Holographic inflation

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Outline

Cosmology

- Over the last two decades, striking new observations have transformed cosmology from a *qualitative* to a *quantitative* science:

A minimal set of just six parameters characterizes the observed universe, all of which are now known to within a few percent.

- Forthcoming experiments (*e.g.* the Planck satellite) promise a wealth of new precision data.
- This presents a unique window to Planck-scale physics and a challenge (and an opportunity) for fundamental theory.

The origin of structure

- All inhomogeneity in the present day universe may be traced back to primordial fluctuations in the matter density and curvature of the early universe.
- These fluctuations may be imaged directly through observations of the cosmic microwave background (*e.g.*, using the WMAP satellite).
- A widely accepted explanation for the origin of these fluctuations is the theory of *inflation*, which postulates the early universe underwent a brief period of rapid, accelerated expansion.
- Despite its successes, the theory of inflation is still unsatisfactory in a number of ways (*e.g.*, fine-tuning, initial conditions, trans-Planckian issues).

With future observations promising an unprecedented era of precision cosmology, the constraints on cosmological parameters are expected to tighten further still, particularly as regards the inflationary sector.

It thus become imperative that

- inflation is embedded in a UV complete theory (indeed there is increasing amount of effort devoted to embedding inflation in string theory),
- alternative scenarios are developed.

The holographic approach that we undertake provides both.

Holography

- The idea of holography ['t Hooft (1993)] emerged from black holes physics as an answer to the question:

  Why is black hole entropy proportional to the area of the horizon rather than the volume the black hole occupies?

Definition

Holography states that a theory which includes gravity can be described by a theory with no gravity is one fewer dimension.

- Holography became a prominent research direction when holographic dualities were found in string theory [Maldacena (1997)], [Gubser, Klebanov, Polyakov (1998)] [Witten (1998)].

Holography

- The examples found in string theory involve spacetimes with negative cosmological constant but the general argument for holography is applicable to any theory of gravity.
- In particular, it should apply to our own universe.
- The purpose of this work is to propose a concrete holographic framework applicable to our own universe and in particular to its cosmological evolution.

Holography for cosmology

- Specifically, I will address the question:

Can a four-dimensional inflationary cosmology be described in terms of a three-dimensional QFT? (*without* gravity!)

References

The talk is based on

- Paul McFadden, KS,
  Holography for Cosmology,
  arXiv:0907.5542
- Paul McFadden, KS,
  The Holographic Universe,
  arXiv:1007.2007
- Adam Bzowski, Paul McFadden, KS
  on-going

Outline

Outline

Form of the primordial perturbations

The primordial scalar power spectrum Δ_S^2 may be characterized by an amplitude $\mathcal{A}$ and a spectral tilt n_s according to

$$\Delta_S^2 = \mathcal{A}\,(q/q_0)^{n_s-1}\,,$$

where q is the wavenumber of the perturbations and q_0 is an arbitrary scale. The WMAP data then yield (for $q_0 = 0.002\mathrm{Mpc}^{-1}$)

$$\mathcal{A} = (2.43 \pm 0.11) \times 10^{-9}, \qquad n_s - 1 = -0.037 \pm 0.014,$$

i.e., the perturbations have small amplitude and are nearly scale invariant.

- These two small numbers should appear naturally in any theory that explains the data.

Scalar running

- In reality, the spectral tilt n_s may be a function of momentum q. This is parametrized by the "running", α_s, where

$$\alpha_s = \mathrm{d}n_s/\mathrm{d}\ln q.$$

- Observationally

$$\alpha_s = -0.022 \pm 0.020 (68\% CL)$$

Data from [Komatsu et al 1001.4538]

Tensor Power spectrum

- Tensor power spectrum Δ_T^2:

$$\Delta_T^2(q) = A_t(q_*)\,(q/q_*)^{n_t(q_*)}$$

  Only upper limits on A_t and n_t.
- Tensor-to-scalar ratio $r = P_t/P_s$. Observationally,

$$r < 0.22(95\% C.L.)$$

Non-Gaussianity

- While the power spectrum is related to 2-point functions (as we will review later), non-gausianities are related to higher-point functions, e.g.

$$\langle \zeta_{\vec{q}_1} \zeta_{\vec{q}_2} \zeta_{\vec{q}_3} \rangle = (2\pi)^3 \delta(\vec{q}_1 + \vec{q}_2 + \vec{q}_3) f_{NL} F(q_1, q_2, q_3)$$

- $F(q_1, q_2, q_3)$ is a function of the momenta.
- Observationally,

$$f_{NL} = 32 \pm 21 (68\% CL)$$

Outline

Results

Our two main results are:

Standard inflation is holographic.

There are holographic models that have different phenomenology than standard inflation but they are nevertheless consistent with current observations.

Standard inflation is holographic

Assuming that the standard gauge/gravity duality is valid and restricting to single-scalar models (*for simplicity*) we give a mathematical proof that:

- Inflationary cosmological observables, such as the power spectrum and non-gausianities, of four dimensional inflationary models are encoded in correlation functions of strongly coupled three dimensional QFT.

$\longrightarrow$ This provides a UV completion for standard inflation: the three dimensional QFTs that enter this discussion are well-defined theories: they are either conformal or super-renormalizable.

New holographic models

Standard inflation is linked to strongly coupled QFTs. There are new models based on weakly coupled QFT.

- In these models gravity is strongly coupled at early times.
- They provide a new mechanism for a scale invariant spectrum.
- They are compatible with current observations, yet they have different phenomenology than standard inflation.

$\longrightarrow$ Alternative scenarios to standard inflation.

Smoking-gun for new holographic models

- In our holographic models the running is minus the deviation from scale invariance (to leading order):

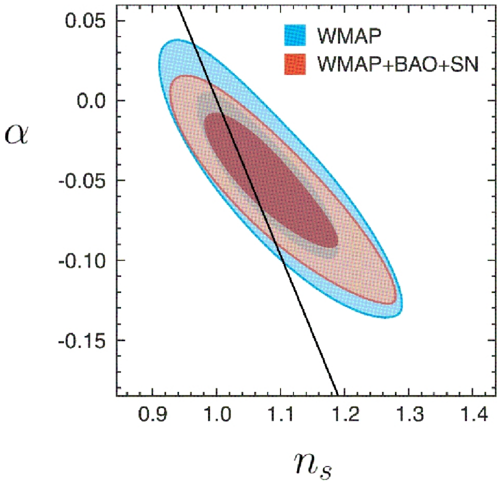
$$\alpha = -(n_s - 1)$$

- In conventional slow-roll inflation, however, $\alpha/(n_s - 1)$ is very small (of first order in slow-roll).

$\Rightarrow$ Predictions of new holographic scenario are different from standard inflation.

WMAP data

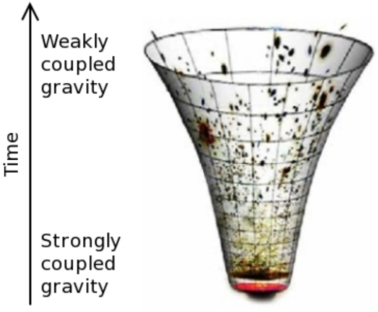
Komatsu et al. arXiv:0803.0547.



Solid line:

$$\alpha = -(n_s - 1)$$

The holographic universe



Outline

Part I: Plan

In the first part we will explain the sense in which inflation is holographic.

- Review standard inflationary computations.
- Review how to compute strong coupling QFT results using standard gauge/gravity duality.
- Show that the inflationary results can be fully expressed in terms of correlators of strongly coupled QFTs.

Cosmological Perturbations

We start by reviewing standard inflationary cosmology.

- We will discuss single field (for simplicity) four dimensional inflationary models,

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}(R - (\partial\Phi)^2 - 2\kappa^2 V(\Phi))$$

- We assume a spatially flat background (for simplicity) and perturb

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t)[\delta_{ij} + h_{ij}(t,\vec{x})]dx^i dx^j \\ \Phi &= \varphi(t) + \delta\varphi(t,\vec{x}) \end{aligned}$$

where $h_{ij} = \psi(z,\vec{x})\delta_{ij} + \partial_i\partial_j\chi(z,\vec{x}) + \gamma_{ij}(z,\vec{x})$

- γ_{ij} is transverse traceless and we form the gauge invariant combination $\zeta = -\psi/2 + (H/\dot{\varphi})\delta\varphi$.

Cosmological perturbations

- The equations for perturbations take the form:

$$
\begin{aligned}
0 &= \ddot{\zeta} + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta} + a^{-2}q^2\zeta \\
0 &= \ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + a^{-2}q^2\gamma_{ij}
\end{aligned}
$$

where H is the Hubble function and $\epsilon = 2(H'/H)^2$ is the slow-roll parameter. We are not assuming that ϵ is small.

Power spectrum

In the inflationary paradigm, cosmological perturbations are assumed to originate at sub-horizon scales as quantum fluctuations.

- Quantising the perturbations in the usual manner,

$$\begin{aligned}\langle \zeta(t,\vec{q})\zeta(t,-\vec{q})\rangle &= |\zeta_q(t)|^2 \\ \langle \gamma_{ij}(t,\vec{q})\gamma_{kl}(t,-\vec{q})\rangle &= 2|\gamma_q(t)|^2\Pi_{ijkl},\end{aligned}$$

  where Π_{ijkl} is the transverse traceless projection operator and $\zeta_q(t)$ and $\gamma_q(t)$ are the mode functions.

- The superhorizon power spectra are obtained by

$$\Delta_S^2(q) = \frac{q^3}{2\pi^2}|\zeta_{q(0)}|^2, \quad \Delta_T^2(q) = \frac{2q^3}{\pi^2}|\gamma_{q(0)}|^2,$$

  where $\gamma_{q(0)}$ and $\zeta_{q(0)}$ are the constant late-time values of the cosmological mode functions. Initial conditions are set by the Bunch-Davies vacuum.

Power spectrum through response functions

In preparation to the holographic discussion, we rewrite the power spectrum as follows.

- We define the response functions as

$$\Pi^{\zeta} = \Omega\zeta, \quad \Pi^{\gamma}_{ij} = E\gamma_{ij},$$

where Π^{ζ} and Π^{γ}_{ij} are the canonical momenta.

- One can show that

$$|\zeta_q|^{-2} = -2\mathrm{Im}[\Omega(q)], \quad |\gamma_q|^{-2} = -4\mathrm{Im}[E(q)].$$

so the power spectra can be expressed in terms of the late time behavior of the response functions.

Domain-wall/cosmology correspondence

The springboard for our discussion is a correspondence between cosmologies and domain-wall spacetimes.

- Domain-wall spacetime:

$$\begin{aligned} ds^2 &= dr^2 + e^{2A(r)} dx^i dx^i \\ \Phi &= \Phi(r) \end{aligned}$$

- This solves the field equations that follow from

$$S_{DW} = \frac{1}{2\bar{\kappa}^2} \int d^4x \sqrt{g} \left[-R + (\partial\Phi)^2 + 2\bar{\kappa}^2 \bar{V}(\Phi) \right],$$

Domain-wall/cosmology correspondence

- One can prove the following:

Domain-wall/Cosmology correspondence

For **every** domain-wall solution of a model with potential $\bar{V}$ there is a FRW solution for a model with potential $(V = -\bar{V})$. [Cvetic, Soleng (1994)], [KS, Townsend (2006)]

- The correspondence also applies to open and closed FRW universes which correspond to curved domain-walls.
- The correspondence can be understood as analytic continuation for the metric. The flip in the sign of V guarantees that the scalar field remains real.
- An equivalent way to state the correspondence is

$$\bar{\kappa}^2 = -\kappa^2$$

Domain-walls and holography

Domain-wall spacetimes enter prominently in holography. They describe holographic RG flows.

- The AdS_{d+1} metric is the unique metric whose isometry group is the same as the conformal group in d dimensions. This is the main reason why the bulk dual of a CFT is AdS.
- The domain-wall spacetimes are the most general solutions whose isometry group is the Poincaré group in d dimensions. Thus, if a QFT has a holographic dual the bulk solution must be of the domain-wall type.

Holographic RG flows

There are two different types of domain-wall spacetimes whose holographic interpretation is fully understood.

1. The domain-wall is asymptotically AdS_{d+1},

$$A(r) \to r, \qquad \Phi(r) \to 0, \qquad \text{as} \quad r \to \infty$$

   This corresponds to a QFT that in the UV approaches a fixed point. The fixed point is the CFT which is dual to the AdS spacetime approached as $r \to \infty$.

Holographic RG flows

2. The domain-wall has the following asymptotics

$$A(r) \to n \log r, \qquad \Phi(r) \to \sqrt{2n} \log r, \qquad \text{as} \quad r \to \infty$$

This case has only been understood recently [Kanitscheider, KS, Taylor (2008)] [Kanitscheider, KS (2009)].

$\longrightarrow$ Specific cases of such spacetimes are ones obtained by taking the near-horizon limit of the non-conformal branes (D0, D1, F1, D2, D4).

$\longrightarrow$ These solutions describe QFTs with a dimensionful coupling constant in the regime where the dimensionality of the coupling constant drives the dynamics.

Domain-wall/cosmology correspondence

Let us see how the correspondence acts on the domain-walls describing holographic RG flows.

1. Asymptotically AdS domain-walls are mapped to inflationary cosmologies that approach de Sitter spacetime at late times,

$$ds^2 \to ds^2 = -dt^2 + e^{2t} dx^i dx^i, \qquad \text{as} \quad t \to \infty$$

2. The second type of domain-walls is mapped to solutions that approach power-law scaling solutions at late times,

$$ds^2 \to ds^2 = -dt^2 + t^{2n} dx^i dx^i, \qquad \text{as} \quad t \to \infty$$

Holography: a primer

The holographic dictionary for cosmology will be based on the standard holographic dictionary, so we now briefly review standard holography:

1. There is 1-1 correspondence between local gauge invariant operators $\mathcal{O}$ of the boundary QFT and bulk supergravity modes Φ.
   - $\longrightarrow$ The bulk metric corresponds to the energy momentum tensor of the boundary theory.
   - $\longrightarrow$ Bulk scalar fields correspond to boundary scalar operators, i.e. $F_{\mu\nu}F^{\mu\nu}, \bar{\psi}\psi$, etc.
2. Correlation functions of gauge invariant operators can be extracted from the asymptotics of bulk solutions.

Asymptotic solutions

To understand the holographic computations we need to know a few things about the structure of solutions of Einstein's theory with a negative cosmological constant.

- For the metric, the most general asymptotic form (in 4 bulk dimensions) looks like [Fefferman, Graham (1985)]

$$ds^2 = dr^2 + e^{2r} g_{ij}(x,r) dx^i dx^j$$

$$g_{ij}(x,r) = \mathbf{g_{(0)ij}}(\mathbf{x}) + e^{-2r} g_{(2)ij}(x) + e^{-3r} g_{(3)ij}(x) + \ldots$$

- $\mathbf{g_{(0)}}(\mathbf{x})$ is the metric of the spacetime where the boundary theory lives and (as such) it is also the source of the boundary energy momentum tensor.

Correlation functions

- Using the formalism of holographic renormalization, we then find a precise relation between correlation functions and asymptotics [de Haro, Solodukhin, KS (2000)]

$$\langle T_{ij} \rangle = \frac{3}{2\kappa^2} g_{(3)ij}.$$

- Higher-point functions are obtained by differentiating the 1-point functions w.r.t. sources and then setting the sources to their background value

$$\langle T_{i_1 j_1}(x_1) T_{i_2 j_2}(x_2) \cdots T_{i_n j_n}(x_n) \rangle \sim \frac{\delta^{(n-1)} g_{(3) i_1 j_1}(x_1)}{\delta g_{(0) i_2 j_2}(x_2) \cdots \delta g_{(0) i_n j_n}(x_n)} \Big|_{g_{(0)}=\eta}$$

- Thus to solve the theory we need to know $g_{(3)}$ as a function of $g_{(0)}$. This can be obtained perturbatively: 2-point functions are obtained by solving linearized fluctuations, 3-point functions by solving quadratic fluctuations etc.

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Correlation functions for holographic RG flows

- To compute 2-point functions we perturb around the domain-wall

$$
\begin{aligned}
ds^2 &= dr^2 + e^{2A(r)}[\delta_{ij} + h_{ij}(r,x^i)]dx^i dx^j \\
\Phi &= \varphi(r) + \delta\varphi(r,x^i)
\end{aligned}
$$

  where $h_{ij} = \psi(r,x^i)\delta_{ij} + \partial_i\partial_j\chi(r,x^i) + \gamma_{ij}(r,x^i)$
- γ_{ij} is transverse traceless and we form the gauge invariant combination $\zeta = -\psi/2 + (H/\dot{\varphi})\delta\varphi$ and $H = -W/2$, with W the fake superpotential.

Correlation functions for holographic RG flows

- The linearized equations are given by [Bianchi, Freedman, KS (2001)], [Papadimitriou, KS (2004)],

$$
\begin{aligned}
0 &= \ddot{\zeta} + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta} - \bar{q}^2 e^{-2A}\zeta \\
0 &= \ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} - \bar{q}^2 e^{-2A}\gamma_{ij},
\end{aligned}
$$

- Comparing with the cosmological perturbations, we find that the equations are mapped to each other provided

$$\bar{q} = -iq$$

Domain-wall/Cosmology correspondence

In other words, we have just establish that:

The Domain-wall/cosmology correspondence maps not only the background solutions but also linear fluctuations around them.

This extends to all orders in fluctuations.

Correlation functions for holographic RG flows

We now want to extract the QFT correlators from the linearized solution.

- Schematically, we must expand the linearized solution near the $r \to \infty$ and extract the piece that scales like e^{-3r}.
- It is convenient to work in terms of response functions [Papadimitriou, KS (2004)]

$$\bar{\Pi}^{\zeta} = -\bar{\Omega}\zeta, \quad \bar{\Pi}^{\gamma}_{ij} = -\bar{E}\gamma_{ij},$$

where $\bar{\Pi}^{\zeta}, \bar{\Pi}^{\gamma}_{ij}$ are radial canonical momenta.

2-point functions for holographic RG flows

The 2-point function of the energy momentum tensor is then given by

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl},$$

where

$$\Pi_{ijkl} = \frac{1}{2}(\pi_{ik}\pi_{lj} + \pi_{il}\pi_{kj} - \pi_{ij}\pi_{kl}), \quad \pi_{ij} = \delta_{ij} - \bar{q}_i \bar{q}_j / \bar{q}^2.$$

$$\begin{aligned} A(\bar{q}) &= 4 \left[\bar{E}(\bar{q})\right]_{(0)} \\ B(\bar{q}) &= \frac{1}{4} \left[\bar{\Omega}(\bar{q})\right]_{(0)} . \end{aligned}$$

The subscript indicates that one should pick the term with appropriate scaling in the asymptotic expansion.

Holography for cosmology

We are now ready to show that the power spectrum is holographic.

- We have shown earlier that

$$\Delta_S^2(q) = \frac{-q^3}{4\pi^2 \mathrm{Im}\Omega_{(0)}(q)}, \quad \Delta_T^2(q) = \frac{-q^3}{2\pi^2 \mathrm{Im}E_{(0)}(q)},$$

- Applying the analytic continuation,

$$\bar{\kappa}^2 = -\kappa^2, \qquad \bar{q} = -iq$$

we find:

$$\Delta_S^2(q) = \frac{q^3}{2\pi^2}\left(\frac{-1}{8\mathrm{Im}B(-iq)}\right), \quad \Delta_T^2(q) = \frac{2q^3}{\pi^2}\left(\frac{-1}{\mathrm{Im}A(-iq)}\right),$$

where the holographic 2-point function is

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q})\rangle = A(\bar{q})\Pi_{ijkl} + B(\bar{q})\pi_{ij}\pi_{kl},$$

Non-gaussianity [McFadden, KS], [Bzowski, McFadden, KS] to appear

A similar type of analysis shows a direct link between non - Gausianities and holographic higher-point functions. The precise holographic dictionary requires technical control over a number of issues. For example, for 3-point functions:

- one needs to know the precise form of the cosmological perturbations to second order (without slow-roll).
- one needs to know the general form of

$$\langle T_{i_1 j_1}(\bar{q}_1) T_{i_2 j_2}(\bar{q}_2) T_{i_3 j_3}(\bar{q}_3) \rangle = \ldots$$

Both of these are near completion ...

Example: power-law cosmology

We have established that any inflationary model is holographic. Let us now see how this works in a simple example.

- Consider the potential

$$V(\varphi) = V_0 \exp(-\sqrt{2/n}\kappa\varphi)$$

- The corresponding solution is

$$ds^2 = -dt^2 + (t/t_0)^n dx^i dx^i, \qquad \kappa\varphi = \sqrt{2n}\ln t/t_0$$

- When $n = 7$ this solution is related via the DW/cosmology correspondence to the near-horizon limit of a stack of D2 branes.

Example: power-law cosmology

- The holographic 2-point functions have been c omputed in [Kanitscheider, KS, Taylor (2008)]

$$A(\bar{q}) = 2nB(\bar{q}) = -\frac{2\pi}{4^\sigma \Gamma^2(\sigma) \sin \pi\sigma} \kappa^{-2} \bar{q}^{2\sigma}.$$

where $\sigma = (3n-1)/(n-1) > 3/2$.
- Using the analytic continuation one obtains

$$\Delta_T^2(q) = \frac{16}{n} \Delta_S^2(q) = \frac{4^\sigma \Gamma^2(\sigma)}{\pi^3} \kappa^2 q^{3-2\sigma},$$

which is the correct answer.

Analytic continuation in QFT variables

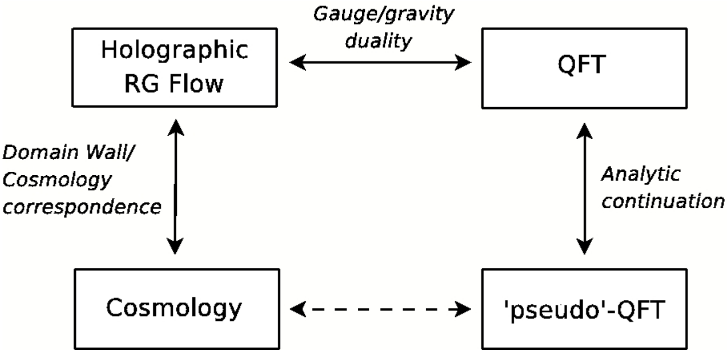
- The analytic continuation

$$\bar{\kappa}^2 = -\kappa^2, \qquad \bar{q} = -iq,$$

translates in QFT language to

$$N^2 \to -N^2, \qquad \bar{q} \to -iq$$

The proposal



Pseudo-QFT

We operationally define the pseudo-QFT as follows:

- we do the computation in the QFT dual to the domain-wall and then analytically continue parameters and momenta appropriately.

Perhaps a more fundamental perspective is to consider the QFT action with complex parameters as the fundamental object.

- Then the results on different real domains will be applicable to DW/cosmology as appropriate.

$\longrightarrow$ The supergravity realization of the DW/cosmology correspondence works this way.

Part II: Beyond the weak gravitational description

- So far the discussion was on the gravitational side.
- We inferred a QFT description using known gauge/gravity dualities and analytic continuation, but all computations were done on the gravitational side.
- When gravity is strongly coupled the QFT description is weakly coupled, so one may use the duality.
- This allows us to compute the late time behavior of the response functions and therefore the power spectra etc when the early time behavior is strongly coupled/stringy.

Holographic phenomenology for cosmology

- The boundary theory will be a combination of gauge fields, fermions and scalars and it should admit a large N expansion.
- To extract predictions we need to compute the coefficients A and B,

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl},$$

  analytically continue the result and insert in the formulae for the power spectra.
- One can then look for a holographic theory that models well the observations.

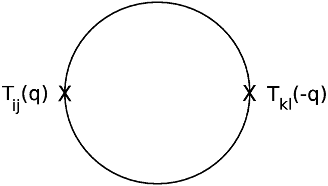
Holographic phenomenology for cosmology

- As a starting point one can consider the strong coupling version of asymptotically dS cosmologies and power-law cosmology.
- In this talk we focus on QFTs dual to the latter. These are super-renormalizable QFTs that depend on a single dimensionful coupling:

$$S = \frac{1}{g_{\rm YM}^2} \int d^3x \mathrm{tr} \left[\frac{1}{2} F^I_{ij} F^{Iij} + \frac{1}{2} (D\phi^J)^2 + \frac{1}{2} (D\chi^K)^2 + \bar{\psi}^L \not{D} \psi^L \right.$$
$$\left. + \lambda_{M_1 M_2 M_3 M_4} \Phi^{M_1} \Phi^{M_2} \Phi^{M_3} \Phi^{M_4} + \mu^{\alpha\beta}_{M L_1 L_2} \Phi^M \psi^{L_1}_\alpha \psi^{L_2}_\beta \right].$$

A new mechanism for scale invariant spectrum

We need to compute the 2-point function of T_{ij}. The leading order computation is at 1-loop:



The answer follows from general considerations:

- The stress energy tensor has dimension 3 in three dimensions.
- 1-loop amplitudes are independent of g^2_{YM}
- There is a factor of N^2 because of the trace over the gauge indices.

$$\langle T_{ij} T_{kl} \rangle \sim N^2 \bar{q}^3$$

A new mechanism for scale invariant spectrum

Recalling the holographic map:

$$\Delta_S^2 \sim \frac{\bar{q}^3}{\langle TT \rangle} \sim \frac{1}{N^2}$$

- Spectrum scale invariant to leading order, independent of the details of the holographic theory.

Furthermore,

- Amplitude of power spectrum $\mathcal{A} \sim 1/N^2$.
- Small $\mathcal{A} \sim 10^{-9} \Rightarrow$ large $N \sim 10^4$, justifying the large N limit.

Power spectra

The complete answer is

$$A(\bar{q}) = C_A \bar{N}^2 \bar{q}^3 + O(g_{\mathrm{YM}}^2), \qquad B(\bar{q}) = C_B \bar{N}^2 \bar{q}^3 + O(g_{\mathrm{YM}}^2),$$

where

$$C_A = (\mathcal{N}_A + \mathcal{N}_\phi + \mathcal{N}_\chi + 2\mathcal{N}_\psi)/256, \qquad C_B = (\mathcal{N}_A + \mathcal{N}_\phi)/256.$$

$\mathcal{N}_A$: # of gauge fields,
$\mathcal{N}_\phi$: # of minimally coupled scalars,
$\mathcal{N}_\chi$: # of conformally coupled scalars,
$\mathcal{N}_\psi$: # of fermions.

$$\Rightarrow \Delta_S^2(q) = \frac{1}{16\pi^2 N^2 C_B} + O(g_{\mathrm{YM}}^2), \qquad \Delta_T^2(q) = \frac{2}{\pi^2 N^2 C_A} + O(g_{\mathrm{YM}}^2).$$

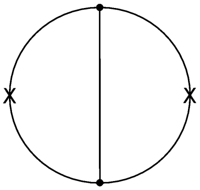
Tensors-to-scalar ratio

- It follows that

$$r = \Delta_T^2/\Delta_S^2 = 32C_B/C_A,$$

- The upper bound on r translates into a constraint on the field content of the dual QFT.
- A smaller upper bound on r requires increasing the number of conformal scalars and massless fermions and/or decreasing the number of gauge fields and minimal scalars.

Subleading corrections



Subleading corrections give small deviations from scale invariance:

$$n_s - 1 \sim g_{\text{eff}}^2 = g_{\text{YM}}^2 N/q.$$

The observational value $(n_s - 1) \sim 10^{-2}$ is then consistent with the QFT being weakly interacting.

- To determine the sign of $(n_s - 1)$ (positive: red-tilted spectrum, negative: blue-tilted spectrum) requires summing all 2-loop graphs, and will in general depend on the field content of the dual QFT.

[Work in progress]

2-loop details

Super-renormalizable theories often have infrared problems. The specific type of theories we consider however are well-defined: g_{YM}^2 acts as an infrared cut-off. [Jackiw, Templeton (1981)] [Appelquist, Pisarski (1981)].

The 2-loop integrals are indeed finite and one obtains:

$$A(\bar{q}) = C_A \bar{N}^2 \bar{q}^3 [1 + D_A g_{\text{eff}}^2 \ln(\bar{q}/\bar{q}_*) + O(g_{\text{eff}}^4)],$$
$$B(\bar{q}) = C_B \bar{N}^2 \bar{q}^3 [1 + D_B g_{\text{eff}}^2 \ln(\bar{q}/\bar{q}_*) + O(g_{\text{eff}}^4)],$$

where $g_{\text{eff}}^2 = g_{\text{YM}}^2 N/q$ and D_A and D_B are numerical constants.
This leads to

$$n_S(q) - 1 = -D_B g_{\text{eff}}^2 + O(g_{\text{eff}}^4), \qquad n_T(q) = -D_A g_{\text{eff}}^2 + O(g_{\text{eff}}^4).$$

Running

- Independent of the details of the theory, the scalar spectral index runs as

$$\alpha_s = \frac{dn_s}{d\ln q} = -(n_s - 1) + O(g_{\text{eff}}^4).$$

- This prediction is qualitatively different from slow-roll inflation, for which $\alpha_s/(n_s - 1)$ is of first-order in slow-roll.

Non-Gaussianity *[in progress]*

- To compute the leading non-Gaussianity one needs to compute

$$\langle T_{i_1 j_1}(\bar{q}_1) T_{i_2 j_2}(\bar{q}_2) T_{i_3 j_3}(\bar{q}_3) \rangle$$

- Leading contribution is 1-loop and does not depends on the interactions of the holographic model.
- Preliminary results show that these holographic models predict $f_{NL} \sim O(10^0)$, in agreement with current expectations!

Outline

Conclusions

- I have presented a holographic description of inflationary cosmology in terms of a 3-dimensional QFT (without gravity!)
- When gravity is weakly coupled, holography correctly reproduces standard inflationary predictions for cosmological observables.
- When gravity is strongly coupled, one finds new models that have a QFT description.
- We initiated a holographic phenomenological approach to cosmology.

Holographic phenomenology

- Generic holographic models lead to a scale invariant spectrum.
- One can find models that fit all current observations. This fixes the parameters of the model, N, g_{YM}^2, and constrains the field content.
- These models have distinct phenomenology than standard inflation.
- Further cosmological observables are computable, essentially with no further adjustable parameters.

Outlook

- Further develop holographic phenomenology and obtain precise predictions for the cosmological observables.
- In next few years, these cosmological observables will be known to a very high accuracy and cosmology may well provide the first observational evidence for the holographic nature of our own universe!