Bouncing Cosmologies

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McGill University

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Outline

Plan

Current Paradigm for Early Universe Cosmology

The Inflationary Universe Scenario is the current paradigm of early universe cosmology.

Successes:

- Solves horizon problem
- Solves flatness problem
- Solves size/entropy problem
- Provides a causal mechanism of generating primordial cosmological perturbations (Chibisov & Mukhanov, 1981).

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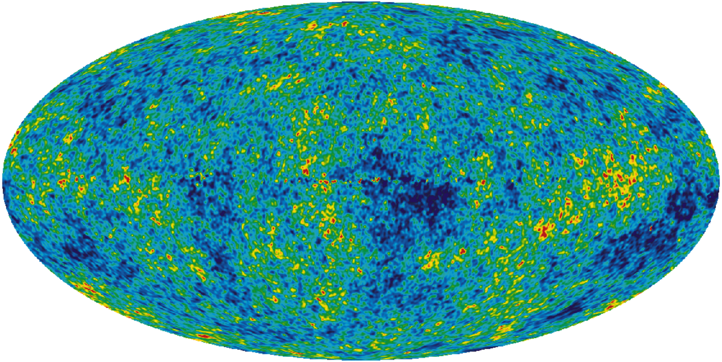
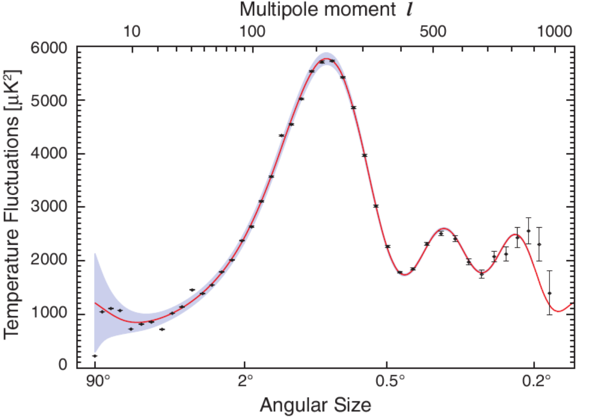


Credit: NASA/WMAP Science Team



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Historical Footnote

1970 paper

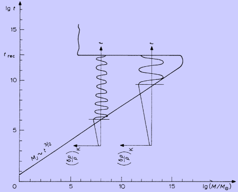

Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

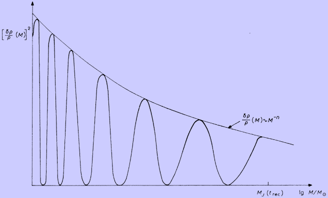

Fig. 1b. The dependence of the square of the amplitude of density perturbations of matter on scale. The fine line designates the usually assumed dependence $(\delta\varrho/\varrho)_M \sim M^{-n}$. It is apparent that fluctuations of relic radiation should depend on scale in a similar manner.

R. Sunyaev & Ya. Zeldovich, Astrophysics and Space Science 7 3-19 (1970)

Challenges for the Current Paradigm

- In spite of the phenomenological successes, current realizations of the inflationary scenario suffer from several conceptual problems.
- One of these problems is the singularity problem.
- A bouncing cosmology would provide a solution to this problem.
- Question: Is it possible to obtain bouncing cosmologies from fundamental physics?
- Question: Can these new paradigms be tested in cosmological observations?

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Review of Inflationary Cosmology

Context:

- General Relativity
- Scalar Field Matter

$$\text{Metric}: \; ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 \quad (1)$$

Inflation:

- phase with $a(t) \sim e^{tH}$
- requires matter with $p \sim -\rho$
- requires a slowly rolling scalar field φ
- - in order to have a potential energy term
- - in order that the potential energy term dominates sufficiently long

Review of Inflationary Cosmology

Bounce

Brandenberger

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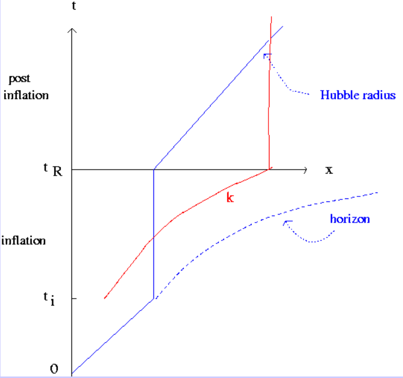
Time line of inflationary cosmology:

- t_i: inflation begins
- t_R: inflation ends, reheating

Review of Inflationary Cosmology II

Space-time sketch of inflationary cosmology:



Note:

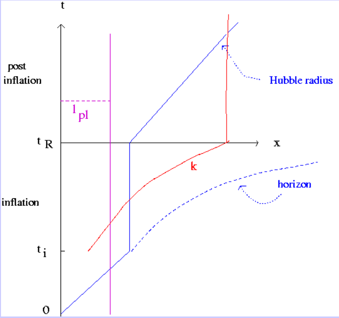
- $H = \frac{\dot{a}}{a}$
- curve labelled by k: wavelength of a fluctuation

- inflation renders the universe large, homogeneous and spatially flat
- classical matter redshifts $\rightarrow$ matter vacuum remains
- quantum vacuum fluctuations: seeds for the observed structure [Chibisov & Mukhanov, 1981]
- sub-Hubble $\rightarrow$ locally causal

Conceptual Problems of Inflationary Cosmology

- Nature of the scalar field φ (the “inflaton")
- Conditions to obtain inflation (initial conditions, slow-roll conditions, graceful exit and reheating)
- Amplitude problem
- Trans-Planckian problem
- Cosmological constant problem
- Applicability of General Relativity
- Singularity problem

Trans-Planckian Problem



- **Success of inflation**: At early times scales are inside the Hubble radius → causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation
- → new physics MUST enter into the calculation of the fluctuations.

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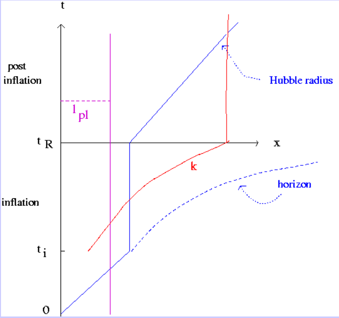

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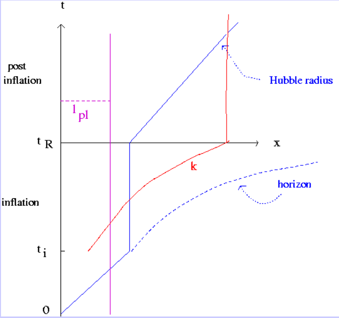

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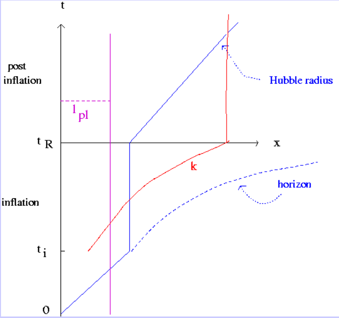

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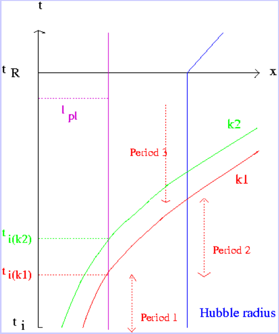
Recent Reference: A. Linde, V. Mukhanov and A. Vikman, arXiv:0912.0944

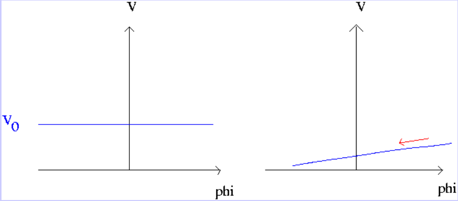
- It is not sufficient to show that the Hubble constant is smaller than the Planck scale.
- The frequencies involved in the analysis of the cosmological fluctuations are many orders of magnitude larger than the Planck mass. Thus, “the methods used in [1] are inapplicable for the description of the .. process of generation of perturbations in this scenario."

Trans-Planckian Window of Opportunity



- If evolution in Period I is non-adiabatic, then scale-invariance of the power spectrum will be lost [J. Martin and RB, 2000]
- $\rightarrow$ Planck scale physics testable with cosmological observations!

Cosmological Constant Problem



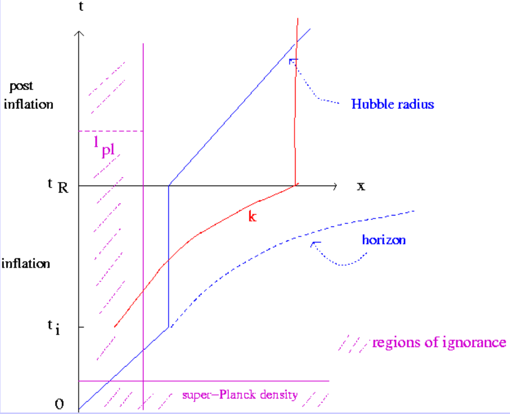
- Quantum vacuum energy does not gravitate.
- Why should the almost constant $V(\varphi)$ gravitate?

$$\frac{V_0}{\Lambda_{obs}} \sim 10^{120} \tag{2}$$

Applicability of GR

- In all approaches to quantum gravity, the Einstein action is only the leading term in a low curvature expansion.
- Correction terms may become dominant at much lower energies than the Planck scale.
- Correction terms will dominate the dynamics at high curvatures.
- The energy scale of inflation models is typically $\eta \sim 10^{16}$GeV.
- $\rightarrow \eta$ too close to m_{pl} to trust predictions made using GR.

Zones of Ignorance



Singularity Problem

- Standard cosmology: Penrose-Hawking theorems $\rightarrow$ initial singularity $\rightarrow$ incompleteness of the theory.
- Inflationary cosmology: In scalar field-driven inflationary models the initial singularity persists [Borde and Vilenkin] $\rightarrow$ incompleteness of the theory.

Penrose-Hawking theorems:

- Ass: 1) Einstein gravitational action,
- Ass: 2) weak energy conditions for matter $\rho > 0, \quad \rho + 3p \geq 0$
- $\rightarrow$ space-time is geodesically incomplete.

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Message

- Current realizations of inflation have conceptual problems.
- We need a new paradigm of very early universe cosmology.
- Bouncing cosmologies may provide an alternative early universe scenario.
- Bouncing cosmologies require new fundamental physics.
- Bouncing cosmologies need not involve a period of inflation.
- Any viable bouncing cosmology must explain the current data and make predictions with which it can be distinguished from those of standard slow-roll inflation.

Plan

Requirements I

Model must yield a successful structure formation scenario:

- Scales of cosmological interest today must originate inside the Hubble radius.
- Long propagation on super-Hubble scales.
- Scale-invariant spectrum of adiabatic cosmological perturbations.

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Requirements II

Model must refer to the problems of Standard Cosmology which the inflationary scenario addresses.

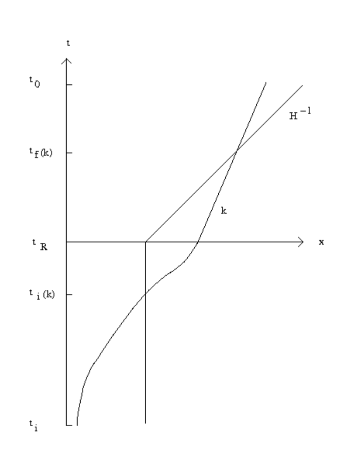
- Solution of the horizon problem.
- Solution of the flatness problem.
- Solution of the size and entropy problems.

Achieving a Bouncing Cosmology

New physics is required in order to obtain a bounce (in order to circumvent the assumptions made in the Hawking-Penrose singularity theorems):

- First option: New form of matter violating the weak energy condition.
- Second option: Corrections to the gravitational action in the ultraviolet.

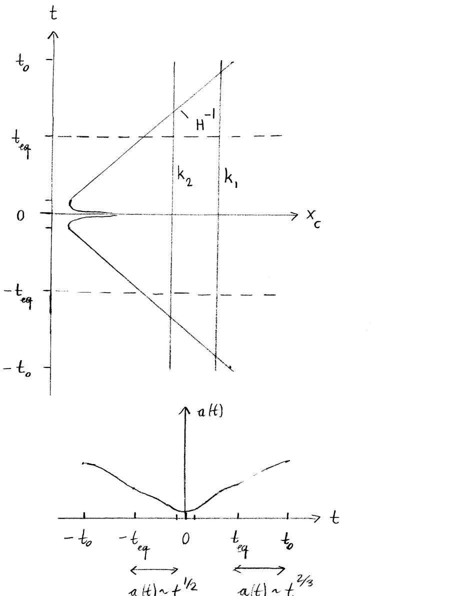
Structure formation in inflationary cosmology



N.B. Perturbations originate as quantum vacuum fluctuations.

Matter Bounce Scenario

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*, D. Wands, *Phys. Rev. D60 (1999)*



Overview of the Matter Bounce

- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.
- Adiabatic fluctuation mode acquires a scale-invariant spectrum of curvature perturbations on super-Hubble scales.
- Horizon problem: absent.
- Flatness problem: weak point.
- Size and entropy problems: not present if we assume that the universe begins cold and large.

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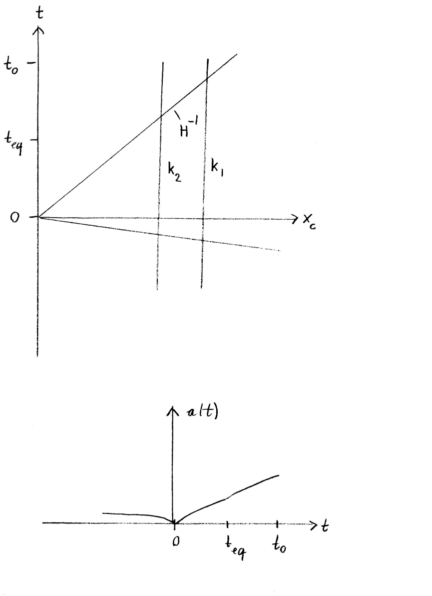
Ekpyrotic Bounce

J. Khoury, B. Ovrut, P. Steinhardt and N. Turok *Phys. Rev. D64, 123522 (2001)*



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- Adiabatic fluctuation mode not scale invariant.
- Entropic fluctuation modes acquire a scale-invariant spectrum of curvature perturbations on super-Hubble scales.
- Transfer of to adiabatic fluctuations on super-Hubble scales (similar to curvaton scenario).
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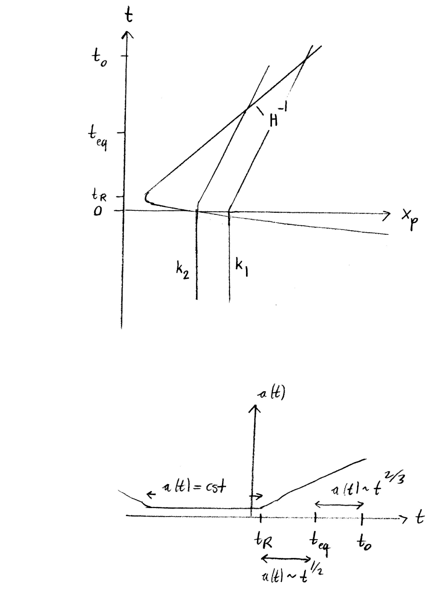
String Gas Bounce

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97, 021302 (2006)*, T. Biswas, R.B., A. Mazumdar and W. Siegel, 2006



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Theory of Cosmological Perturbations: Basics

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter $\rightarrow$ large-scale structure
- Fluctuations of metric $\rightarrow$ CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- 1. Fluctuations are small today on large scales
- $\rightarrow$ fluctuations were very small in the early universe
- $\rightarrow$ can use linear perturbation theory
- 2. Sub-Hubble scales: matter fluctuations dominate
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Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

$$ds^2 = a^2[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2] \quad (3)$$

$$\varphi = \varphi_0 + \delta\varphi \quad (4)$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$S^{(2)} = \frac{1}{2}\int d^4x\left((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2\right) \quad (5)$$

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Step 3: Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0 \quad (8)$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

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More on Perturbations I

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$$\zeta = z^{-1} v \tag{10}$$

Its physical meaning: curvature perturbation in comoving gauge.

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More on Perturbations II

- In the case of entropy fluctuations there are more than one degrees of freedom for the scalar metric inhomogeneities. Example: extra scalar field.
- Entropy fluctuations seed an adiabatic mode even on super-Hubble scales.

$$\dot{\zeta} = \frac{\dot{p}}{p + \rho} \delta S \tag{11}$$

- Example: topological defect formation in a phase transition.
- Example: Axion perturbations when axions acquire a mass at the QCD scale (M. Axenides, R.B. and M. Turner, 1983).

More on Perturbations II

- In the case of entropy fluctuations there are more than one degrees of freedom for the scalar metric inhomogeneities. Example: extra scalar field.
- Entropy fluctuations seed an adiabatic mode even on super-Hubble scales.

$$\dot{\zeta} = \frac{\dot{p}}{p+\rho}\delta S \qquad (11)$$

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Plan

Models for a Matter Bounce I

By modifying the matter sector:

- Quintom matter: extra matter field with negative kinetic term in the action (Y. Cai et al, 2007)
- Lee-Wick matter: higher derivative matter action with an extra pole to cancel quadratic loop divergences in scattering amplitudes (Y. Cai et al, 2008).

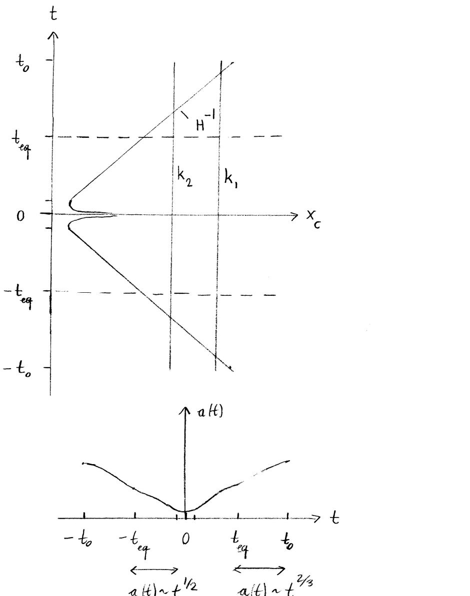
Models for a Matter Bounce II

By modifying the gravitational action:

- Nonsingular Universe construction: Special invariant added to the Einstein action constructed such that at large curvatures all solutions tend to de Sitter (R.B., V. Mukhanov and A. Sornborger, 1993).
- Mirage Cosmology: Dynamics induced by the motion of a brane in a non-singular bulk (Kehagias and Kiritsis, 1999).
- Horava-Lifshitz gravity: in the presence of spatial curvature the extra spatial derivative terms in the gravitational action act as ghost radiation and ghost anisotropic stress (R.B., 2009).

Matter Bounce Scenario

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*, D. Wands, *Phys. Rev. D60 (1999)*



Origin of Scale-Invariant Spectrum

- The initial vacuum spectrum is blue:

$$P_\zeta(k) \,=\, k^3|\zeta(k)|^2 \,\sim\, k^2 \tag{12}$$

- The curvature fluctuations grow on super-Hubble scales in the contracting phase:

$$v_k(\eta) \,=\, c_1\eta^2 + c_2\eta^{-1}\,, \tag{13}$$

- For modes which exit the Hubble radius in the matter phase the resulting spectrum is scale-invariant:

$$\begin{aligned} P_\zeta(k,\eta) &\sim k^3|v_k(\eta)|^2 a^{-2}(\eta) \\ &\sim k^3|v_k(\eta_H(k))|^2\left(\frac{\eta_H(k)}{\eta}\right)^2 \sim k^{3-1-2} \\ &\sim \text{const}\,, \end{aligned} \tag{14}$$

Transfer of the Spectrum through the Bounce

- In a nonsingular background the fluctuations can be tracked through the bounce explicitly (both numerically in an exact manner and analytically using matching conditions at times when the equation of state changes).
- Explicit computations have been performed in the case of quintom matter (Y. Cai et al, 2008), mirage cosmology (R.B. et al, 2007), Horava-Lifshitz bounce (X. Gang et al, 2009).
- Result: On length scales larger than the duration of the bounce the spectrum of v goes through the bounce unchanged.

Signature in the Bispectrum: formalism

$$
\begin{aligned}
& < \zeta(t, \vec{k}_1)\zeta(t, \vec{k}_2)\zeta(t, \vec{k}_3) > \\
& = i \int_{t_i}^{t} dt' < [\zeta(t, \vec{k}_1)\zeta(t, \vec{k}_2)\zeta(t, \vec{k}_3), L_{int}(t')] > ,
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
< \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) > \quad = \quad & (2\pi)^7 \delta(\sum \vec{k}_i) \frac{P_\zeta^2}{\prod k_i^3} \\
& \times \mathcal{A}(\vec{k}_1, \vec{k}_2, \vec{k}_3) ,
\end{aligned}
\tag{16}
$$

$$
|\mathcal{B}|_{NL}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{10}{3} \frac{\mathcal{A}(\vec{k}_1, \vec{k}_2, \vec{k}_3)}{\sum_i k_i^3} . \tag{17}
$$

Signature in the Bispectrum: Results

Y. Cai, W. Xue, R.B. and X. Zhang, *JCAP 0905:011 (2009)*

If we project the resulting shape function $\mathcal{A}$ onto some popular shape masks we get

$$|\mathcal{B}|_{NL}^{\mathrm{local}} = -\frac{35}{8}\,, \tag{18}$$

for the local shape ($k_1 \ll k_2 = k_3$). This is negative and of order $O(1)$.
For the equilateral form ($k_1 = k_2 = k_3$) the result is

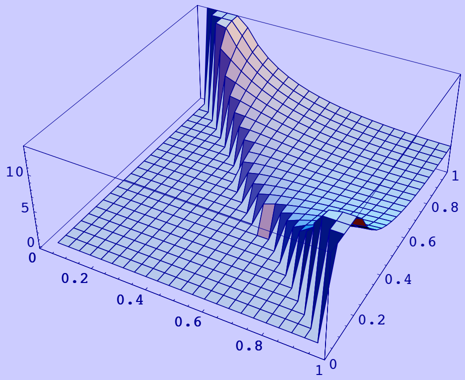
$$|\mathcal{B}|_{NL}^{\mathrm{equil}} = -\frac{255}{64}\,, \tag{19}$$

For the folded form ($k_1 = 2k_2 = 2k_3$) one obtains the value

$$|\mathcal{B}|_{NL}^{\mathrm{folded}} = -\frac{9}{4}\,. \tag{20}$$

Bispectrum of the Matter Bounce Scenario

Y. Cai, W. Xue, R.B. and X. Zhang, *JCAP 0905:011 (2009)*



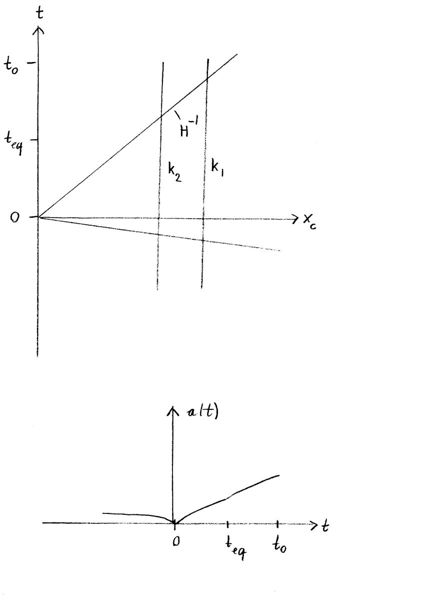
Challenges for the Matter Bounce Scenario

- Obtaining a matter bounce in a model free of ghosts and other unwanted degrees of freedom.
- Instability to anisotropic stress.
- Initial conditions for fluctuations?

Plan

Ekpyrotic Bounce

J. Khoury, B. Ovrut, P. Steinhardt and N. Turok *Phys. Rev. D64, 123522 (2001)*



Obtaining a Phase of Ekpyrotic Contraction

Introduce a scalar field with negative exponential potential and AdS minimum:

$$V(\phi) = -V_0 exp(-(\frac{2}{p})^{1/2}\frac{\phi}{m_{pl}}) \quad 0 < p \ll 1 \tag{21}$$

Motivated by potential between branes in heterotic M-theory
In the homogeneous and isotropic limit, the cosmology is given by

$$a(t) \sim a(t)^p \tag{22}$$

and the equation of state is

$$w \equiv \frac{p}{\rho} = \frac{2}{3p} - 1 \gg 1 . \tag{23}$$

Solution to the flatness problem

The energy density in the Ekpyrotic field scales as

$$\rho(a) \, = \, \rho_0 a^{-3(1+w)} \tag{24}$$

and thus dominates all other forms of energy density (including anisotropic stress) as the universe shrinks $\rightarrow$ quasi-homogeneous bounce, no chaotic mixmaster behavior.

Spectrum of Adiabatic Fluctuations

If $a(t) \sim t^p$ then conformal time scales as $\eta \sim t^{1-p}$.

The solution of the mode equation for v is

$$v_k(\eta) \, = \, c_1 \eta^{-\alpha} + c_2 \eta \, , \tag{25}$$

where c_1 and c_2 are constant coefficients and $\alpha \simeq p$ for $p \ll 1$.

Hence, the power spectrum in not scale invariant:

$$\begin{aligned} P_\zeta(k, t) \quad &= \quad \left(\frac{z(t)}{v(t_H(k))}\right)^2 k^3 |v_k(t_H(k))|^2 \\ &\sim \quad k^3 k^{-1} k^{-2p} \sim k^{2(1-p)} \, . \end{aligned} \tag{26}$$

Spectrum of Entropy Fluctuations I

Consider a second scalar field χ with the same negative exponential potential

$$\delta\ddot{\chi}_k + \left(k^2 + V''\right)\delta\chi_k \, = \, 0 \, . \tag{27}$$

$$\delta\ddot{\chi}_k + \left(k^2 - \frac{2}{t^2}\right)\delta\chi_k \, = \, 0 \, . \tag{28}$$

Vacuum initial conditions

$$\delta\chi_k \, \rightarrow \, \frac{1}{\sqrt{2k}} e^{ikt} \text{ as } k(-t) \rightarrow \infty \tag{29}$$

Spectrum of Entropy Fluctuations II

Solution:

$$\delta\chi_k \sim H^{(1)}_{3/2}(-kt) \sim k^{-3/2} \tag{30}$$

in the super-Hubble limit.

Hence

$$P_\chi(k) \sim k^3 k^{-3} \sim k^0 \,, \tag{31}$$

i.e. a scale-invariant power spectrum.

Origin of the Entropy Mode

- New Ekpyrotic Scenario (Buchbinder, Khoury and Ovrut (2007); Creminelli and Senatore (2007); Lehners et al (2007)) Assume a second scalar field χ with the same Ekpyrotic potential.
- Extra metric degrees of freedom which arise when the Ekpyrotic scenario is considered in terms of its 5-d M-theoretic origin (T. Battefeld, RB and S. Patil (2005)).

Challenges for the Ekpyrotic Scenario

- Description of the bounce.
- Initial conditions for fluctuations.

Plan

Principles

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

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T-Duality

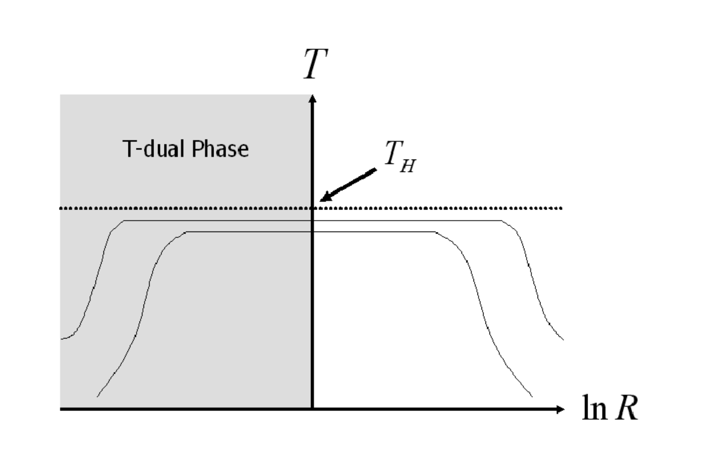
T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\rightarrow$ existence of D-branes

Adiabatic Considerations

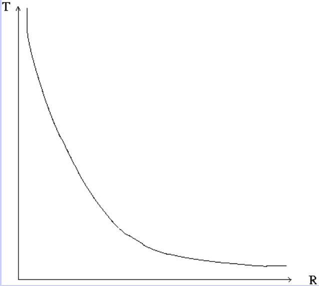
R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology



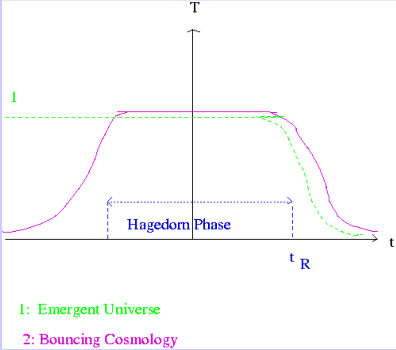
Singularity Problem in Standard and Inflationary Cosmology

Temperature-size relation in standard cosmology



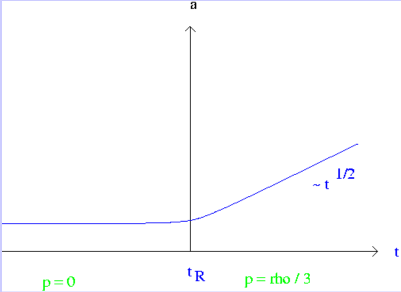
Dynamics

Assume some action gives us $R(t)$



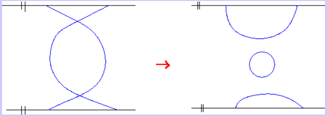
Dynamics II

We will thus consider the following background dynamics for the scale factor $a(t)$:



Dynamics III

The transition from the Hagedorn phase to the radiation phase of standard cosmology is given by the unwinding of winding modes:



Moduli Stabilization in SGC

Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow$ $V_{eff}(R)$ has a minimum at a finite value of R, $\rightarrow$ R_{min}
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

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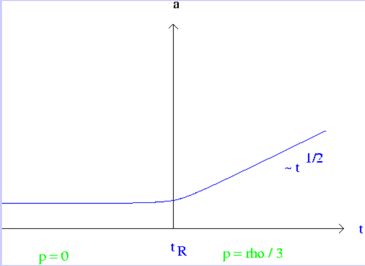
Dilaton stabilization in SGC

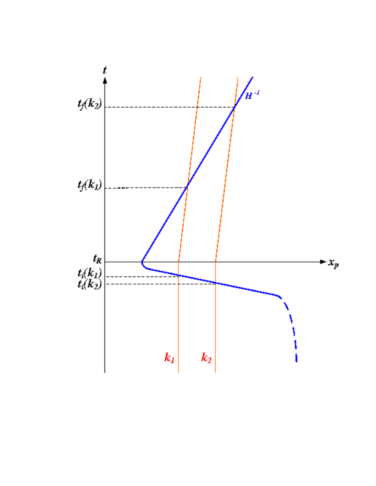
- The only remaining modulus is the dilaton
- Make use of gaugino condensation to give the dilaton a potential with a unique minimum
- $\rightarrow$ diltaton is stabilized
- Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., 2008]

Background for string gas cosmology



Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big)\,. \quad (32)$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^0{}_0(k)\delta T^0{}_0(k)\rangle\,, \quad (33)$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^i{}_j(k)\delta T^i{}_j(k)\rangle\,. \quad (34)$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V . \quad (35)$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} . \quad (36)$$

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Power spectrum of cosmological fluctuations

$$\begin{aligned} P_\Phi(k) &= 8G^2 k^{-1} < |\delta\rho(k)|^2 > \\ &= 8G^2 k^2 < (\delta M)^2 >_R \\ &= 8G^2 k^{-4} < (\delta\rho)^2 >_R \\ &= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H} \end{aligned} \tag{37}$$

Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

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Comments

- Evolution for $t > t_i(k)$: $\Phi \simeq \mathrm{const}$ since the equation of state parameter $1 + w$ stays the same order of magnitude unlike in inflationary cosmology.
- Squeezing of the fluctuation modes takes place on super-Hubble scales like in inflationary cosmology $\rightarrow$ acoustic oscillations in the CMB angular power spectrum
- In a dilaton gravity background the dilaton fluctuations dominate $\rightarrow$ different spectrum [R.B. et al, 2006; Kaloper, Kofman, Linde and Mukhanov, 2006]

Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

$$\begin{aligned} P_h(k) &= 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \\ &= 16\pi^2 G^2 k^{-4} < |T_{ij}(R)|^2 > \\ &\sim 16\pi^2 G^2 \frac{T}{\ell_s^3}(1 - T/T_H) \end{aligned} \tag{38}$$

Key ingredient for string thermodynamics

$$< |T_{ij}(R)|^2 > \sim \frac{T}{l_s^3 R^4}(1 - T/T_H) \tag{39}$$

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Requirements

- 1. static Hagedorn phase (including static dilaton) $\rightarrow$ new physics required.
- 2. $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.
- 3. Cosmological fluctuations in the IR are described by Einstein gravity.

Note: Specific higher derivative toy model: T. Biswas, R.B., A. Mazumdar and W. Siegel, 2006

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Network of cosmic superstrings

- Remnant of the Hagedorn phase: network of cosmic superstrings
- This string network will be present at all times and will achieve a scaling solution like cosmic strings forming during a phase transition.
- Scaling Solution: The network of strings looks statistically the same at all times when scaled to the Hubble radius.

Kaiser-Stebbins Effect

Space perpendicular to a string is conical with deficit angle

$$\alpha = 8\pi G\mu \,, \tag{40}$$

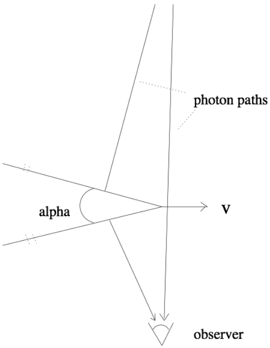
Photons passing by the string undergo a relative Doppler shift

$$\frac{\delta T}{T} = 8\pi\gamma(v)vG\mu \,, \tag{41}$$

$\rightarrow$ network of line discontinuities in CMB anisotropy maps

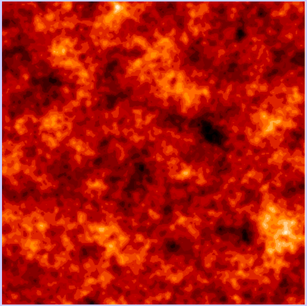
.

N.B. characteristic scale: comoving Hubble radius at the time of recombination $\rightarrow$ need good angular resolution to detect these edges.


photon paths
alpha
V
observer

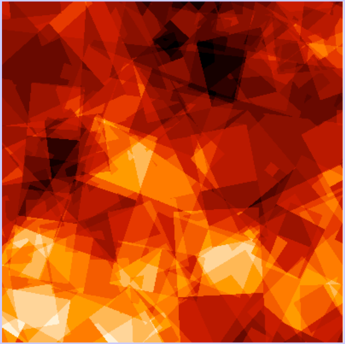
Gaussian temperature map

$10'^0 \times 10^0$ map of the sky at 1.5' resolution (South Pole Telescope specifications)



Cosmic string temperature map

10^0 x 10^0 map of the sky at 1.5' resolution

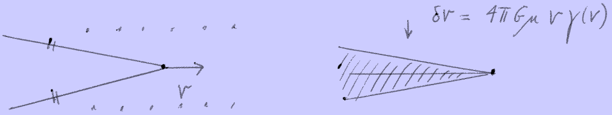


This signal is superimposed on the Gaussian map. The relative power of the string signature depends on $G\mu$ and is bound to contribute less than 10% of the power.

B-Mode Polarization Signal

R. Danos, R.B. and G. Holder, arXiv:1003.0905

Cosmic string → wake



- Wake: overdensity of free electrons
- $\rightarrow$ rectangle in the sky with extra polarization.
- Since the direction of the string is uncorrelated with the axis of the CMB quadrupole, statistically an equal contribution of E and B modes is predicted.
- Signal is strongest from wakes produced by strings close to t_{rec}
- $\rightarrow$ typical length scale is 1^o

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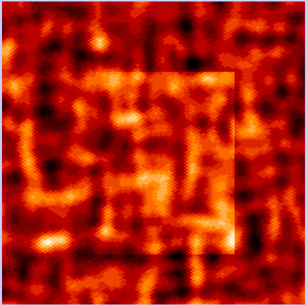
- $\rightarrow$ typical length scale is 1^o

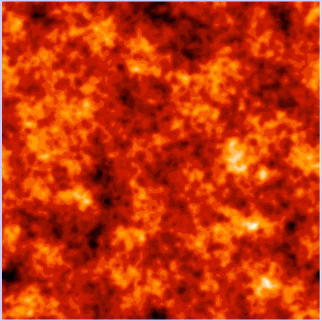
Q Mode Polarization Sky Map

$G\mu = 3 \times 10^{-7}$, string signal multiplied by 10^2, “noise" is due to the (dominant) Gaussian fluctuations.



Temperature map Gaussian + strings

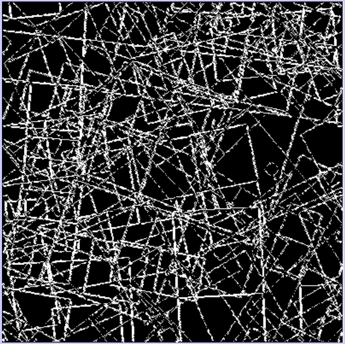
Challenge: pick out the string signature from the Gaussian "noise" which has a much larger amplitude



CANNY edge detection algorithm

- New technique: use CANNY edge detection algorithm [Canny, 1986]
- Idea: find edges across which the gradient is in the correct range to correspond to a Kaiser-Stebbins signal from a string
- Step 1: generate "Gaussian" and "Gaussian plus strings" CMB anisotropy maps: size and angular resolution of the maps are free parameters, flat sky approximation, cosmic string toy model in which a fixed number of straight string segments is laid down at random in each Hubble volume in each Hubble time step between t_{rec} and t_0.
- Step 2: run the CANNY algorithm on the temperature maps to produce edge maps.
- Step 3: Generate histogram of edge lengths
- Step 4: Use Fisher combined probability test.

Edge map



Preliminary Results

- For South Pole Telescope (SPT) specification: limit $G\mu < 2 \times 10^{-8}$ can be set [A. Stewart and R.B., 2008, R. Danos and R.B., 2008]
- Anticipated SPT instrumental noise only insignficantly effects the limits [A. Stewart and R.B., 2008]
- WMAP data: limit $G\mu < 2 \times 10^{-7}$ can be set [E. Thewalt, in prep.]

Challenges for the String Gas Cosmology Bounce

- Description of the Hagedorn phase.
- Length of the Hagedorn phase.
- Ensuring that the dominant source of fluctuations has holographic scaling.

Plan

Conclusions I

Conclusions :

- The Inflationary Cosmology scenario, although phenomenologically very successful, suffers from conceptual problems, in particular the singularity problem.
- Non-singular bounces can provide alternative scenarios.
- Three bounce scenarios presented: Matter Bounce, Ekpyrotic Bounce and String Gas Cosmology Bounce.

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Conclusions II

- Matter bounce scenario: time-symmetric background evolution. Quantum vacuum perturbations on sub-Hubble scales which exit the Hubble radius in the matter-dominated contracting phase develop into a scale-invariant spectrum of curvature fluctuations on super-Hubble scales in the expanding phase.
- Note: The evolution of fluctuations breaks the time symmetry which the background satisfies (R.B., 2009).
- Note: Distinctive shape and amplitude of the bispectrum.
- Ekpyrotic bounce scenario: Quantum vacuum perturbations on sub-Hubble scales in the contracting phase lead to a scale-invariant spectrum of entropy fluctuations which in turn can induce a scale-invariant spectrum of curvature perturbations.

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- String gas cosmology (SGC): a bouncing scenario characterized by a long-lived loitering Hagedorn phase.
- Thermal fluctuations in the Hagedorn phase have holographic scaling of thermodynamic correlation functions.
- Scale invariant spectrum of cosmological fluctuations (like in inflationary cosmology).
- Spectrum of gravitational waves has a small blue tilt (unlike in inflationary cosmology).
- Possibly distinctive signatures from cosmic superstrings in position space maps of CMB temperature and polarization.

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Action

Bounce
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Introduction
Motivation
Inflation
Problems
Message
Overview
Perturbations
Matter Bounce
Models
Structure Formation
Ekpyrotic Bounce
String Gas Bounce
Principles
Structure
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Conclusions

Action: Dilaton gravity plus string gas matter

$$S = \frac{1}{\kappa}\left(S_g + S_\phi\right) + S_{SG}\,, \tag{42}$$

$$S_{SG} = -\int d^{10}x\sqrt{-g}\sum_\alpha \mu_\alpha \epsilon_\alpha\,, \tag{43}$$

where

- μ_α: number density of strings in the state α
- ϵ_α: energy of the state α.

Introduce comoving number density:

$$\mu_\alpha = \frac{\mu_{0,\alpha}(t)}{\sqrt{g_s}}\,, \tag{44}$$

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Energy-Momentum Tensor

Ansatz for the metric:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 + \sum_{a=1}^{6} b_a(t)^2 dy_a^2 \, , \quad (45)$$

Contributions to the energy-momentum tensor

$$\rho_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \epsilon_\alpha^2 \, , \quad (46)$$

$$p_\alpha^i = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \frac{p_d^2}{3} \, , \quad (47)$$

$$p_\alpha^a = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}\alpha'} \left(\frac{n_a^2}{b_a^2} - w_a^2 b_a^2 \right) \, . \quad (48)$$

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Single string energy

ϵ_α is the energy of the string state α:

$$\epsilon_\alpha = \frac{1}{\sqrt{\alpha'}}\Big[\alpha' p_d^2 + b^{-2}(n,n) + b^2(w,w) + 2(n,w) + 4(N-1)\Big]^{1/2}, \quad (49)$$

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Background equations of motion

Radion equation:

$$\ddot{b} + \dot{b}(3\frac{\dot{a}}{a} + 5\frac{\dot{b}}{b}) = \frac{8\pi G \mu_{0,\alpha}}{\alpha' \sqrt{\hat{G}_{a\epsilon_\alpha}}} \times \left[\frac{n_a^2}{b^2} - w_a^2 b^2 + \frac{2}{(D-1)}[b^2(w,w) + (n,w) + 2(N-1)]\right] \tag{50}$$

Scale factor equation:

$$\ddot{a} + \dot{a}(2\frac{\dot{a}}{a} + 6\frac{\dot{b}}{b}) = \frac{8\pi G \mu_{0,\alpha}}{\sqrt{\hat{G}_{i\epsilon_\alpha}}} \times \left[\frac{p_d^2}{3} + \frac{2}{\alpha'(D-1)}[b^2(w,w) + (n,w) + 2(N-1)]\right], \tag{51}$$

Special states

Enhanced symmetry states

$$b^2(w,w) + (n,w) + 2(N-1) \; = \; 0\,. \tag{52}$$

Stable radion fixed point:

$$\frac{n_a^2}{b^2} - w_a^2 b^2 \; = \; 0\,. \tag{53}$$

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Gaugino condensation

Add a single non-perturbative ingredient - gaugino condensation - in order to fix the remaining modulus, the dilaton

Kähler potential: (standard)

$$\mathcal{K}(S) = -\ln(S + \bar{S}), \quad S = e^{-\Phi} + ia. \tag{54}$$

where $\Phi = 2\phi - 6\ln b$ is the 4-d dilaton, b is the radion and a is the axion.

Non-perturbative superpotential (from gaugino condensation):

$$W = M_P^3\left(C - Ae^{-a_0 S}\right) \tag{55}$$

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Dilaton potential I

Yields a potential for the dilaton (and radion)

$$V = \frac{M_P^4}{4} b^{-6} e^{-\Phi} \left[\frac{C^2}{4} e^{2\Phi} + ACe^{\Phi} \left(a_0 + \frac{1}{2} e^{\Phi} \right) e^{-a_0 e^{-\Phi}} + A^2 \left(a_0 + \frac{1}{2} e^{\Phi} \right)^2 e^{-2a_0 e^{-\Phi}} \right] . \quad (56)$$

Expand the potential about its minimum:

$$V = \frac{M_P^4}{4} b^{-6} e^{-\Phi_0} a_0^2 A^2 \left(a_0 - \frac{3}{2} e^{\Phi_0} \right)^2 e^{-2a_0 e^{-\Phi_0}} \times \left(e^{-\Phi} - e^{-\Phi_0} \right)^2 . \quad (57)$$

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Dilaton potential II

Lift the potential to 10-d, redefining b to be in the Einstein frame.

$$V(b,\phi) = \frac{M_{10}^{16}\hat{V}}{4} e^{-\Phi_0} a_0^2 A^2 \left(a_0 - \frac{3}{2}e^{\Phi_0}\right)^2 e^{-2a_0 e^{-\Phi_0}} \times e^{-3\phi/2}\left(b^6 e^{-\phi/2} - e^{-\Phi_0}\right)^2 . \quad (58)$$

Dilaton potential in 10d Einstein frame

$$V \simeq n_1 e^{-3\phi/2}\left(b^6 e^{-\phi/2} - n_2\right)^2 \quad (59)$$

Analysis including both string matter and dilaton potential I

Worry: adding this potential will mess up radion stablilization

Thus: consider dilaton and radion equations resulting from the action including both the dilaton potential and string gas matter.

Step 1: convert the string gas matter contributions to the 10-d Einstein frame

$$g^E_{\mu\nu} = e^{-\phi/2} g^s_{\mu\nu} \quad (60)$$

$$b_s = e^{\phi/4} b_E \quad (61)$$

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Joint analysis II

Step 2: Consider both dilaton and radion equations:

$$
\begin{aligned}
- & \frac{M_{10}^8}{2}\Big(3a^2\dot{a}b^6\dot{\phi} + 6a^3b^5\dot{b}\dot{\phi} + a^3b^6\ddot{\phi}\Big) \\
+ & \frac{3}{2}n_1a^3b^6e^{-3\phi/2}\Big(b^6e^{-\phi/2} - n_2\Big)^2 \\
+ & a^3b^{12}n_1e^{-2\phi}\Big(b^6e^{-\phi/2} - n_2\Big) \\
+ & \frac{1}{2\epsilon}e^{\phi/4}\Big(-\mu_0\epsilon^2 + \mu_0|p_d|^2 \\
+ & 6\mu_0\Big[\frac{n_a^2}{\alpha'}e^{-\phi/2}b^{-2} - \frac{w^2}{\alpha'}e^{\phi/2}b^2\Big]\Big) \\
= & \; 0\,,
\end{aligned}
\tag{63}
$$

Joint analysis III

$$
\begin{aligned}
\ddot{b} \quad + \quad & 3\frac{\dot{a}}{a}\dot{b} + 5\frac{\dot{b}^2}{b} = -\frac{n_1 b}{M_{10}^8} e^{-3\phi/2}\left(b^6 e^{-\phi/2} - n_2\right)^2 \\
& -\frac{2n_1}{M_{10}^8} b^7 e^{-2\phi}\left(b^6 e^{-\phi/2} - n_2\right) \\
& +\frac{1}{2-D}\left[-\frac{10b}{M_{10}^8} n_1 e^{-3\phi/2}\left(b^6 e^{-\phi/2} - n_2\right)^2\right. \\
& \left.-\frac{12n_1}{M_{10}^8} b^7 e^{-2\phi}\left(b^6 e^{-\phi/2} - n_2\right)\right] \\
& +\frac{8\pi G_D \mu_0}{\alpha'\sqrt{\hat{G}_{a\epsilon}}} e^{2\phi}\left[n_a^2 b^{-2} e^{-\phi/2} - w_a^2 b^2 e^{\phi/2}\right. \\
& \left.+\frac{2}{D-1}\left(e^{\phi/2} b^2 w^2 + n \cdot w + 2(N-1)\right)\right]
\end{aligned} \tag{64}
$$

Joint analysis IV

Step 3: Identifying extremum

- Dilaton at the minimum of its potential and
- Radion at the enhanced symmetry state

Step 4: Stability analysis

- Consider small fluctuations about the extremum
- show stability (tedious but straightforward)

Result: Dilaton and radion stabilized simultaneously at the enhanced symmetry point.

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