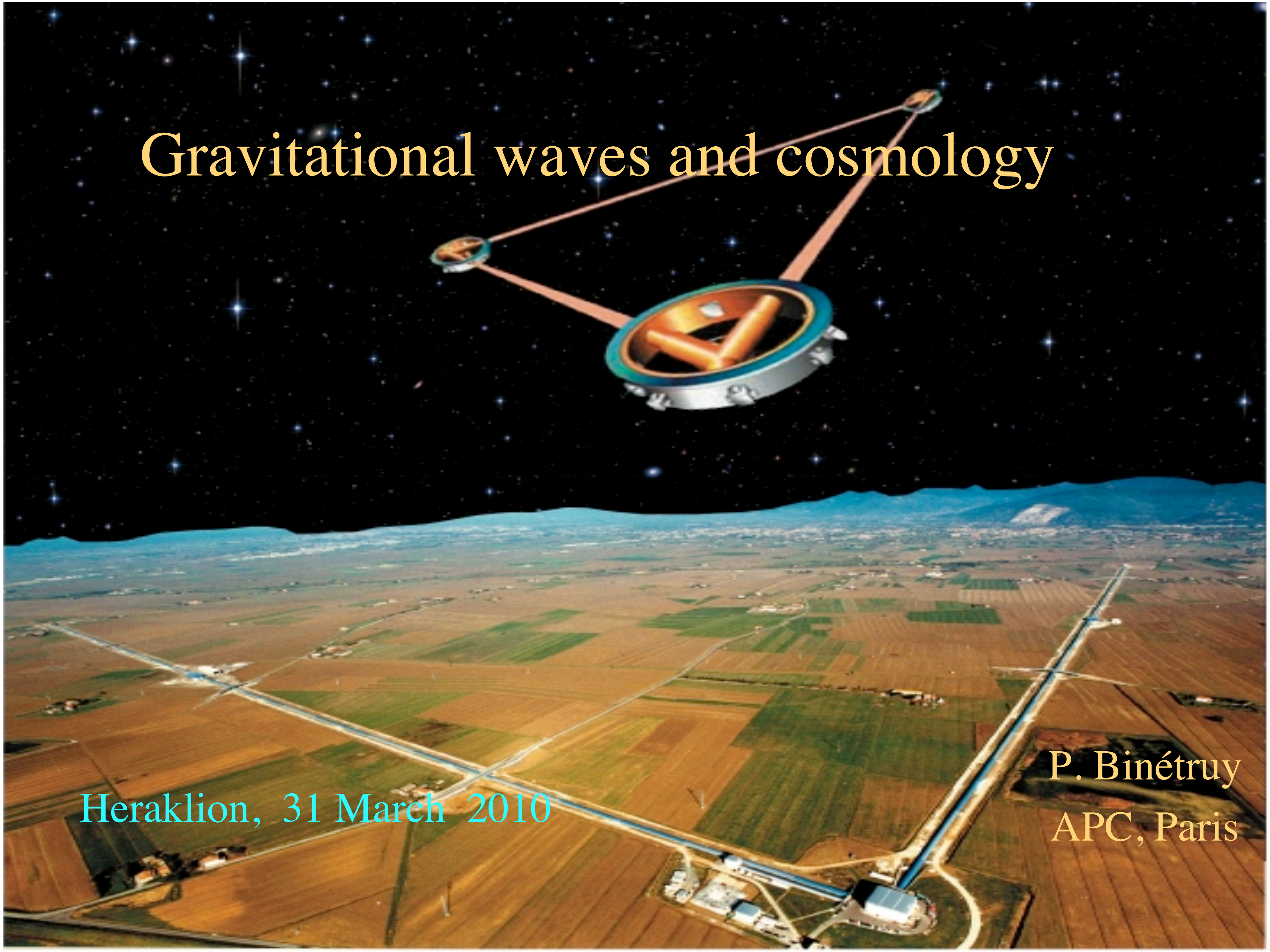

Gravitational waves and cosmology
P. Binétruy
APC, Paris
Heraklion, 31 March 2010

Plan

Ground vs space interferometers

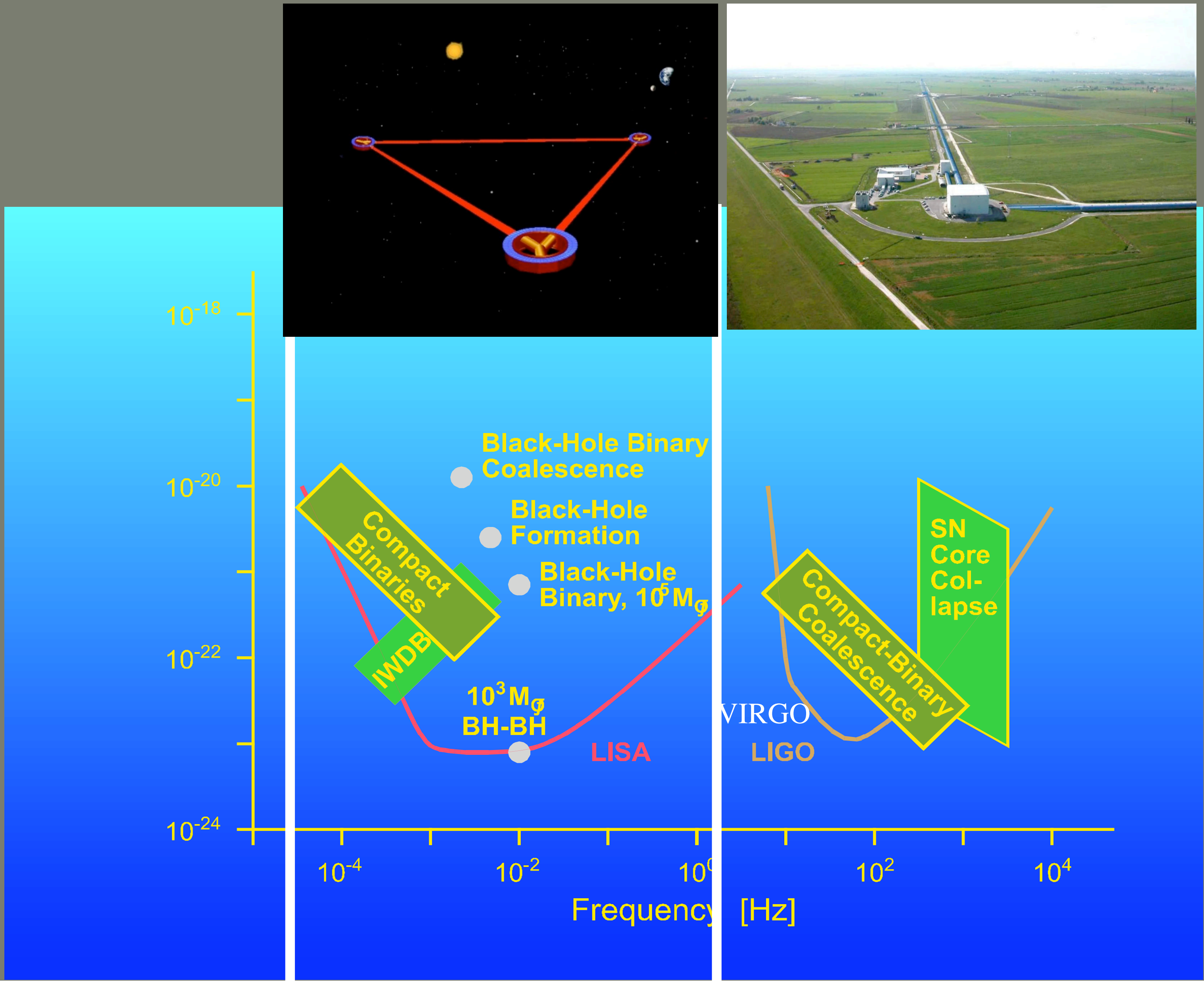
Gravitational waves and cosmology

Cosmic strings: cusps and kinks for strings with junctions

w/ **A. Bohé**, T. Hertog, D. Steer

Ground vs space interferometers

Gravitational wave detection



How to choose the size of an interferometer?

Size ~ Wavelength of the gravitational wave

~ c / f

Frequency f of gravitational waves $\sim \sqrt{M/R^3}$

(Kepler law for binary systems)

Neutron stars ($M \sim 1{,}4 M_{*}$) : $f \sim 100$ Hz

$\Rightarrow$ size ~ 3000 km

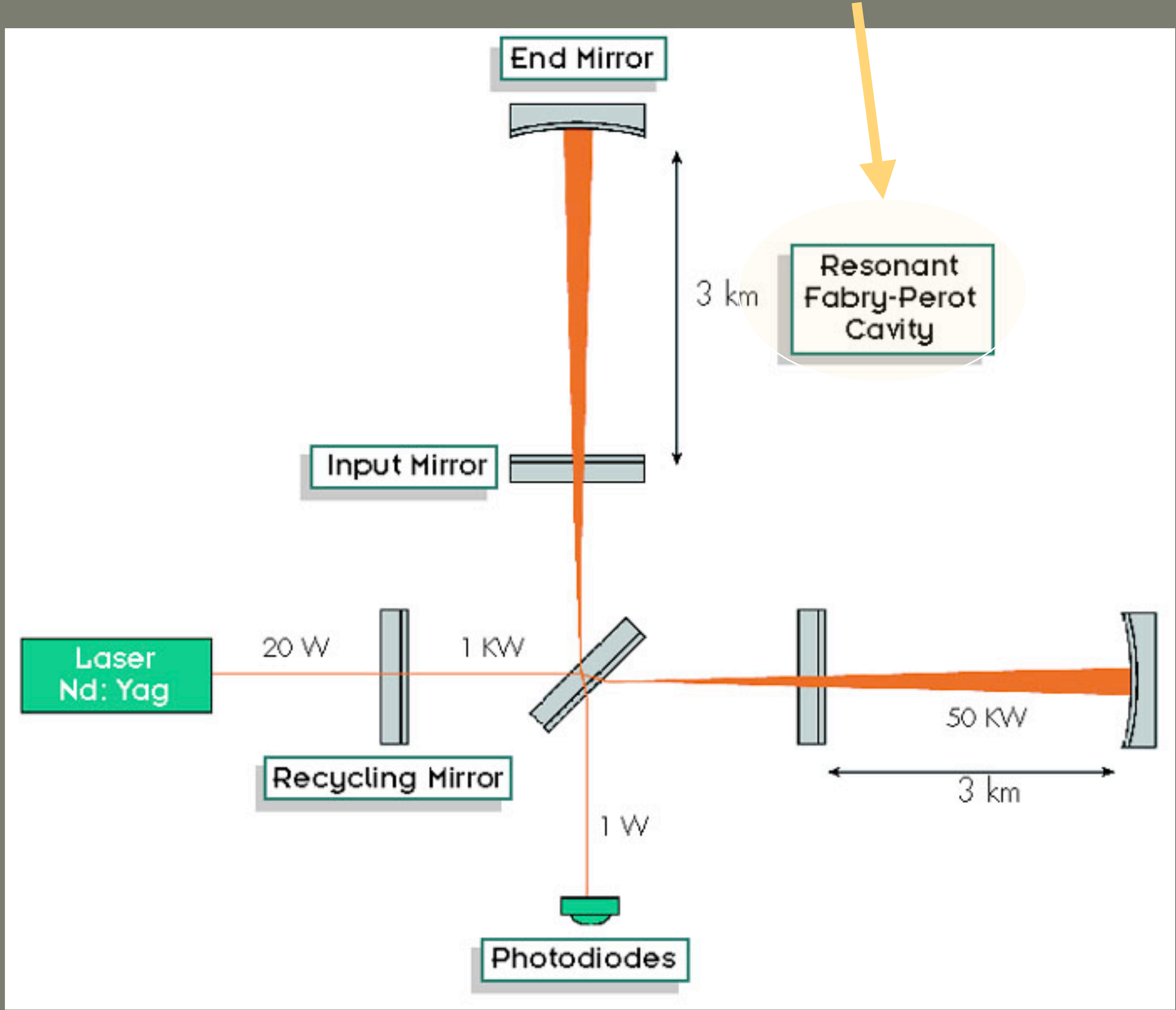
Ground interferometers?

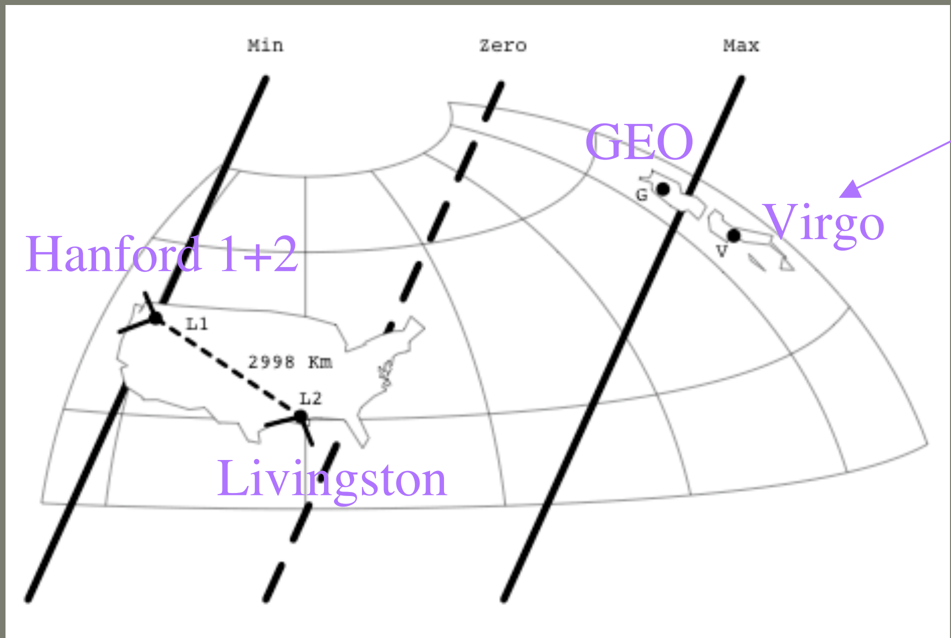
Supermassive black holes ($M \sim 10^6\, M_{*}$) : $f \sim 10^{-4}$ à 10^{-2} Hz

$\Rightarrow$ size ~ 30 million km

LISA

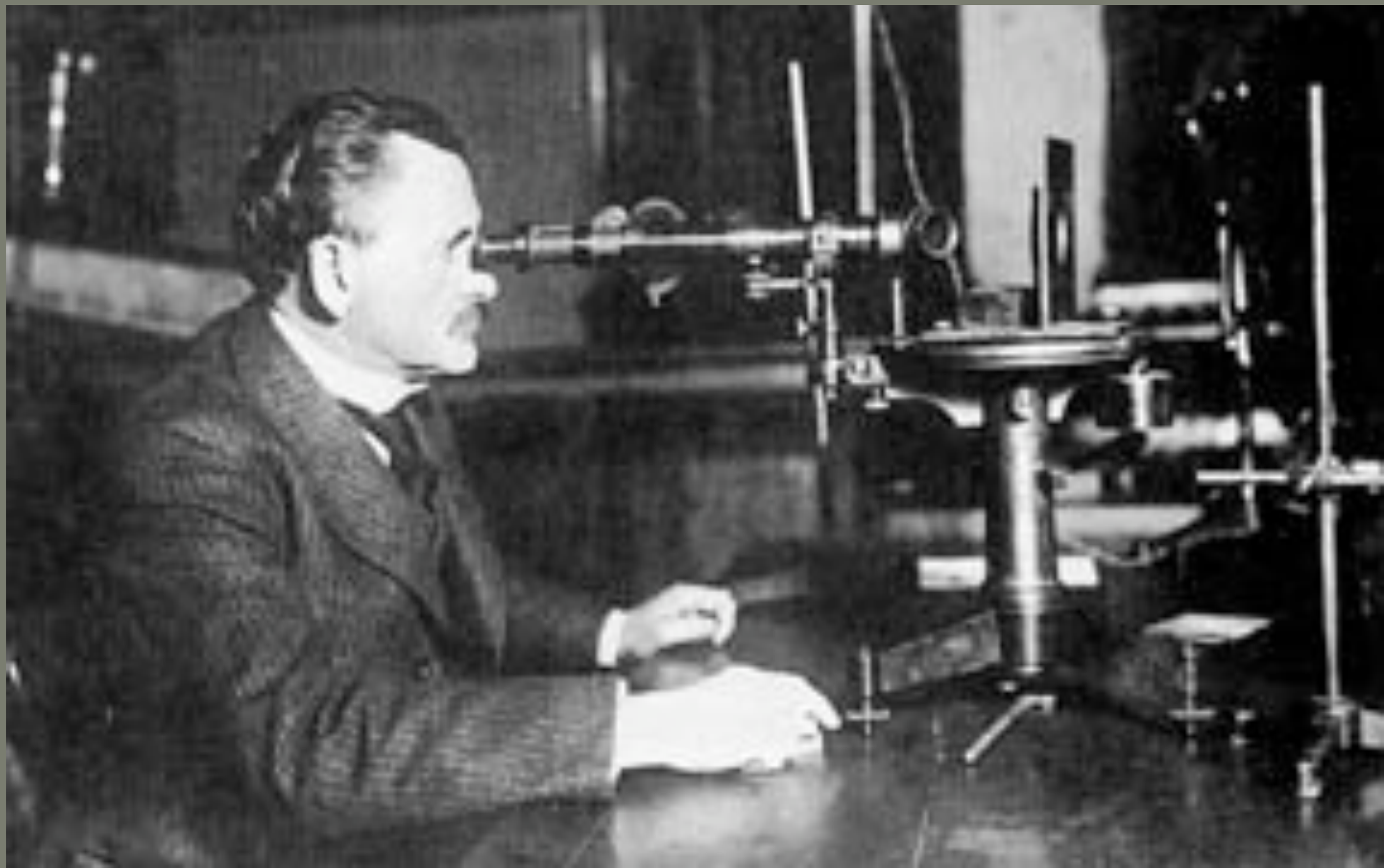
On Earth, how to obtain the 3000 km necessary?



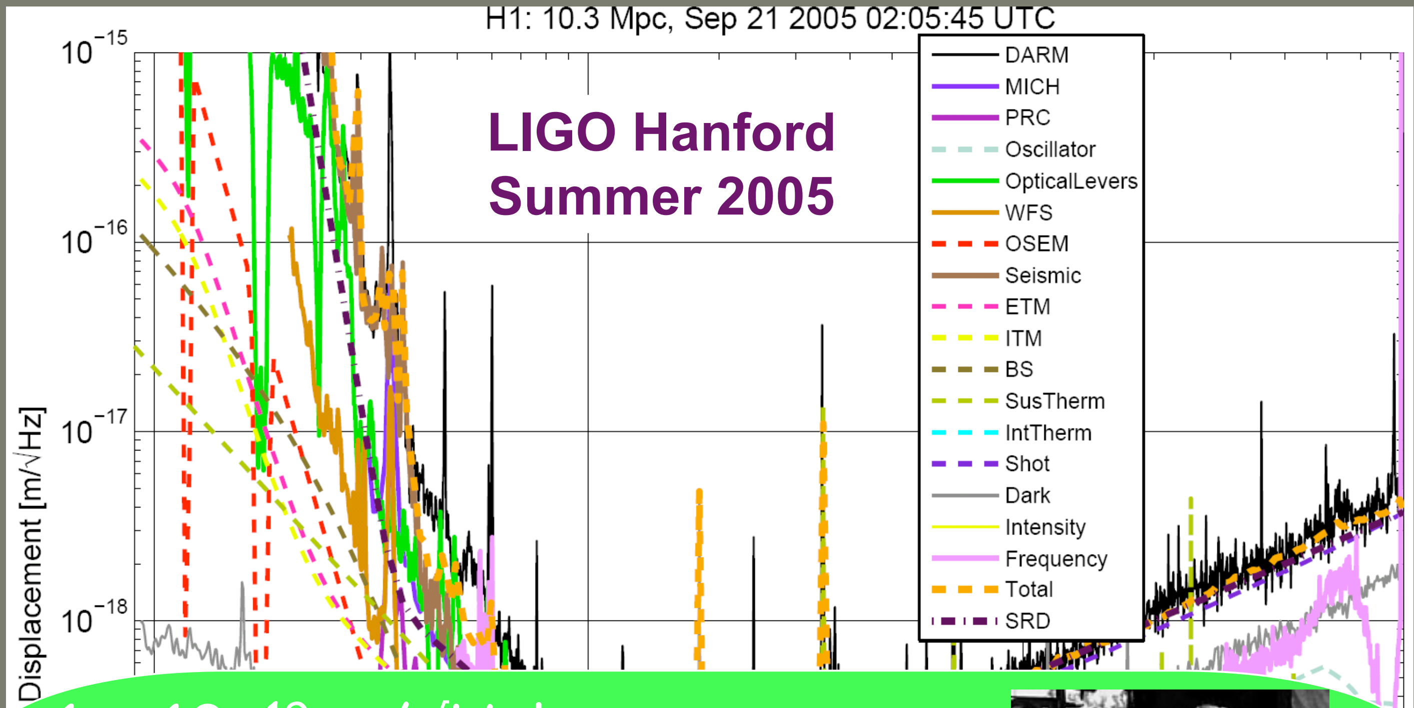


An international array of ground interferometers is ready to observe the gravitational wave sky

Albert Michelson counting interference fringes

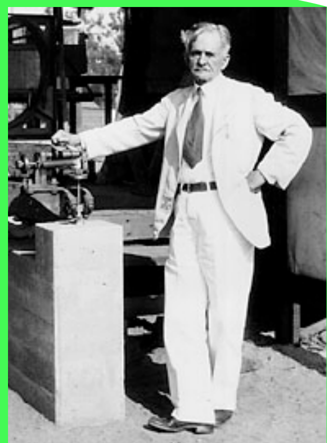


Sensitivity of ground interferometers

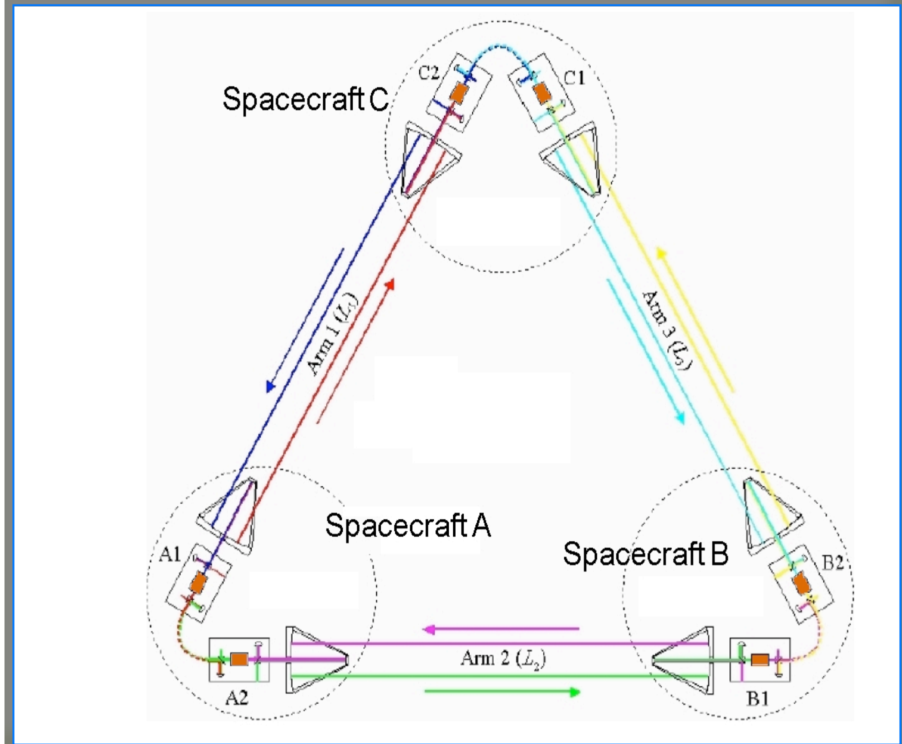


1×10^{-19} m/√Hz!

6×10^{9} better than Michelson/Morley 1887!

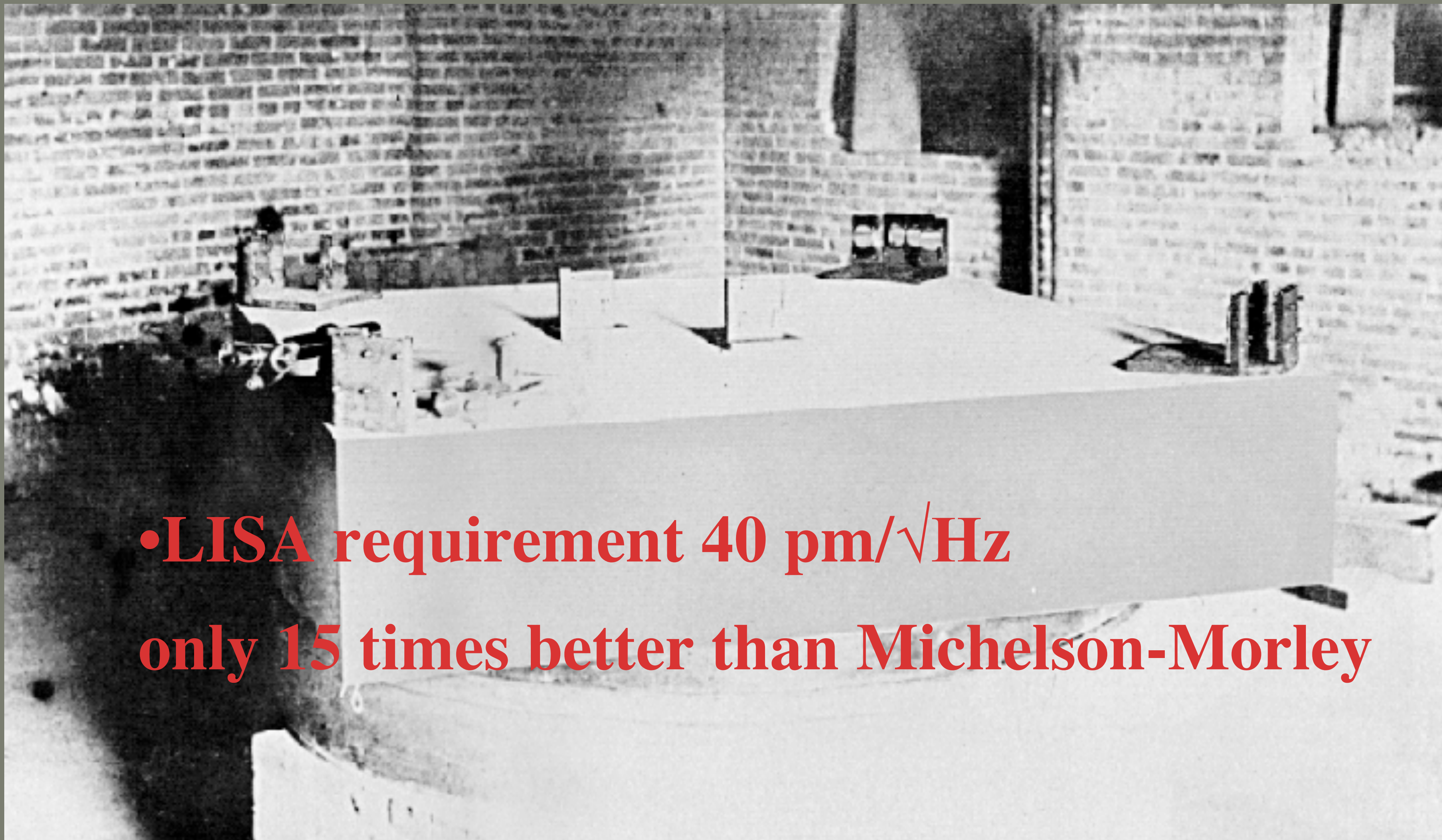


The LISA interferometer

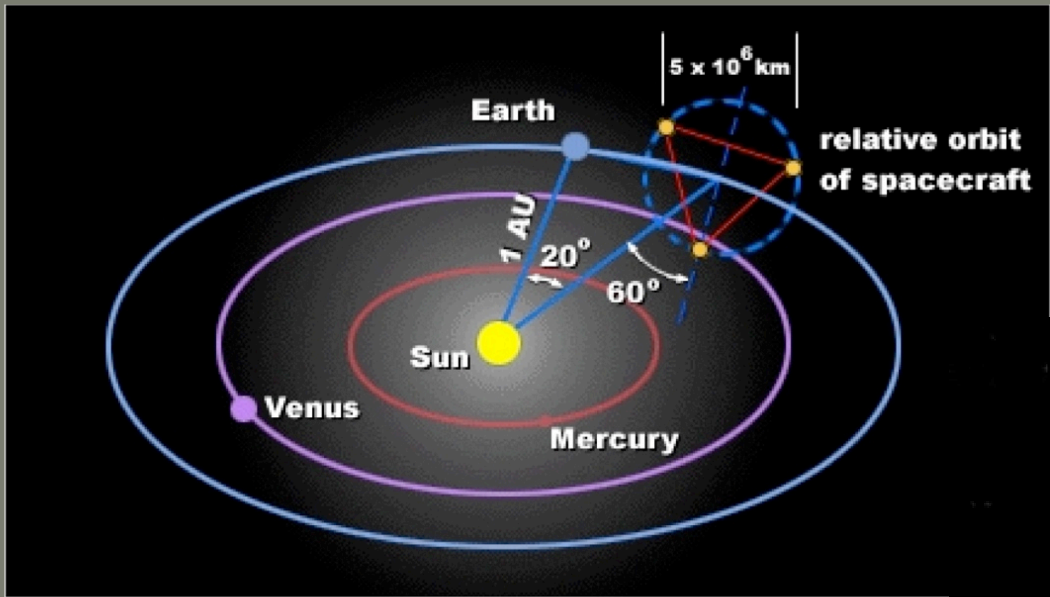


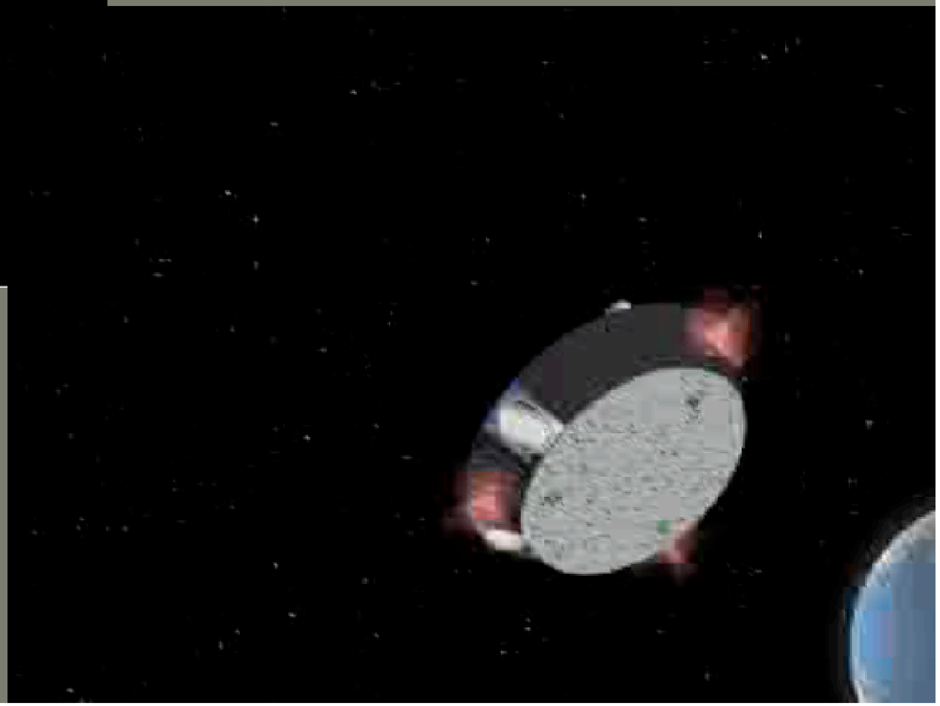
- 1 W emission 70 pW reception.
- Interference between the internal beam and the incident laser beam
- 12 laser beams:
  - 6 lasers between satellites
  - 6 internal reference lasers

$5 \ 10^6$ km

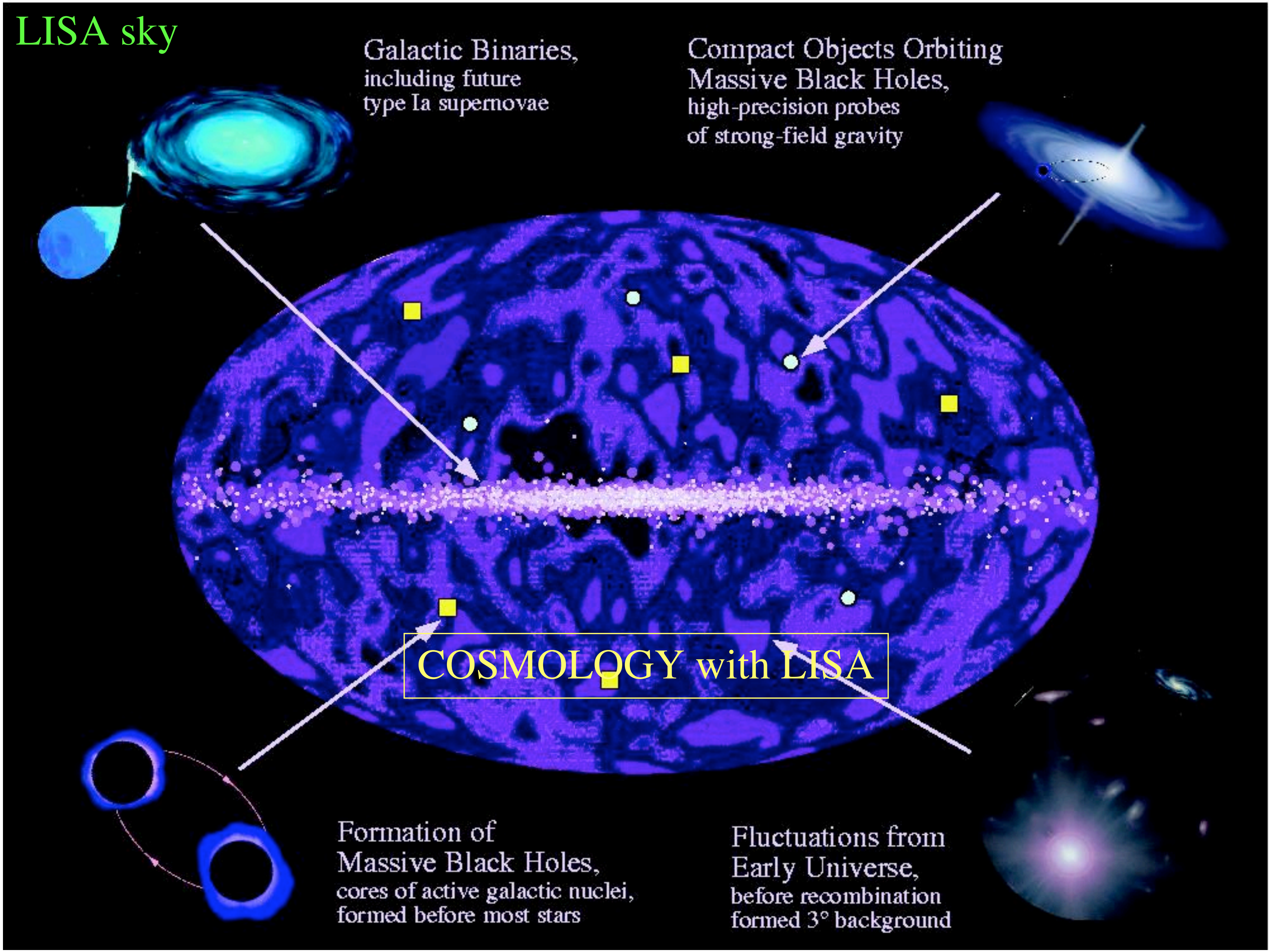

•LISA requirement 40 pm/√Hz
only 15 times better than Michelson-Morley

LISA position





Earth-LISA distance
50 million kms

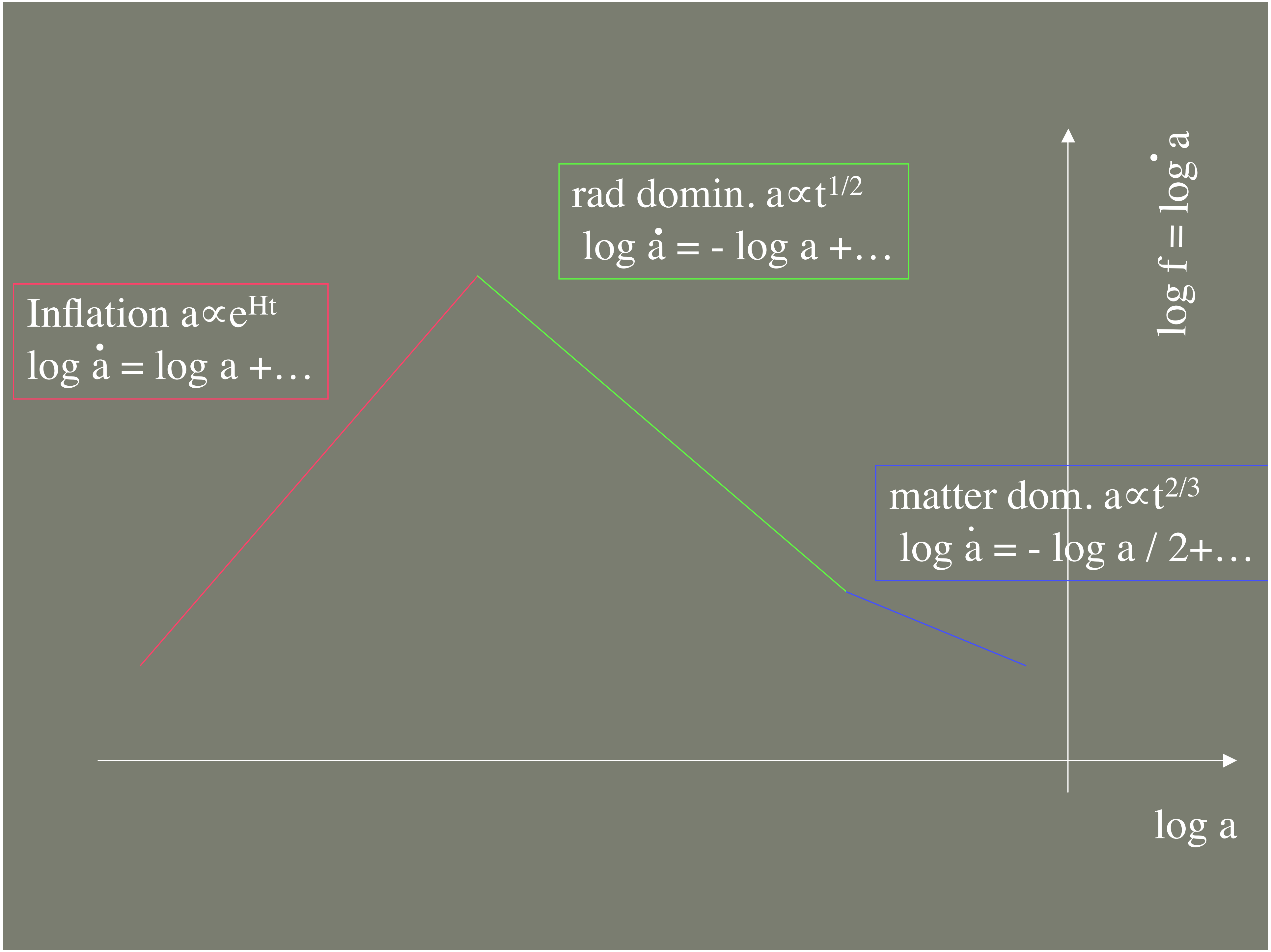

LISA sky
Galactic Binaries,
including future
type Ia supernovae
Compact Objects Orbiting
Massive Black Holes,
high-precision probes
of strong-field gravity
COSMOLOGY with LISA
Formation of
Massive Black Holes,
cores of active galactic nuclei,
formed before most stars
Fluctuations from
Early Universe,
before recombination
formed 3° background

The cosmological evolution of the Universe in the « redshifted Hubble frequency » diagram

Inverse apparent horizon size at scale factor a : $H = \dot{a} / a$

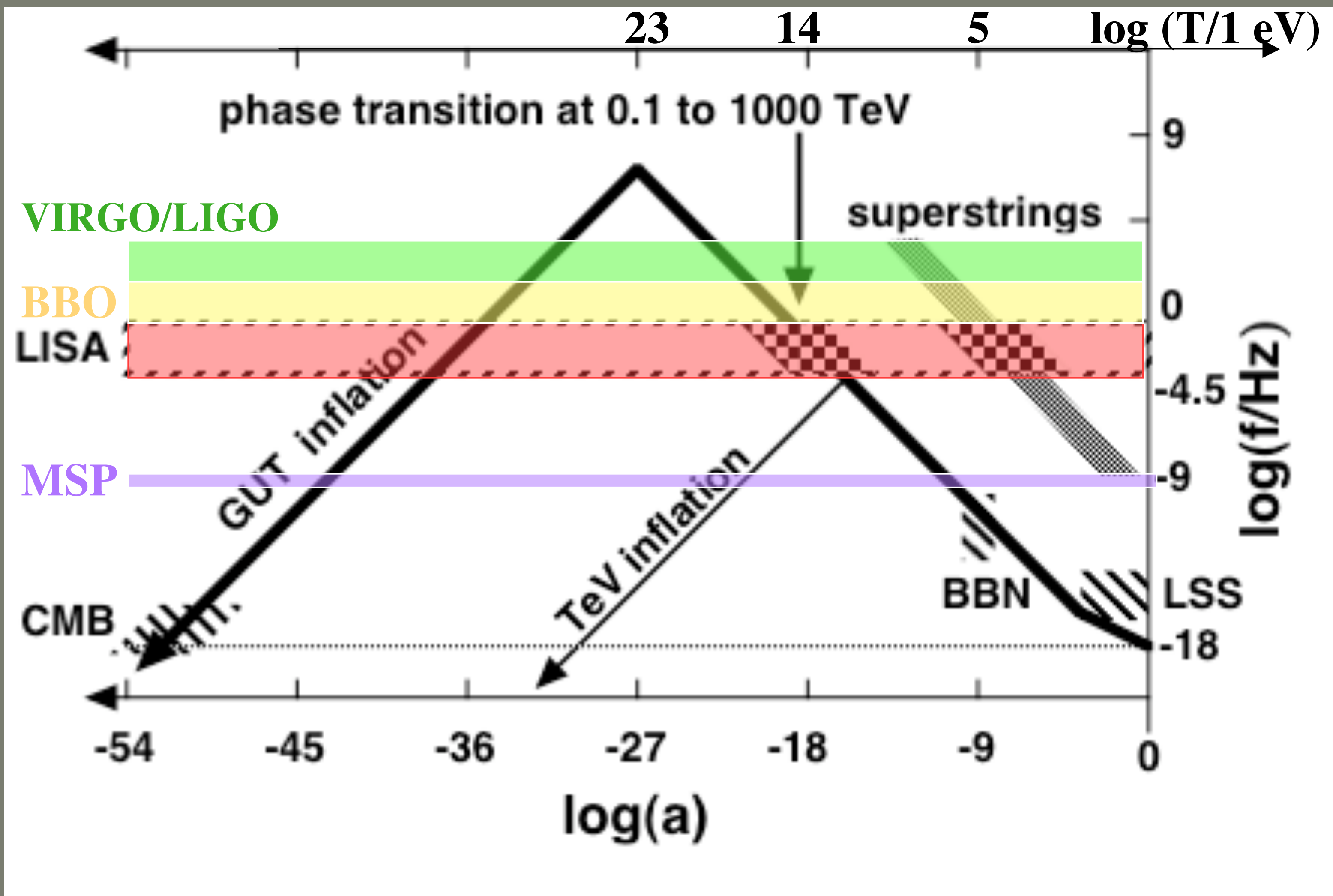
$$\Big\downarrow \text{ redshift}$$

Frequency observed today $f = aH = \dot{a}$



Inflation a∝e^Ht
log ȧ = log a +…
rad domin. a∝t^1/2
log ȧ = - log a +…
matter dom. a∝t^2/3
log ȧ = - log a / 2+…
log f = log ȧ
log a

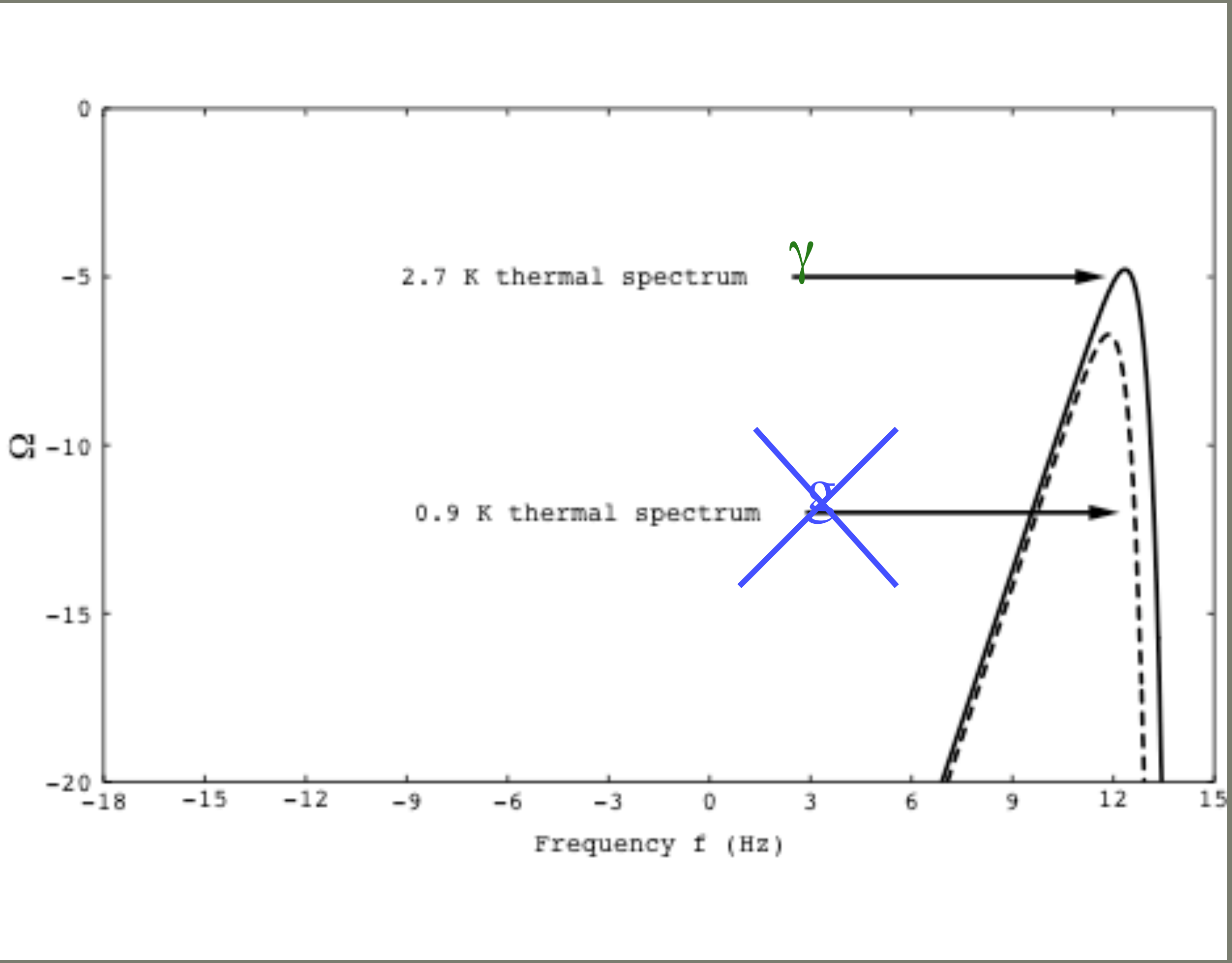
cosmic time



23
14
5
log (T/1 eV)
phase transition at 0.1 to 1000 TeV
9
superstrings
VIRGO/LIGO
BBO
0
LISA
-4.5
GUT inflation
MSP
-9
log(f/Hz)
TeV inflation
BBN
LSS
CMB
-18
-54
-45
-36
-27
-18
-9
0
log(a)

If gravitons were in thermal equilibrium in the primordial universe

$\Omega = \rho^{-1}\, d\rho/d\log f$



When do gravitons decouple?

Interaction rate $\Gamma \sim G_N^2\, T^5 \sim \dfrac{T^5}{M_{Pl}^4}$

Expansion rate $H \sim \dfrac{T^2}{M_{Pl}}$ (radiation dominated era)

$$\boxed{\frac{\Gamma}{H} \sim \frac{T^3}{M_{Pl}^3}}$$

Gravitons decouple at the Planck era : fossile radiation

Gravitons produced at temperature T_* with frequency f_* :

$$f_* = H_*/\varepsilon \text{ i.e. } \lambda_* = \varepsilon H_*^{-1}$$

Wavelength Horizon length

provide a background observed at a redshifted frequency :

$$\boxed{f = 1.65\ 10^{-7}\,\mathrm{Hz}\ \frac{1}{\varepsilon}\left(\frac{T_*}{1\mathrm{GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}}$$

The electroweak phase transition

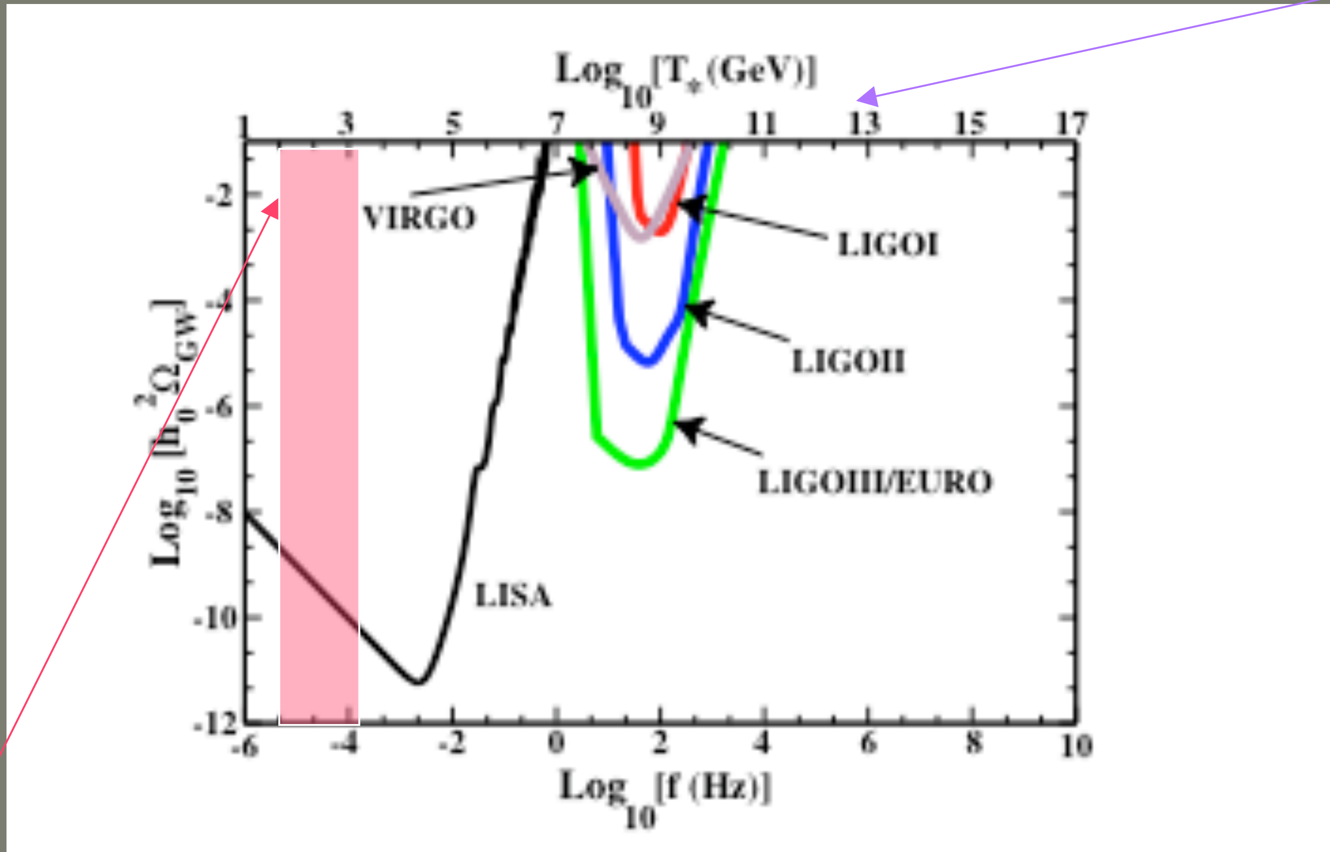
$$f = 1.65\ 10^{-7}\,\mathrm{Hz}\ \frac{1}{\varepsilon}\ \left(\frac{T_*}{1\mathrm{GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}$$

The electroweak phase transition

$$f = 1.65\ 10^{-7}\,\mathrm{Hz}\ \frac{1}{\varepsilon}\ \left(\frac{T_*}{1\mathrm{GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}$$

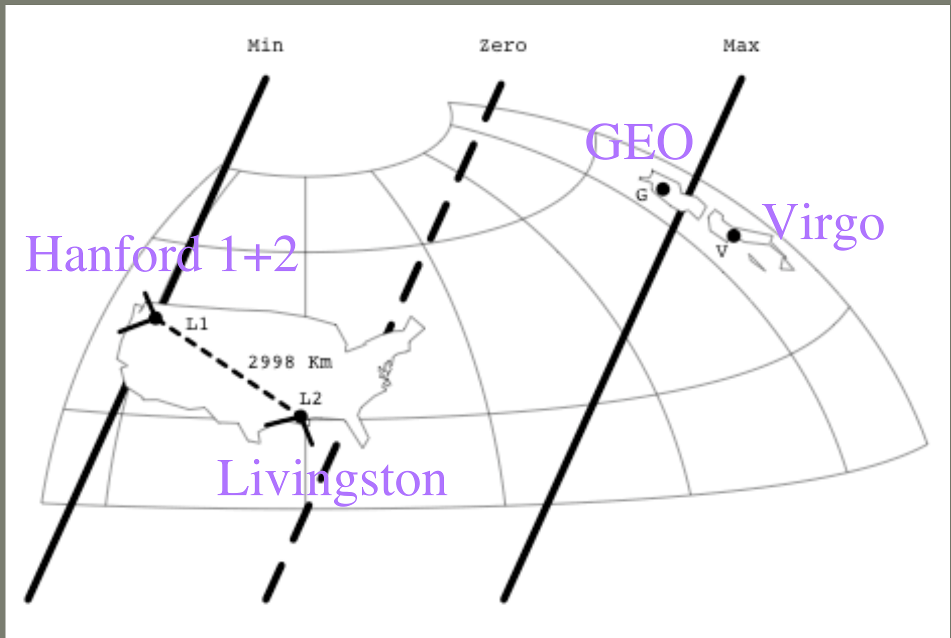
for ε=1

Ω_{GW}



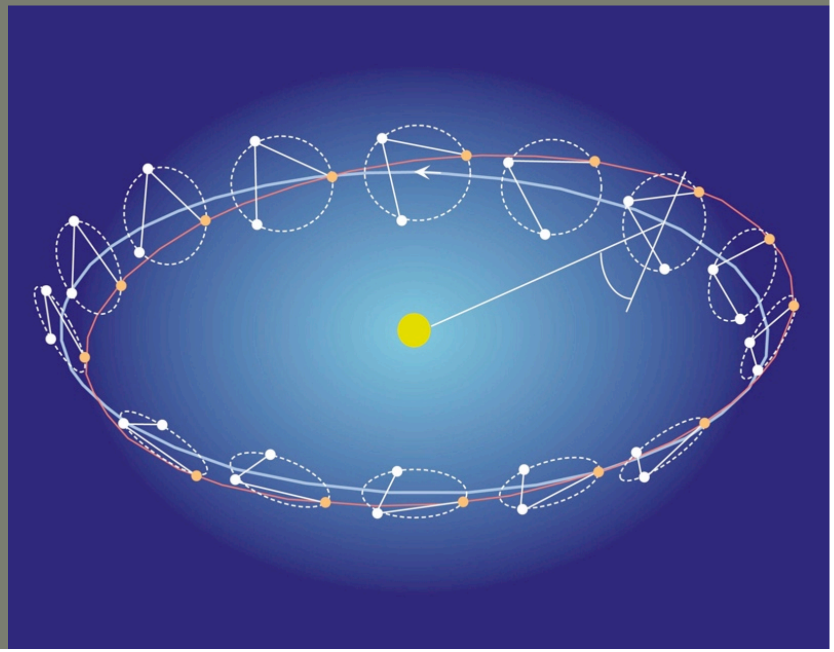
Gravitons produced at the electroweak phase transition would be observed in the LISA window.

How to measure a stochastic background?



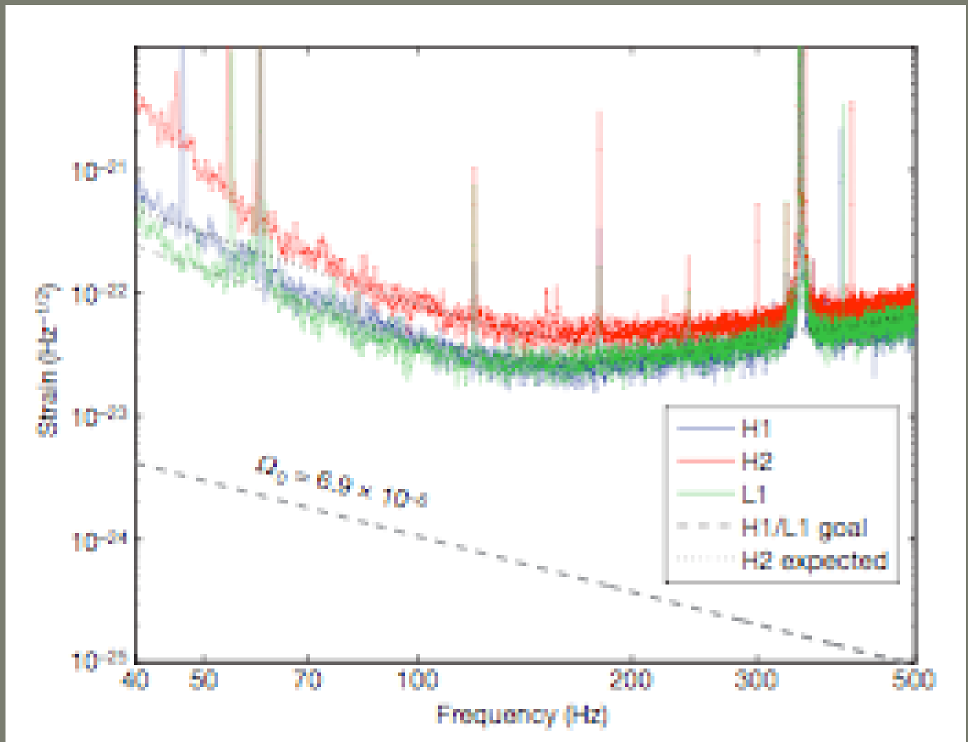
Cross correlate
ground interferometers

Let LISA move around the Sun



A milestone recently passed by ground interferometers

The sensitivity achieved by the LIGO detectors at f = 100 Hz has allowed to constrain GW backgrounds in yet unchartered territories.



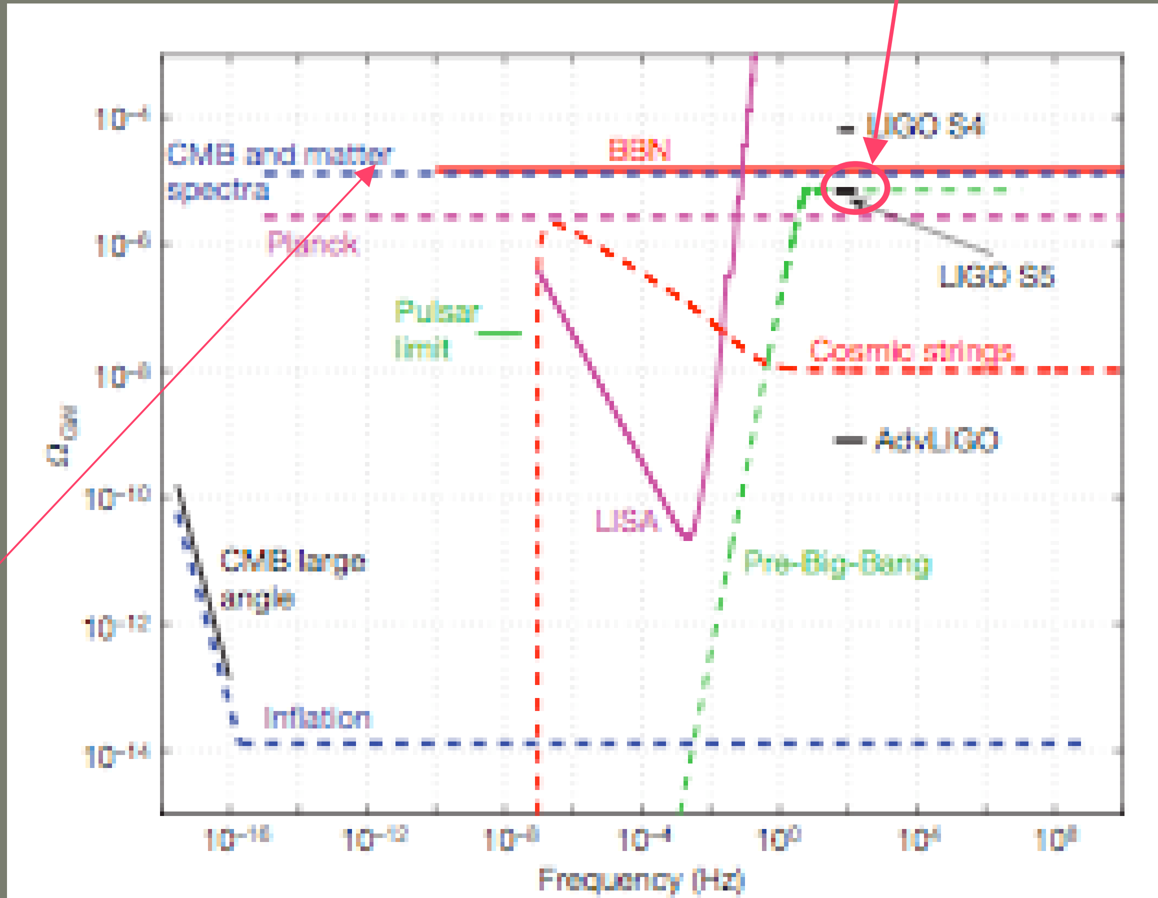
Hanford 1
Hanford 2
Livingston Parish

LIGO-VirgoNature 460, August 2009

$$\Omega_{GW} = \frac{1}{\rho_c} \frac{d\,\rho_{GW}}{d\,\log f}$$

, $\rho_c = 3H_0/(8\pi G_N)$

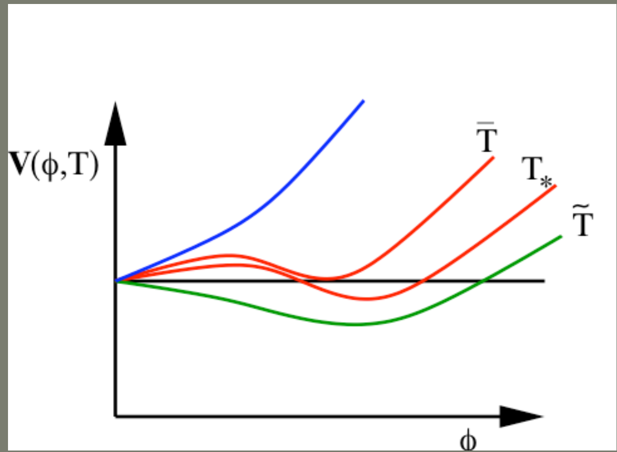
New LIGO scientific run



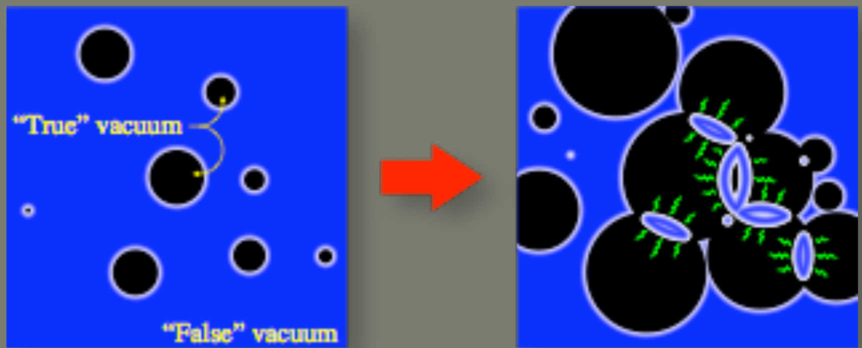
Gravitons present at the time of decoupling (CMB) or at nucleosynthesis (BBN) may alter the rate of expansion.

Studying the electroweak phase transition with gravitational waves

Are gravitons produced in sufficient numbers at the electroweak phase transition?



If the transition is first order, nucleation of true vacuum bubbles inside the false vacuum



Collision of bubbles and (MHD) turbulence
$\longrightarrow$ production of gravitational waves

Pros and cons for a 1st order phase transition at the Terascale:

- in the Standard Model, requires $m_h < 72$ GeV (ruled out)
- MSSM requires too light a stop but generic in NMSSM
- possible to recover a strong 1st order transition by including H^6 terms in SM potential
- other symmetries than SU(2)xU(1) at the Terascale ($\longrightarrow$ baryogenesis)

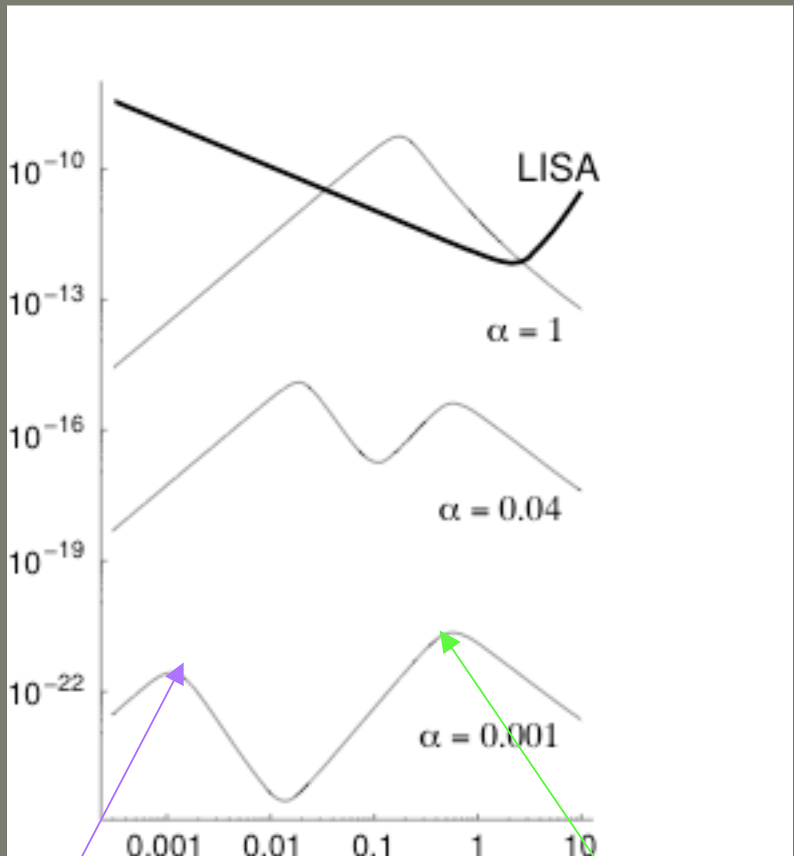
Two basic parameters :

$$\alpha = \frac{E_{\text{false vac}}}{aT_*^4}$$

β= time variation of bubble nucleation rate

radiation energy at transition

$h_0^2\,\Omega_{GW}$



f in mHz

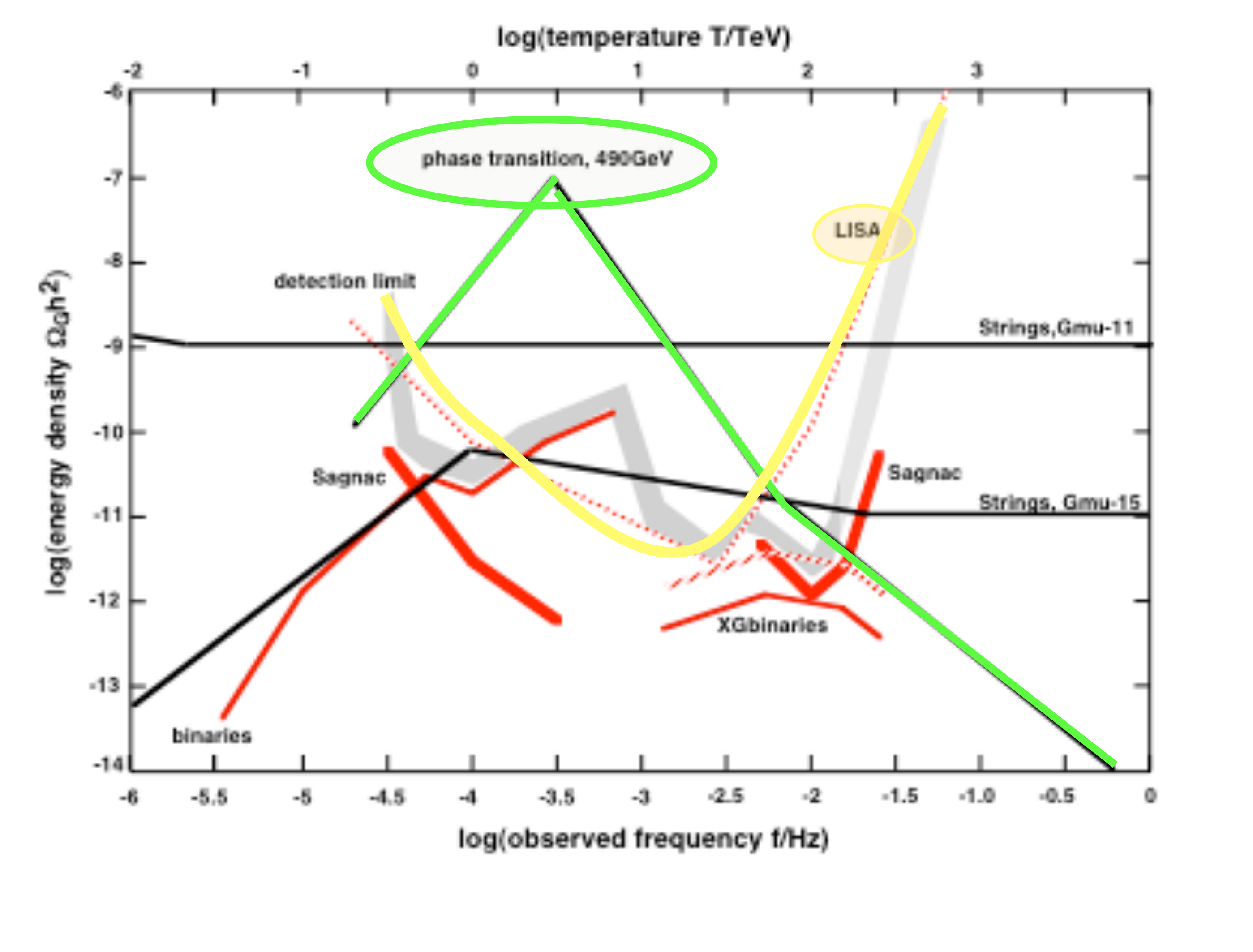
$\beta^{-1} \sim 10^{-2}\,H^{-1}$

duration of phase transition

Nicolis
gr-qc/0303084

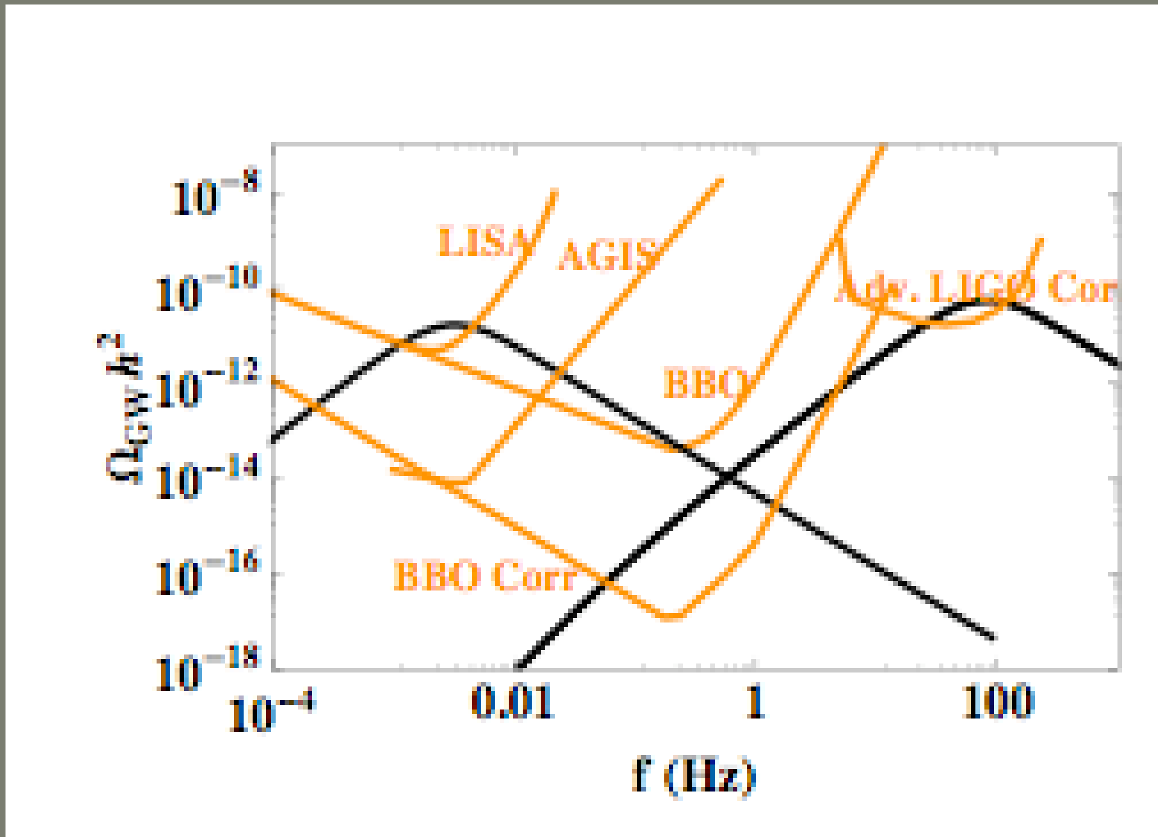
turbulence

bubble collision



log(temperature T/TeV)
-2
-1
0
1
2
3
phase transition, 490GeV
LISA
detection limit
Strings,Gmu-11
Sagnac
Sagnac
Strings, Gmu-15
XGbinaries
binaries
log(energy density $\Omega_G h^2$)
-6
-7
-8
-9
-10
-11
-12
-13
-14
-6
-5.5
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1.0
-0.5
0
log(observed frequency f/Hz)

Caprini, Durrer and Servant in 0909.0622 [astro-ph.CO] model more realistically the spectrum of MHD turbulence



$T_* = 100$ GeV, $\beta^{-1} \sim H^{-1}/100$

$T_* = 5.10^6$ GeV, $\beta^{-1} \sim H^{-1}/50$

LISA potential reach is even higher than LHC

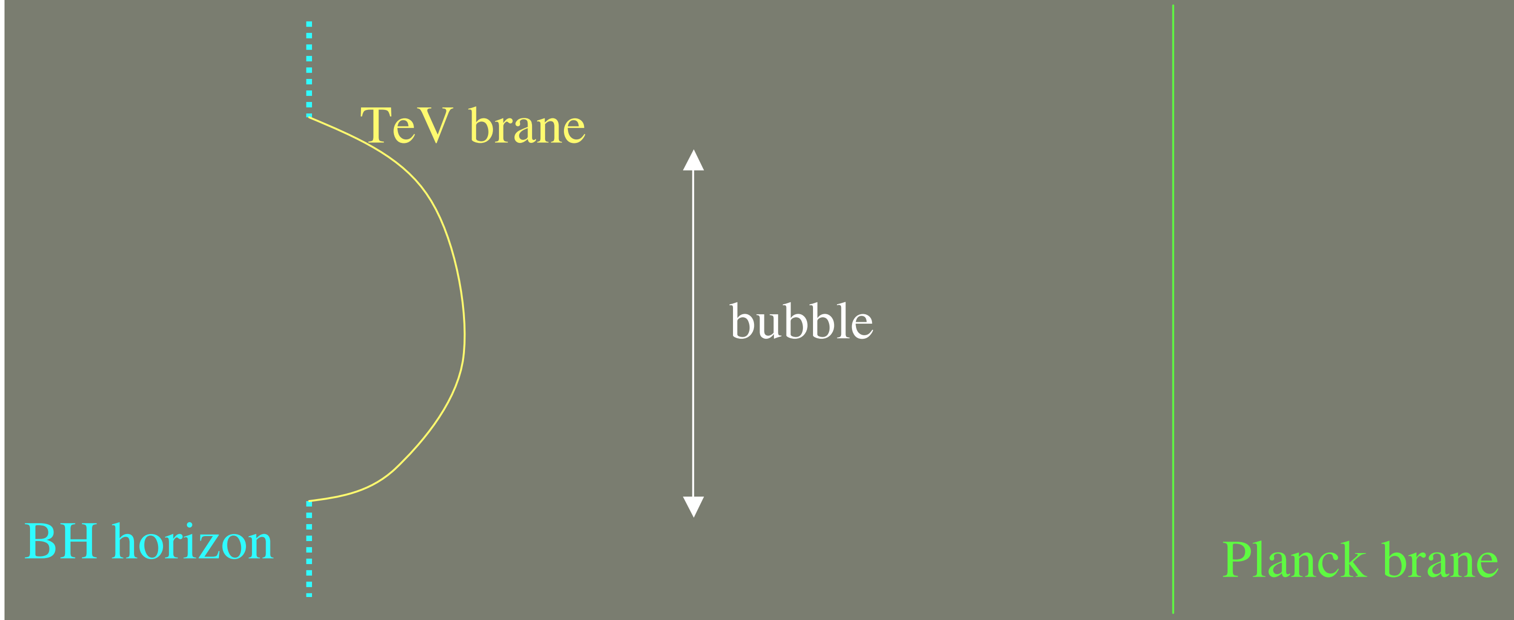
e.g. phase transition in RS1 model expected to be strongly first order

L. Randall, G. Servant hep-ph/0607158

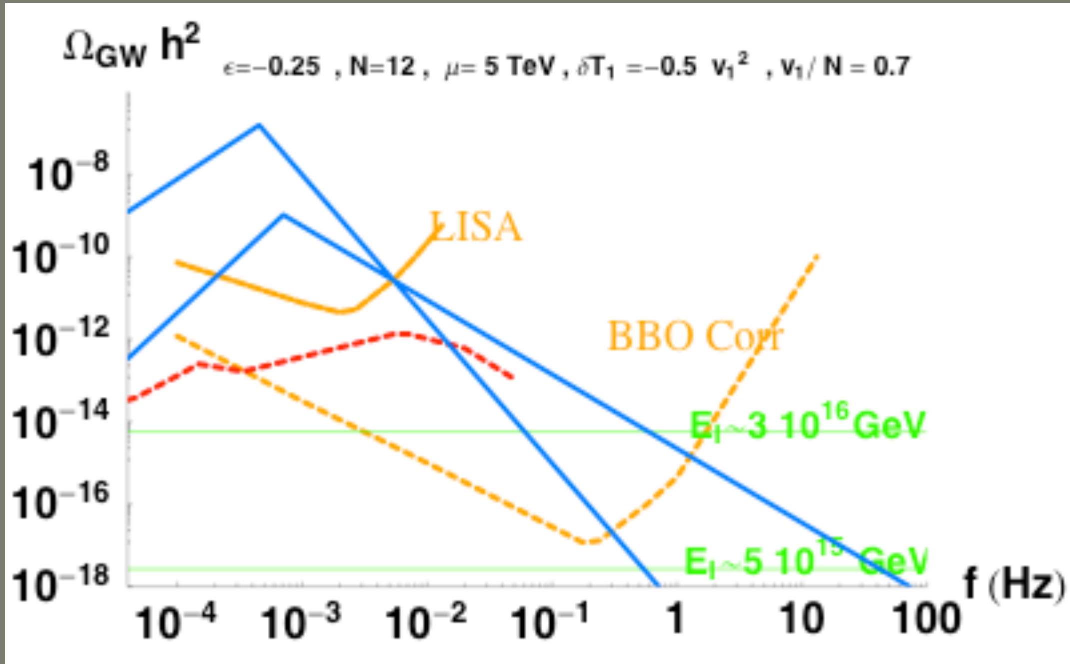
Low temperature phase: Planck brane and TeV brane

High temperature phase: AdS_5-Schwarzschild with black hole horizon

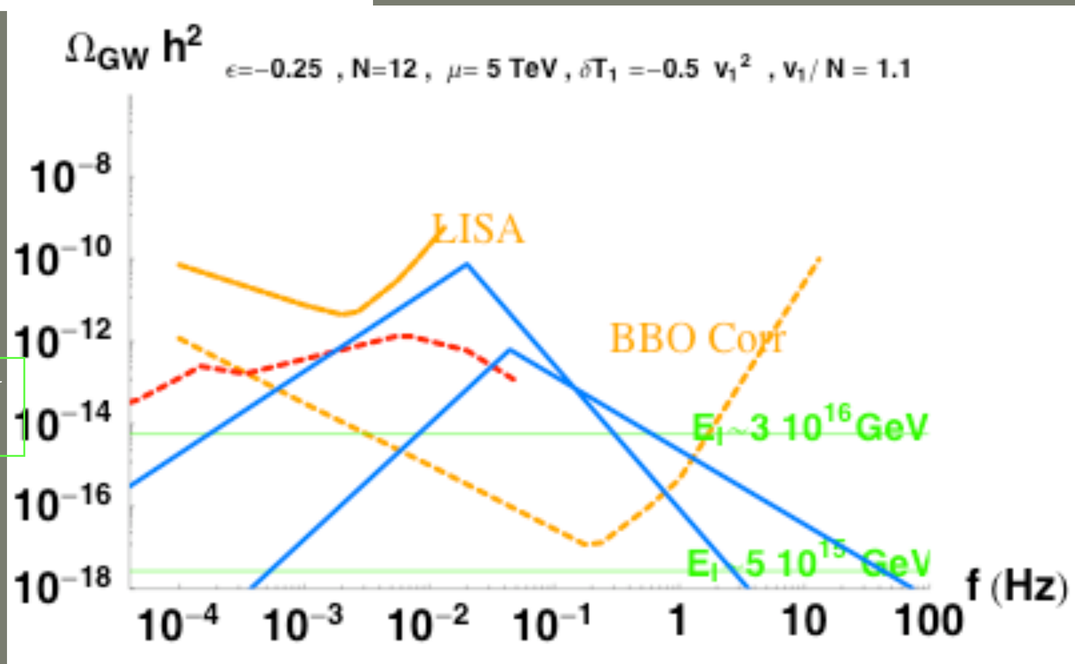
Creminelli, Nicolis, Rattazzi



hep-ph/0607158



α=26, β/H=21, T=490GeV

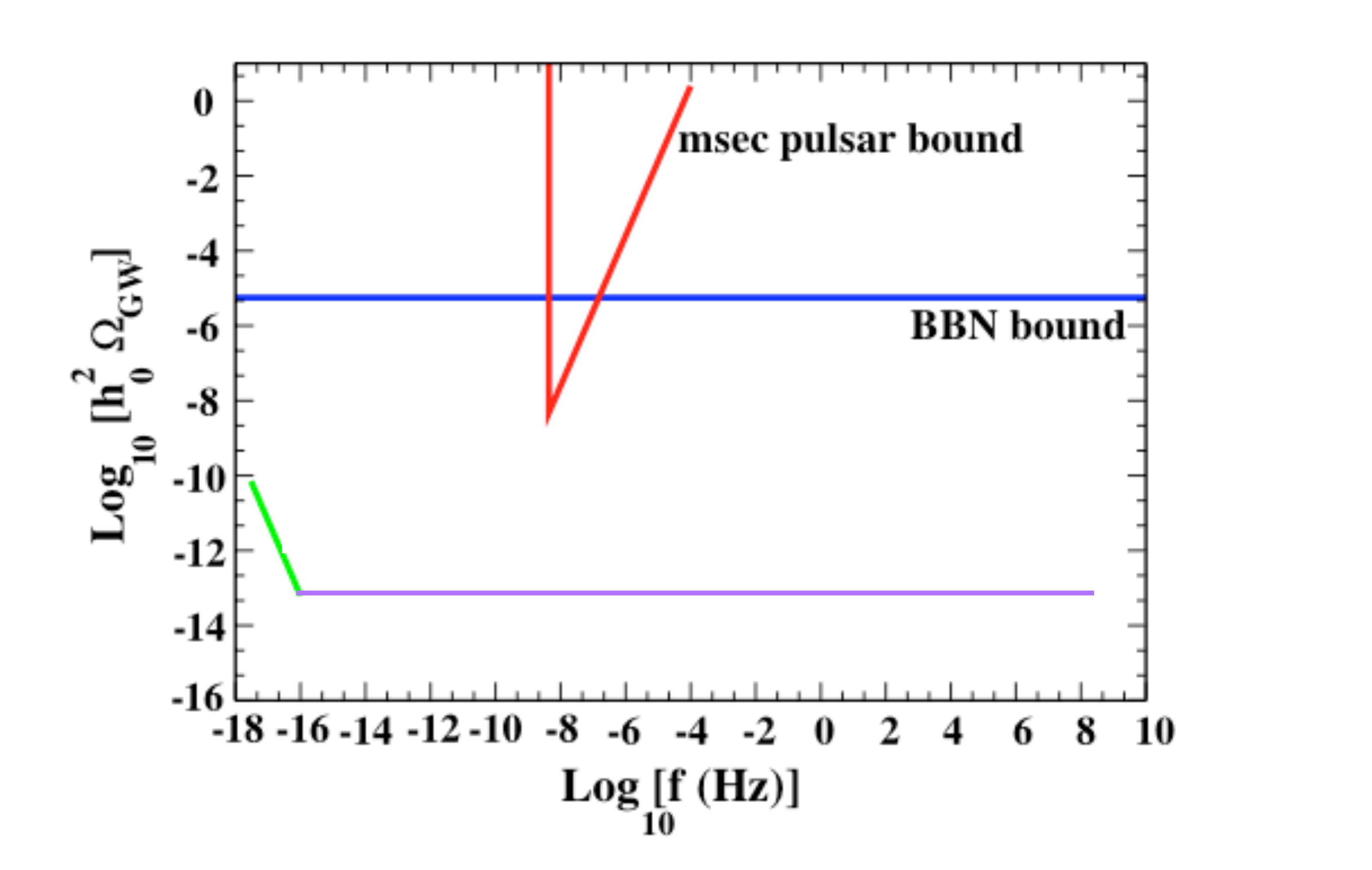


α=1.6,β/H=350,T=18300GeV

Studying flat directions with gravitational waves

Inflation

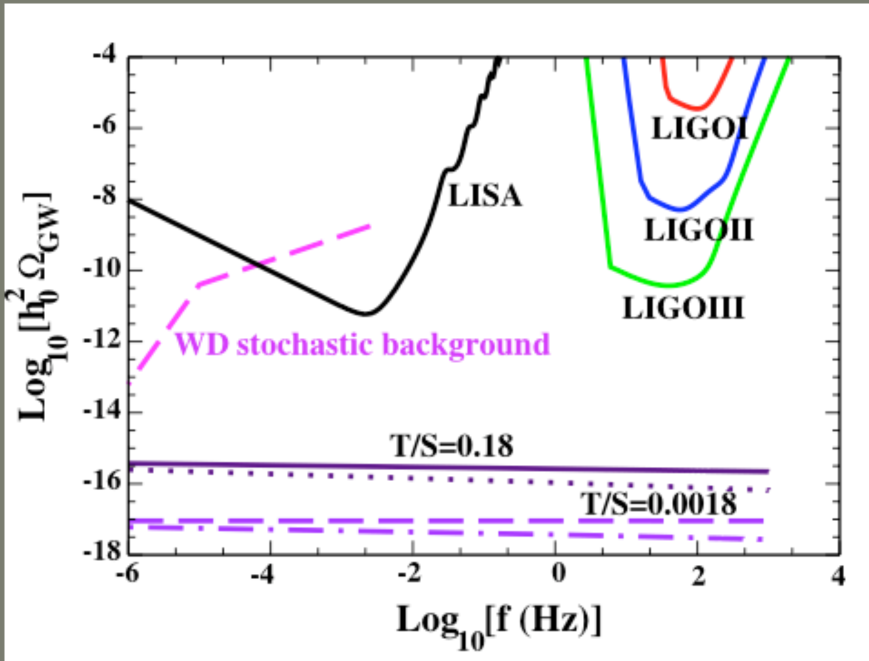
Vacuum fluctuations : de Sitter phase

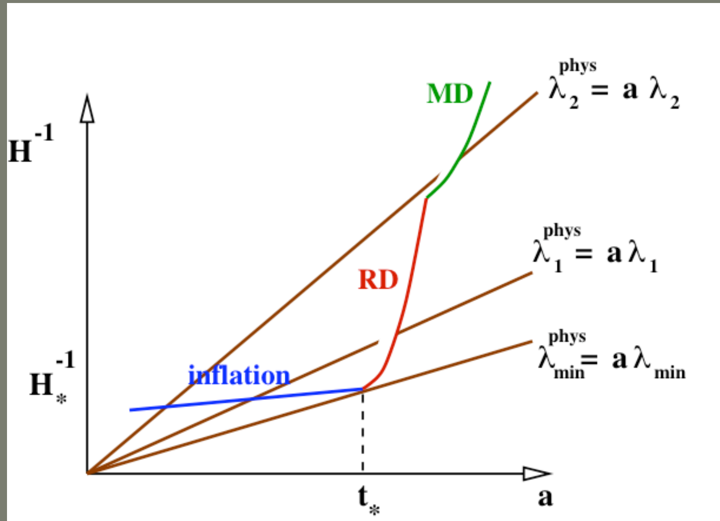


$h_0^2\,\Omega_{GW} = 10^{-13}(f_{eq}/f)^{2}(H/10^{-4}M_{Pl})^2$ $\quad$ $h_0^2\,\Omega_{GW} = 10^{-13}\,(H/10^{-4}M_{Pl})^2$

Fluctuations reenter horizon during matter era $\quad$ radiation era

Inflation models

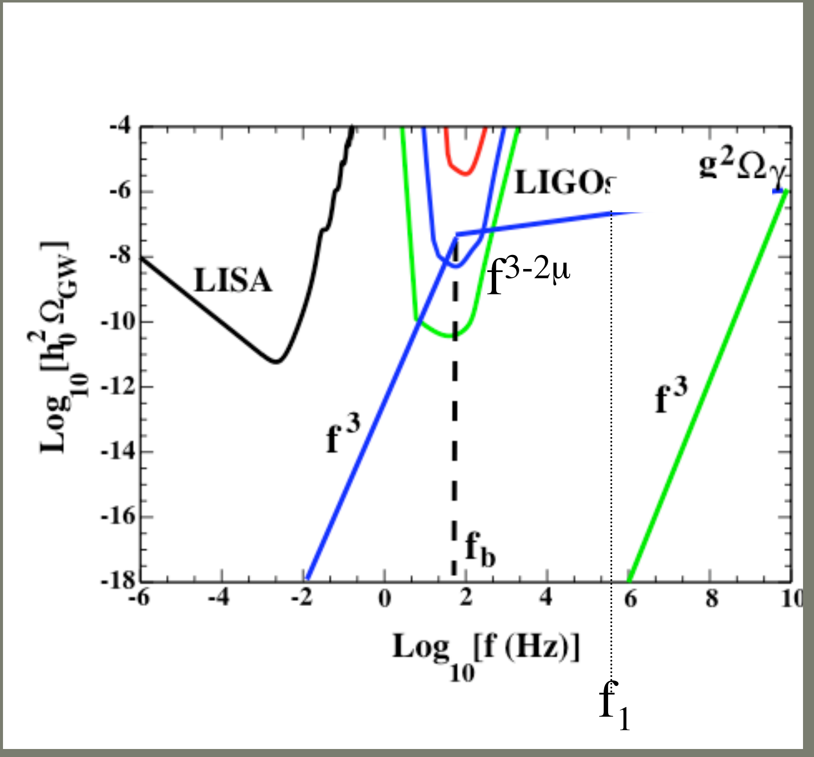


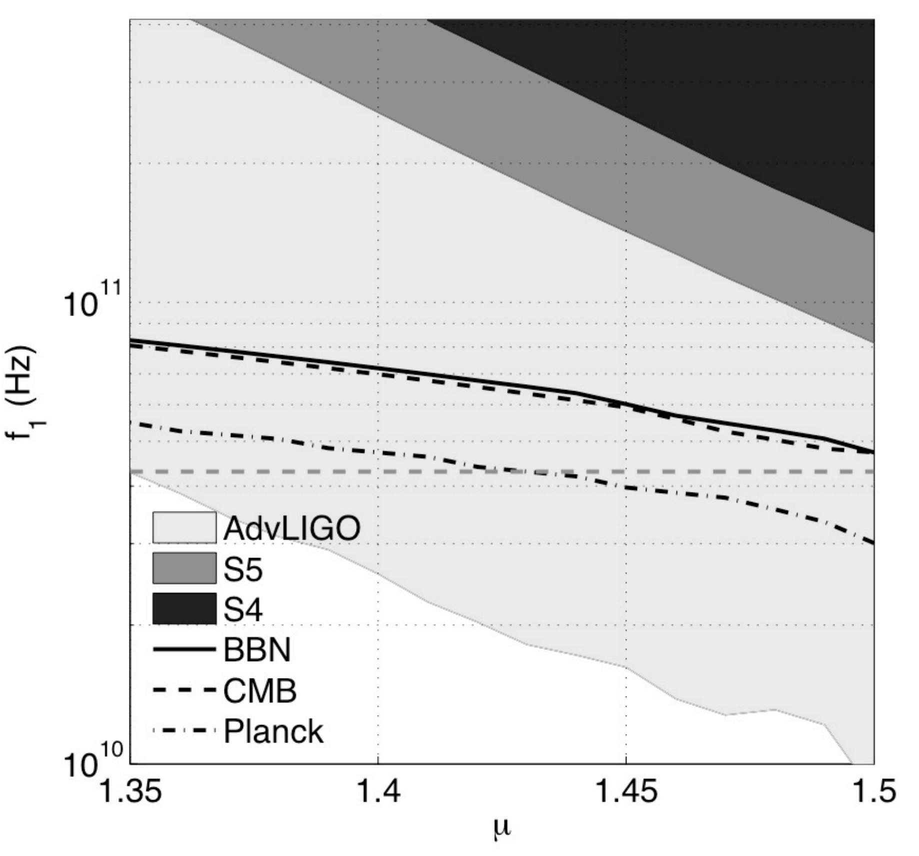


$$h_0^2\,\Omega_{GW} \sim V\, f^{\,n_T}$$

$$n_T = -\,(V'/V)^2\, M_{Pl}^2/8\pi = -T/7S$$

Pre-bigbang scenario





LIGO-VirgoNature 460, August 2009

Production of gravitational waves during reheating

- chaotic inflation

Khlebnikov, Tkachev; Easther, Lim

preheating $\xrightarrow[\text{resonance}]{\text{Parametric}}$ population of highly occupied modes
~ matter waves

collisions → GW with f ~ 10^7 to 10^9 Hz

- hybrid inflation

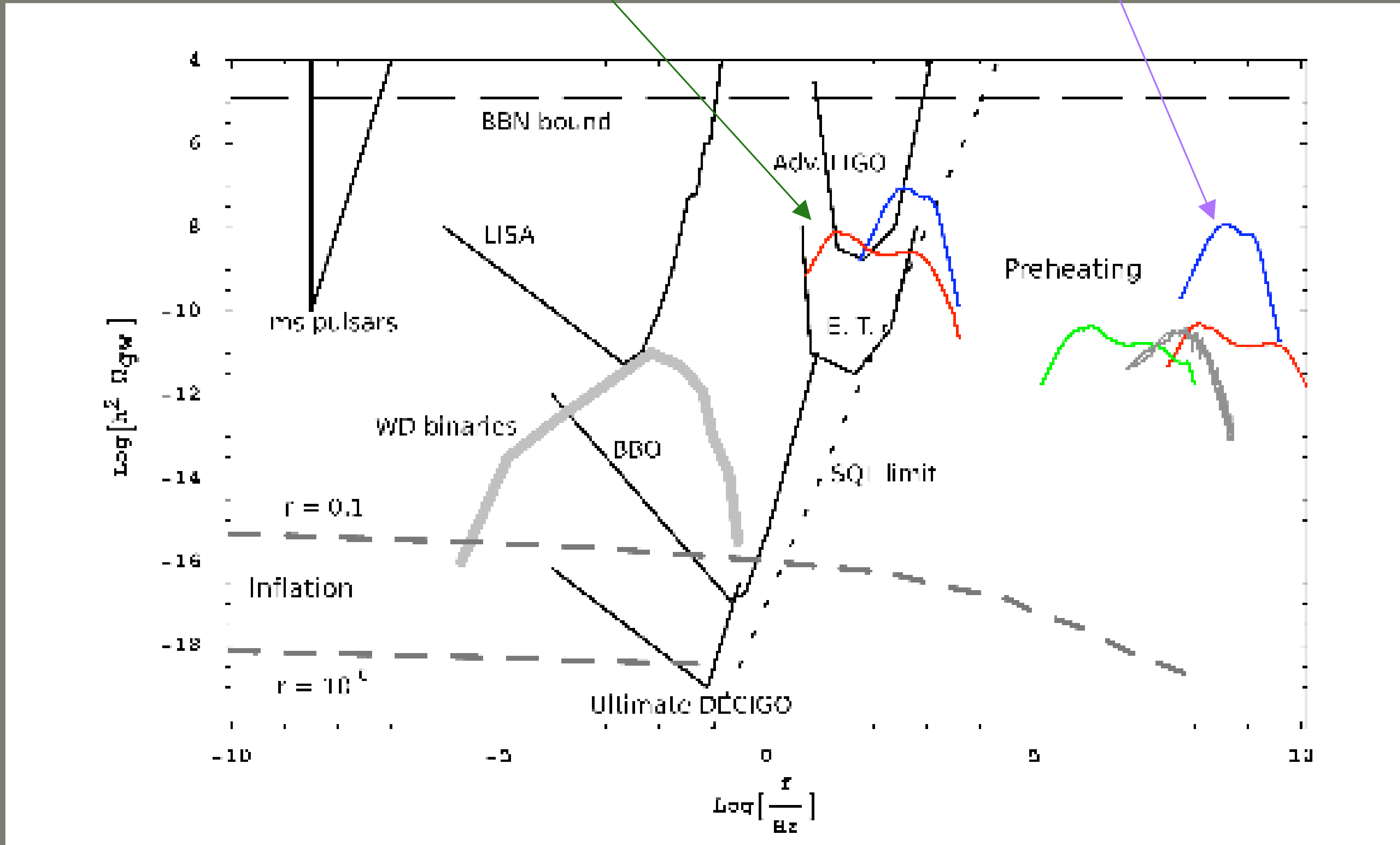
tachyonic preheating
Felder, Garcia-Bellido, Greene, Kofman, Linde, Tkachev

expect lower frequencies
Easther, Giblin, Lim; Garcia-Bellido, Figueroa

H ~ 100 GeV

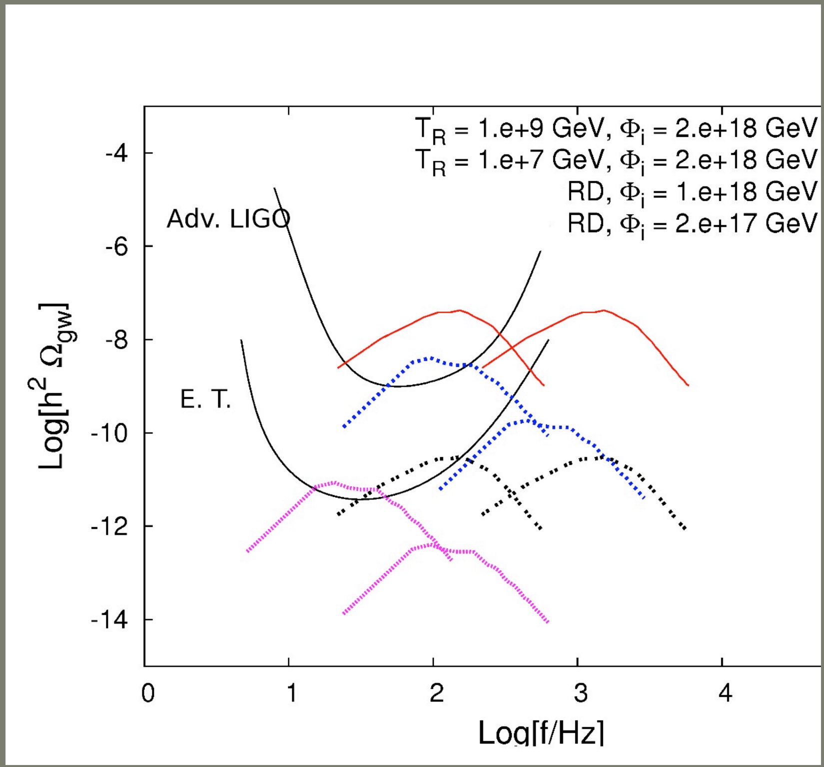
Low scale hybrid inflation
$v = 3.10^{-7} M_P$, $\lambda \sim g^2 \sim 10^{-14}$

High scale hybrid inflation
$v = 10^{-3} M_P$, $\lambda \sim g^2 \sim 10^{-5}$



Dufaux, Felder, Kofman, Navros 0812.2917 [astro-ph]

Probing flat directions of scalar potential: non-perturbative resonant effects may lead to explosive decays of flat directions.



J.F. Dufaux 0902.2574 [astro-ph.CO]

Strings

Cosmic strings

String energy does not grow with expansion because strings form loops which lose energy through gravitational (or other) radiation

Interconnection

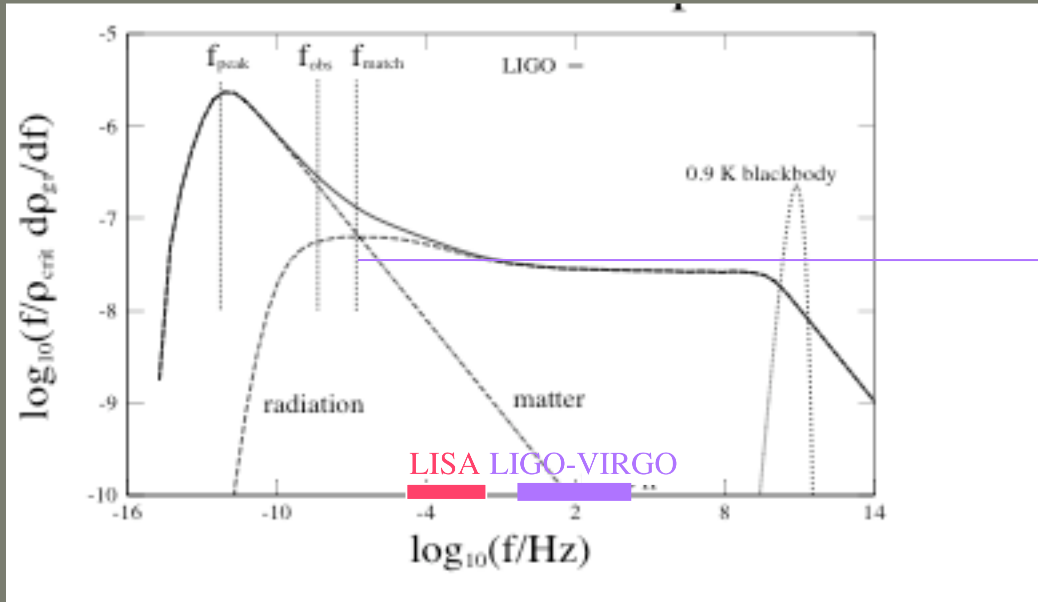
probability

$p \sim 1$

A string network evolves toward a scaling regime where

$L_{string} \sim \xi \sim t$ $\qquad L_{wiggles} \sim \alpha t$ $\qquad \alpha \propto G\mu$

$L_{loops} \equiv L = \alpha t$ $\qquad$ number density of loops $n(t) = \alpha^{-1}\, t^{-3}$

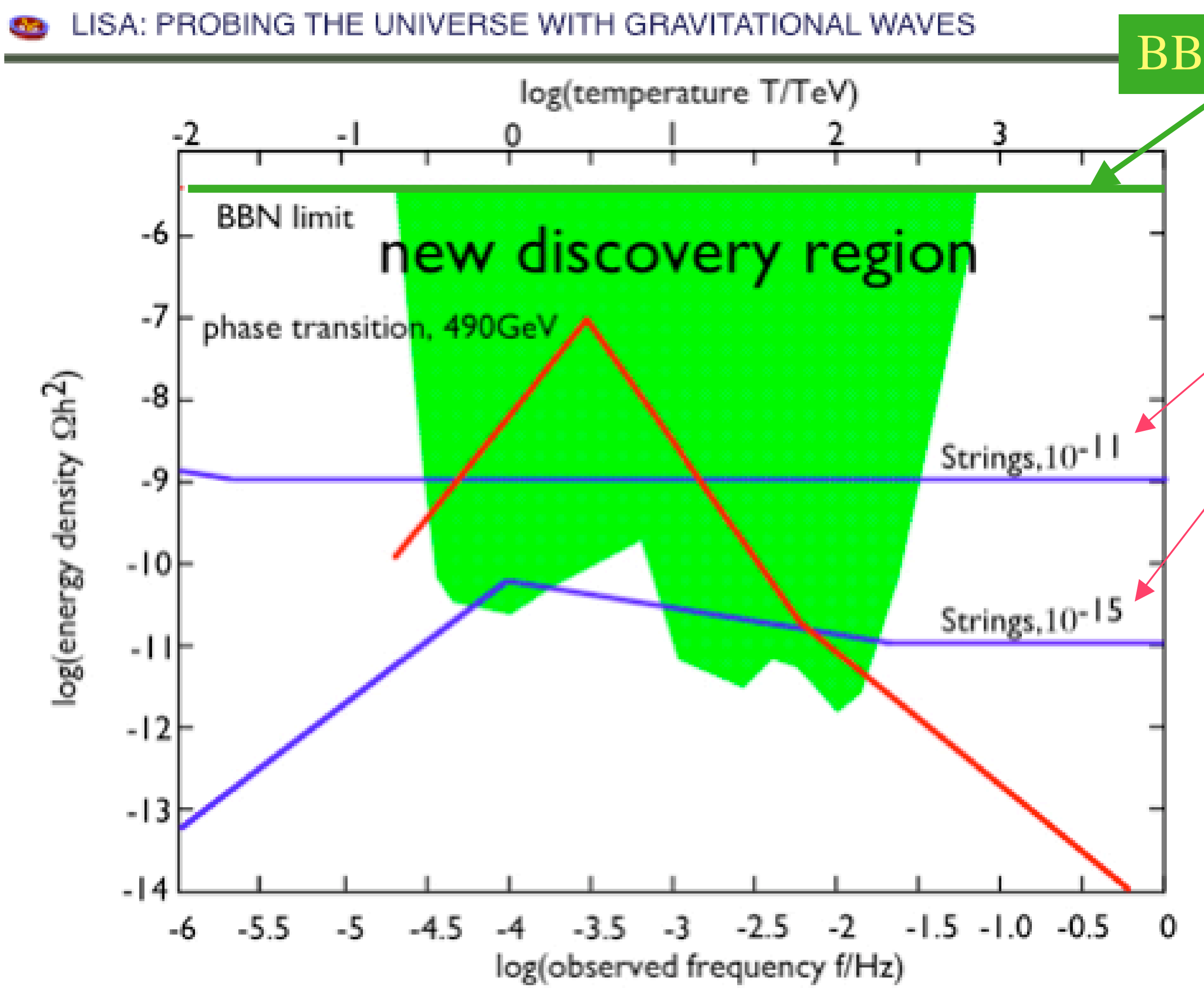


Flat spectrum
height $\propto G\mu$

loops which died
during matter dom.

loops which died
during radiation dom.

Gravitational radiation comes predominantly from the loops and the distribution of the size of the loops is still controversial (Polchinski…)



$G\mu<10^{-6}$

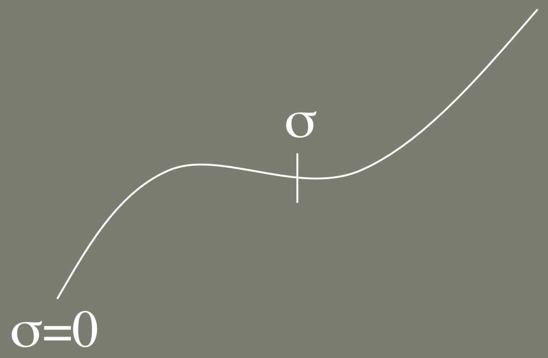
BBN limit

value of $G\mu$

Cusps and kinks

Damour-Vilenkin

Presence of cusps and kinks enhances the production of gravitational waves (frequencies much larger than the frequency of the fundamental mode of the string)



$$\mathbf{x}(\sigma,t) = \frac{1}{2}\left(\mathbf{a}(\sigma+t) + \mathbf{b}(\sigma-t)\right)$$

gauge conditions : $\mathbf{a}'^2 = \mathbf{b}'^2 = 1$

Gravitational radiation enhanced for:

direction of emission

cusps $\quad \exists\, (\sigma_c,t_c), \quad \mathbf{a}'\,(\sigma_c+t_c) = -\,\mathbf{b}'\,(\sigma_c-t_c) \equiv \mathbf{n}$

($|\,\dot{\mathbf{x}}\,(\sigma_c,t_c)|=1$: moves at speed of light)

kinks $\quad \mathbf{a}'$ or $\mathbf{b}'$ discontinuous

Why?

Linearized Einstein equations :

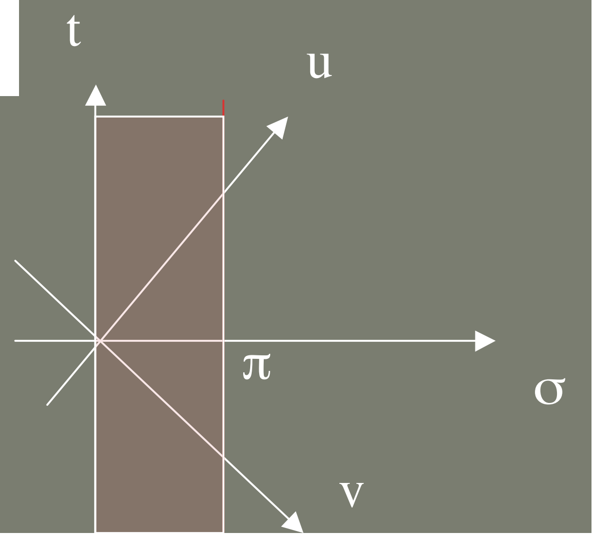
$\boxed{\square\, h_{ij}^{(TT)} = -16\pi G\, T_{ij}^{(TT)}}$

$$h_{ij}^{(TT)}(\mathbf{x},\omega) = \frac{4G}{r} e^{i\omega r}\, T_{ij}^{(TT)}(\omega\mathbf{n}, \omega) \qquad \mathbf{k} = \omega\mathbf{n}$$

$$\begin{aligned} T_{ij}(\mathbf{k},\omega) &= \mu \int \mathrm{d}t \int_{s_A(t)}^{s_B(t)} \left(\dot{x}_i \dot{x}_j - x'_i x'_j\right)\Big|_{(\sigma,t)} e^{i(\omega t - \mathbf{k}\cdot\mathbf{x}(\sigma,t))} \mathrm{d}\sigma \\ &= \frac{\mu}{2} \int \mathrm{d}t \int_{s_A(t)}^{s_B(t)} a'_{(i}(\sigma + t)\, b'_{j)}(\sigma - t) e^{i(\omega t - \mathbf{k}\cdot\mathbf{x}(\sigma,t))} \mathrm{d}\sigma \end{aligned}$$

$u=\sigma+t,\ v=\sigma-t$

$$T_{ij} \propto \left(\int a'_{(i}(u) e^{\frac{i}{2}(\omega u - \mathbf{k}\cdot\mathbf{a}(u))} \mathrm{d}u\right)\left(\int b'_{j)}(v) e^{-\frac{i}{2}(\omega v + \mathbf{k}\cdot\mathbf{b}(v))} \mathrm{d}v\right).$$



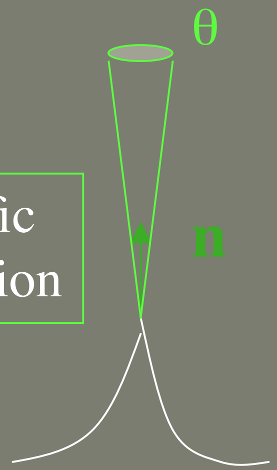
$$I(\omega) = \int_a^b f(t)\, e^{-i\omega\phi(t)}\, dt$$

$I(\omega) \rightarrow 0$ as $\omega \rightarrow \infty$ faster than any power of $1/\omega$ unless:

- there is a saddle point t_c where $\dot{\phi}(t_c) = 0$
- f or ϕ are discontinuous

With the previous integrals, this means:

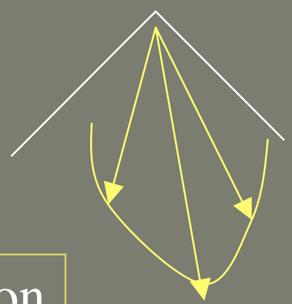
- $\phi(u) = u - \mathbf{n}.\mathbf{a}(u) \Rightarrow 1 = \mathbf{n}.\mathbf{a}'(u_c)$
  and $\phi(v) = v + \mathbf{n}.\mathbf{b}(u) \Rightarrow 1 = -\,\mathbf{n}.\,(v_c)\, b'$
- $\mathbf{a}'$ or $\mathbf{b}'$ discontinuous



specific direction

cusps

kinks



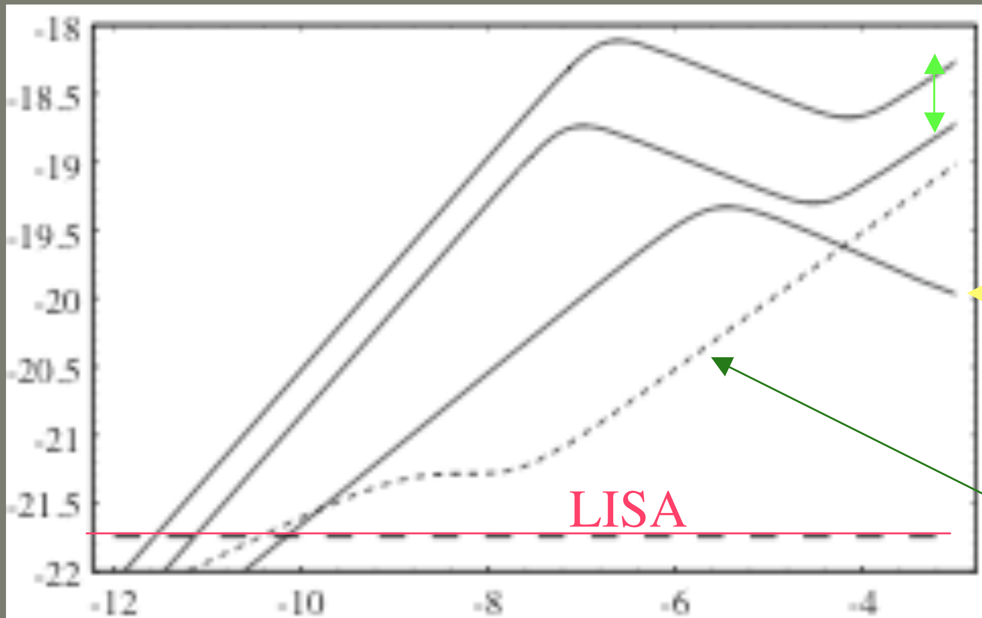
fan-like emission

$$\text{Amplitude } h(f) \sim \frac{G\mu L}{[(1+z)Lf]^{\delta}} \; \frac{1+z}{t_0 z} \; H[\theta_m(f,L,z) - \theta]$$

t_0 age of Universe

$\theta_m(f,L,z) = [f(1+z)L]^{-1/3}$

log h



log 50 Gμ

cusps $\delta = 1/3$ 1 cusp/loop osc.

0.1 cusp/loop osc

kinks $\delta = 2/3$

kink-kink $\delta = 1$

Confusion noise

Damour-Vilenkin

Choice $\varepsilon \equiv \alpha/(50G\mu) = 1$

$L=\alpha t$

Fundamental strings

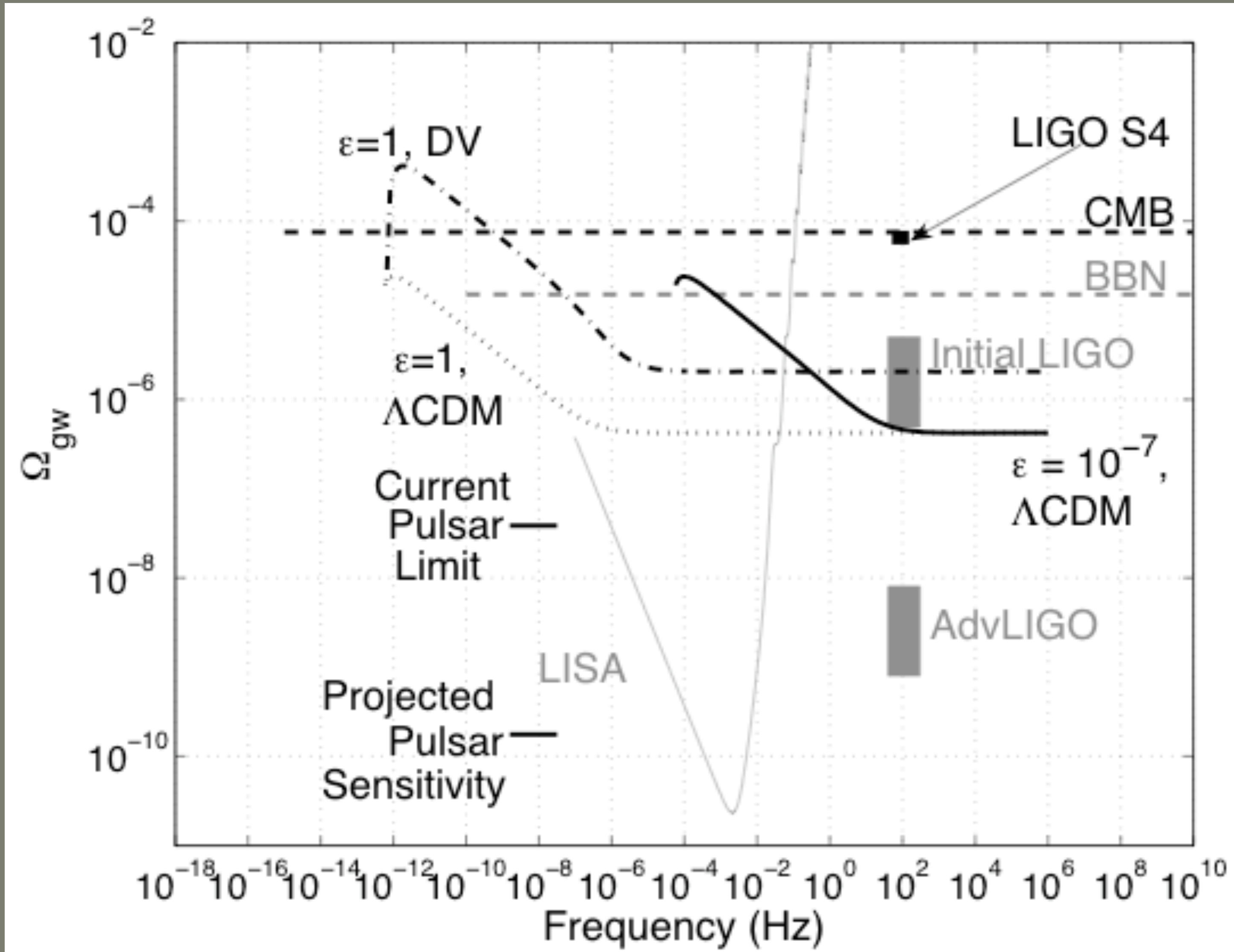
• reconnection probability p: when 2 strings colllide, they either pass through each other (proba 1-p) or reconnect (proba p).

<u>Reason</u>: they may miss each other in the higher dimensions

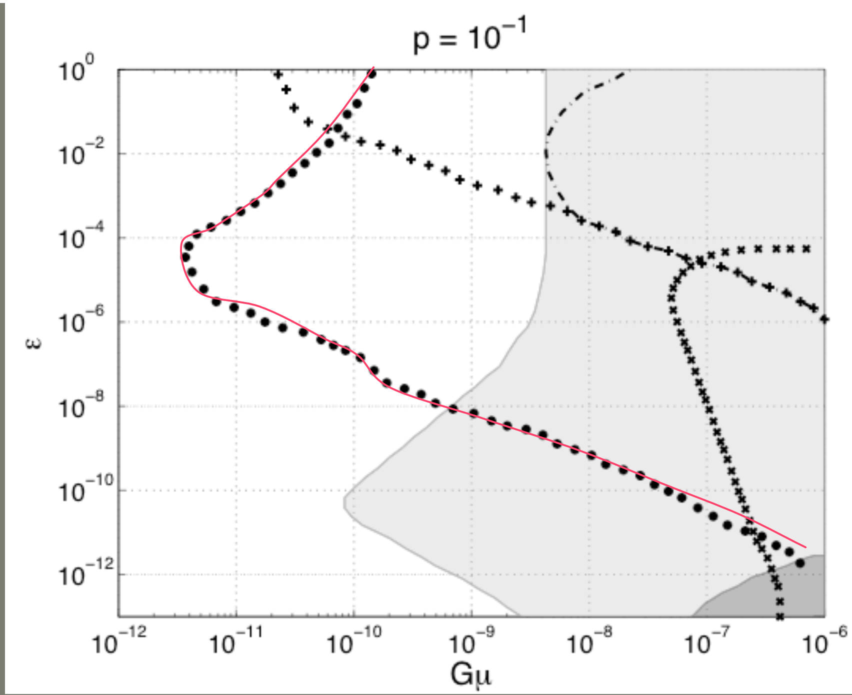
• existence of multi-string junctions

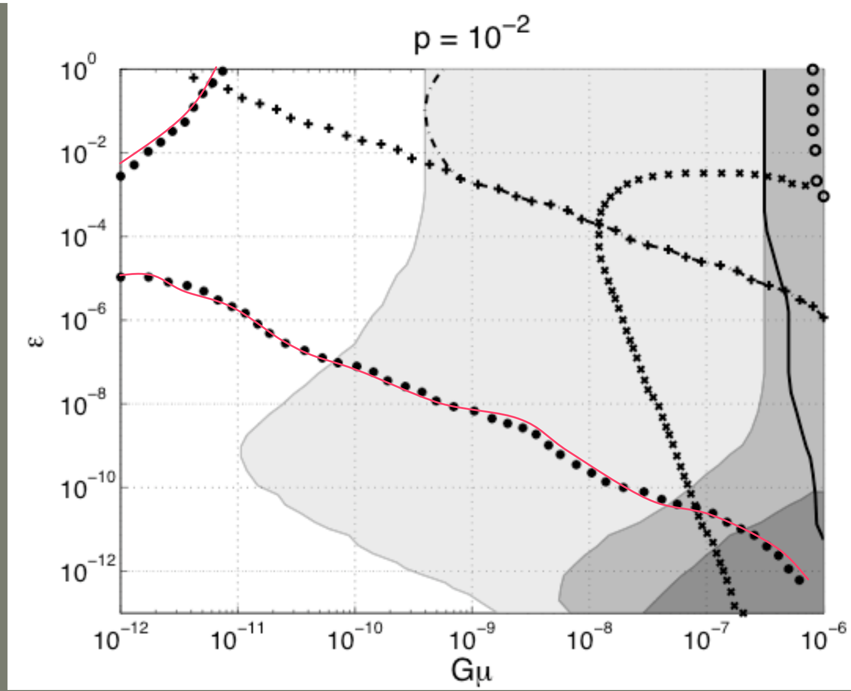
Copeland, Kibble, Steer, hep-th/0601153,061243
Salmi, Achucarro, Copeland, Kibble, de Putter, Steer, 0712.1204 [hep-th],
Brandenberger, Firouzjahi, Karouby 0710.1636 [hep-th]
Suyama 0807.4355 [astro-ph]
Davis, Nelson, Rajamanoharan, Sakellariadou 0809.2263 [hep-th]
Brandenberger, Firouzjahi, Karouby, Khosravi 0810.4521 [hep-th]

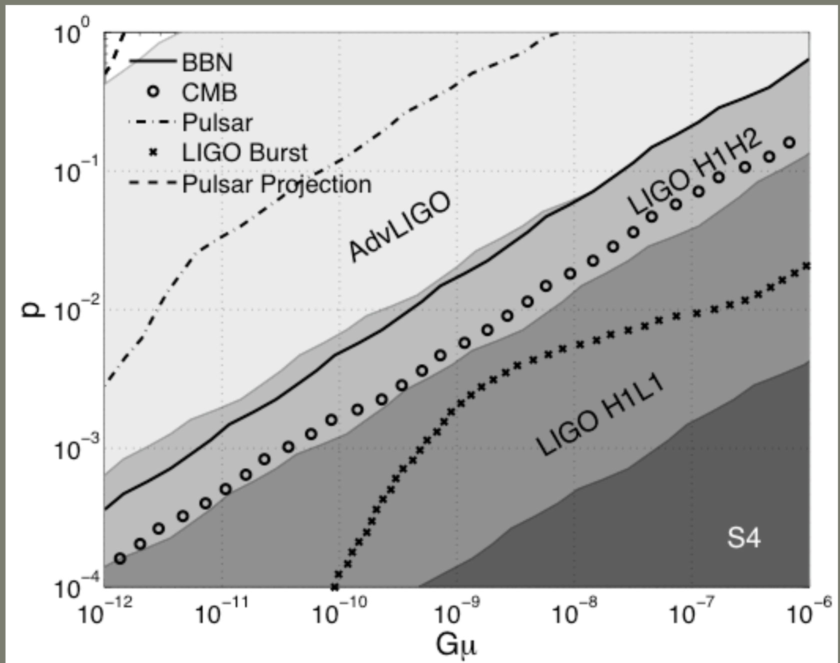
$p = 5.10^{-3}$ $G\mu = 10^{-7}$



Siemens, Mandic, Creighton astro-ph/0610920







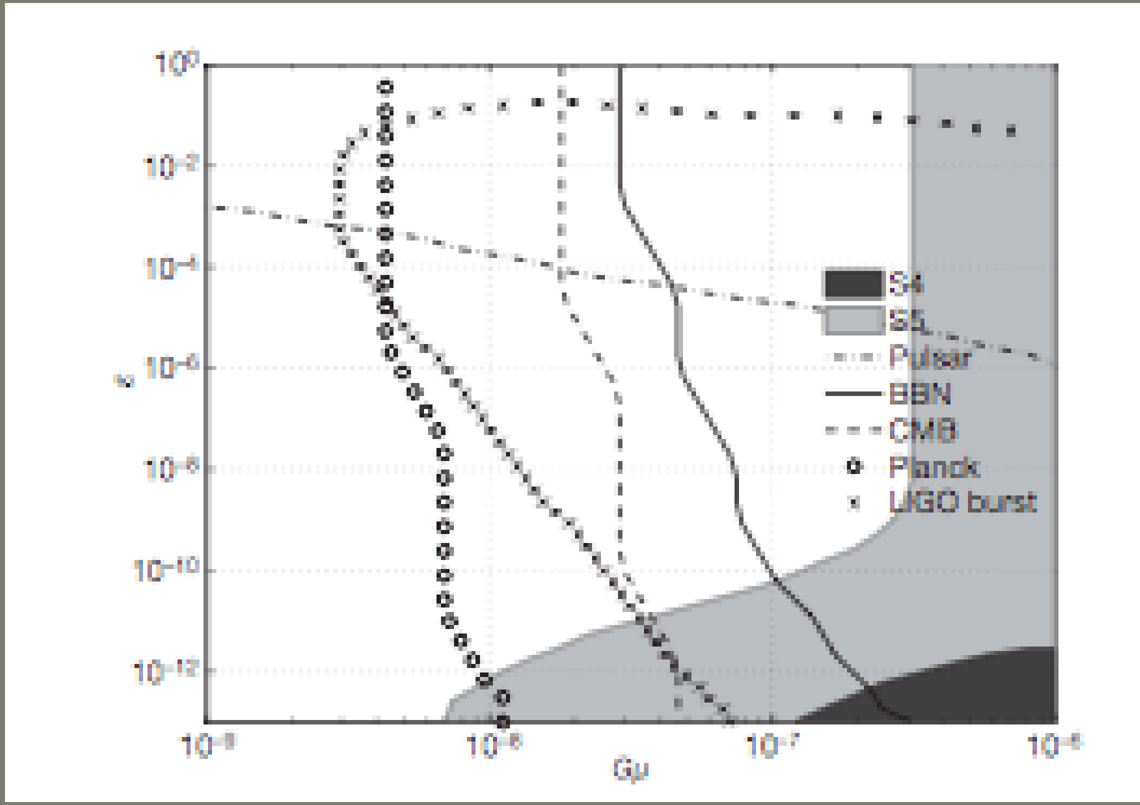
large long-lived loops

p reconnection probability

Siemens, Mandic, Creighton
astro-ph/0610920

LISA

$p=10^{-3}$



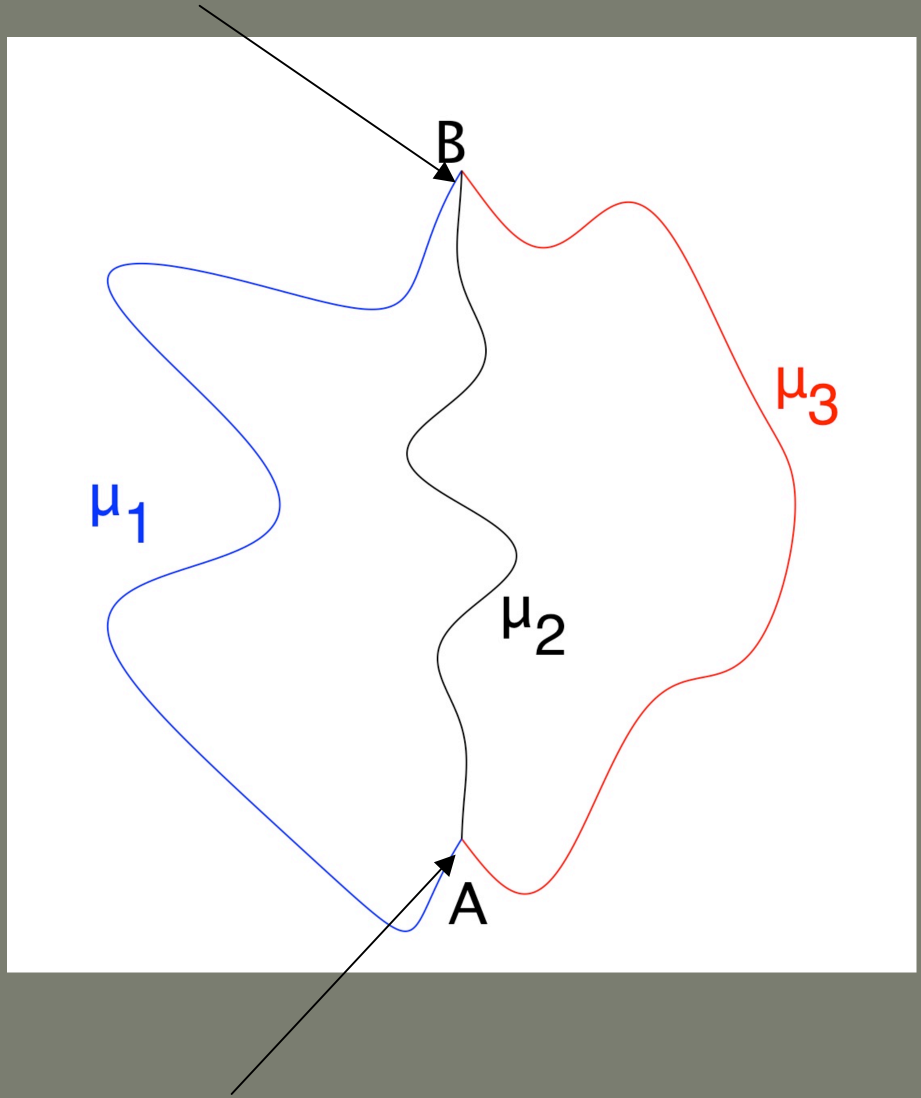
LIGO-VirgoNature 460, August 2009

Strings with junction

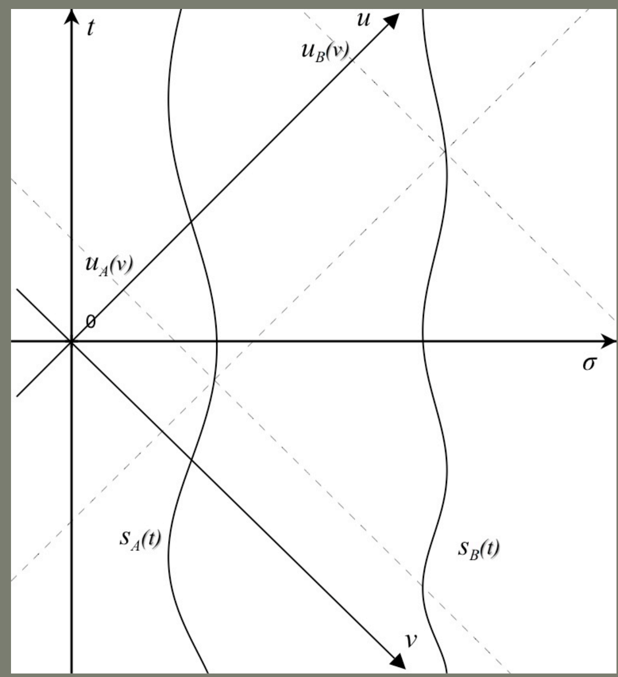
A. Bohé, T. Hertog, D. Steer, P.B.

arXiv 0907.4522 [hep-th]
+ 2 papers in preparation

$\sigma_1 = s_{B,1}(t)$

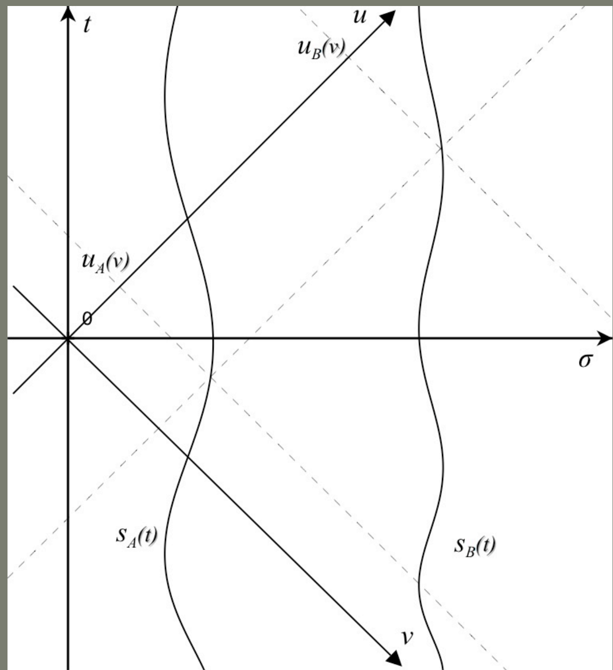


$\sigma_1 = s_{A,1}(t)$



string worlsheet

$$T_{ij}(\omega, \omega \mathbf{n}) = \frac{\mu}{4} \int_{-\infty}^{\infty} b'_j(v) e^{-\frac{i\omega}{2}(v+\mathbf{n}\cdot\mathbf{b}(v))} \left(\int_{u_A(v)}^{u_B(v)} a'_i(u) e^{\frac{i\omega}{2}(u-\mathbf{n}\cdot\mathbf{a}(u))} \mathrm{d}u \right) \mathrm{d}v.$$



Results: $h(f) \propto f^{-\delta}$

- $\delta = 1/3$: cusps (emitting in a given direction)

- $\delta = 2/3$: kinks (emitting in a fan-like direction)

- $\delta = 2/3$: string expanding at speed of light at a junction (given direction)

- $\delta = 1$: R-moving kink and R-moving kinks passing through each other (all directions)

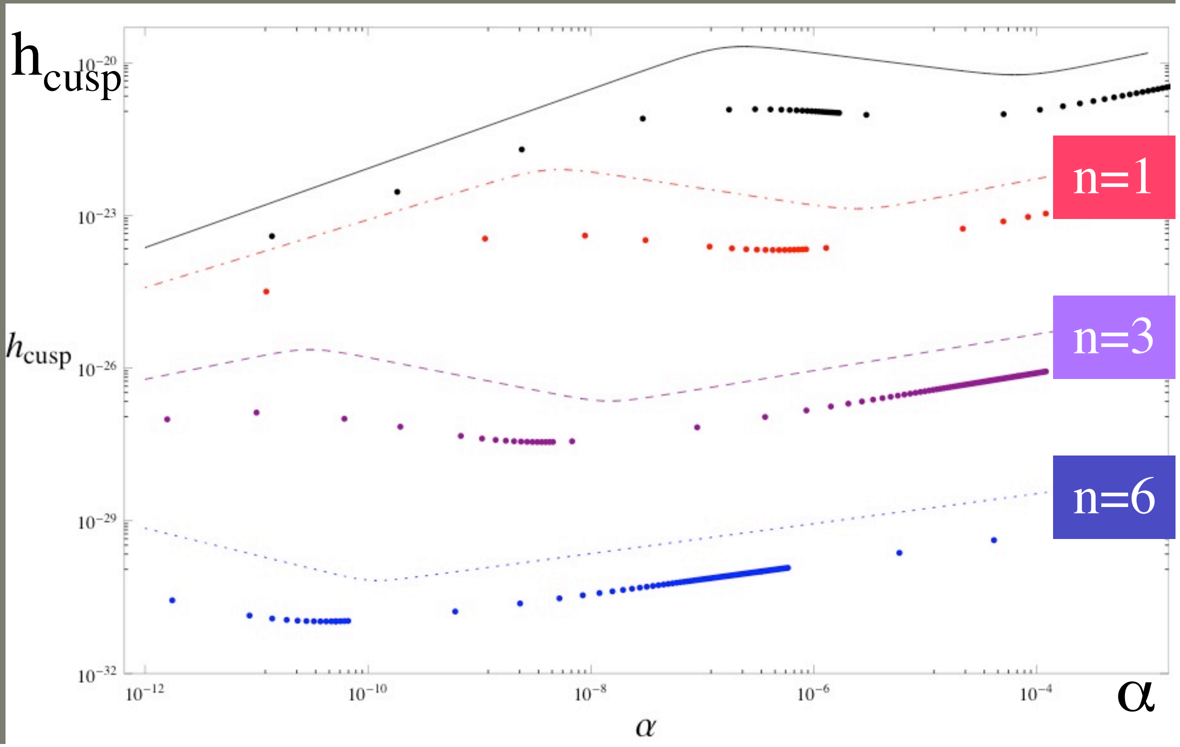
- $\delta = 1$: kink passing through a junction (all directions)

Note: gravity wave emission may not be as large for cusps with fundamental strings

O'Callaghan, Chadburn, Geshnizjani, Gregory, Zavala 10034395 [hep-th].

Extra dimensions round off cusps and reduce the probability of their formation

n extra dimensions



Proliferation of kinks through junctions

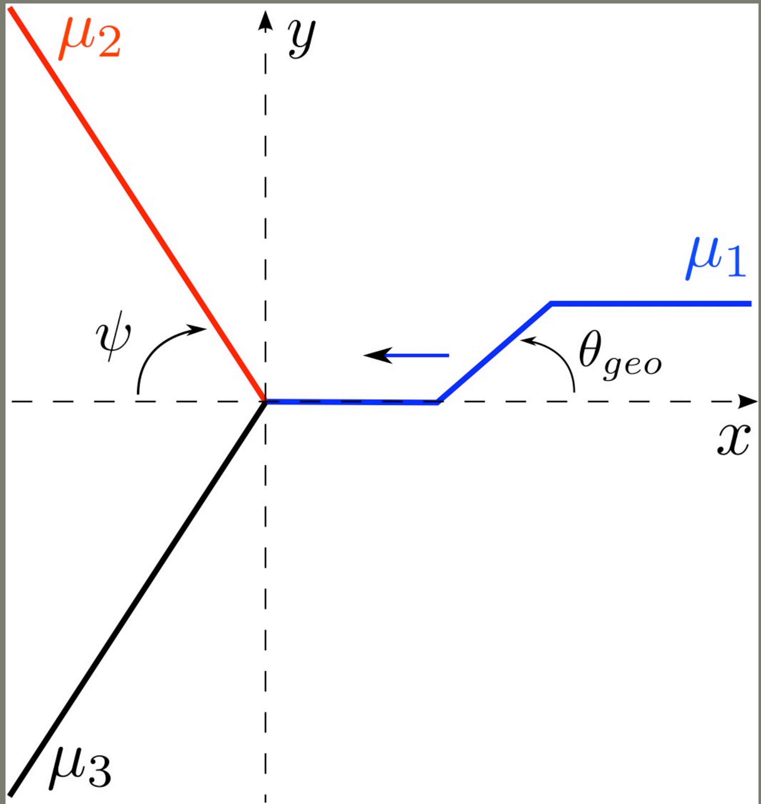
$$\mathbf{a'}_j(s_j(t) + t) = \frac{1}{1+\dot{s}_j} \left[(1- \dot{s}_j)\, \mathbf{b'}_j\, (s_j(t)-t) + 2\, \dot{\mathbf{X}} \right]$$

where $\dot{\mathbf{X}} = - \frac{1}{\sum \mu_k} \sum \mu_k\, (1-\dot{s}_k)\, \mathbf{b'}_k\, (s_k(t) - t)$ is the velocity of the junction

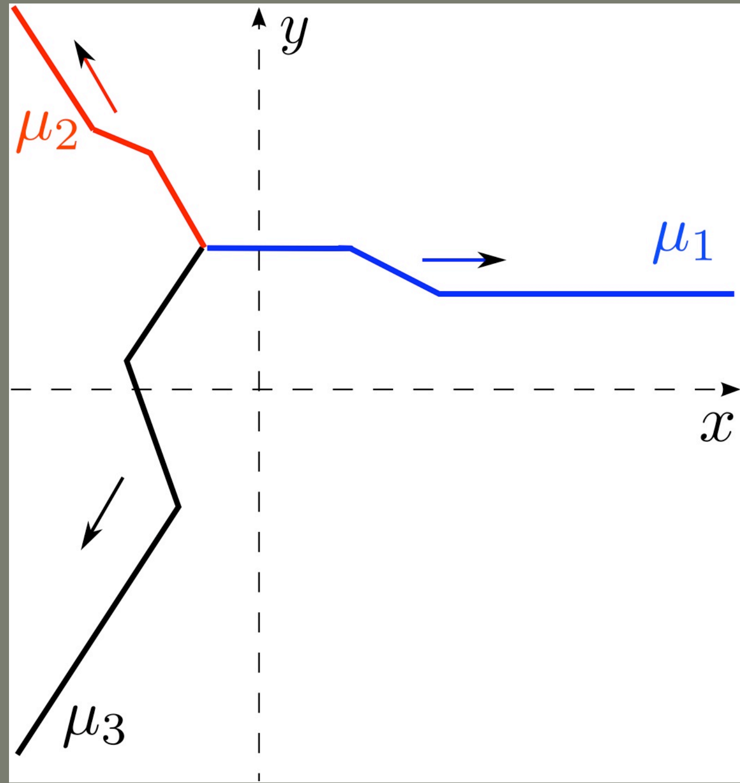
Hence kink propagating towards the junction (disc. in $\mathbf{b'}_j$)

$\rightarrow$ outgoing kinks on the 3 strings (disc. in $\mathbf{a'}_i$, i=1,2,3)

Example of a static junction

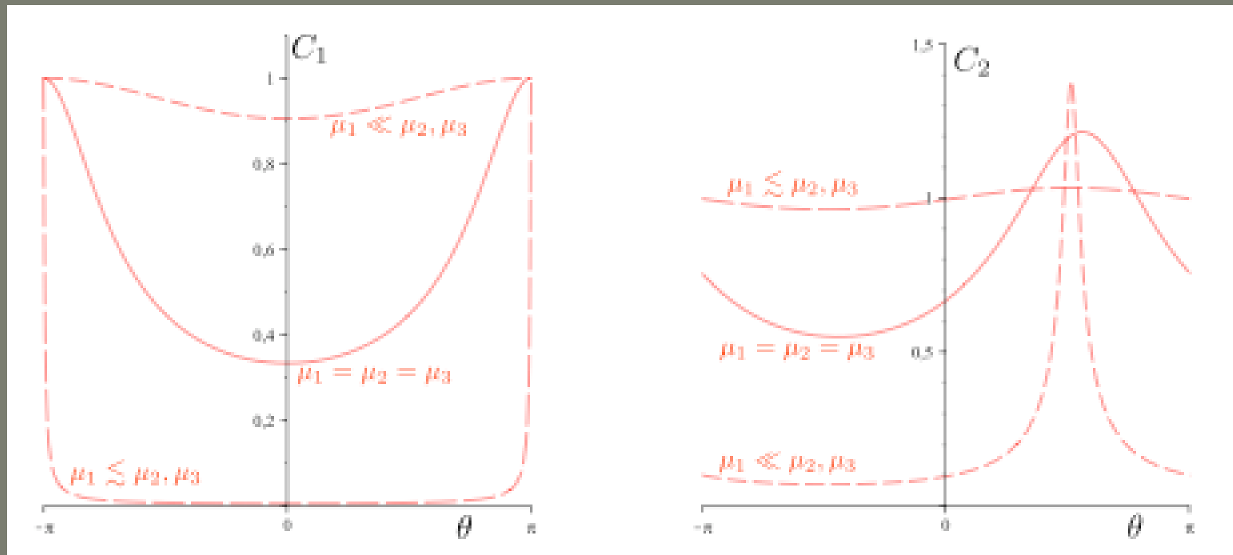


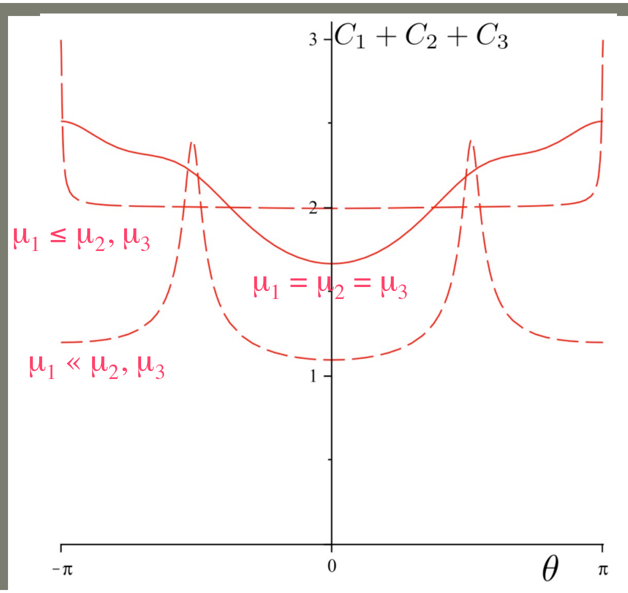
Initial state



Final state

Transmission coefficients: $C_i = \dfrac{\mathcal{A}[\mathbf{a'}_j]}{\mathcal{A}[\mathbf{b'}_1]}$ ← gravitational wave amplitude

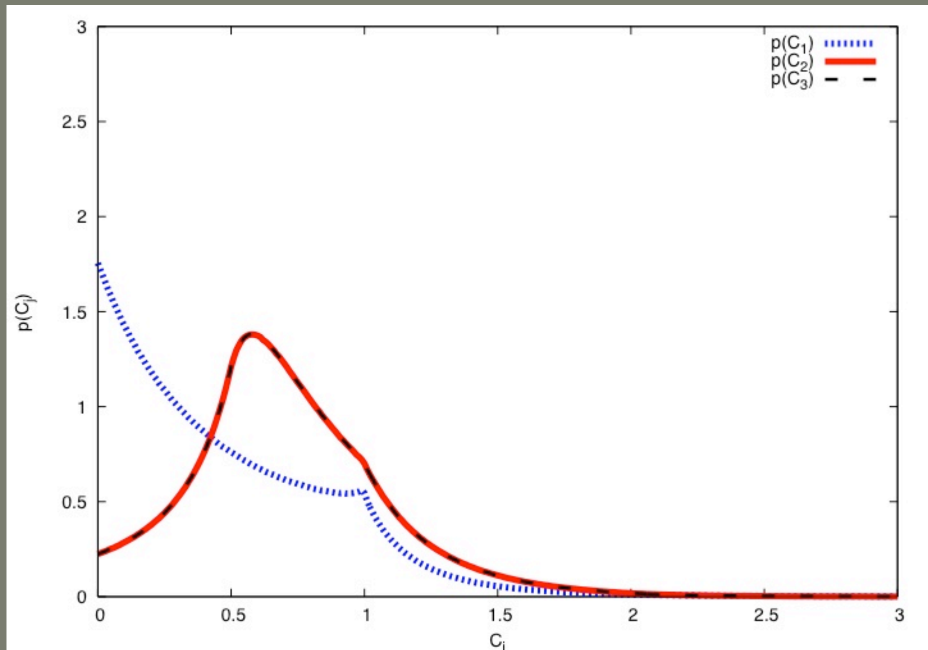




Dynamical junction

$\mu_1 = \mu_2 = \mu_3$

probability distribution $P(C_i)$



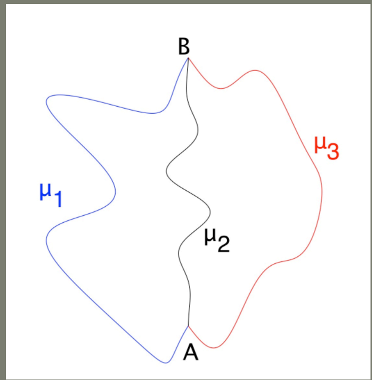
C_1

$<C_1>=0.49, <C_2>=0.72, <C_1>=0.72$

P(1 amplification) = 0.43, P(2 amplifications) = 0.07

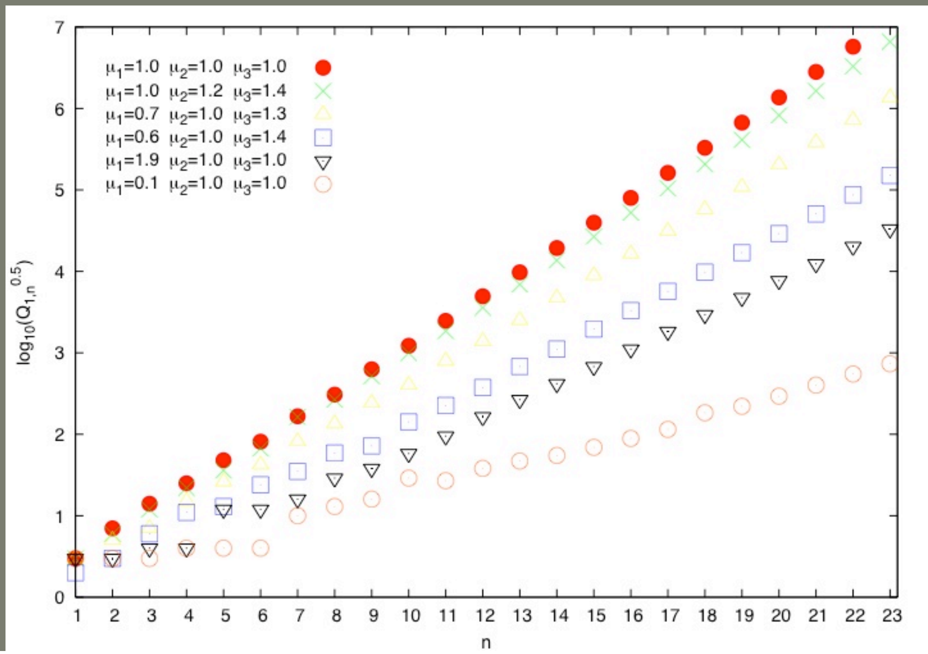
Amplification is sufficient to sustain the exponential growth of the number of large amplitude kinks

Example:



Define $Q^{a}_{j,n}$ the number of kinks on string j of amplitude larger than A at the n-th generation

$\log Q^{0.5}_{1,n}$

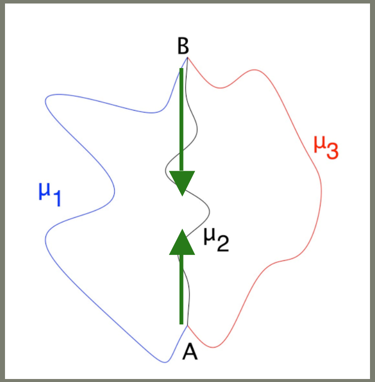


j=1
A=0.5

n

Limiting processes?

- loop dynamics: junction collision



- backreaction

Many observable consequences of this proliferation

e.g. kink-kink events are not favoured at large frequencies but proliferate even more quickly: they form a background that might swamp the cusps or kinks signals.

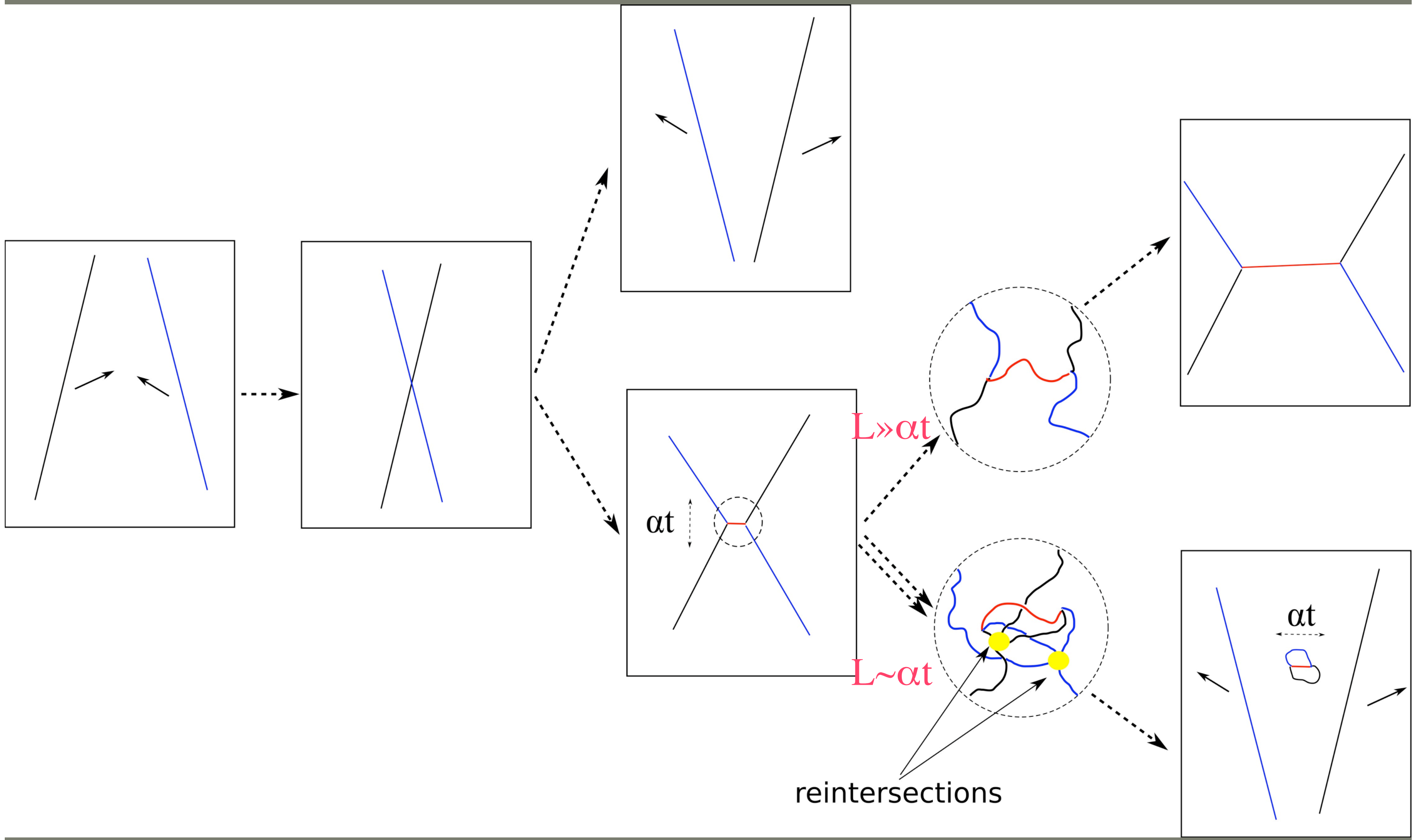
To show this, need a model of network evolution for strings with junctions:

Loops with no junctions $n(t) = \alpha^{-1}\ t^{-3}$

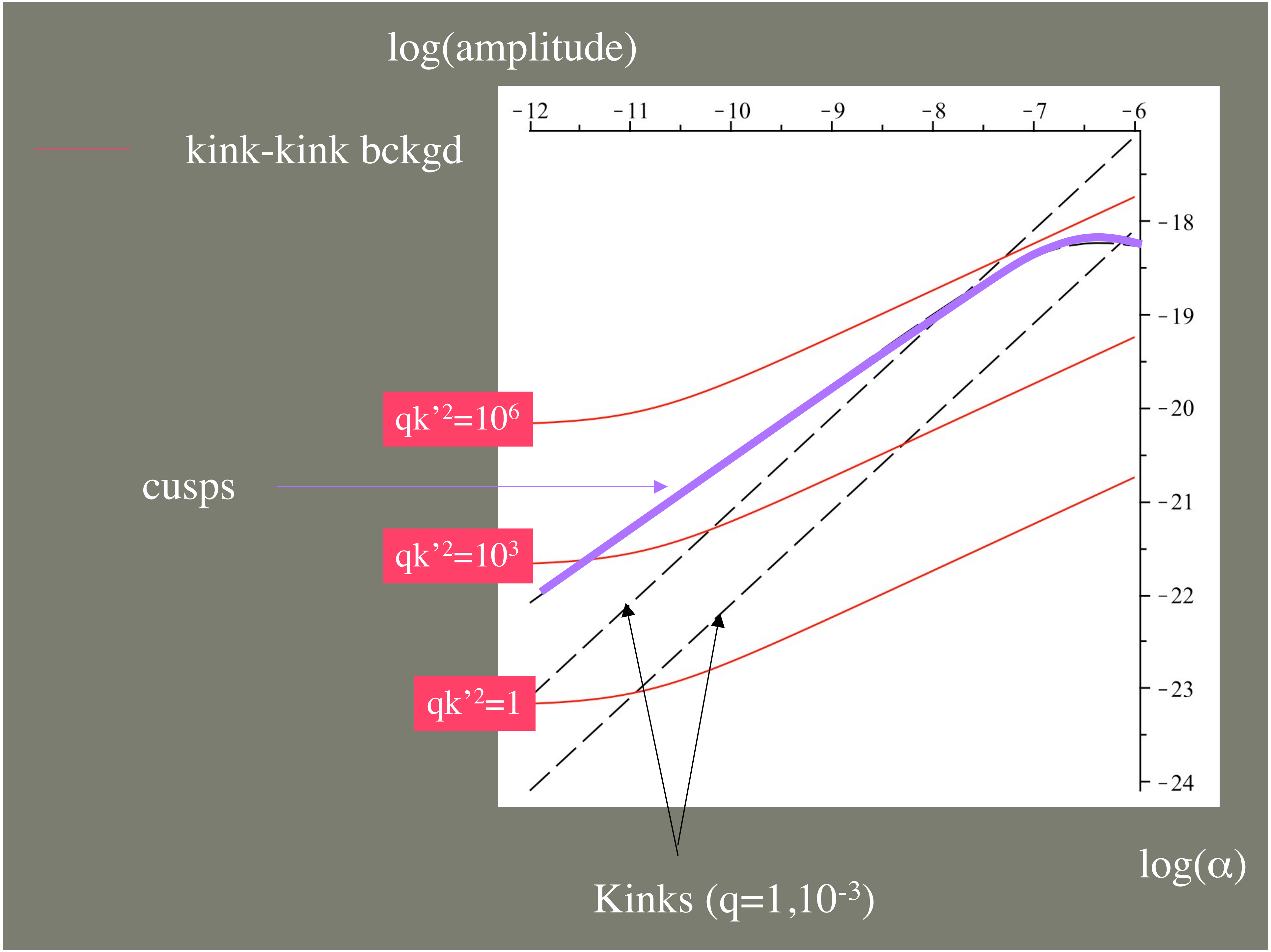
Loops with junctions $n'(t) = q\ n(t)$

Number of kinks formed by interval of time L:

$$\nu'_{kinks} = (k'/L)\ n'(t)$$

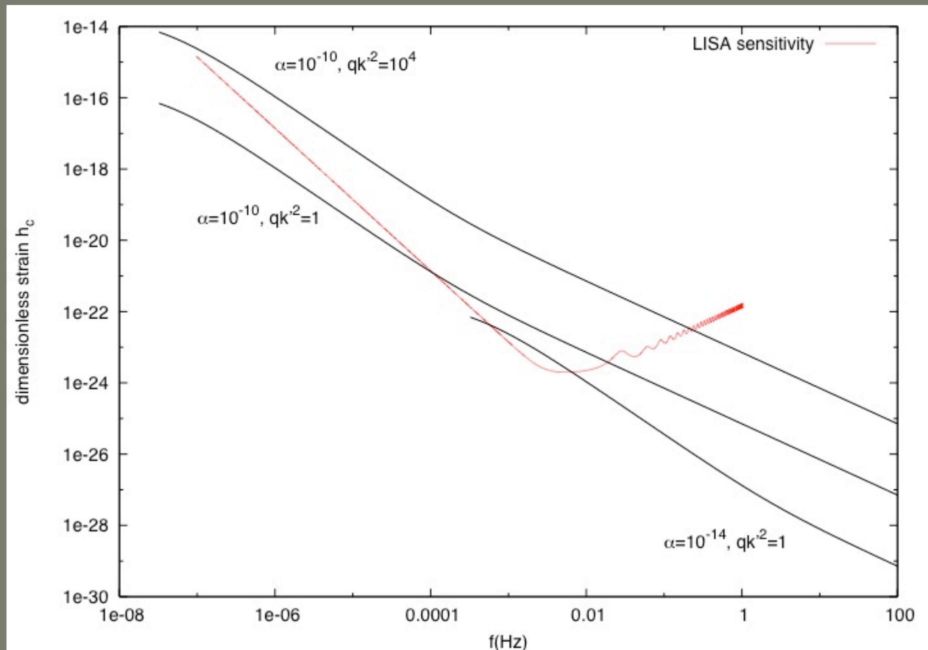


αt
L»αt
L~αt
αt
reintersections



log(amplitude)
kink-kink bckgd
qk'²=10⁶
cusps
qk'²=10³
qk'²=1
Kinks (q=1,10⁻³)
log(α)
- 12
- 11
- 10
- 9
- 8
- 7
- 6
- 18
- 19
- 20
- 21
- 22
- 23
- 24

The kink-kink background for LISA



Conclusions

Detection of gravitational waves very complementary tool to probe the Terascale region.

Ground interferometers are now prospecting unchartered territory in their frequency window. Pulsar timing arrays as well.

LISA has a very promising programme for a large set of the cosmological issues relevant today.

Search for ways to probe fundamental scales: kink dynamics may provide this.

Kink proliferation through junctions may change the expected signatures : kink-kink background…