Plan

Ground vs space interferometers

Gravitational waves and cosmology

Cosmic strings: cusps and kinks for strings with junctions

w/ A. Bohé, T. Hertog, D. Steer
Ground vs space interferometers
Gravitational wave detection
How to choose the size of an interferometer?

Size ~ Wavelength of the gravitational wave

~ \( \frac{c}{f} \)
Frequency $f$ of gravitational waves $\sim \sqrt{M/R^3}$

(Kepler law for binary systems)

Neutron stars ($M \sim 1.4M_\odot$) : $f \sim 100$ Hz

$\Rightarrow$ size $\sim 3000$ km

Supermassive black holes ($M \sim 10^6 M_\odot$) : $f \sim 10^{-4}$ to $10^{-2}$ Hz

$\Rightarrow$ size $\sim 30$ million km

Ground interferometers? LISA
On Earth, how to obtain the 3000 km necessary?
An international array of ground interferometers is ready to observe the gravitational wave sky
Albert Michelson counting interference fringes
Sensitivity of ground interferometers

LIGO Hanford Summer 2005

$1 \times 10^{-19} \text{ m/\sqrt{Hz}}$

$6 \times 10^9$ better than Michelson/Morley 1887!
The LISA interferometer

- 1 W emission 70 pW reception.
- Interference between the internal beam and the incident laser beam
- 12 laser beams:
  - 6 lasers between satellites
  - 6 internal reference lasers

5 $10^6$ km
• LISA requirement $40 \text{ pm/} \sqrt{\text{Hz}}$

only 15 times better than Michelson-Morley
LISA position

Earth-LISA distance
50 million kms
Galactic Binaries, including future type Ia supernovae

Compact Objects Orbiting Massive Black Holes, high-precision probes of strong-field gravity

Formation of Massive Black Holes, cores of active galactic nuclei, formed before most stars

Fluctuations from Early Universe, before recombination formed 3° background

COSMOLOGY with LISA
The cosmological evolution of the Universe in the « redshifted Hubble frequency » diagram

Inverse apparent horizon size at scale factor $a : H = \dot{a} / a$

\[ f = aH = \dot{a} \]
Inflation $a \propto e^{Ht}$
$log \dot{a} = \log a + ...$

Rad domin. $a \propto t^{1/2}$
$log \dot{a} = - \log a + ...$

Matter dom. $a \propto t^{2/3}$
$log \dot{a} = - \log a / 2 + ...$
MSP

VIRGO/LIGO

BBO

LISA

MSP

phase transition at 0.1 to 1000 TeV

superstrings

CMB

TeV inflation

log (a)

log (T/1 eV)

log (f/Hz)

cosmic time

-54 -45 -36 -27 -18 -9 0

0 9

-4.5 -3 -2 -1 0 1 2 3 4 5 6 7 8 9

-18 -12 -6 0

-15 -10 -5 0

-15 -10 -5 0
If gravitons were in thermal equilibrium in the primordial universe

\[ \Omega = \rho^{-1} \frac{d\rho}{d\log f} \]
When do gravitons decouple?

Interaction rate
\[ \Gamma \sim G_N^2 T^5 \sim \frac{T^5}{M_{Pl}^4} \]

Expansion rate
\[ H \sim \frac{T^2}{M_{Pl}} \] (radiation dominated era)

Gravitons decouple at the Planck era: fossil radiation
Gravitons produced at temperature $T_*$ with frequency $f_*$:

$$f_* = H_*/\varepsilon \text{ i.e. } \lambda_* = \varepsilon H_*^{-1}$$

Wavelength $\lambda_*$ and Horizon length $H_*$ provide a background observed at a redshifted frequency:

$$f = 1.65 \times 10^{-7} \text{ Hz} \quad \frac{1}{\varepsilon} \left( \frac{T_*}{1\text{GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6}$$
The electroweak phase transition

\[ f = 1.65 \times 10^{-7} \text{ Hz} \cdot \frac{1}{\epsilon} \left( \frac{T_\ast}{1 \text{ GeV}} \right) \left( \frac{g_\ast}{100} \right)^{1/6} \]
Gravitons produced at the electroweak phase transition would be observed in the LISA window.

The electroweak phase transition

\[
\Omega_{GW} \quad \text{(for } \varepsilon = 1) 
\]

\[
f = 1.65 \times 10^{-7} \text{ Hz} \quad \frac{1}{\varepsilon} \left( \frac{T_*}{1 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6}
\]
How to measure a stochastic background?

Cross correlate ground interferometers

Let LISA move around the Sun
A milestone recently passed by ground interferometers

The sensitivity achieved by the LIGO detectors at $f = 100$ Hz has allowed to constrain GW backgrounds in yet unchartered territories.
\[ \Omega_{GW} = \frac{1}{\rho_c} \frac{d \rho_{GW}}{d \log f} \]

, \( \rho_c = \frac{3H_0}{8\pi G_N} \)

Gravitons present at the time of decoupling (CMB) or at nucleosynthesis (BBN) may alter the rate of expansion.
Studying the electroweak phase transition with gravitational waves
Are gravitons produced in sufficient numbers at the electroweak phase transition?

If the transition is first order, nucleation of true vacuum bubbles inside the false vacuum

Collision of bubbles and (MHD) turbulence → production of gravitational waves
Pros and cons for a 1st order phase transition at the Terascale:

• in the Standard Model, requires $m_h < 72$ GeV (ruled out)
• MSSM requires too light a stop but generic in NMSSM
• possible to recover a strong 1st order transition by including $H^6$ terms in SM potential
• other symmetries than $SU(2) \times U(1)$ at the Terascale ($\rightarrow$ baryogenesis)
Two basic parameters:

\[ \alpha = \frac{E_{\text{false vac}}}{aT_*^4} \]

\[ \beta = \text{time variation of bubble nucleation rate} \]

\[ h_0^2 \Omega_{GW} \]

\[ \beta^{-1} \sim 10^{-2} H^{-1} \]

\[ f \text{ in mHz} \]

\[ \text{turbulence} \]

\[ \text{bubble collision} \]

\[ \text{duration of phase transition} \]

\[ \text{radiation energy at transition} \]

Nicolis gr-qc/0303084
Caprini, Durrer and Servant in 0909.0622 [astro-ph.CO] model more realistically the spectrum of MHD turbulence

\[ T_\ast = 100 \text{ GeV}, \beta^{-1} \sim H^{-1}/100 \]

\[ T_\ast = 5 \times 10^6 \text{ GeV}, \beta^{-1} \sim H^{-1}/50 \]
LISA potential reach is even higher than LHC

e.g. phase transition in RS1 model expected to be strongly first order

L. Randall, G. Servant hep-ph/0607158

Low temperature phase: Planck brane and TeV brane

High temperature phase: AdS$_5$-Schwarzschild with black hole horizon

Creminelli, Nicolis, Rattazzi
\( \alpha = 26, \beta/H = 21, T = 490 \text{GeV} \)

\( \alpha = 1.6, \beta/H = 350, T = 18300 \text{GeV} \)
Studying flat directions with gravitational waves
Inflation

Vacuum fluctuations: de Sitter phase

\[ h_0^2 \Omega_{GW} = 10^{-13} (f_{eq}/f)^2 \left(\frac{H}{10^{-4}M_{Pl}}\right)^2 \]

Fluctuations reenter horizon during **matter era**

\[ h_0^2 \Omega_{GW} = 10^{-13} \left(\frac{H}{10^{-4}M_{Pl}}\right)^2 \]

**radiation era**
\[ h_0^2 \Omega_{GW} \sim V f^{n_T} \]

\[ n_T = -\left(\frac{V'}{V}\right)^2 M_{Pl}^2 / 8\pi = -T/7S \]
Pre-bigbang scenario
Production of gravitational waves during reheating

- chaotic inflation

Khlebnikov, Tkachev; Easther, Lim

preheating $\xrightarrow{\text{Parametric resonance}}$ population of highly occupied modes

$\sim$ matter waves

collisions $\rightarrow$ GW with $f \sim 10^7$ to $10^9$ Hz

- hybrid inflation

tachyonic preheating

Felder, Garcia-Bellido, Greene, Kofman, Linde, Tkachev

expect lower frequencies

Easther, Giblin, Lim; Garcia-Bellido, Figueroa
High scale hybrid inflation
\[ v = 10^{-3} \text{M}_P, \lambda \sim g^2 \sim 10^{-5} \]

Low scale hybrid inflation
\[ v = 3.10^{-7} \text{M}_P, \lambda \sim g^2 \sim 10^{-14} \]

H ~ 100 GeV

Dufaux, Felder, Kofman, Navros 0812.2917 [astro-ph]
Probing flat directions of scalar potential: non-perturbative resonant effects may lead to explosive decays of flat directions.

J.F. Dufaux 0902.2574 [astro-ph.CO]
Strings
Cosmic strings

String energy does not grow with expansion because strings form loops which lose energy through gravitational (or other) radiation

A string network evolves toward a scaling regime where

$$L_{\text{string}} \sim \xi \sim t$$
$$L_{\text{wiggles}} \sim \alpha t$$
$$\alpha \propto G\mu$$

$$L_{\text{loops}} \equiv L = \alpha t$$

number density of loops $n(t) = \alpha^{-1} t^3$
Gravitational radiation comes predominantly from the loops and the distribution of the size of the loops is still controversial (Polchinski…)

loops which died during matter dom. loops which died during radiation dom.

Flat spectrum height $\propto G_\mu$
\[ G_\mu < 10^{-6} \]

Value of \( G_\mu \)
Cusps and kinks

Presence of cusps and kinks enhances the production of gravitational waves (frequencies much larger than the frequency of the fundamental mode of the string)

\[ x(\sigma,t) = \frac{1}{2} \left( a(\sigma+t) + b(\sigma-t) \right) \]

gauge conditions: \( a'^2 = b'^2 = 1 \)

Gravitational radiation enhanced for:

cusps \( \exists (\sigma_c,t_c), \ a' (\sigma_c+t_c) = - b' (\sigma_c-t_c) = n \)

\( (|x(\sigma_c,t_c)|=1 : \text{moves at speed of light}) \)

kinks \( a' \) or \( b' \) discontinuous
Why?

Linearized Einstein equations:

$$h_{ij}^{(TT)}(x, \omega) = \frac{4G}{r} e^{i\omega r} T_{ij}^{(TT)}(\omega n, \omega)$$

$$k = \omega n$$

$$u = \sigma + t, \quad v = \sigma - t$$

$$T_{ij}(k, \omega) = \mu \int dt \int_{s_B(t)}^{s_H(t)} (\dot{x}_i \dot{x}_j - x'_i x'_j) e^{i(\omega t - k \cdot x(\sigma, t))} d\sigma$$

$$= \frac{\mu}{2} \int dt \int_{s_A(t)}^{s_H(t)} a'_i(\sigma + t) b'_j(\sigma - t) e^{i(\omega t - k \cdot x(\sigma, t))} d\sigma$$

$$T_{ij} \propto (\int a'_i(u) e^{\frac{i}{2}(\omega u - k \cdot a(u))} du) (\int b'_j(v) e^{-\frac{i}{2}(\omega v + k \cdot b(v))} dv).$$
\[
I(\omega) = \int_a^b f(t) \ e^{-i\omega\phi(t)} \ dt
\]

I(\omega) \rightarrow 0 \text{ as } \omega \rightarrow \infty \text{ faster than any power of } 1/\omega \text{ unless:}

- there is a saddle point \( t_c \) where \( \phi(t_c) = 0 \)
- \( f \) or \( \phi \) are discontinuous

With the previous integrals, this means:

- \( \phi(u) = u - n.a(u) \Rightarrow 1 = n.a'(u_c) \)
  and \( \phi(v) = v + n.b(u) \Rightarrow 1 = -n. (v_c) b' \)
- \( a' \) or \( b' \) discontinuous
Amplitude \( h(f) \sim \frac{G \mu L}{[(1+z)Lf]^\delta} \frac{1+z}{t_0 z} H[\theta_m(f,L,z) - \theta] \)

\[ \theta_m(f,L,z) = [f(1+z)L]^{-1/3} \]

\( t_0 \) age of Universe

Cusps \( \delta = 1/3 \)

Kinks \( \delta = 2/3 \)

Kink-kink \( \delta = 1 \)

Confusion noise

Choice \( \epsilon \equiv \alpha/(50G\mu) = 1 \)

L = \alpha t
Fundamental strings

- reconnection probability $p$: when 2 strings collide, they either pass through each other (proba $1-p$) or reconnect (proba $p$).

**Reason**: they may miss each other in the higher dimensions

- existence of multi-string junctions

Copeland, Kibble, Steer, hep-th/0601153,061243
Salmi, Achucarro, Copeland, Kibble, de Putter, Steer, 0712.1204 [hep-th],
Brandenberger, Firouzjahi, Karouby 0710.1636 [hep-th]
Suyama 0807.4355 [astro-ph]
Davis, Nelson, Rajamanoharan, Sakellariadou 0809.2263 [hep-th]
Brandenberger, Firouzjahi, Karouby, Khosravi 0810.4521 [hep-th]
\[ p = 5 \times 10^{-3} \quad \text{and} \quad G_{\mu} = 10^{-7} \]
large long-lived loops

$p$ reconnection probability

Siemens, Mandic, Creighton
astro-ph/0610920

LISA
p=10^{-3}
Strings with junction

A. Bohé, T. Hertog, D. Steer, P.B.

arXiv 0907.4522 [hep-th]
+ 2 papers in preparation
\[ \sigma_1 = s_{B,1}(t) \]

\[ \sigma_1 = s_{A,1}(t) \]

string worksheet
$T_{ij}(\omega, \omega n) = \frac{\mu}{4} \int_{-\infty}^{\infty} b'_j(v)e^{-\frac{\omega}{2}(v+n\cdot b(v))} \left( \int_{u_A(v)}^{u_B(v)} a'_i(u)e^{\frac{\omega}{2}(u-n\cdot a(u))} du \right) dv.$
Results: $h(f) \propto f^{-\delta}$

- $\delta = 1/3$: cusps (emitting in a given direction)
- $\delta = 2/3$: kinks (emitting in a fan-like direction)
- $\delta = 2/3$: string expanding at speed of light at a junction (given direction)
- $\delta = 1$: R-moving kink and R-moving kinks passing through each other (all directions)
- $\delta = 1$: kink passing through a junction (all directions)
Note: gravity wave emission may not be as large for cusps with fundamental strings

O’Callaghan, Chadburn, Geshnizjani, Gregory, Zavala 10034395 [hep-th].

Extra dimensions round off cusps and reduce the probability of their formation

n extra dimensions
Proliferation of kinks through junctions

\[
\mathbf{a}'_j(s_j(t) + t) = \frac{1}{1 + \dot{s}_j} \left[ (1 - \dot{s}_j) \mathbf{b}'_j (s_j(t) - t) + 2 \dot{\mathbf{X}} \right]
\]

where \( \dot{\mathbf{X}} = -\frac{1}{\sum \mu_k} \sum \mu_k (1 - \dot{s}_k) \mathbf{b}'_k (s_k(t) - t) \) is the velocity of the junction

Hence kink propagating towards the junction (disc. in \( \mathbf{b}'_j \))

→ outgoing kinks on the 3 strings (disc. in \( \mathbf{a}'_i, i=1,2,3 \))
Example of a static junction

Initial state

Final state
Transmission coefficients: \[ C_i = \frac{\mathcal{A}[a'_{ij}]}{\mathcal{A}[b'_{1j}]} \]

gravitational wave amplitude
Dynamical junction

\[ \mu_1 = \mu_2 = \mu_3 \]

probability distribution

\[ P(C_i) \]

\[ \langle C_1 \rangle = 0.49, \langle C_2 \rangle = 0.72, \langle C_1 \rangle = 0.72 \]

\[ P(1 \text{ amplification}) = 0.43, P(2 \text{ amplifications}) = 0.07 \]
Amplification is sufficient to sustain the exponential growth of the number of large amplitude kinks

Example:

Define $Q_{j,n}^{a}$ the number of kinks on string $j$ of amplitude larger than $A$ at the $n$-th generation

$$\log Q_{1,n}^{0.5}$$
Limiting processes?

- loop dynamics: junction collision

- backreaction
Many observable consequences of this proliferation

E.g. kink-kink events are not favoured at large frequencies but proliferate even more quickly: they form a background that might swamp the cusps or kinks signals.

To show this, need a model of network evolution for strings with junctions:

Loops with no junctions $n(t) = \alpha^{-1} t^{-3}$

Loops with junctions $n'(t) = q n(t)$

Number of kinks formed by interval of time $L$:

$v'_{kinks} = (k'/L) n'(t)$
log(amplitude)

kink-kink bckgd

q_{k'}^2 = 10^6

q_{k'}^2 = 10^3

q_{k'}^2 = 1

cusps

Kinks (q=1, 10^{-3})

log(\alpha)
The kink-kink background for LISA
Conclusions

Detection of gravitational waves very complementary tool to probe the Terascale region.

Ground interferometers are now prospecting unchartered territory in their frequency window. Pulsar timing arrays as well.

LISA has a very promising programme for a large set of the cosmological issues relevant today.

Search for ways to probe fundamental scales: kink dynamics may provide this.

Kink proliferation through junctions may change the expected signatures: kink-kink background…