Review of
Small Field Models of Inflation

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אוניברסיטת בן-גוריון

I. Ben-Dayan 0907.2384
+ in progress
I. Ben-Dayan, S. de Alwis 0802.3160

- Small field models of inflation
  - Designing small field SUGRA models
  - Relevance to string theory
- Predictions for the CMB:
  - Simplest models: $n_S<1$, $r_{0.01}<<1$, $\alpha_{0.05}<<1$
  - New class: n_S , $r_{0.01}$, $\alpha_{0.05}$ spans allowed values

Models of inflation: Background

de Sitter phase $\rho + p << \rho$ ➔ $H \sim const.$

Parametrize the deviation from constant H

by the value of the field

$$\varepsilon(\varphi) = \frac{m_p^{\;2}}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta(\varphi) = m_p^{\;2}\frac{V''}{V}$$

$$\xi^2(\varphi) = m_p^{\;4}\frac{V'''V'}{V^2}$$

Or by the number of e-folds

$$N(\varphi) = \int_t^{t_{ei}} d\log a(t) = \int_t^{t_{ei}} H dt = \int_\varphi^{\varphi_{ei}} \frac{H}{\dot{\varphi}} d\varphi = \frac{1}{\sqrt{2} m_p}\int_{\varphi_{ei}}^{\varphi} \frac{1}{\sqrt{\varepsilon(\varphi)}} d\varphi$$

Inflation ends when $\varepsilon = 1$

Models of inflation:Perturbations

- Spectrum of scalar perturbations

$$P_{\Re}(k) = \frac{2}{\pi}\left(\frac{H}{m_p}\right)^2 \frac{1}{\varepsilon}\Big|k = aH$$

$$n - 1 \equiv \frac{d\ln P_{\mathcal{R}}}{d\ln k}$$

$$\alpha = \frac{dn_S}{d\ln k}$$

- Spectrum of tensor perturbations

$$P_T(k) = \frac{2}{\pi}\left(\frac{H}{m_p}\right)^2 \Big|k = aH$$

Spectral indices

$$n_S = 1 - 6\epsilon_{CMB} + 2\eta_{CMB}$$

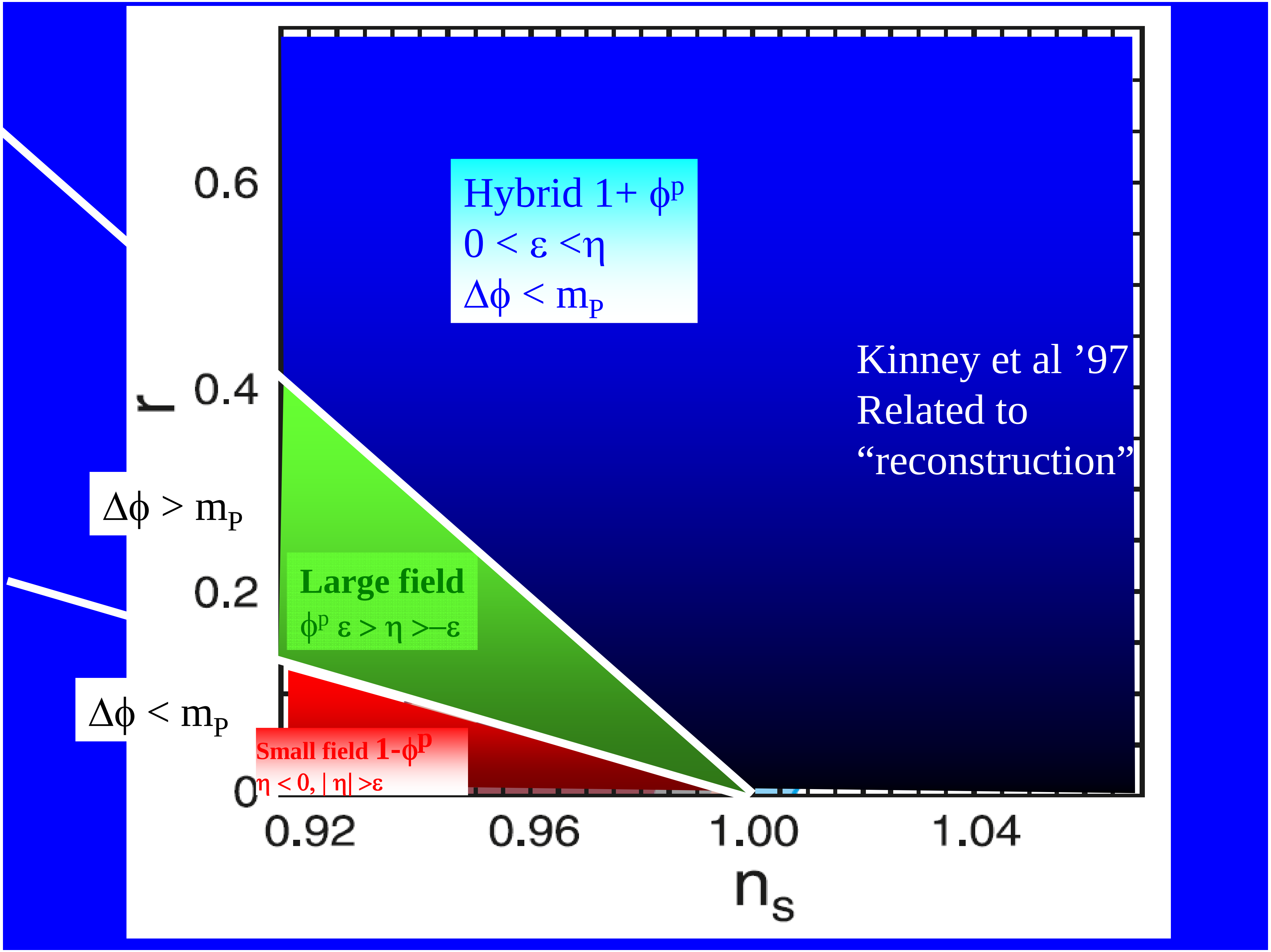
$$n_T \simeq -2\epsilon = -2\frac{P_T}{P_{\mathcal{R}}}$$

$$\alpha = 16\epsilon\eta - 24\epsilon^2 - 2\xi^2$$

α - RUN

Tensor to scalar ratio (many definitions)
r is determined by P_T/P_R
("current canonical" r =16 ε)

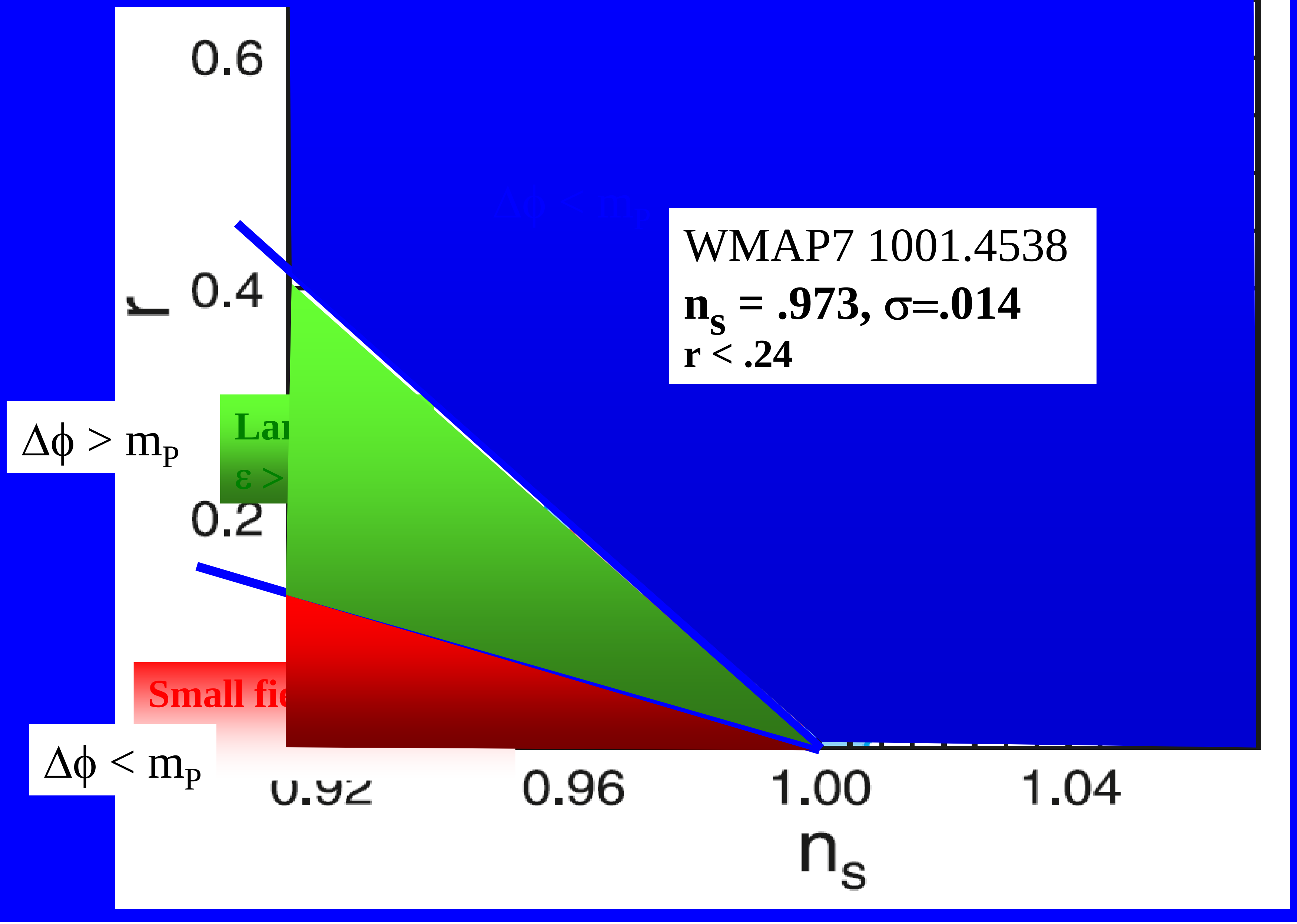
CMB observables determined by quantities ~ 60 efolds before the end of inflation



Hybrid 1+ ϕ^p
0 < ε < η
Δϕ < m_P
Kinney et al ’97
Related to
“reconstruction”
Δϕ > m_P
Large field
ϕ^p ε > η >−ε
Δϕ < m_P
Small field 1-ϕ^p
η < 0, | η| >ε
r
0.6
0.4
0.2
0
0.92
0.96
1.00
1.04
n_s

WMAP5

No evidence for running

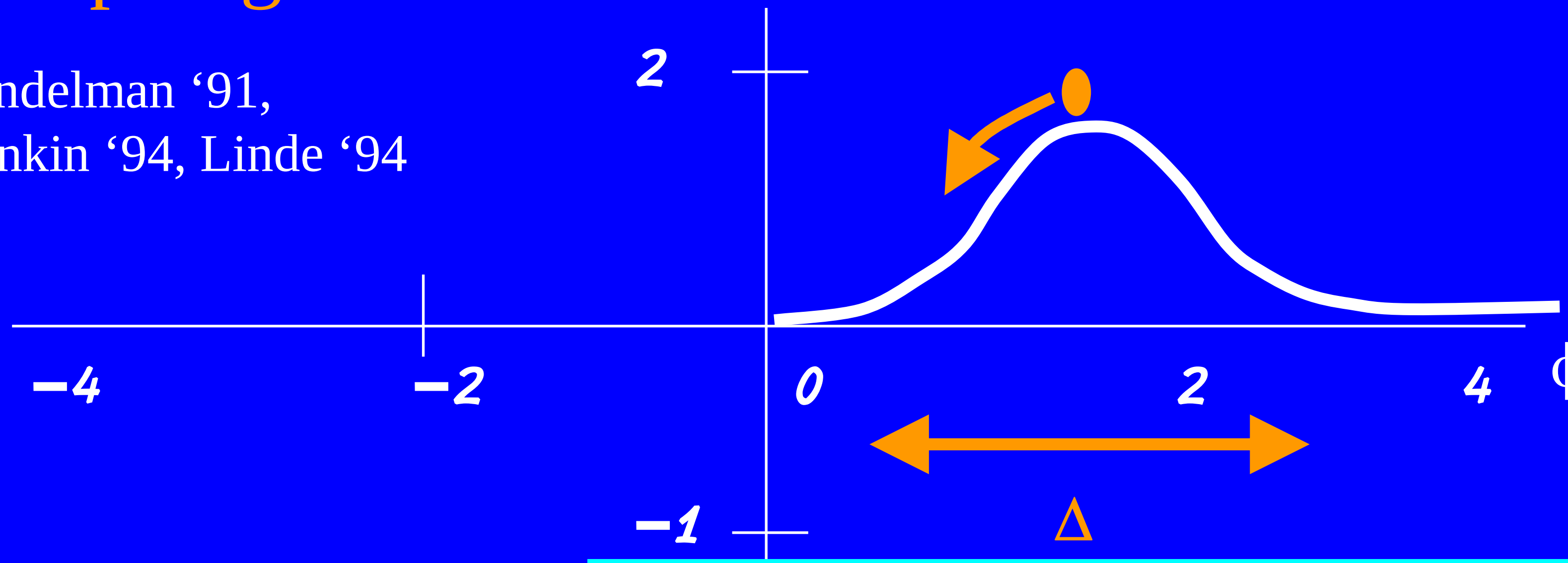


(My) preferred models of modular inflation: small field models

hep-th/0205042
hep-th/0212344
S. de Alwis, E. Novak

- "Topological inflation": inflation off a flat feature

Guendelman '91,
Vilenkin '94, Linde '94



Enough inflation ⇔ *V''/V<1/50*

δ – wall thickness in space

$(\Delta/\delta)^2 \sim \Lambda^4$

$H^2 \sim 1/3\ \Lambda^4/m_p^2$

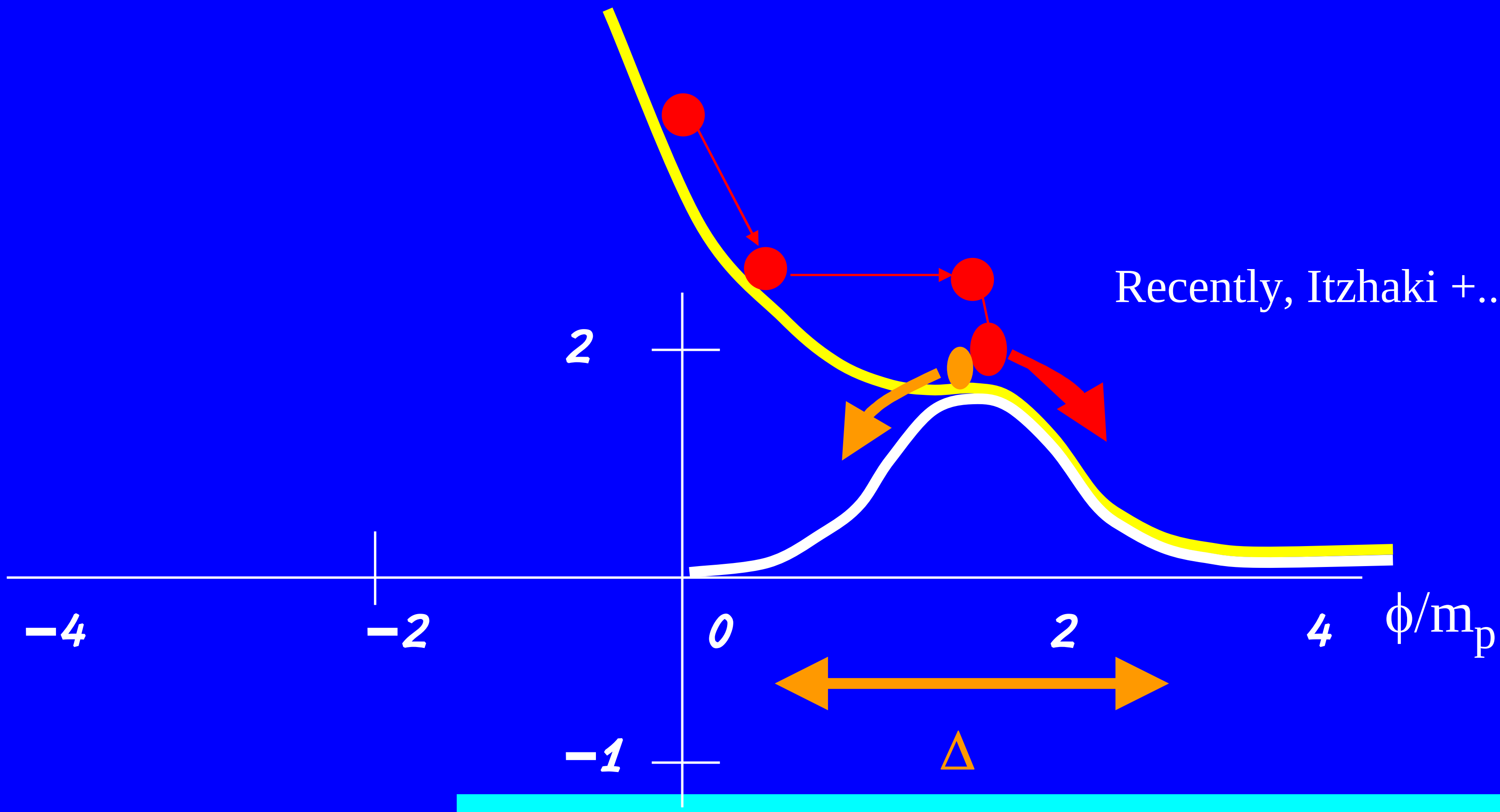
Inflation ⇔ *δH > 1* ⇔ *Δ > m_p*

$$m_p \equiv \frac{1}{8\pi G_N} = 2.4 \times 10^{18} GeV$$

hep-th/0408160
S. de Alwis, P. Martens

(My) preferred models of modular inflation: small field models

- Another version of inflation off a flat feature



Recently, Itzhaki +...

Enough inflation ⇔ *V''/V<1/50*

Designing flat features for single field modular SUGRA inflation

I. Ben-Dayan, S. de Alwis
0802.3160

$$V = e^K \left(|D_T W|^2 K^{T\overline{T}} - 3|W|^2 \right)$$

$$\partial_T V = e^K \left(D_T^2 W \, D_{\overline{T}} \overline{W} \, K^{T\overline{T}} - 2 D_T W \overline{W} \right)$$

$$\eta = \begin{pmatrix} \frac{K^{\overline{T}T} V_{T\overline{T}}}{V} & \frac{K^{\overline{T}T} V_{TT}}{V} \\ \frac{K^{\overline{T}T} V_{\overline{TT}}}{V} & \frac{K^{\overline{T}T} V_{\overline{T}T}}{V} \end{pmatrix}$$

At an extremum with $V_{|T_0} > 0$

$$|D_T^2 W| K^{T\overline{T}} = 2|W|$$

$$\partial_{\overline{T}} \partial_T V|_{T_0} = e^{K(T_0, \overline{T}_0)} \left(-R_{T\overline{T}T\overline{T}} (K^{T\overline{T}})^2 \; D_T W \, D_{\overline{T}} \overline{W} + 2 K_{T\overline{T}} |W|^2 \right) |_{T_0}$$

$$R_{T\overline{T}T\overline{T}} = K_{T\overline{T}T\overline{T}} - K^{T\overline{T}} K_{T\overline{T}T} K_{\overline{T}T\overline{T}}$$

A single field with a logarithmic Kaehler potential

$$K = -A \ln\left(T + \bar{T}\right)$$

$$W = \sum b_i (T - T_0)^i$$

$$\partial_{\overline{T}} \partial_T V|_{T_0} = e^{K(T_0, \overline{T}_0)} \left(-R_{T\overline{T}T\overline{T}} (K^{T\overline{T}})^2 \; D_T W \, D_{\overline{T}} \overline{W} + 2 K_{T\overline{T}} |W|^2 \right) |_{T_0}$$

$$R_{T\overline{T}T\overline{T}} = K_{T\overline{T}T\overline{T}} - K^{T\overline{T}} K_{T\overline{T}T} K_{\overline{T}T\overline{T}}$$

$$(K^{T\overline{T}})^2 R_{T\overline{T}T\overline{T}} = \frac{2}{A}$$

$$\mathrm{Tr}\ \eta = -\frac{4}{A} \left(1 + (3 - A) \frac{|b_0|^2}{|B|^2 - 3|b_0|^2} \right)$$

$$\mathrm{Tr}\ \eta \leqslant -\frac{4}{A} \quad \text{for} \quad 0 < A \leqslant 3$$

Cannot design a flat feature !

Also:
Gomez-Reino and Scrucca, 0706.2785
Badziak, Olechowski 0802.1014
Covi et al 0805.3290

Take the simplest Kahler potential and superpotential in the vicinity of an extremum

$$\phi = T - T_0$$

$$K = \phi\bar{\phi}; \quad W = \sum_{i=0}^{N} b_i \phi^i$$

Always a good approximation when expanding in a small region ($\phi < 1$)

$$V = e^K [K^{\phi\bar{\phi}} |D_\phi W|^2 - 3|W|^2]$$

$$\simeq (1 + \phi\bar{\phi})[(\bar{\phi}W + W_\phi)(\phi\bar{W} + \bar{W}_{\bar{\phi}}) - 3W\bar{W}]$$

For the purpose of finding local properties
V can be treated as a polynomial

Designing flat features for single field SUGRA modular inflation

$$V_T(T_0,\overline{T}_0), V_{\overline{T}}(T_0,\overline{T}_0) = 0$$
$$V(T_0,\overline{T}_0) > 0$$
$$|\eta| < \mathcal{O}(10^{-2})$$
$$|\Delta T| \gtrsim m_p.$$

Not a local equation

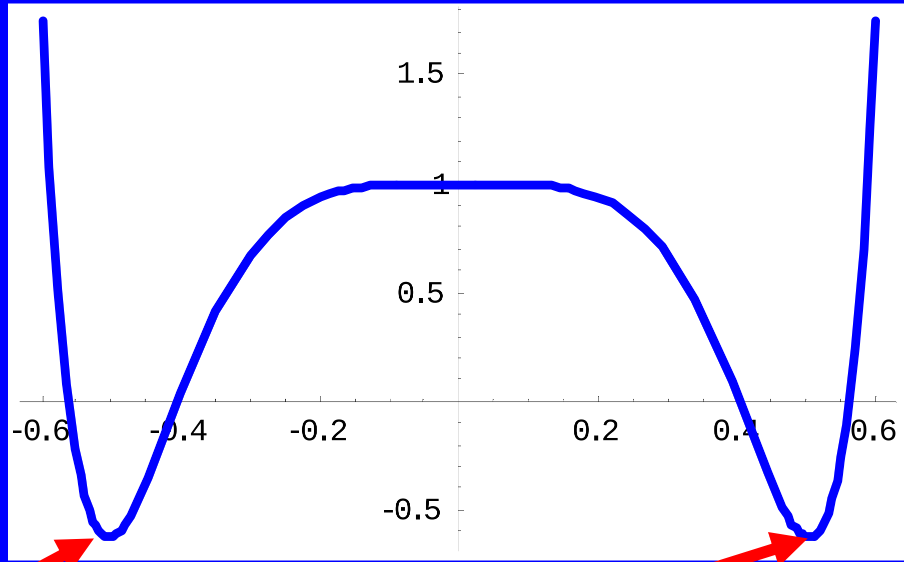
$$V_T(0) = 0$$
$$V(0) = 1$$
$$|\eta| < \mathcal{O}(10^{-2})$$
$$D_T W(\pm y) = 0, \quad y \sim 1$$

A local equation

T – a complex scalar field

A numerical example:

The potential is not sensitive to small changes in coefficients Including adding small higher order terms, inflation is indeed 1/100 of tuning away



$b_2=0$, $b_4=0$, $b_1=1$, $b_3=\eta/6$,

$b_5\, y^4(y^2+5) + y^2+1=0$

$\eta = 6\, b_1\, b_3 - 2(b_0)^2$

Need 5 parameters:

$V'(0)=0$, $V(0)=1$, $V''/V=\eta$

$D_T W(-y)$, $D_T W(+y) = 0$

$$K = \phi\bar{\phi}; \quad W = \sum_{i=0}^{N} b_i \phi^i$$

If one wishes to tune the CC @ min to be small enough replace D_T W=0 by V_T=0, V=0 (one more condition)

Relevance to string theory

- Small field models, High scale of inflation,

  central region of moduli space $g_s \lesssim 1$, $V_{compact} \gtrsim 1$
- Relatively small separation of scales

  $m_p \gtrsim M_s \gtrsim \Lambda_{Inflation}$

  Some hope for stringy physics in anisotropies!

Small field models, standard lore: No Observable GW

$$N(\varphi) = \int_t^{t_{ei}} d\log a(t) = \int_t^{t_{ei}} H dt = \int_\varphi^{\varphi_{ei}} \frac{H}{\dot{\varphi}} d\varphi = \frac{1}{\sqrt{2} m_p} \int_{\varphi_{ei}}^{\varphi} \frac{1}{\sqrt{\varepsilon(\varphi)}} d\varphi$$

$r = 16\,\varepsilon$ ➔ $\frac{dN}{d\phi} = \sqrt{\frac{8}{r}}$

If ε ~ const. ➔ $r \simeq 8 \left(\frac{\Delta\phi}{N_{CMB}} \right)^2$

"Lyth theorem" $\Delta\phi \sim 1$ ➔ $r_{0.01} > 1$ (depending on "choice" of N_{CMB})

In practice need $\Delta\phi \sim 10$

$r_{0.01} = (r/0.01)$

"dream" sensitivity

Small field models, standard lore: No Observable RUN

Easthe Simple example: $V(\phi) = \Lambda^4 (1 - a_p \phi^p)$ $\phi_{END} \lesssim 1$

"Thus, a definitive obser ... ply

n ... of

$$(n_S)_{CMB} = 1 - \frac{2}{N_{CMB}} \frac{p-1}{p-2}$$

s ... ct

$$\eta_{CMB} = -\frac{1}{N_{CMB}} \frac{p-1}{p-2}$$

i ... t small

$$\epsilon_N = \frac{1}{2} \left(\frac{1}{p a_p}\right)^{\frac{2}{p-2}} \left(\frac{1}{(p-2)N}\right)^{2\frac{p-1}{p-2}}$$

$$\epsilon_{N,max} = e^{-2\frac{(p-1)^2}{p-2}}$$

Dodelson, Kolb+Kinney, 9702166:

I. Ben-Dayan, S. de Alwis 0802.3160

$$\alpha = -16\epsilon\eta + 24\epsilon^2 + 2\xi^2$$

calar spectrum (here, blue implies n > 1) ... ybrid models."

$$\alpha_N = 2\frac{(p-1)}{(p-2)} \frac{1}{N^2}$$

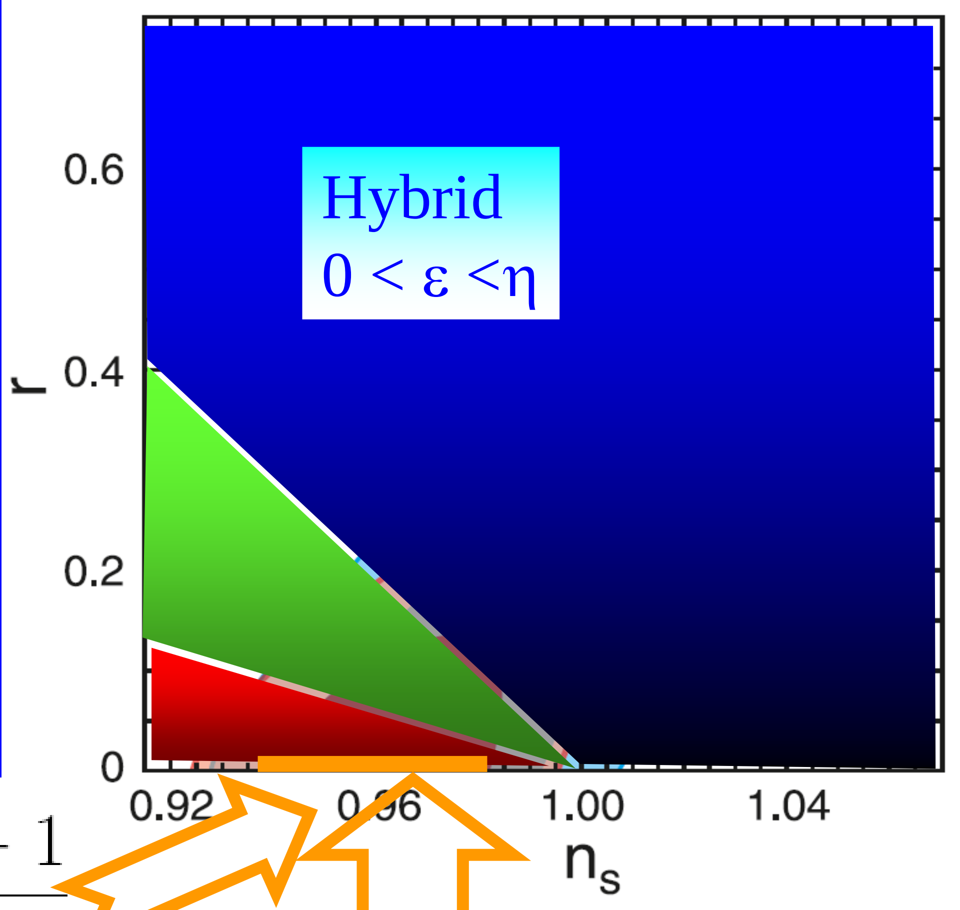
$$\alpha_{CMB} = 2.8 \times 10^{-4} \frac{(p-1)}{(p-2)} \left(\frac{60}{N_{CMB}}\right)^2$$

Simple example:

$$V(\phi) = \Lambda^4 \left(1 - a_p \phi^p\right)$$

$$\varepsilon_{CMB} \ll \eta^2_{CMB}$$

$$\alpha \approx \eta^2_{CMB}$$

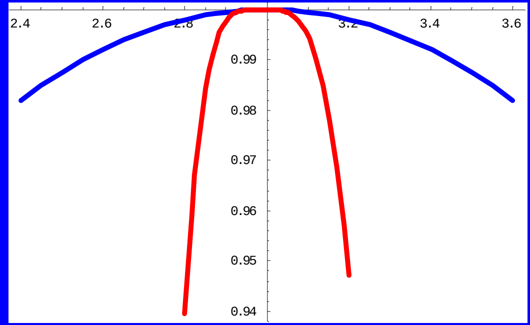


$$(n_S)_{CMB} = 1 - \frac{2}{N_{CMB}} \frac{p-1}{p-2}$$

p	3	4	5	7	10	$p \to \infty$
r	3.1×10^{-7}	3.3×10^{-6}	6.1×10^{-6}	7.9×10^{-6}	6.8×10^{-6}	0
α	5.6×10^{-4}	4.2×10^{-4}	3.7×10^{-4}	3.4×10^{-4}	3.4×10^{-4}	2.8×10^{-4}

- The "minimal" model:
  - Quadratic maximum
  - End of inflation determined by higher order terms

$$V(\phi) = \Lambda^4 \left(1 - a_2\phi^2 - a_p\phi^p\right)$$



- Results:
  - Suppression of GW

$$r_{max} = 16\left[\left(\frac{1}{60e\sqrt{2}}\right)\frac{p-1}{p-2}\right]^{2\frac{p-1}{p-2}} (\phi_{END})^{2\frac{p-1}{p-2}} \left(\frac{60}{N_{CMB}}\right)^{2\frac{p-1}{p-2}}$$

  - Suppression of running

$$\alpha_{max} = 3\times 10^{-4}\,\frac{p-1}{p-2}\left(\frac{60}{N_{CMB}}\right)^2$$

$\phi_{END}=1$

p	3	4	5	7	10	$p\to\infty$
r_{max}	9.0×10^{-8}	4.4×10^{-6}	1.7×10^{-5}	5.3×10^{-5}	1.0×10^{-4}	3.0×10^{-4}
α	6.0×10^{-4}	3.7×10^{-4}	2.1×10^{-4}	6.0×10^{-5}	6.2×10^{-6}	0
α_{max}	6.0×10^{-4}	4.5×10^{-4}	4.0×10^{-4}	3.6×10^{-4}	3.4×10^{-4}	3.0×10^{-4}
r	9.0×10^{-8}	3.5×10^{-6}	1.0×10^{-5}	1.9×10^{-5}	2.0×10^{-5}	0

Observational consequences

- Observation of GW signal in the CMB ➔

  ~~small field models~~ ?

- Observation of RUN in the CMB ➔

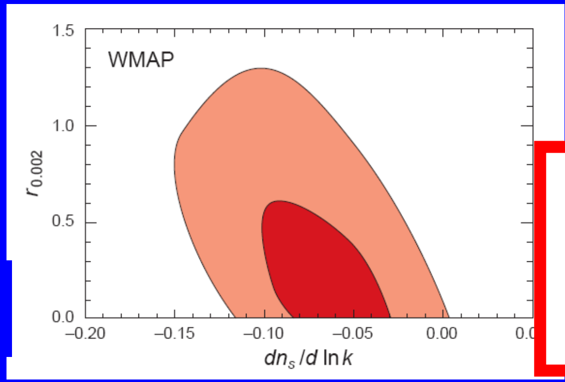
  ~~small field models~~ ? ~~single field models~~ ?

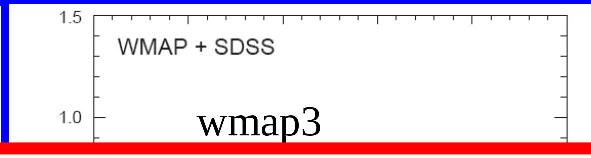
CMB measured parameters relevant to inflation physics

- Amplitude of temperature fluctuations TT
- Polarization EE, EB, BB
- Non-Gaussianity
- 5 parameters relevant to inflation physics:
  - ✓ A_s
  - ☑ n_s
  - ☒ r
  - ☑ run
  - ☒ f_{NL}

RUN observations*

* A *hard* measurement





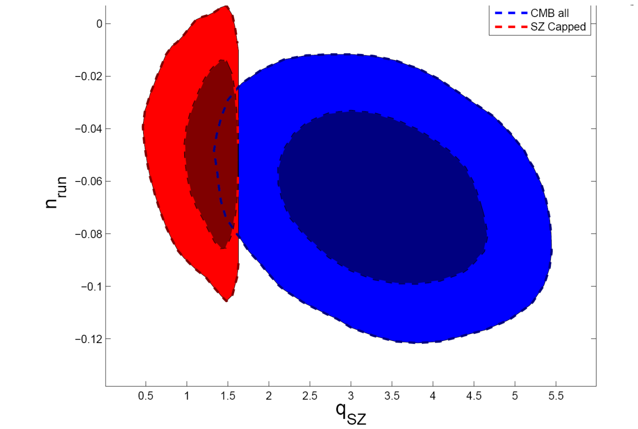
WMAP7 1001.4538

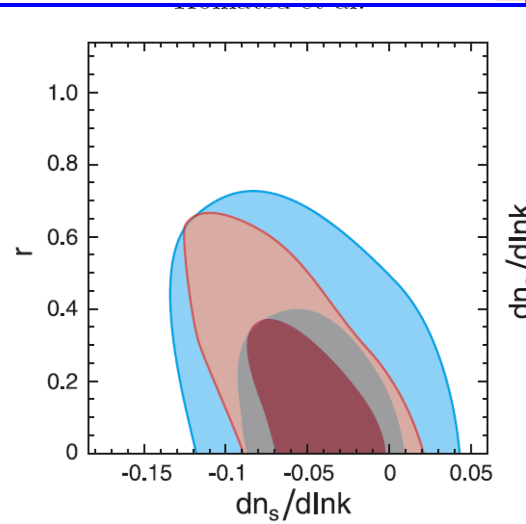
$n_s = 1.076$, $\sigma=.065$

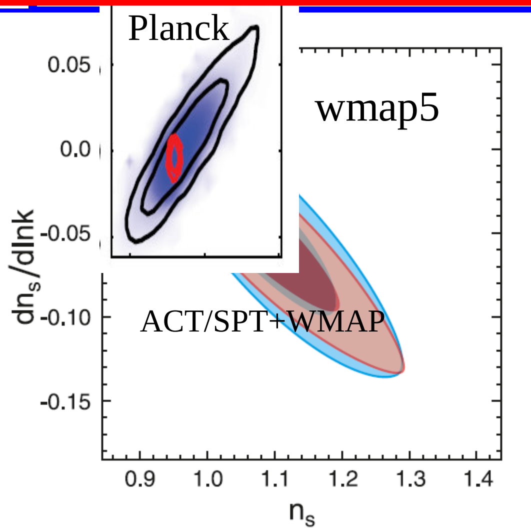
$r < .49$, $\alpha = -.049$, $\sigma=.029$

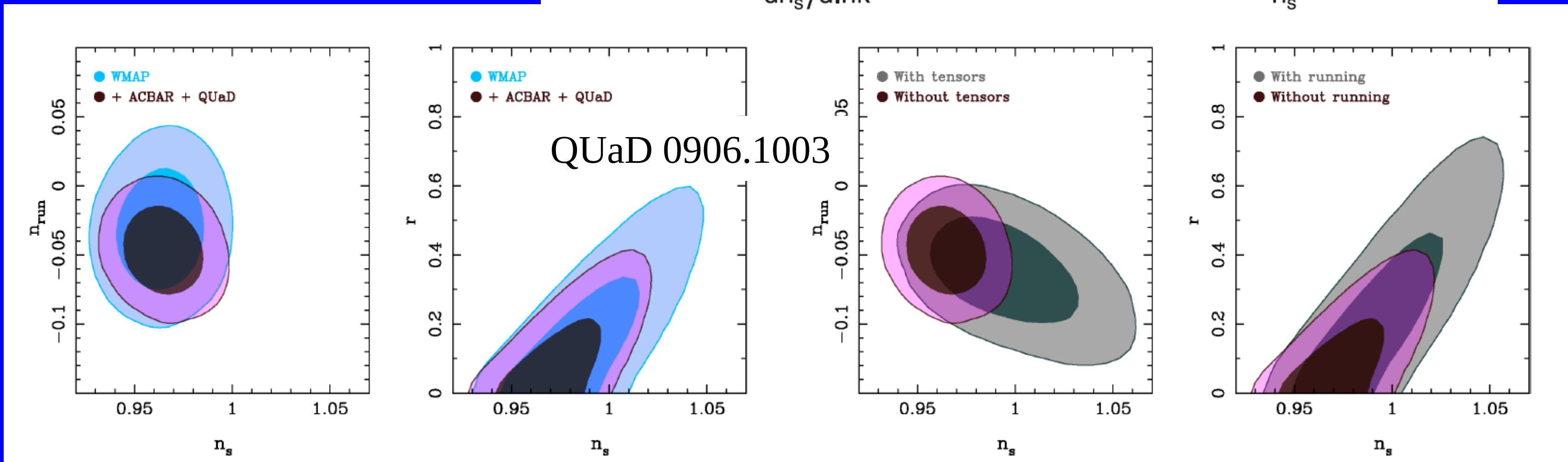
CBI 0901.4540 Wmap5,+CBI,+All

$\frac{dn_s}{d\log k} = (-0.041 \pm 0.031, -0.048 \pm 0.028, -0.066 \pm 0.022)$









New class of small field models

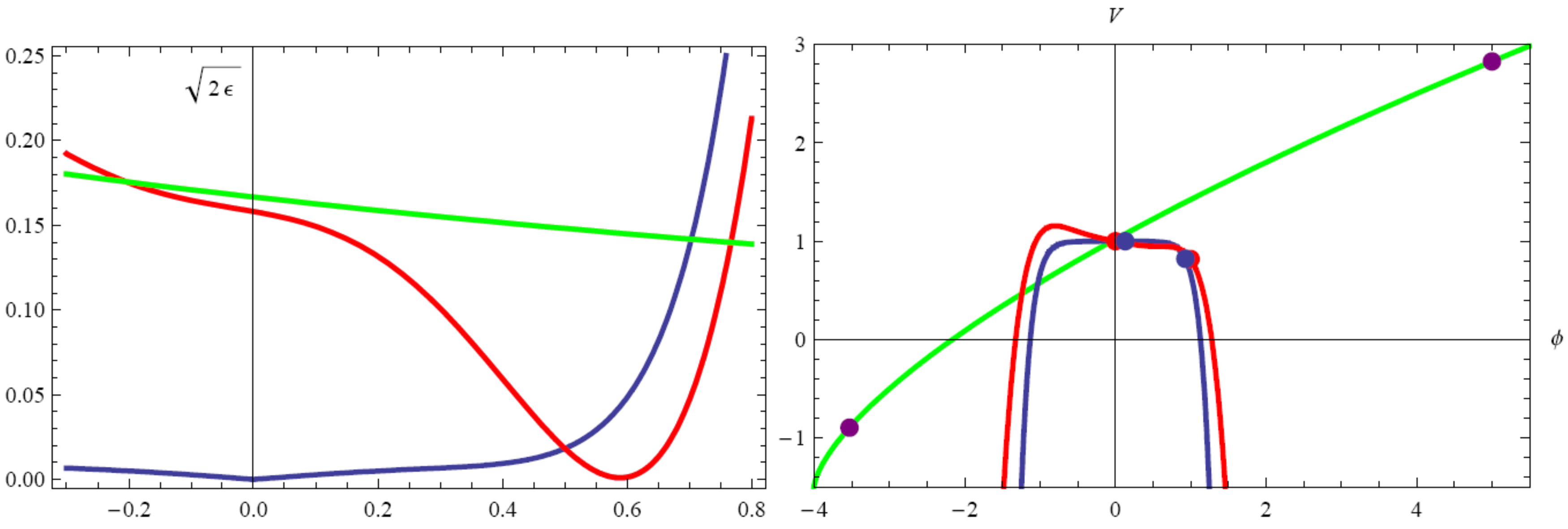


FIG. 1: Shown is a graph of $\sqrt{2\epsilon} = V'/V$ (left) and V (right) for a small field canonical SUGRA model (blue), a large field model (green) and a model of the new class with non-monotonic ϵ (red). The new model interpolates between the two others. For the small field model (blue) the CMB point is at $\phi_{CMB} = 0.13$ and inflation ends at $\phi_{END} = 0.93$. For the large field model (green) ($\phi_{CMB} = 5, \phi_{END} = -3.53$) and for the new model (red) ($\phi_{CMB} = 0, \phi_{END} = 1.0$). The large field model is offset $V \to V - 1.5$. Additionally, to demonstrate the similarity between the small field model(blue) and the new model (red) a symmetric example was chosen, i.e. $a_5 = 0, a_6 = 0.3911$. The CMB observables are $n_s = 1.03, r = 0.2, \alpha = -0.07$.

$\sqrt{\;\;}$ ... $+ B(\phi - \phi_{\min})^3$

... $^3 - a_4\phi^4 - a_5\phi^5$

New class of small field models

$$\sqrt{\varepsilon} \sim \frac{1}{N} + A(\phi - \phi_{\min})^2 \Rightarrow \frac{V}{V_0} \sim 1 + \frac{1}{N}\phi + B(\phi - \phi_{\min})^3$$

$$\frac{V(\phi)}{V(0)} = 1 - \sqrt{\frac{r_0}{8}}\phi + \frac{\eta_0}{2}\phi^2 + \frac{\alpha_0}{3\sqrt{2r_0}}\phi^3 - a_4\phi^4 - a_5\phi^5$$

$$\frac{1}{2}\left(\frac{-\sqrt{\frac{r_0}{8}} + \eta_0\phi_{END} + \frac{\alpha_0}{\sqrt{2r_0}}\phi_{END}^2 - 4a_4\phi_{END}^3 - 5a_5\phi_{END}^4}{1 - \sqrt{\frac{r_0}{8}}\phi_{END} + \frac{\eta_0}{2}\phi_{END}^2 + \frac{\alpha_0}{3\sqrt{2r_0}}\phi_{END}^3 - a_4\phi_{END}^4 - a_5\phi_{END}^5}\right)^2 = 1$$

$$N_{CMB} = \int_0^{\phi_{END}} \frac{d\phi}{\sqrt{2\epsilon(\phi;a_5)}}$$

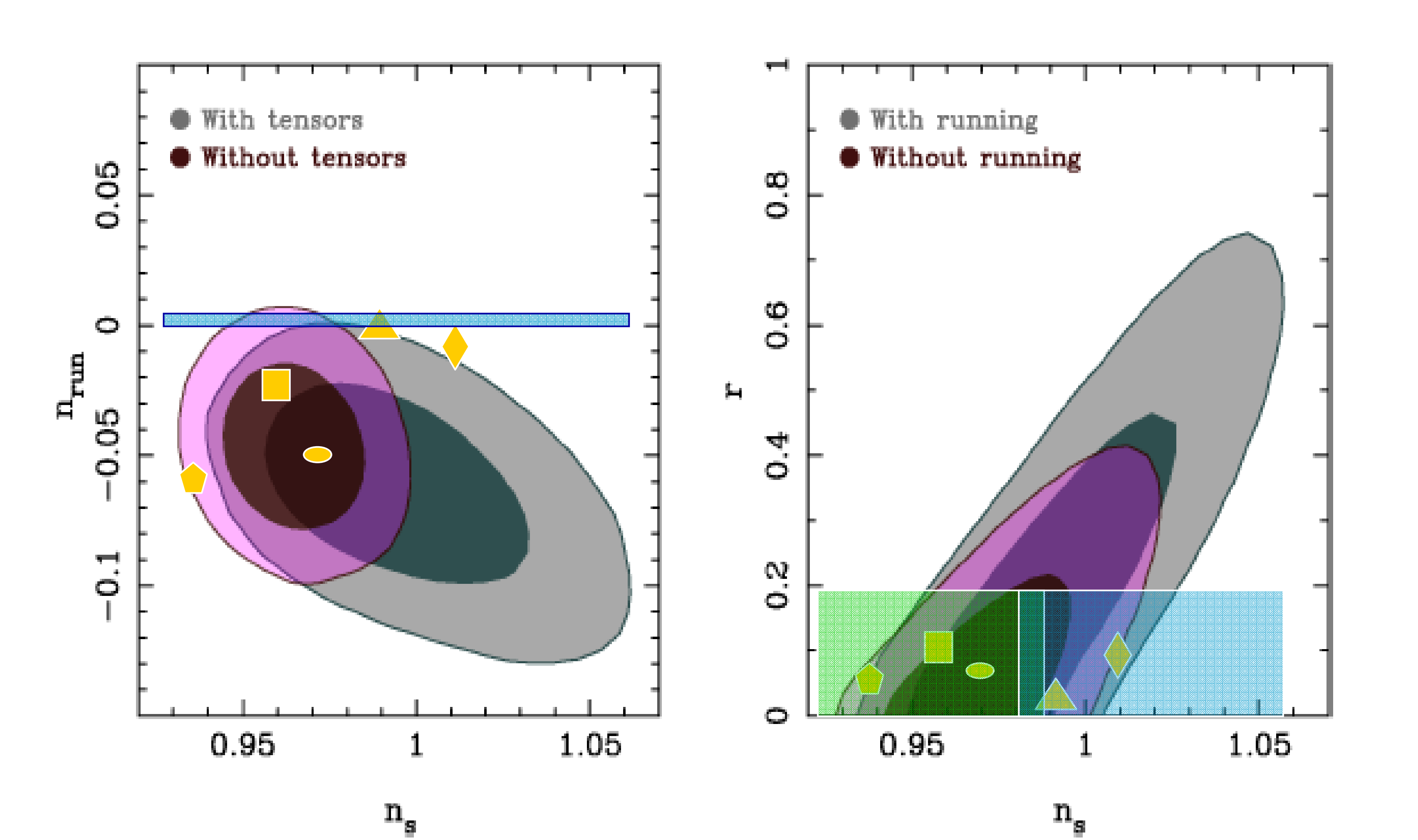
2 initial conditions,
5 equations,
1 non-linear :N = 60
1 non-linear constraint $\phi_{END}<1$

New class of models: "Predictions"

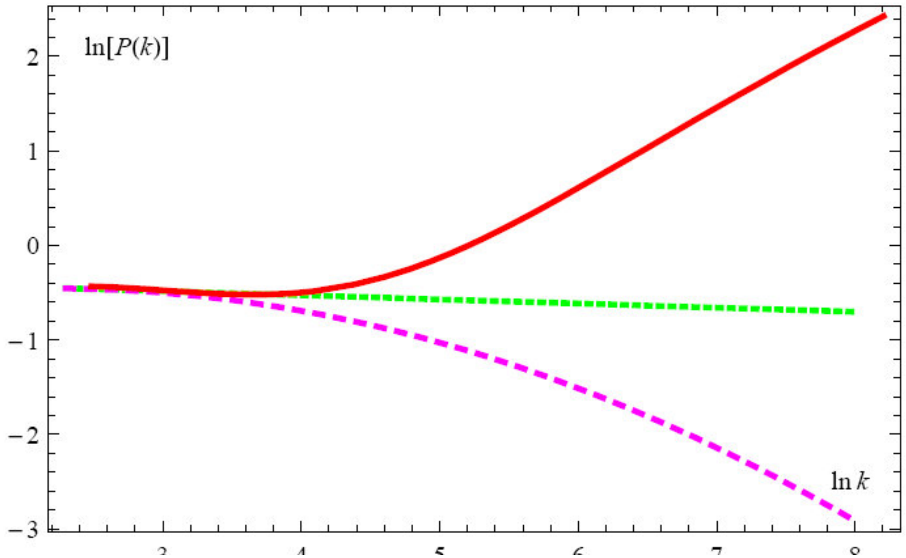
Potential parameters					Range		CMB observables		
r_0	η_0	α_0	a_4	a_5	$\Delta\phi_{50}$	$\Delta\phi_{60}$	n_s	r	α
$*$ 0.10	0.015	0.03	-0.6102	0.709	0.567	1.0	0.96	0.10	-0.07
$*$ 0.02	0.01	0.005	-0.875	1.451	0.4	0.8	0.99	0.02	0.001
$*$ 0.08	-0.005	-0.02	-0.695	0.7567	0.566	1.0	0.97	0.08	-0.05
$*$ 0.10	0	0	-0.688	0.7591	0.573	1.0	1.01	0.10	-0.006
$*$ 0.05	-0.02	-0.03	-0.6834	0.7405	0.555	1.0	0.94	0.05	-0.06
0.01	0	0	-0.3919	0.538	0.485	1.0	0.99	0.01	0.001
0.02	0.108	0.003	0.0341	0	0.8	2.0	1.21	0.02	0.0054

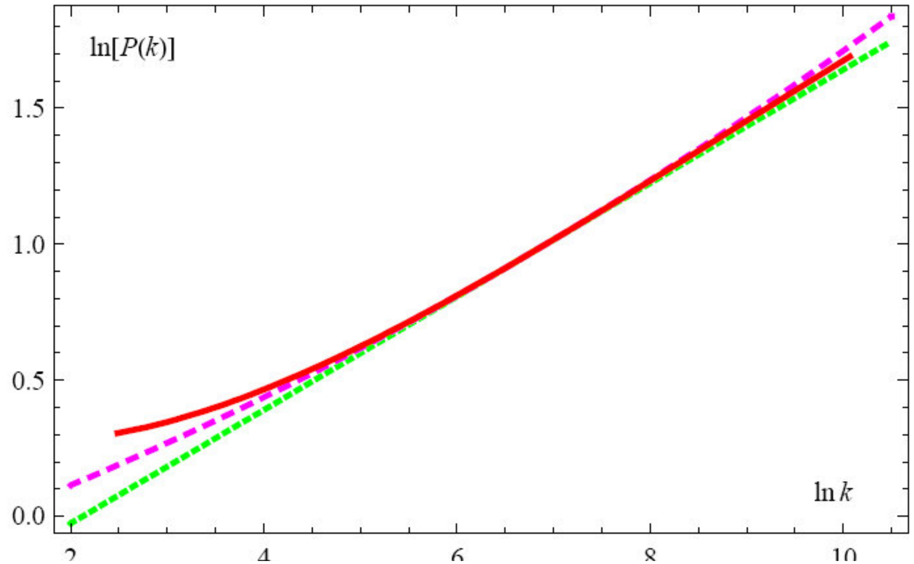
TABLE I: Listed are the values of the potential parameters, the range of inflaton motion after 50 and 60 e-folds and the values of the CMB observables, assuming that $N_{CMB} = 60$. The models appearing in Fig. 2 are marked with an asterisk. The last model is a renormalizable model with $a_5 = 0$.

New class of models: "Predictions"



New class of models: "Predictions"





Potential parameters					Range		CMB observables		
r_0	η_0	α_0	a_4	a_5	$\Delta\phi_{50}$	$\Delta\phi_{60}$	n_s	r	α
$*$ 0.10	0.015	0.03	−0.6102	0.709	0.57	1.0	0.96	0.10	−0.07

Potential parameters					Range		CMB observables		
r_0	η_0	α_0	a_4	a_5	$\Delta\phi_{50}$	$\Delta\phi_{60}$	n_s	r	α
0.02	0.108	0.003	0.0341	0	0.80	2.0	1.21	0.02	0.005

Table I. The calculated spectra are the red curves. The observables in the table are at the pivot scale of $\ln(k/k_0) = 2.5$ (left panel) and $\ln(k/k_0) = 6.7$ (right panel). The green dashed curves assumes just constant n_S, the dashed purple curves constant n_S and α. The red curves are the numerical spectra.

New class of small field models: EFT considerations

$$V = \Lambda^4 \left(1 + \sum_{n=1} \lambda_n (\phi/m_P)^n\right)$$

$$\Lambda \simeq 1 \times 10^{16}\ \mathrm{GeV}\ (r/.01)^{1/4}$$

$(E/\Lambda)^{+ve}$ $\lambda_n \ll 1,\ n \geq 4$

Small scale-separation

$$\min(\sqrt{\varepsilon}, \sqrt{\eta}) \frac{\Lambda}{m_p} \Lambda = \min(\sqrt{\varepsilon}, \sqrt{\eta}) H < E < \Lambda$$

λ_i i=1,2,3, special. For example $\lambda_1 = -.035(r/.01)^{1/2}$

$(E/\Lambda)^{-ve}$, $\lambda_3 << 1$

$$r_{0.01} = .5 \ \alpha_{0.05}^2 \widehat{\lambda}_3^{-2}$$

λ_i i=1,2,3, special. For example $\lambda_1 = -.035(r/.01)^{1/2}$

$\widehat{\lambda}_3 \equiv 3!\lambda_3$ Assume $\lambda_3 << 1$ to allow the energy range E > H $\eta^{1/2}$

$$\alpha \simeq 2\xi^2 = 2m_p^4 \frac{V'''V'}{V^2} \qquad m_p^3 \frac{V'''}{V} = 3! \ \lambda_3$$

$$r < 1, \alpha > .001 \Rightarrow 3\times10^{16} GeV > \Lambda > 1\times10^{15} GeV$$

$$\Lambda \simeq 1 \times 10^{16} \text{ GeV } (r/.01)^{1/4}$$

Non-Gaussianity

- The running does not scale as $1/(N_{CMB})^2$
- Integrating over the trajectory:

$$f_{NL} \simeq \frac{5}{6}\sqrt{2\epsilon_{CMB}} \int dM \frac{V'''(M)}{V(M)} = \frac{5}{6}\sqrt{2\epsilon_{CMB}} \int d\psi \frac{V'''(\psi)}{V'(\psi)}$$

- HOWEVER: Maldacena '03

single field ➔ boundary term (verified explicitly)

➔ Need additional fields? (as in SUGRA)

Conclusions for models of inflation

- Small field models of inflation are interesting

  (in my opinion most relevant to string/SUGRA)

- Predictions for the CMB:

  - Simplest models: $n_S<1$, $r_{0.01}<<1$, $\alpha_{0.05}<<1$

  - New class: n_S , $r_{0.01}$, $\alpha_{0.05}$ spans all allowed values

- RUN has a strong discriminating power among cosmological models, linked with high *r* in our models

Conclusions for inflation physics

- Simple "Reconstruction" = finding the inflaton potential from cosmological observables, is practically impossible

- Identifying "The Inflaton" is extremely hard

  - High scale inflation, inflaton an arbitrary direction n field space, moving over a limited range

  - Only window to inflaton dynamics through cosmological observables