Bouncing Cosmologies

Robert Brandenberger
McGill University

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Outline

1. Introduction
   - Motivation
   - Review of Inflationary Cosmology
   - Problems of Inflationary Cosmology
   - Message

2. Overview of Bouncing Cosmologies

3. Review of the Theory of Cosmological Perturbations

4. Matter Bounce
   - Models for a Matter Bounce
   - Structure Formation

5. Ekpyrotic Bounce

6. String Gas Bounce
   - Principles
   - String Gas Cosmology and Structure Formation
   - Signatures in CMB anisotropy maps

7. Conclusions
Plan

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7. Conclusions
The Inflationary Universe Scenario is the current paradigm of early universe cosmology. Successes:

- Solves horizon problem
- Solves flatness problem
- Solves size/entropy problem
- Provides a causal mechanism of generating primordial cosmological perturbations (Chibisov & Mukhanov, 1981).
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Bounce
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Inflation Problems
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Principles Structure CMB Signatures

Conclusions

Credit: NASA/WMAP Science Team
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Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

Fig. 1b. The dependence of the square of the amplitude of density perturbations of matter on scale. The fine line designates the usually assumed dependence $\delta^2 a^2 \sim M^{-n}$. It is apparent that fluctuations of relic radiation should depend on scale in a similar manner.
In spite of the phenomenological successes, current realizations of the inflationary scenario suffer from several conceptual problems.

One of these problems is the singularity problem.

A bouncing cosmology would provide a solution to this problem.

Question: Is it possible to obtain bouncing cosmologies from fundamental physics?

Question: Can these new paradigms be tested in cosmological observations?
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Review of Inflationary Cosmology

**Context:**
- General Relativity
- Scalar Field Matter

**Metric:** \( ds^2 = dt^2 - a(t)^2 dx^2 \) (1)

**Inflation:**
- phase with \( a(t) \sim e^{tH} \)
- requires matter with \( p \sim -\rho \)
- requires a slowly rolling scalar field \( \varphi \)
- in order to have a potential energy term
- in order that the potential energy term dominates sufficiently long
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Time line of inflationary cosmology:

\[ a(t) = e^{tH} \]

- \( t_i \): inflation begins
- \( t_R \): inflation ends, reheating
Review of Inflationary Cosmology II

Space-time sketch of inflationary cosmology:

Note:
- $H = \frac{\dot{a}}{a}$
- curve labelled by $k$: wavelength of a fluctuation
- inflation renders the universe large, homogeneous and spatially flat
- classical matter redshifts $\rightarrow$ matter vacuum remains
- quantum vacuum fluctuations: seeds for the observed structure [Chibisov & Mukhanov, 1981]
- sub-Hubble $\rightarrow$ locally causal
Conceptual Problems of Inflationary Cosmology

- Nature of the scalar field $\varphi$ (the “inflaton”)
- Conditions to obtain inflation (initial conditions, slow-roll conditions, graceful exit and reheating)
- Amplitude problem
- Trans-Planckian problem
- Cosmological constant problem
- Applicability of General Relativity
- Singularity problem
Success of inflation: At early times scales are inside the Hubble radius → causal generation mechanism is possible.

Problem: If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_p$ at the beginning of inflation

→ new physics MUST enter into the calculation of the fluctuations.
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Trans-Planckian Problem

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**Problem**: If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation

→ new physics MUST enter into the calculation of the fluctuations.

- It is not sufficient to show that the Hubble constant is smaller than the Planck scale.
- The frequencies involved in the analysis of the cosmological fluctuations are many orders of magnitude larger than the Planck mass. Thus, “the methods used in [1] are inapplicable for the description of the .. process of generation of perturbations in this scenario.”
If evolution in Period I is non-adiabatic, then scale-invariance of the power spectrum will be lost [J. Martin and RB, 2000]

→ Planck scale physics testable with cosmological observations!
Cosmological Constant Problem

Quantum vacuum energy does not gravitate.

Why should the almost constant $V(\phi)$ gravitate?

\[
\frac{V_0}{\Lambda_{obs}} \sim 10^{120}
\] (2)
In all approaches to quantum gravity, the Einstein action is only the leading term in a low curvature expansion. Correction terms may become dominant at much lower energies than the Planck scale. Correction terms will dominate the dynamics at high curvatures. The energy scale of inflation models is typically $\eta \sim 10^{16}\text{GeV}$. $\eta$ too close to $m_{pl}$ to trust predictions made using GR.
Zones of Ignorance
Singularity Problem

- **Standard cosmology:** Penrose-Hawking theorems $\rightarrow$ initial singularity $\rightarrow$ incompleteness of the theory.
- **Inflationary cosmology:** In scalar field-driven inflationary models the initial singularity persists [Borde and Vilenkin] $\rightarrow$ incompleteness of the theory.

Penrose-Hawking theorems:

- Ass: 1) Einstein gravitational action,
- Ass: 2) weak energy conditions for matter $\rho > 0, \rho + 3p \geq 0$

$\rightarrow$ space-time is geodesically incomplete.
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Penrose-Hawking theorems:
- Ass: 1) Einstein gravitational action,
- Ass: 2) weak energy conditions for matter $\rho > 0, \quad \rho + 3p \geq 0$
- $\rightarrow$ space-time is geodesically incomplete.
Current realizations of inflation have conceptual problems.

We need a new paradigm of very early universe cosmology.

Bouncing cosmologies may provide an alternative early universe scenario.

Bouncing cosmologies require new fundamental physics.

Bouncing cosmologies need not involve a period of inflation.

Any viable bouncing cosmology must explain the current data and make predictions with which it can be distinguished from those of standard slow-roll inflation.
Model must yield a successful structure formation scenario:

- Scales of cosmological interest today must originate inside the Hubble radius.
- Long propagation on super-Hubble scales.
- Scale-invariant spectrum of adiabatic cosmological perturbations.
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- Scale-invariant spectrum of adiabatic cosmological perturbations.
Model must refer to the problems of Standard Cosmology which the inflationary scenario addresses.

- Solution of the **horizon problem**.
- Solution of the **flatness problem**.
- Solution of the **size and entropy problems**.
New physics is required in order to obtain a bounce (in order to circumvent the assumptions made in the Hawking-Penrose singularity theorems):

- First option: New form of matter violating the weak energy condition.
- Second option: Corrections to the gravitational action in the ultraviolet.
N.B. Perturbations originate as quantum vacuum fluctuations.
Matter Bounce Scenario
Overview of the Matter Bounce

- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.
- Adiabatic fluctuation mode acquires a scale-invariant spectrum of curvature perturbations on super-Hubble scales.
- Horizon problem: absent.
- Flatness problem: weak point.
- Size and entropy problems: not present if we assume that the universe begins cold and large.
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Ekpyrotic Bounce
J. Khoury, B. Ovrut, P. Steinhardt and N. Turok Phys. Rev. D64, 123522 (2001)
Overview of the Ekpyrotic Bounce

- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.

- Adiabatic fluctuation mode not scale invariant.

- Entropic fluctuation modes acquire a scale-invariant spectrum of curvature perturbations on super-Hubble scales.

- Transfer of to adiabatic fluctuations on super-Hubble scales (similar to curvaton scenario).

- Horizon problem: absent.

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- Fluctuations originate as thermal string perturbations on sub-Hubble scales in the Hagedorn (loitering) phase.
- Adiabatic fluctuation mode acquires a scale-invariant spectrum of curvature perturbations on super-Hubble scales.
- Horizon problem: absent if the loitering phase lasts sufficiently long.
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- Fluctuations originate as **thermal string perturbations** on sub-Hubble scales in the Hagedorn (loitering) phase.

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Cosmological fluctuations connect early universe theories with observations

- Fluctuations of **matter** → large-scale structure
- Fluctuations of **metric** → CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

**Key facts:**

1. Fluctuations are small today on large scales
   → fluctuations were very small in the early universe
   → can use linear perturbation theory
2. Sub-Hubble scales: matter fluctuations dominate
   Super-Hubble scales: metric fluctuations dominate
Theory of Cosmological Perturbations: Basics

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Step 1: Metric including fluctuations

\[ ds^2 = a^2 [(1 + 2\Phi) d\eta^2 - (1 - 2\Phi) dx^2] \]  \hspace{1cm} (3)

\[ \varphi = \varphi_0 + \delta\varphi \]  \hspace{1cm} (4)

Note: \( \Phi \) and \( \delta\varphi \) related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

\[ S^{(2)} = \frac{1}{2} \int d^4 x \left( (v')^2 - v_i v^i + \frac{z''}{z} v^2 \right) \]  \hspace{1cm} (5)

\[ v = a (\delta\varphi + \frac{z}{a} \Phi) \]  \hspace{1cm} (6)

\[ z = a \frac{\varphi_0'}{\mathcal{H}} \]  \hspace{1cm} (7)
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Quantum Theory of Linearized Fluctuations
Step 3: Resulting equation of motion (Fourier space)

\[ v''_k + (k^2 - \frac{Z''}{Z})v_k = 0 \]  \hspace{1cm} (8)

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales \( v_k \sim z \)

Quantum vacuum initial conditions:

\[ v_k(\eta_i) = (\sqrt{2k})^{-1} \]  \hspace{1cm} (9)
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\[ \zeta = z^{-1} v \]  

(10)

Its physical meaning: curvature perturbation in comoving gauge.

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In the case of entropy fluctuations there are more than one degrees of freedom for the scalar metric inhomogeneities. Example: extra scalar field.

Entropy fluctuations seed an adiabatic mode even on super-Hubble scales.

\[
\dot{\zeta} = \frac{\dot{\rho}}{\rho + \rho} \delta S
\]  

Example: topological defect formation in a phase transition.

Example: Axion perturbations when axions acquire a mass at the QCD scale (M. Axenides, R.B. and M. Turner, 1983).
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More on Perturbations II

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7. Conclusions
By modifying the matter sector:

- **Quintom matter**: extra matter field with negative kinetic term in the action (Y. Cai et al, 2007)
- **Lee-Wick matter**: higher derivative matter action with an extra pole to cancel quadratic loop divergences in scattering amplitudes (Y. Cai et al, 2008).
Models for a Matter Bounce II

By modifying the gravitational action:

- **Nonsingular Universe construction**: Special invariant added to the Einstein action constructed such that at large curvatures all solutions tend to de Sitter (R.B., V. Mukhanov and A. Sornborger, 1993).

- **Mirage Cosmology**: Dynamics induced by the motion of a brane in a non-singular bulk (Kehagias and Kiritsis, 1999).

- **Horava-Lifshitz gravity**: in the presence of spatial curvature the extra spatial derivative terms in the gravitational action act as ghost radiation and ghost anisotropic stress (R.B., 2009).
Matter Bounce Scenario

Origin of Scale-Invariant Spectrum

- The initial vacuum spectrum is blue:

\[
P_\zeta(k) = k^3 |\zeta(k)|^2 \sim k^2 \quad (12)
\]

- The curvature fluctuations grow on super-Hubble scales in the contracting phase:

\[
v_k(\eta) = c_1 \eta^2 + c_2 \eta^{-1}, \quad (13)
\]

- For modes which exit the Hubble radius in the matter phase the resulting spectrum is scale-invariant:

\[
P_\zeta(k, \eta) \sim k^3 |v_k(\eta)|^2 a^{-2}(\eta) \sim k^3 \left( \frac{\eta H(k)}{\eta} \right)^2 \sim k^{3-1-2} \sim \text{const},
\]
Transfer of the Spectrum through the Bounce

In a nonsingular background the fluctuations can be tracked through the bounce explicitly (both numerically in an exact manner and analytically using matching conditions at times when the equation of state changes).

Explicit computations have been performed in the case of quintom matter (Y. Cai et al, 2008), mirage cosmology (R.B. et al, 2007), Horava-Lifshitz bounce (X. Gang et al, 2009).

Result: On length scales larger than the duration of the bounce the spectrum of $\nu$ goes through the bounce unchanged.
Signature in the Bispectrum: formalism

\[
\langle \zeta(t, \vec{k}_1)\zeta(t, \vec{k}_2)\zeta(t, \vec{k}_3) \rangle = i \int_{t_i}^{t} \, dt' \, \langle [\zeta(t, \vec{k}_1)\zeta(t, \vec{k}_2)\zeta(t, \vec{k}_3), L_{int}(t')] \rangle , \tag{15}
\]

\[
\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \rangle = (2\pi)^7 \delta(\sum \vec{k}_i) \frac{P^2_\zeta}{\prod k^3_i} \times A(\vec{k}_1, \vec{k}_2, \vec{k}_3) , \tag{16}
\]

\[
|B|_{NL}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{10}{3} \frac{A(\vec{k}_1, \vec{k}_2, \vec{k}_3)}{\sum_i k^3_i} . \tag{17}
\]
If we project the resulting shape function $A$ onto some popular shape masks we get

$$|B|_{NL}^{\text{local}} = -\frac{35}{8},$$

(18)

for the local shape ($k_1 \ll k_2 = k_3$). This is negative and of order $O(1)$.

For the equilateral form ($k_1 = k_2 = k_3$) the result is

$$|B|_{NL}^{\text{equil}} = -\frac{255}{64},$$

(19)

For the folded form ($k_1 = 2k_2 = 2k_3$) one obtains the value

$$|B|_{NL}^{\text{folded}} = -\frac{9}{4}.$$  

(20)
Bispectrum of the Matter Bounce Scenario
Challenges for the Matter Bounce Scenario

- Obtaining a matter bounce in a model free of ghosts and other unwanted degrees of freedom.
- Instability to anisotropic stress.
- Initial conditions for fluctuations?
Plan

1. Introduction
   - Motivation
   - Review of Inflationary Cosmology
   - Problems of Inflationary Cosmology
   - Message

2. Overview of Bouncing Cosmologies

3. Review of the Theory of Cosmological Perturbations

4. Matter Bounce
   - Models for a Matter Bounce
   - Structure Formation

5. Ekpyrotic Bounce

6. String Gas Bounce
   - Principles
   - String Gas Cosmology and Structure Formation
   - Signatures in CMB anisotropy maps

7. Conclusions
Ekpyrotic Bounce
J. Khoury, B. Ovrut, P. Steinhardt and N. Turok Phys. Rev. D64, 123522 (2001)
Obtaining a Phase of Ekpyrotic Contraction

Introduce a scalar field with negative exponential potential and AdS minimum:

\[ V(\phi) = -V_0 \exp(-\left(\frac{2}{p} \right)^{1/2} \frac{\phi}{m_{pl}}) \quad 0 < p \ll 1 \]  \hspace{1cm} (21)

Motivated by potential between branes in heterotic M-theory in the homogeneous and isotropic limit, the cosmology is given by

\[ a(t) \sim a(t)^p \]  \hspace{1cm} (22)

and the equation of state is

\[ w \equiv \frac{\rho}{p} = \frac{2}{3p} - 1 \gg 1. \]  \hspace{1cm} (23)
Solution to the flatness problem

The energy density in the Ekpyrotic field scales as

$$\rho(a) = \rho_0 a^{-3(1+w)}$$

and thus dominates all other forms of energy density (including anisotropic stress) as the universe shrinks → quasi-homogeneous bounce, no chaotic mixmaster behavior.
If $a(t) \sim t^p$ then conformal time scales as $\eta \sim t^{1-p}$.

The solution of the mode equation for $v$ is

$$v_k(\eta) = c_1 \eta^{-\alpha} + c_2 \eta,$$

(25)

where $c_1$ and $c_2$ are constant coefficients and $\alpha \simeq p$ for $p \ll 1$.

Hence, the power spectrum is not scale invariant:

$$P_\zeta(k, t) = \left( \frac{Z(t)}{v(t_H(k))} \right)^2 k^3 |v_k(t_H(k))|^2$$

$$\sim k^3 k^{-1} k^{-2p} \sim k^{2(1-p)}.$$  (26)
Consider a second scalar field $\chi$ with the same negative exponential potential

$$\ddot{\delta\chi}_k + (k^2 + V'') \delta\chi_k = 0. \quad (27)$$

$$\ddot{\delta\chi}_k + \left(k^2 - \frac{2}{t^2}\right) \delta\chi_k = 0. \quad (28)$$

Vacuum initial conditions

$$\delta\chi_k \rightarrow \frac{1}{\sqrt{2k}} e^{ikt} \text{ as } k(-t) \rightarrow \infty \quad (29)$$
Solution:

$$\delta \chi_k \sim H_{3/2}^1(-kt) \sim k^{-3/2}$$  \hspace{1cm} (30)

in the super-Hubble limit.

Hence

$$P_\chi(k) \sim k^3 k^{-3} \sim k^0,$$  \hspace{1cm} (31)

i.e. a scale-invariant power spectrum.
New Ekpyrotic Scenario (Buchbinder, Khoury and Ovrut (2007); Creminelli and Senatore (2007); Lehners et al (2007)) Assume a second scalar field $\chi$ with the same Ekpyrotic potential.

Extra metric degrees of freedom which arise when the Ekpyrotic scenario is considered in terms of its 5-d M-theoretic origin (T. Battefeld, RB and S. Patil (2005)).
Challenges for the Ekpyrotic Scenario

- Description of the bounce.
- Initial conditions for fluctuations.
Principles

Idea: make use of the **new symmetries and new degrees of freedom** which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings.
Assumption: Space is compact, e.g. a torus.

Key points:

- **New degrees of freedom**: string oscillatory modes
- Leads to a **maximal temperature** for a gas of strings, the Hagedorn temperature
- **New degrees of freedom**: string winding modes
- Leads to a **new symmetry**: physics at large $R$ is equivalent to physics at small $R$
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T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R$ \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\rightarrow$ existence of D-branes
Temperature-size relation in string gas cosmology

$T$-dual Phase

$T_H$

$\ln R$
Singularity Problem in Standard and Inflationary Cosmology

Temperature-size relation in standard cosmology
Assume some action gives us $R(t)$.
We will thus consider the following background dynamics for the scale factor $a(t)$:
The transition from the Hagedorn phase to the radiation phase of standard cosmology is given by the unwinding of winding modes:
Moduli Stabilization in SGC


- winding modes prevent expansion
- momentum modes prevent contraction

\[
\rightarrow V_{\text{eff}}(R) \text{ has a minimum at a finite value of } R, \quad \rightarrow R_{\min}
\]

- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at \( R_{\min} \)

\[
\rightarrow V_{\text{eff}}(R_{\min}) = 0
\]

- \( \rightarrow \) size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states

\[
\rightarrow \text{harmonic oscillator potential for } \theta
\]

\[
\rightarrow \text{shape moduli stabilized}
\]
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Shape Moduli [E. Cheung, S. Watson and R.B., 2005]
- enhanced symmetry states
- $\rightarrow$ harmonic oscillator potential for $\theta$
- $\rightarrow$ shape moduli stabilized
Moduli Stabilization in SGC

**Size Moduli** [S. Watson, 2004; S. Patil and R.B., 2004, 2005]
- winding modes prevent expansion
- momentum modes prevent contraction
- $V_{\text{eff}}(R)$ has a minimum at a finite value of $R$, $\rightarrow R_{\text{min}}$
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at $R_{\text{min}}$
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**Shape Moduli** [E. Cheung, S. Watson and R.B., 2005]
- enhanced symmetry states
- $\rightarrow$ harmonic oscillator potential for $\theta$
- $\rightarrow$ shape moduli stabilized
Dilaton stabilization in SGC

- The only remaining modulus is the dilaton
- Make use of \textit{gaugino condensation} to give the dilaton a potential with a unique minimum
- \(\rightarrow\) dilaton is stabilized
- Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., 2008]
Background for string gas cosmology
Structure formation in string gas cosmology

N.B. Perturbations originate as thermal string gas fluctuations.
Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed $k$, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations
Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

\[ ds^2 = a^2(\eta)((1 + 2\Phi)d\eta^2 - [(1 - 2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j) \]. 

(32)

Inserting into the perturbed Einstein equations yields

\[ \langle |\Phi(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0_0(k)\delta T^0_0(k) \rangle, \]

(33)

\[ \langle |h(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i_j(k)\delta T^i_j(k) \rangle. \]

(34)
Key ingredient: For thermal fluctuations:

\[ \langle \delta \rho^2 \rangle = \frac{T^2}{R^6} C_V . \] (35)

Key ingredient: For string thermodynamics in a compact space

\[ C_V \approx 2 \frac{R^2/\ell_S^3}{T \left(1 - T/T_H \right)} . \] (36)
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Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T (1 - T/T_H)}.$$  \hfill (36)
Power spectrum of cosmological fluctuations

\[ P_\Phi(k) = 8G^2k^{-1} < |\delta \rho(k)|^2 > \]
\[ = 8G^2k^2 < (\delta \mathcal{M})^2 >_R \]
\[ = 8G^2k^{-4} < (\delta \rho)^2 >_R \]
\[ = 8G^2 \frac{T \ell_s^3}{1 - T/T_H} \]

Key features:
- scale-invariant like for inflation
- slight red tilt like for inflation
Power spectrum of cosmological fluctuations

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Key features:
- scale-invariant like for inflation
- slight red tilt like for inflation
Evolution for $t > t_i(k)$: $\Phi \sim \text{const}$ since the equation of state parameter $1 + w$ stays the same order of magnitude unlike in inflationary cosmology.

Squeezing of the fluctuation modes takes place on super-Hubble scales like in inflationary cosmology $\rightarrow$ acoustic oscillations in the CMB angular power spectrum.

In a dilaton gravity background the dilaton fluctuations dominate $\rightarrow$ different spectrum [R.B. et al, 2006; Kaloper, Kofman, Linde and Mukhanov, 2006]
\[ P_h(k) = 16\pi^2 G^2 k^{-1} \langle |T_{ij}(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle |T_{ij}(R)|^2 \rangle \sim 16\pi^2 G^2 \frac{T}{\ell^3} T_s (1 - T / T_H) \] (38)

Key ingredient for string thermodynamics

\[ \langle |T_{ij}(R)|^2 \rangle \sim \frac{T}{\ell^3 R^4} (1 - T / T_H) \] (39)

Key features:
- scale-invariant (like for inflation)
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Key features:

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1. static Hagedorn phase (including static dilaton) \(\rightarrow\) new physics required.

2. \(C_V(R) \sim R^2\) obtained from a thermal gas of strings provided there are winding modes which dominate.

3. Cosmological fluctuations in the IR are described by Einstein gravity.

Note: Specific higher derivative toy model: T. Biswas, R.B., A. Mazumdar and W. Siegel, 2006
1. static Hagedorn phase (including static dilaton) $\rightarrow$ new physics required.

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Network of cosmic superstrings

- Remnant of the Hagedorn phase: network of cosmic superstrings
- This string network will be present at all times and will achieve a scaling solution like cosmic strings forming during a phase transition.
- Scaling Solution: The network of strings looks statistically the same at all times when scaled to the Hubble radius.
Kaiser-Stebbins Effect

Space perpendicular to a string is conical with deficit angle

\[ \alpha = 8\pi G\mu , \] (40)

Photons passing by the string undergo a relative Doppler shift

\[ \frac{\delta T}{T} = 8\pi \gamma(v) v G\mu , \] (41)

→ network of line discontinuities in CMB anisotropy maps.

\textit{N.B. characteristic scale: comoving Hubble radius at the time of recombination} \textit{→ need good angular resolution to detect these edges.}
Gaussian temperature map

10° x 10° map of the sky at 1.5’ resolution (South Pole Telescope specifications)
Cosmic string temperature map

$10^0 \times 10^0$ map of the sky at 1.5’ resolution
This signal is superimposed on the Gaussian map. The relative power of the string signature depends on $G_\mu$ and is bound to contribute less than 10% of the power.
B-Mode Polarization Signal

Cosmic string $\rightarrow$ wake

\[ \delta v = 4\pi G \mu \chi \gamma(v) \]
Wake: overdensity of free electrons

- rectangle in the sky with extra polarization.

Since the direction of the string is uncorrelated with the axis of the CMB quadrupole, statistically an equal contribution of E and B modes is predicted.

Signal is strongest from wakes produced by strings close to \( t_{\text{rec}} \)

→ typical length scale is 1\(^{\circ} \)
**Wake**: overdensity of free electrons

→ rectangle in the sky with extra polarization.

Since the direction of the string is uncorrelated with the axis of the CMB quadrupole, statistically an equal contribution of E and B modes is predicted.

Signal is strongest from wakes produced by strings close to $t_{rec}$

→ typical length scale is 1°
Wake: overdensity of free electrons

→ rectangle in the sky with extra polarization.

Since the direction of the string is uncorrelated with the axis of the CMB quadrupole, statistically an equal contribution of E and B modes is predicted.

Signal is strongest from wakes produced by strings close to $t_{\text{rec}}$

→ typical length scale is $1^\circ$
Wake: overdensity of free electrons

→ rectangle in the sky with extra polarization.

Since the direction of the string is uncorrelated with the axis of the CMB quadrupole, statistically an equal contribution of E and B modes is predicted.

Signal is strongest from wakes produced by strings close to $t_{rec}$

→ typical length scale is $1^o$
$G_\mu = 3 \times 10^{-7}$, string signal multiplied by $10^2$, “noise" is due to the (dominant) Gaussian fluctuations.
Challenge: pick out the string signature from the Gaussian "noise" which has a much larger amplitude
New technique: use CANNY edge detection algorithm [Canny, 1986]

Idea: find edges across which the gradient is in the correct range to correspond to a Kaiser-Stebbins signal from a string

Step 1: generate "Gaussian" and "Gaussian plus strings" CMB anisotropy maps: size and angular resolution of the maps are free parameters, flat sky approximation, cosmic string toy model in which a fixed number of straight string segments is laid down at random in each Hubble volume in each Hubble time step between $t_{rec}$ and $t_0$.

Step 2: run the CANNY algorithm on the temperature maps to produce edge maps.

Step 3: Generate histogram of edge lengths

Step 4: Use Fisher combined probability test.
Edge map
Preliminary Results

- For South Pole Telescope (SPT) specification: limit $G_\mu < 2 \times 10^{-8}$ can be set [A. Stewart and R.B., 2008, R. Danos and R.B., 2008]
- Anticipated SPT instrumental noise only insignificantly effects the limits [A. Stewart and R.B., 2008]
- WMAP data: limit $G_\mu < 2 \times 10^{-7}$ can be set [E. Thewalt, in prep.]
Challenges for the String Gas Cosmology

- **Description of the Hagedorn phase.**
- **Length of the Hagedorn phase.**
- **Ensuring that the dominant source of fluctuations has holographic scaling.**
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Conclusions I

Conclusions:

- The **Inflationary Cosmology** scenario, although phenomenologically very successful, suffers from conceptual problems, in particular the *singularity problem*.

- Non-singular bounces can provide alternative scenarios.

- Three bounce scenarios presented: Matter Bounce, *Ekpyrotic Bounce* and *String Gas Cosmology Bounce*. 
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Matter bounce scenario: time-symmetric background evolution. Quantum vacuum perturbations on sub-Hubble scales which exit the Hubble radius in the matter-dominated contracting phase develop into a scale-invariant spectrum of curvature fluctuations on super-Hubble scales in the expanding phase.

Note: The evolution of fluctuations breaks the time symmetry which the background satisfies (R.B., 2009).

Note: Distinctive shape and amplitude of the bispectrum.

Ekpyrotic bounce scenario: Quantum vacuum perturbations on sub-Hubble scales in the contracting phase lead to a scale-invariant spectrum of entropy fluctuations which in turn can induce a scale-invariant spectrum of curvature perturbations.
Conclusions II

- **Matter bounce scenario:** time-symmetric background evolution. Quantum vacuum perturbations on sub-Hubble scales which exit the Hubble radius in the matter-dominated contracting phase develop into a scale-invariant spectrum of curvature fluctuations on super-Hubble scales in the expanding phase.

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- Thermal fluctuations in the Hagedorn phase have holographic scaling of thermodynamic correlation functions.
- Scale invariant spectrum of cosmological fluctuations (like in inflationary cosmology).
- Spectrum of gravitational waves has a small blue tilt (unlike in inflationary cosmology).
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**Action**

**Action: Dilaton gravity plus string gas matter**

\[ S = \frac{1}{\kappa} \left( S_g + S_\phi \right) + S_{SG}, \quad (42) \]

\[ S_{SG} = - \int d^{10}x \sqrt{-g} \sum_\alpha \mu_\alpha \epsilon_\alpha, \quad (43) \]

where

- \( \mu_\alpha \): number density of strings in the state \( \alpha \)
- \( \epsilon_\alpha \): energy of the state \( \alpha \).

Introduce comoving number density:

\[ \mu_\alpha = \frac{\mu_{0,\alpha}(t)}{\sqrt{g_s}}, \quad (44) \]
Action: Dilaton gravity plus string gas matter

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Introduce comoving number density:

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\mu_\alpha = \frac{\mu_{0,\alpha}(t)}{\sqrt{g_s}},
\]
Energy-Momentum Tensor

Ansatz for the metric:

\[ ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 + \sum_{a=1}^{6} b_a(t)^2 dy_a^2 , \]  

Contributions to the energy-momentum tensor

\[ \rho_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \epsilon_\alpha^2 , \]  
\[ p^i_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \frac{p^2_d}{3} , \]  
\[ p^a_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}\alpha'} \left( \frac{n_a^2}{b_a^2} - \frac{w_a^2 b_a^2}{b_a^2} \right) . \]
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Contributions to the energy-momentum tensor

\[ \rho_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \epsilon_\alpha^2, \] (46)

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\[ p^a_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g} \epsilon_\alpha'} \left( \frac{n_a^2}{b_a^2} - w_a b_a^2 \right). \] (48)
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(48)
Single string energy

\( \epsilon_\alpha \) is the energy of the string state \( \alpha \):

\[
\epsilon_\alpha = \frac{1}{\sqrt{\alpha'}} \left[ \alpha' p_d^2 + b^{-2}(n, n) + b^2(w, w) + 2(n, w) + 4(N - 1) \right]^{1/2}, \tag{49}
\]

where

- \( \vec{n} \) and \( \vec{w} \): momentum and winding number vectors in the internal space
- \( \vec{p}_d \): momentum in the large space
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$$

where

- $\vec{n}$ and $\vec{w}$: momentum and winding number vectors in the internal space
- $\vec{p}_d$: momentum in the large space
Background equations of motion

**Radion equation:**

\[
\ddot{b} + b \left(3 \frac{\ddot{a}}{a} + 5 \frac{\dot{b}}{b} \right) = \frac{8\pi G_{\mu_0,\alpha}}{\alpha' \sqrt{\hat{G}_{\alpha}}}
\times \left[ \frac{n^2_a}{b^2} - w^2_a b^2 + \frac{2}{(D-1)} \left[ b^2(w, w) + (n, w) + 2(N - 1) \right] \right]
\]

**Scale factor equation:**

\[
\ddot{a} + \dot{a} \left(2 \frac{\ddot{a}}{a} + 6 \frac{\dot{b}}{b} \right) = \frac{8\pi G_{\mu_0,\alpha}}{\sqrt{\hat{G}_i}}
\times \left[ \frac{p^2_0}{3} + \frac{2}{\alpha'(D-1)} \left[ b^2(w, w) + (n, w) + 2(N - 1) \right] \right]
\]
Special states

Enhanced symmetry states

\[ b^2(w, w) + (n, w) + 2(N - 1) = 0. \]  \hspace{1cm} (52)

Stable radion fixed point:

\[ \frac{n_a^2}{b^2} - w_a^2 b^2 = 0. \]  \hspace{1cm} (53)
**Enhanced symmetry states**

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**Stable radion fixed point:**

\[ \frac{n_a^2}{b^2} - w_a^2 b^2 = 0. \]  \hspace{1cm} (53)
Add a single non-perturbative ingredient - gaugino condensation - in order to fix the remaining modulus, the dilaton.

Kähler potential: (standard)

\[ K(S) = -\ln(S + \bar{S}) , \quad S = e^{-\Phi} + ia . \] (54)

where \( \Phi = 2\phi - 6\ln b \) is the 4-d dilaton, \( b \) is the radion and \( a \) is the axion.

Non-perturbative superpotential (from gaugino condensation):

\[ W = M_P^3 \left( C - Ae^{-a_0 S} \right) \] (55)
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Dilaton potential I

Yields a potential for the dilaton (and radion)

\[ V = \frac{M_P^4}{4} b^{-6} e^{-\Phi} \left[ \frac{C^2}{4} e^{2\Phi} + A C e^\Phi \left( a_0 + \frac{1}{2} e^\Phi \right) e^{-a_0 e^{-\Phi}} \right. \\
\left. + A^2 \left( a_0 + \frac{1}{2} e^\Phi \right)^2 e^{-2a_0 e^{-\Phi}} \right] . \]  

(56)

Expand the potential about its minimum:

\[ V = \frac{M_P^4}{4} b^{-6} e^{-\Phi_0} a_0^2 A^2 \left( a_0 - \frac{3}{2} e^{\Phi_0} \right)^2 e^{-2a_0 e^{-\Phi_0}} \times \left( e^{-\Phi} - e^{-\Phi_0} \right)^2 . \]  

(57)
Yields a potential for the dilaton (and radion)

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\times \left( e^{-\Phi} - e^{-\Phi_0} \right)^2 .
\]

(57)
Lift the potential to 10-d, redefining $b$ to be in the Einstein frame.

$$\begin{align*}
V(b, \phi) &= \frac{M_{10}^{16}}{4} \hat{V} e^{-\phi_0} a_0^2 A^2 \left( a_0 - \frac{3}{2} e^{\phi_0} \right)^2 e^{-2a_0 e^{-\phi_0}} \\
&\quad \times e^{-3\phi/2} \left( b^6 e^{-\phi/2} - e^{-\phi_0} \right)^2 .
\end{align*}$$

(58)

Dilaton potential in 10d Einstein frame

$$V \simeq n_1 e^{-3\phi/2} \left( b^6 e^{-\phi/2} - n_2 \right)^2$$

(59)
Analysis including both string matter and dilaton potential I

Worry: adding this potential will mess up radion stabilization

Thus: consider dilaton and radion equations resulting from the action including both the dilaton potential and string gas matter.

Step 1: convert the string gas matter contributions to the 10-d Einstein frame

\[
g_{\mu\nu}^E = e^{-\phi/2} g_{\mu\nu}^S \tag{60}
\]

\[
b_s = e^{\phi/4} b_E \tag{61}
\]

\[
T_{\mu\nu}^E = e^{2\phi} T_{\mu\nu}^S \tag{62}
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\[
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g^{E}_{\mu\nu} & = e^{-\phi/2} g^{S}_{\mu\nu} \\
b_{s} & = e^{\phi/4} b_{E} \\
T^{E}_{\mu\nu} & = e^{2\phi} T^{S}_{\mu\nu}.
\end{align*}
\]
Joint analysis II

Step 2: Consider both dilaton and radion equations:

\[-\frac{M_{10}^8}{2} \left( 3a^2 \dot{a} b^6 \dot{\phi} + 6a^3 b^5 b \dot{\phi} + a^3 b^6 \ddot{\phi} \right) + \frac{3}{2} n_1 a^3 b^6 e^{-3\phi/2} \left( b^6 e^{-\phi/2} - n_2 \right)^2 + a^3 b^{12} n_1 e^{-2\phi} \left( b^6 e^{-\phi/2} - n_2 \right) + \frac{1}{2\epsilon} e^{\phi/4} \left( -\mu_0 \epsilon^2 + \mu_0 |p_d|^2 \right) + 6\mu_0 \left[ \frac{n_2^2}{\alpha'} e^{-\phi/2} b^{-2} - \frac{w^2}{\alpha'} e^{\phi/2} b^2 \right] \]  

\[= 0, \quad (63)\]
\[ \ddot{b} + 3\frac{\dot{a}}{a} \dot{b} + 5\frac{\dot{b}^2}{b} = -\frac{n_1 b}{M_{10}^8} e^{-3\phi/2} \left( b^6 e^{-\phi/2} - n_2 \right)^2 \\
- \frac{2n_1}{M_{10}^8} b^7 e^{-2\phi} \left( b^6 e^{-\phi/2} - n_2 \right) \\
+ \frac{1}{2 - D} \left[ -10b \frac{n_1 e^{-3\phi/2}}{M_{10}^8} \left( b^6 e^{-\phi/2} - n_2 \right)^2 \\
- \frac{12n_1}{M_{10}^8} b^7 e^{-2\phi} \left( b^6 e^{-\phi/2} - n_2 \right) \right] \\
+ \frac{8\pi G_D \mu_0}{\alpha' \sqrt{\hat{G}_{a\epsilon}}} e^{2\phi} \left[ n_2^2 b^{-2} e^{-\phi/2} - w_2^2 b^2 e^{\phi/2} \right] \\
+ \frac{2}{D - 1} \left( e^{\phi/2} b^2 w^2 + n \cdot w + 2(N - 1) \right) \]
Step 3: Identifying extremum

- Dilaton at the minimum of its potential and
- Radion at the enhanced symmetry state

Step 4: Stability analysis

- Consider small fluctuations about the extremum
- Show stability (tedious but straightforward)

Result: Dilaton and radion stabilized simultaneously at the enhanced symmetry point.
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