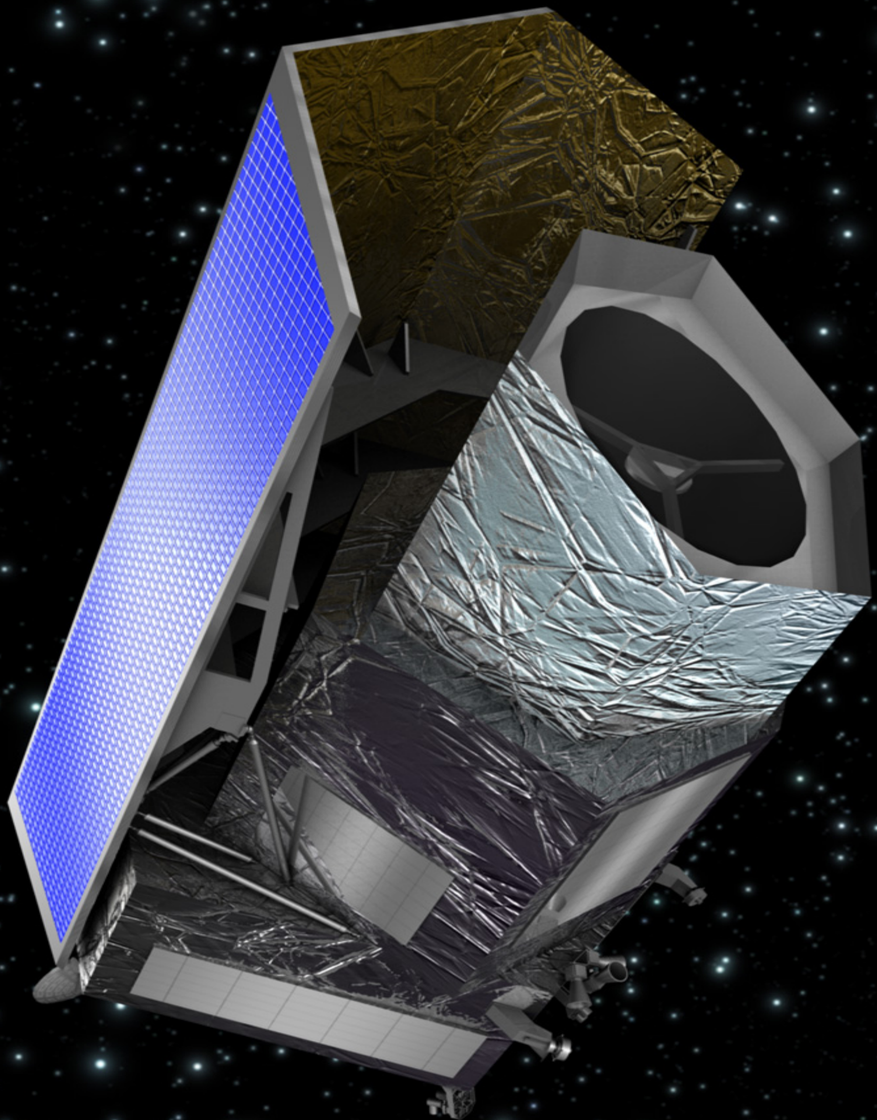



Cosmic shear: $\Phi + \Psi$




**De la Torre, Jullo & VIPERS Team, in preparation**

# Euclid



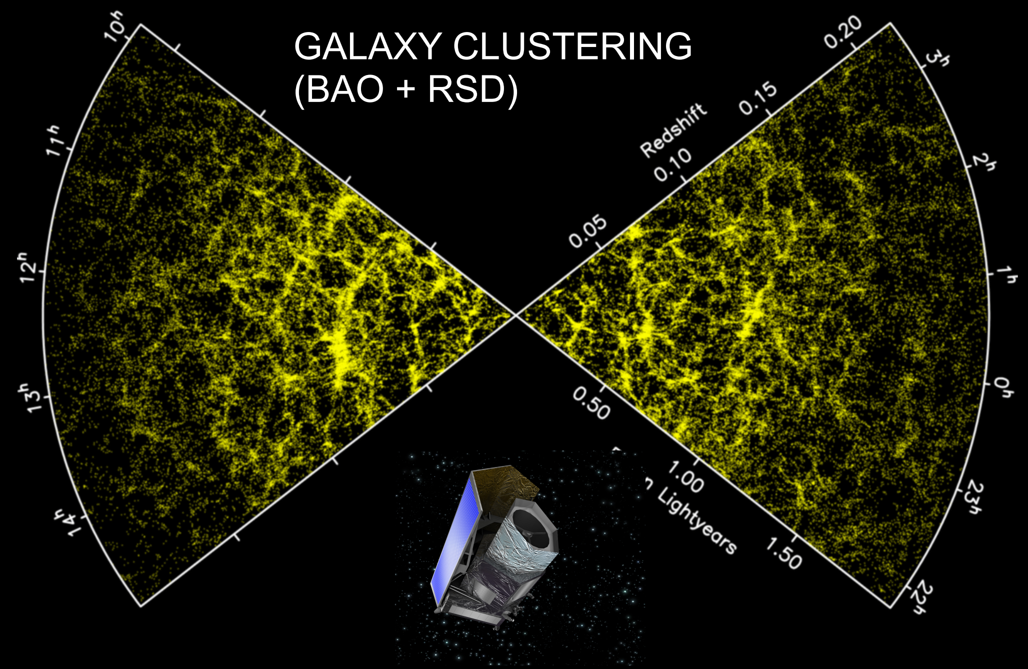
- ESA mission + extra contribution by national agencies (legacy of parent DUNE+SPACE projects)

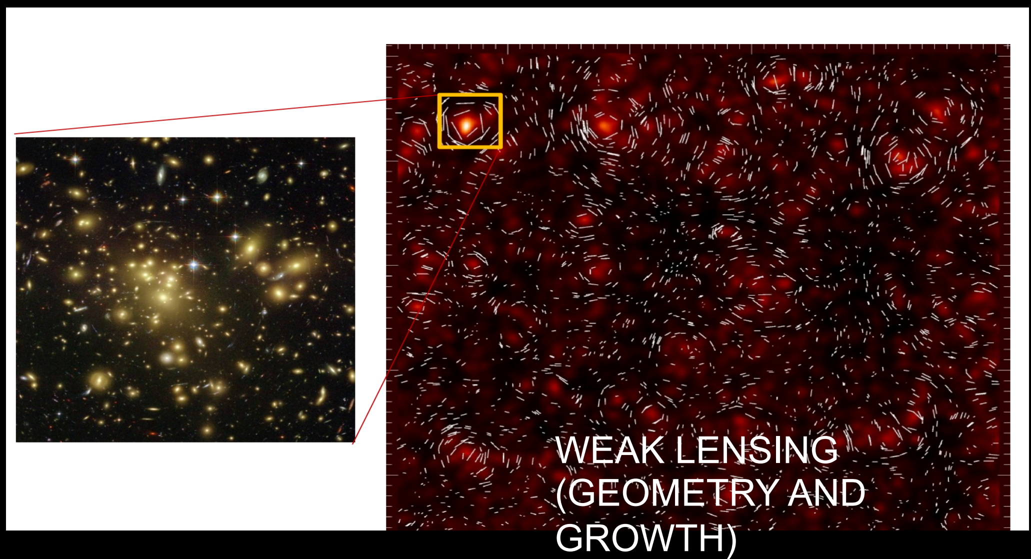
- Euclid Consortium Lead: Yannick Mellier (IAP)

- 1.2 m telescope
- Visible imaging (1 band)
- Infrared imaging (Y,J,H)
- Infrared slitless spectroscopy
- Launch 2020

- 15,000 $deg^2$ survey
- Images for $2x10^9$ galaxies
- Spectra for ~$5 \times 10^7$ galaxies ($0.9<z<1.8$)

# Euclid




**OBJECTIVES**:

- Build a map of dark and luminous matter over 1/3 of the sky and to z~2
- Unveil the nature of dark matter
- Solve the mystery of dark energy (cosmic acceleration)
- Use multiple probes → max control over systematic errors

**The Euclid "Red Book"**

http://sci.esa.int/science-e/www/object/index.cfm?fobjectid=48983#

# Summary

- An exciting future for cosmology from galaxy clustering: galaxy redshift surveys can measure both w(z) and f(z) using BAOs/P(k) and z-distortions → test dark energy vs modified gravity
- A renaissance for redshift-space distortions: not considered in this context before 2008, now a key "dark energy probe" (EUCLID)

**More and more data will push statistical errors into 1% regime:**

- Over the past 3 years new RSD results from WiggleZ, BOSS, VIPERS
- VIPERS fills a specific niche, thanks to its high sampling, allowing complementary approaches (multi-population still atractive?)
- EUCLID will couple a ~30 million galaxy (slitless) redshift survey with a high-resolution imaging survey, to combine galaxy clustering and weak lensing (launch 2020)
- Other ground-based surveys, like DESI, are planned in the 10-million z regime

**Need to increase control over systematic effects:**

- Improve modelling of RSD: rapid and promising development after 2008 renaissance (e.g. building upon Scoccimarro 2004)
- Streaming model approach yields promising results (Reid+, Bianchi+, Uhllemann+)
- Use different tracers of RSD, possibly with reduced weight of nonlinear effects (e.g. Mohammad et al., group-galaxy correlations)
- New probes (e.g. voids) / new statistics / improved corrections of observational biases
- All existing components need to be accounted for (e.g. neutrinos cannot be neglected…)

# EXTRA MATERIAL

# Models: improved dispersion approach

$$[1+\delta^s(\boldsymbol{s})]\, d^3\boldsymbol{s} = [1+\delta(\boldsymbol{r})]\, d^3\boldsymbol{r}$$

$$\delta^s(\boldsymbol{s}) = \left(\delta(\boldsymbol{r}) + f\partial_\parallel^2\Delta^{-1}\theta(\boldsymbol{r})\right)\left(1 - f\partial_\parallel^2\Delta^{-1}\theta(\boldsymbol{r})\right)^{-1}$$

$$P^s(k,\mu) = \int \frac{d^3\boldsymbol{r}}{(2\pi)^3} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \Big\langle e^{-ikf\mu\Delta u_\parallel} \times [\delta(\boldsymbol{x}) + \mu^2 f\theta(\boldsymbol{x})][\delta(\boldsymbol{x}') + \mu^2 f\theta(\boldsymbol{x}')]\Big\rangle$$

$$P^s_g(k,\mu) = D(k\mu\sigma_v)P_K(k,\mu,b)$$

$$D(k\mu\sigma_v) = \begin{cases} \exp(-(k\mu\sigma_v)^2) \\ 1/(1+(k\mu\sigma_v)^2) \end{cases}$$

$$P_K(k,\mu,b) = \begin{cases} \text{A}: \; b^2(k)P_{\delta\delta}(k) + 2\mu^2 fb(k)P_{\delta\delta}(k) + \mu^4 f^2 P_{\delta\delta}(k) \\ \text{B}: \; b^2(k)P_{\delta\delta}(k) + 2\mu^2 fb(k)P_{\delta\theta}(k) + \mu^4 f^2 P_{\theta\theta}(k) \\ \text{C}: \; b^2(k)P_{\delta\delta}(k) + 2\mu^2 fb(k)P_{\delta\theta}(k) + \mu^4 f^2 P_{\theta\theta}(k) + C_A(k,\mu;f,b) + C_B(k,\mu;f,b) \end{cases}$$

A: "NL" Kaiser 1987

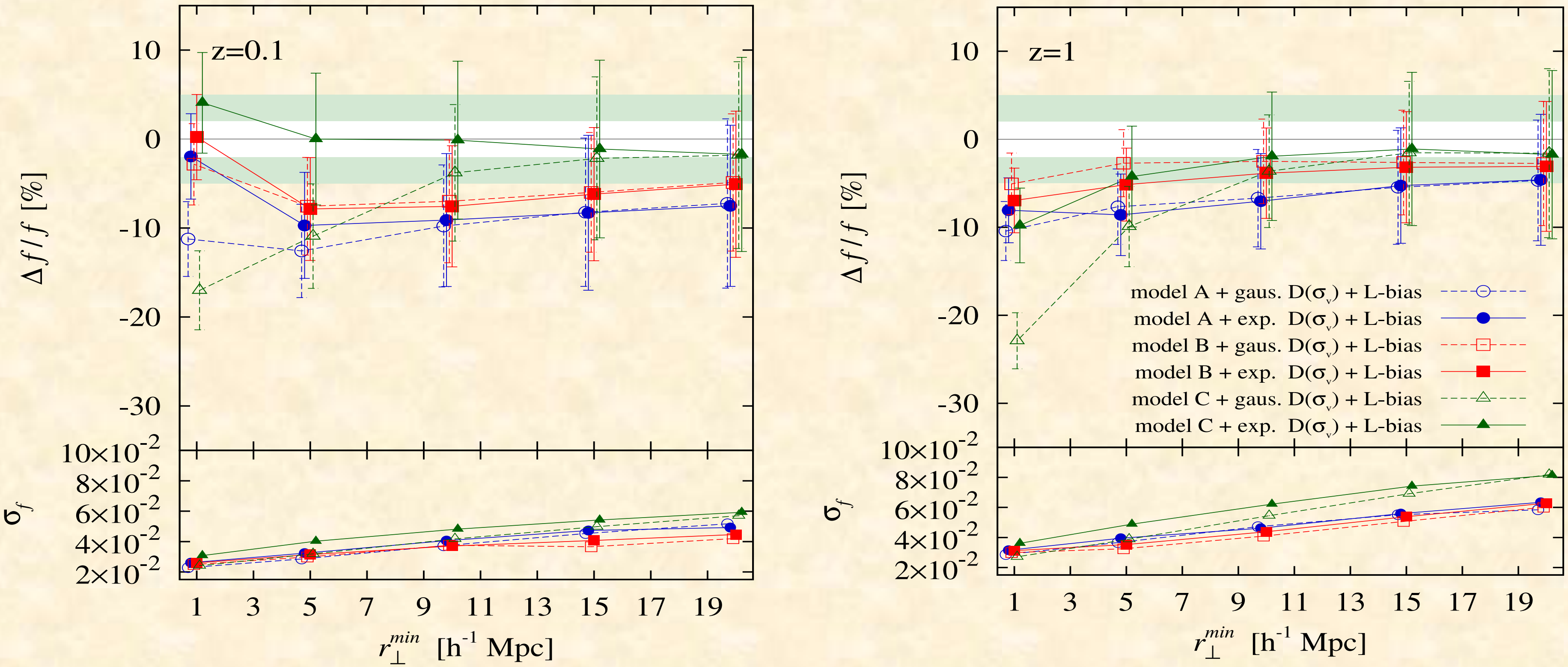
B: Scoccimarro 2004

C: Taruya et al. 2010

Building an accurate RSD non-linear model for galaxies: importance of galaxy biasing

(de la Torre & Guzzo 2012)

# Improved dispersion approach




*Taruya et al.* 2010 model allows recovering ***f*** **at the 5% percent level**, *Scoccimarro* 2004 and dispersion models performing worst (3-10%)

(de la Torre & Guzzo 2012)

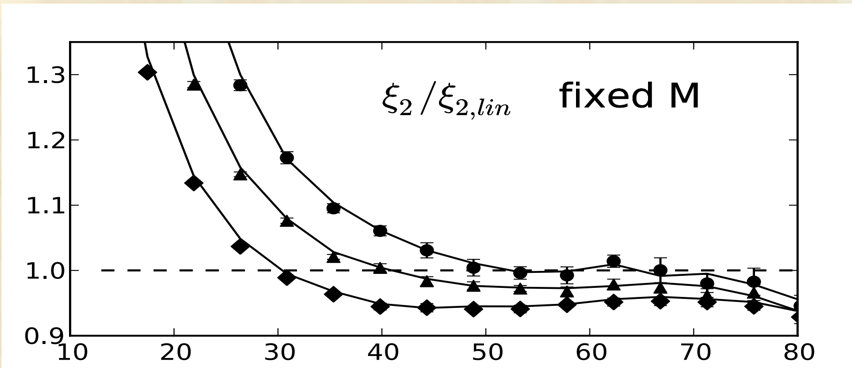
# Models: streaming approach

- Gaussian (scale-dependent) streaming model:

$$1 + \xi_g^s(r_\sigma, r_\pi) = \int \left[1 + \xi_g^r(r)\right] e^{-[r_\pi - y - \mu v_{12}(r)]^2/2\sigma_{12}^2(r,\mu)} \frac{dy}{\sqrt{2\pi\sigma_{12}^2(r,\mu)}}$$

$$\begin{cases} \xi^r(r) \\ v_{12}(r)\hat{r} = \dfrac{\langle[1 + b\delta(\mathbf{x})][1 + b\delta(\mathbf{x}+\mathbf{r})][\mathbf{v}(\mathbf{x}+\mathbf{r}) - \mathbf{v}(\mathbf{x})]\rangle}{\langle[1 + b\delta(\mathbf{x})][1 + b\delta(\mathbf{x}+\mathbf{r})]\rangle} \\ \sigma_{12}^2(r,\mu^2) = \dfrac{\left\langle(1 + b\delta(\mathbf{x}))(1 + b\delta(\mathbf{x}+\mathbf{r}))(v^\ell(\mathbf{x}+\mathbf{r}) - v^\ell(\mathbf{x}))^2\right\rangle}{\langle(1 + b\delta(\mathbf{x}))(1 + b\delta(\mathbf{x}+\mathbf{r}))\rangle} \end{cases}$$

Approximation from SPT or CLPT




(Reid & White 2011; Wang, Reid & White 2013)

# Models summary

| Model | Type | Parameters | Input | Rel. accuracy |
|---|---|---|---|---|
| Dispersion | Linear theory + damping | f, $\sigma_v$, b | $P_{\delta\delta}(k)$ | 10% for galaxies |
| Scoccimarro 2004 | Standard approach | f, $\sigma_v$, b | $P_{\delta\delta}(k)$, $P_{\delta\theta}(k)$, $P_{\theta\theta}(k)$ | 5-8% for galaxies |
| Taruya et al. 2010 | Standard approach + PT | f, $\sigma_v$, b | $P_{\delta\delta}(k)$, $P_{\delta\theta}(k)$, $P_{\theta\theta}(k)$, $C_A(k)$, $C_B(k)$ | 5% for galaxies |
| Seljak & McDonald 2011 | Distribution function + PT | f, $\sigma_v$, b | $T_{ij, \ldots}$ | ? |
| Reid & White 2011, Wang et al. 2013 | Gaussian streaming model + PT | f, b | $P_{gg}(k)$ | Few percent for LRG<br>? for other galaxies |
| Kwan, Lewis & Linder 2012 | Empirical | f, B, b | $P_{\delta\delta}$, A(k), (B(k)), C(k) | 5% for haloes<br>? for galaxies |
| Linder & Samsing 2013 | Empirical | f, A(k), B(k), C(k), b | $P_{\delta\delta}$ | Few percent for DM<br>? for galaxies |
| Zhang et al. 2013 | Standard approach | f, b | $W^2(k)=P_{\theta\theta}(k)/P_{\delta\theta}(k)$ | ? |