Superstring Cosmologies

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April 1, 2010

Observational evidence supports a very rich, but highly involved, version of hot Big Bang cosmology.

Some of its main features are:

an early period of slow-roll inflation (or alternatives), during which the Universe grew to a large macroscopic size

very high temperature, and symmetry breaking phase transitions

a proportionally large amount of dark matter and dark energy, dominating the late time evolution ...

This largely phenomenological model, presents some of the greatest challenges to fundamental physics.

Two in particular:

If we extrapolate the cosmological evolution arbitrarily back in time, using the equations of General Relativity and Quantum Field Theory $\rightarrow$

we are driven to an initial singularity, the Big Bang, where these descriptions are breaking down.

Inflation solves some of the problems of the standard hot Big Bang model, such as the flatness, the large size and the horizon problems,

$\rightarrow$ but it is not past complete . . .

A second concerns the nature of the dark energy,
the simplest explanation for it being a positive,
however unnaturally small, cosmological constant $\rightarrow$

many orders of magnitude smaller than the Planck scale . . .

To date no symmetry principle or mechanism is known to explain its value.

Moreover, if the dark energy persists arbitrarily long, it would imply that
the Universe approaches de Sitter space in the far future $\rightarrow$

and so portions of space will remain out of causal reach of a single observer ...

The observable universe is in a highly mixed state.

So within the context of General Relativity and the Standard Model we lack a coherent framework to analyze the cosmology of our Universe, from beginning to end.

If string theory is a complete theory of quantum gravity, it should eventually provide us with a coherent framework for studying cosmology.

The hope is that by incorporating fundamental duality symmetries and new degrees of freedom of string theory in time-dependent, cosmological settings $\rightarrow$

some of the greatest cosmological puzzles will find a natural resolution with important implications and new tools for cosmological model building.

Indeed, string dualities give us profound insights into the nature of *Spacetime*,

with many surprising phenomena arising when we try to probe features of spacetime and geometry at short distances, of order the string scale l_s or the Planck length l_p.

These lead to important properties and consequences such as

- UV finiteness (Asymptotic safety)
- Stringy spacetime uncertainty principles: $\Delta x \Delta t \sim l_s^2$, $\Delta x \Delta t \sim l_p^2$
- S,T,U dualities
- Resolution of orbifold and conifold singularities
- Holographic gauge/gravity dualities ...

$\rightarrow$ illustrating how String Theory can provide concrete answers to many of the puzzles one has to face in trying to quantize Einstein's theory of general relativity . . .

An important lesson (from weakly coupled strings) is as follows
When the size of space is of order the string scale
classical notions such as geometry, topology and even spacetime dimensionality are ambiguous

the underlying worldsheet CFT system may lead to several equivalent descriptions
with KK momentum, winding and string oscillator states interchanging roles.

E.g. Consider string theory on a circle of radius R.

By T-duality, this is equivalent to string theory on a circle of radius $\alpha'/R \rightarrow$ ambiguous geometry

At $R = 2\sqrt{\alpha'}$, the system is equivalent to string theory on a line segment $\rightarrow$ obtained as a Z_2 orbifold of the circle at the self-dual radius

even the underlying topology is ambiguous

Similar statements hold for compact manifolds with fluxes

E.g. Consider the $SU(2)_k$ WZW model

For large level k, the description is in terms of a large S^3 with k units of NSNS 3-form flux

At $k = 1$, the system is equivalent to string theory on a circle at the self-dual radius

In both cases, a clear geometrical picture involving macroscopic space, along with an effective field theory description, arises in the large moduli limit
$\rightarrow$ obtained via large marginal deformations of the current-current type

Starting with a non-geometrical stringy system, macroscopic space is created via large marginal deformations

Exact, non-singular cosmological solutions to classical superstring theory already exist $\rightarrow$

described by a two-dimensional CFT of the form
$SL(2,R)_{-|k|}/U(1) \ \times \ K$ [C. Kounnas, D. Lust]
K is an internal, compact CFT.

The sigma-model metric is given by

$$ds^2 = |k|\alpha' \frac{dudv}{1-uv} = |k|\alpha' \frac{-dT^2 + dX^2}{1 + T^2 - X^2}$$

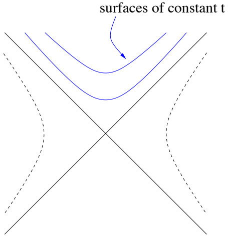
where $u = -T + X$ and $v = T + X$.

There is also a non-trivial dilaton

$$e^{2\Phi} = \frac{e^{2\Phi_0}}{1-uv}$$

The geometry consists of a singularity-free light-cone region, and there are time-like curvature singularities in the regions outside the light-cone horizons.

The singularities occur at $X = \pm\sqrt{1+T^2}$, where the dilaton field is also singular.



If we perform a double analytic continuation, we obtain Witten's 2d black hole.

This is equivalent to changing the sign of the level k.

At the singularities the sigma-model geometric description breaks down. As we will see there is a well defined CFT prescription to describe them.

The cosmological region of interest is the future part of the lightcone region.
It is an expanding, asymptotically flat geometry with the string coupling vanishing at late times:

$$ds^2 = |k|\alpha' \frac{-dt^2 + t^2 dx^2}{1+t^2}, \qquad e^{2\Phi} = \frac{e^{2\Phi_0}}{1+t^2}$$

Asymptotically we get a timelike linear dilaton background.

The cosmological observer never encounters the singularities, as these are hidden behind the visible horizons at $T = \pm X$.

However signals from the singularities can propagate into the lightcone region, and therefore influence its future evolution.

Is there a well defined initial state?

Are there four dimensional models?

The central charge of the superconformal $SL(2,R)/U(1)$ model at negative level k, is given by

$$c = 3 - \frac{6}{|k|+2}, \quad \hat{c} = 2 - \frac{4}{|k|+2}$$

To obtain a 4d model we add two large free super-coordinates together with a compact, superconformal system of central charge $\delta\hat{c} = 6 + 4/(|k|+2)$.

$$\hat{c}_{tot} = 10$$

Internal CFT:

In the large k limit, corresponds to a six dimensional space with curvature of order $1/k$.

For small k, this description is no longer valid. Eg at k=-2, the system can be taken to be a 7 torus.

The metric in Einstein frame is given by

$$ds_E^2 = |k|\alpha'(-dt^2 + t^2 dx^2) + (1 + t^2)(R_y^2 dy^2 + R_z^2 dz^2).$$

This is an anisotropic cosmology. At late times however, and for large $R_y \sim R_z$, it asymptotes to an isotropic flat Friedmann cosmology.

The cosmological region is non-compact, and when $R_{y,z}$ are large it has the desired four-dimensional interpretation. This is so *irrespective of how small the level k is.*

Rotating to Euclidean signature we obtain a disk.
It is parameterized by a complex coordinate $Z = \rho e^{i\phi}$ such that $|Z|^2 \leq 1$.
The Euclidean metric and dilaton are given by

$$ds^2 = |k|\alpha' \frac{d\rho^2 + \rho^2 d\phi^2}{1-\rho^2}, \qquad e^{2\Phi} = \frac{e^{2\Phi_0}}{1-\rho^2}$$

The singularity now occurs at the boundary circle $\rho = 1$.

The radial distance of the center to the boundary of the disk is finite, but the circumference of the boundary circle at $\rho = 1$ is infinite. Geometrically the space looks like a bell.

This Euclidean background corresponds to a well defined worldsheet CFT based on an $SU(2)/U(1)$ gauged WZW model at level $|k|$.
The interesting feature is that the Euclidean CFT is compact.

The worldsheet CFT is perfectly well behaved at $\rho = 1$.

Notice that worldsheet instanton configurations break the $U(1)$ symmetry corresponding to shifts of the angle ϕ to a discrete symmetry $Z_{|k|+2}$ ($\rightarrow$ There is a description in terms of $Z_{|k|+2}$ parafermions.)

We argue now that the non-singular description of the theory is an almost-geometrical one, in terms of a "T-fold".

To obtain it, we perform T-duality along the angular direction ϕ.

The resulting sigma model is based on the metric and dilaton

$$ds'^2 = \frac{\alpha'}{1-\rho'^2}\left(|k|d\rho'^2 + \frac{\rho'^2}{|k|}d\phi'^2\right), \quad e^{2\Phi'} = \frac{e^{2\Phi_0}}{|k|(1-\rho'^2)}$$

$$\rho' = (1-\rho^2)^{\frac{1}{2}}$$

Note that the transformation on ρ exchanges the boundary of the disk and its center.

The T-dual description is weakly coupled near $\rho = 1$ or $\rho' = 0$.

The only curvature singularity there is a benign orbifold singularity.

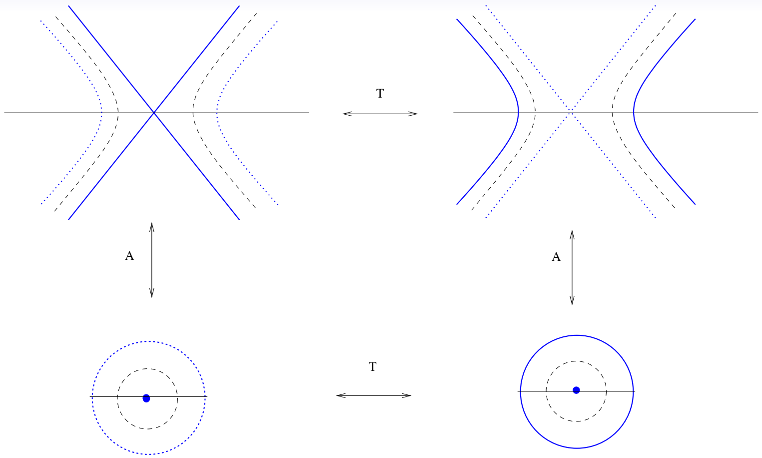
In fact, we can identify the T-dual as a $Z_{|k|+2}$ orbifold of the original model.

By gluing the two T-duals along a non-singular circle (e.g. at $\rho = \rho' = 1/\sqrt{2}$) we obtain a compact T-fold. This has no boundaries or singularities.
The gluing is non-geometrical as it involves a T-duality transformation on the fields.

In the case of the cosmology as well, we can obtain a regular T-fold description as the target space of the CFT.

T-duality interchanges the light-cone and the singularities. We must glue the T-duals along a hyperbola in between the lightcone and the singularities.

The gluing are shown in the following figure.



The resulting almost-geometrical description is very much like 2d de-Sitter space, which we can think of as a hyperboloid embedded in three-dimensional space.

We may think of the Euclidean T-fold (or the corresponding compact CFT) as describing a string field theory instanton.

The non-singular nature of the T-fold allows us to define a Hartle-Hawking wavefunction for the cosmology [C. Kounnas, J. Troost, NT].

As the underlying Euclidean super CFT is tachyon free, there is a finite calculable quantity, namely the (connected) string partition function Z_{string}.

$$||\Psi_{cosm.}||^2 = e^{Z_{string}}$$

Euclidean path integral

As in the case of de-Sitter space, the norm of the wavefunction can be given an interpretation in terms of a thermal ensemble.
$\rightarrow Z_{string}$ corresponds to a thermal CFT amplitude.

Effective temperature: $T = 1/2\pi R$, where

$$R = \sqrt{(|k|+2)\alpha'},$$

below Hagedorn for all $|k| \geq 0$.

$||\Psi||^2$ is a function of the moduli associated to the internal CFT K.

In the rest of this talk, I will be interested in non-trivial string theory cosmological solutions, which in a large region of moduli space are characterized by an underlying "no scale" structure. [Catelin-Jullien, Kounnas, Partouche, NK]

Common feature with the previous example:
The existence of a well defined Euclidean string background, where the string partition function can be systematically computed at the full (perturbative) string level.

Start with a weakly coupled supersymmetric string theory, on an initially flat background:

$$R^4 \times T^6$$

We then introduce sources of supersymmetry breaking, by utilizing geometrical fluxes, threading some cycles of the internal toroidal manifold.

The introduction of these fluxes can be easily adapted at the full string level in the framework of Freely Acting Orbifolds,

a generalization in string theory of Scherk-Schwarz compactification.

Later on we will discuss models where all nine (or at least eight) spatial directions are compact.

Notice that by using (non-perturbative) string dualities, we can also map this system into a dual one, where the sources of supersymmetry breaking are due to wrapped branes and other non-geometrical fluxes

Provided that the radii of moduli participating in the SUSY breaking mechanism are large enough, as compared to the string scale, these models are free of tachyonic instabilities.

At low energies, we get a "no-scale" supergravity theory with spontaneously broken SUSY.

Namely:
At tree level, the moduli participating in the SUSY breaking mechanism are flat directions, while many other moduli get soft breaking masses proportional to the gravitino mass scale.

Non-trivial time dependence arises when we take into account the thermal and quantum corrections.

To analyze it, we first identify a regime of computational control:

$$T, m_{3/2} \ll M_s$$

In this regime,

- The thermal effective potential is calculable, at the full string level, and it is free of UV and IR ambiguities
- When the VEVs of moduli that are not participating in the SUSY breaking mechanism are of order unity, they give exponentially suppressed contributions to the thermal effective potential
- Complex structure moduli of the form R_x/R_y, (the corresponding radii are involved in the SUSY breaking mechanism) are stabilized by the geometrical fluxes

The gravitino mass scale is set by a single running modulus, the no-scale modulus.

Thermal Effective Potential

$$P \sim T^4 F\left(\frac{m_{3/2}}{T}\right),$$

The function F can be expressed neatly in terms of Eisenstein series.

Notice that we do not include exponentially suppressed terms of the form e^{-S}, e^{-R_0}, ...

but we keep all corrections involving the ratio of the two SUSY breaking scales, $m_{3/2}$, T.

Adding estimated exponentially suppressed terms randomly destroys the no-scale structure.

Incorporating the backreaction on the initially flat background, we obtain in several cases a cosmological solution that follows the critical trajectory:

$$m_{3/2} = uT = \frac{1}{\gamma a}$$

a is the scale factor of the universe; u, γ are model dependent constants.

The T, a relation is characteristic of a radiation dominated evolution.

This phase persists, but it is eventually interrupted at both ends of the temperature scale $\rightarrow$

1.) by a symmetry breaking phase transition at lower temperatures, such as the electroweak phase transition

2.) by the onset of Hagedorn instabilities at higher temperatures, temperatures of order the string scale, before the “Big Bang”.

It is important to look for rich enough models$\rightarrow$
to provide a mechanism that stabilizes the SUSY breaking no-scale modulus (and other relevant moduli), at least just after the electroweak symmetry breaking scale.

At around the Hagedorn temperature, new string theoretic degrees of freedom, oscillators and string winding states, become relevant.

Clearly to understand the very early history of these cosmologies, we need to be able to handle the instabilities of string theory at high temperature

obtaining a concrete realization of the String gas cosmological scenario [Brandenberger, Vafa]

I will describe some new ideas towards this direction.

Before I do so let me mention another important result.

The ambiguities of the Hagedorn transition exit can be parameterized in terms of initial time boundary conditions $\rightarrow$

"capping off" the cosmology at an early time just after the temperature has dropped below Hagedorn.

It can be shown that our critical solutions are attractors for the dynamics;

there are large basins of such initial conditions where the resulting evolution is always attracted to the critical cosmological solution with stabilized complex structures. [Bourliot, Estes, Kounnas, Partouche]

In string theory there is an exponential growth in the density of single particle states as a function of the mass.

As a result the canonical ensemble

$$Z = \mathrm{Tr} e^{-\beta H}, \quad \beta = \frac{1}{T}$$

converges only for temperatures below the Hagedorn temperature:

$$T_H \sim \frac{1}{l_s}$$

It has been argued by many authors that at $T \sim T_H$, the system undergoes a phase transition

The partition function can be computed via a Euclidean path integral on $S^1 \times \mathcal{M}$ (S^1 is the Euclidean time circle with period β)

At $T > T_H$ certain stringy winding modes ($n \neq 0$) become tachyonic.

$\rightarrow$ divergence can be thought of as an IR instability and the phase transition is driven by tachyon condensation.

The IR instability can be removed by deforming appropriately the underlying Euclidean background.

This can be achieved either by

1.) Condensing the thermal winding tachyon (hard)

OR

2.) By introducing discrete gravito-magnetic fluxes associated to the graviphoton, and axial vector gauge field, the latter being associated to the $B_{\mu\nu}$ field of string theory.

These can be described in terms of gauge field condensates of zero field strength but with a non-zero value of the Wilson line

$$U = P \exp(i \int_0^{\beta} A_0 dX^0)$$

As the winding tachyons are charged under the graviphoton and axial vector gauge field the tachyonic instabilities are lifted. [Angelantonj, Kounnas, Partouche, NK]

These Wilson lines *refine* the canonical ensemble, and in certain cases render it finite. When this happens the system admits *thermal duality symmetry*

$$R_0 \to R_c^2 / R_0$$

The MSDS Vacua - Creation of Spacetime

A large class of non-singular stringy vacua, suitable for describing the very early non-geometrical era of these cosmologies consists of $\rightarrow$

the recently discovered vacua characterized by a novel Massive boson-fermion Spectrum Degeneracy Symmetry. [Kounnas, Florakis]

These vacua have at least 8 compact directions with radii close to the string scale $\rightarrow d \leq 2$ target space.

All compact supercoordinates are expressed in terms of free worldsheet fermions, free fermionic construction.

E.g. In the type II models, the 8 compact supercoordinates are replaced in terms of 24 left-moving and 24 right-moving worldsheet fermions.

Following the rules of the fermionic construction, these must transform in the adjoint representation of a semi-simple gauge group $H_L \times H_R$, $\dim H_{L,R} = 24$.

The simplest choice of H is $SU(2)^8$.

When the boundary conditions respect the existence of the $H_L \times H_R$ global worldsheet symmetry, the latter is promoted to a local spacetime gauge symmetry.

$\rightarrow$ extended symmetry points.

We can construct very special tachyon free vacua with left/right holomorphic factorization of the partition function

$$Z_{\rm II} = \frac{1}{2^2} \sum_{a,b=0,1} (-)^{a+b} \frac{\theta[^a_b]^{12}}{\eta^{12}} \sum_{\bar a,\bar b=0,1} (-)^{\bar a+\bar b} \frac{\bar\theta[^{\bar a}_{\bar b}]^{12}}{\bar\eta^{12}}$$

$$= (V_{24} - S_{24})(\bar V_{24} - \bar S_{24}) = 576.$$

- The massless level consists of 24×24 bosons only. All fermions are massive.
- The massive levels exhibit boson-fermion degeneracy symmetry.
- The one-loop partition function can be computed exactly in terms of the volume of the fundamental domain.

Similar constructions can be carried out in the heterotic case.

In fact starting with the maximally symmetric type II and heterotic vacua, a large class of MSDS vacua can be constructed in terms of Z_2^N orbifolds [Florakis, Kounnas].

The connection of the MSDS vacua with higher dimensional, macroscopic ones can be achieved via large marginal deformations of the current-current type: $M_{IJ}\ J_L^I \times J_R^J$.

Moduli Space

$$\mathcal{M} = \frac{SO(r_L,\ r_R)}{SO(r_L) \times SO(r_R)}$$

r_L (r_R) are the ranks of the H_L (H_R) gauge groups.

To analyze these, we first locate the MSDS vacua in the moduli space of type II (and heterotic) orbifold compactifications to two dimensions

$$Z_{\rm II} = \frac{1}{2^2} \sum_{a,b=0,1} \sum_{\bar a,\bar b=0,1} (-)^{a+b} \frac{\theta[^a_b]^4}{\eta^{12}} \Gamma_{(8,8)}[^{a,\bar a}_{b,\bar b}] \frac{\bar\theta[^{\bar a}_{\bar b}]^4}{\bar\eta^{12}} (-)^{(\bar a+\bar b)}$$

where the asymmetrically half-shifted $(8,8)$ lattice is given by

$$\Gamma_{(8,8)}[^{a\,,\,\bar a}_{b\,,\,\bar b}] = \frac{\sqrt{\det G_{\mu\nu}}}{(\sqrt{\tau_2})^8} \sum_{\tilde m^\mu, n^\nu} e^{-\frac{\pi}{\tau_2}(G+B)_{\mu\nu}(\tilde m+\tau n)^\mu(\tilde m+\bar\tau n)^\nu + i\pi\mathcal{T}}$$

The phase $\mathcal{T}$ can be written in the form

$$\mathcal{T} = \left[\tilde{m}^0(a+\bar{a}) + n^0(b+\bar{b})\right] + \left(\tilde{m}^1 n^1 + \tilde{m}^1\bar{a} + n^1\bar{b}\right).$$

It describes the couplings of the lattice to the R-symmetry charges (a, b) and $(\bar{a}, \bar{b})$ $\rightarrow$ spontaneous breaking of ordinary supersymmetry.

We see that only two of the eight internal cycles couple to them. In fact the X^0 cycle is "thermally" coupled to the total spacetime fermion number $F_L + F_R$, whereas the X^1 direction is "thermally" coupled to the right-moving fermion number F_R.

At the MSDS point the $G_{\mu\nu}$ and $B_{\mu\nu}$ tensors take very special values.

As a result, starting with the two dimensional maximally symmetric stringy vacuum, and de-compactifying two directions, we get a maximally supersymmetric type II vacuum.
At low energies the effective description is in terms of $d = 4$, $\mathcal{N} = 8$ gauged supergravity, where the gauging is induced by the internal T^6 fluxes.

The creation of the four dimensional macroscopic space is achieved via marginal deformations of the current-current type.

Tachyon free trajectories, and classes of initially MSDS vacua that remain tachyon free under arbitrary marginal deformations have been identified [Florakis, Kounnas, NT].

In fact the full space of MSDS vacua (including the MSDS orbifolds) is in correspondence with the space of four dimensional gauged supergravities with $\mathcal{N} \leq 8$ supersymmetries:

$$[\mathrm{Z}_{\mathrm{orb}} : \ \textit{MSDS} \ ]_{d=2} \quad \longleftrightarrow \quad [\ \mathcal{N} \leq 8 : \ \textit{SUGRA} \ ]_{d=4,\,\mathrm{fluxes}} \ .$$

For large but not infinite deformations, the obtained vacua are those of spontaneously broken supersymmetric vacua in the presence of geometrical fluxes.

Furthermore some Euclidean versions of the models naturally admit a thermal interpretation, where the corresponding canonical ensemble is deformed by gravito-magnetic fluxes.
These fluxes render the partition function free of Hagedorn-like instabilities.

These generic properties suggest that the MSDS vacua are suitable candidates to describe the very early stringy phase of the Universe.

The high degree of symmetry characterizing these vacua could open a window in analyzing the highly stringy dynamics.

In a dynamical setting, the moduli M_{IJ} acquire non-trivial time dependence. It would be interesting to identify initially MSDS vacua which spontaneously decompactify towards four dimensional ones.

In this respect, it is possible to construct (at least heterotic) MSDS vacua whose massless spectra are characterized by an abundance of fermionic (rather than bosonic) degrees of freedom $n_F > n_B$ [Florakis, Kounnas, Rizos, NT].

There are strong indications that this configuration induces a quantum instability (at the one-loop level) that could trigger the desired cosmological evolution, providing the spontaneous exit from the early MSDS era.

Once some of the moduli become sufficiently large, so that a conventional spacetime description emerges, the subsequent evolution in the intermediate cosmological regime can be unambiguously described,

thanks to the attractor cosmological solutions outlined above.

A lot of work is necessary to select initial MSDS vacua that can lead to phenomenologically viable cosmological vacua at late times.

On the other hand the qualitative behavior of the underlying effective no-scale supergravity theories indicates that we are in a good direction.