IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Horava-Lifshitz gravity

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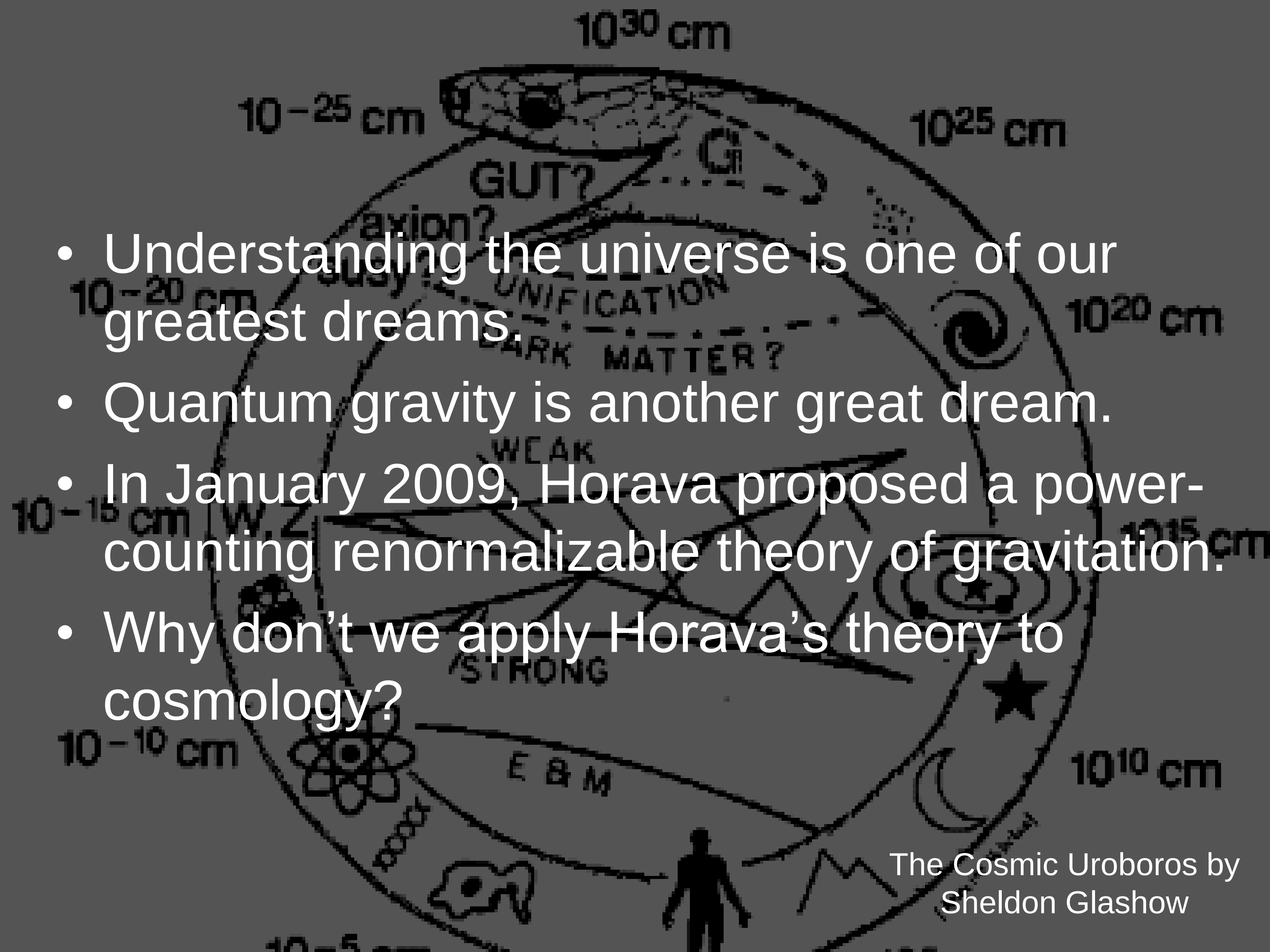
- Understanding the universe is one of our greatest dreams.
- Quantum gravity is another great dream.
- In January 2009, Horava proposed a power-counting renormalizable theory of gravitation.
- Why don't we apply Horava's theory to cosmology?



The Cosmic Uroboros by Sheldon Glashow

Horava-Lifshitz cosmology

- Higher curvature terms lead to regular bounce (Calcagni 2009, Brandenberger 2009).
- Higher curvature terms ($1/a^6$, $1/a^4$) might make the flatness problem milder (Kiritsis&Kofinas 2009).
- The z=3 scaling solves the horizon problem and leads to scale-invariant cosmological perturbations without inflation (Mukohyama 2009).
- Absence of local Hamiltonian constraint leads to CDM as integration "constant" (Mukohyama 2009).
- New mechanism for generation of primordial magnetic seed field (Maeda, Mukohyama, Shiromizu 2009).

Contents of this talk

- Basics of Horava-Lifshitz gravity
- Generation of scale-invariant cosmological perturbation
- Dark matter as integration “constant”
- Comments on scalar graviton
- Non-Gaussianity

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

- **Scaling dim of ϕ**

  t → b t (E → b^{-1}E)

  x → b x

  ϕ → b^s ϕ

  1+3-2+2s = 0

  s = -1

- Renormalizability

$$n \le 4$$

- Gravity is highly non-linear and thus non-renormalizable

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(z+3+ns)/z}$$

- Anisotropic scaling
  $t \rightarrow b^z t$ ($E \rightarrow b^{-z} E$)
  $x \rightarrow b x$
  $\phi \rightarrow b^s \phi$
  $z+3-2z+2s = 0$
  $s = -(3-z)/2$
- $s = 0$ if $z = 3$
- For z = 3, any nonlinear interactions are renormalizable!
- **Gravity becomes renormalizable!?**

Scalar with z=3

free part $$I_\phi = \frac{1}{2}\int dt d^3x \left(\dot{\phi}^2 + \phi \mathrm{O} \phi\right)$$

$$\mathrm{O} = \underbrace{\frac{\Delta^3}{M^4}}_{\text{UV: z=3}} - \frac{\kappa \Delta^2}{M^2} + \underbrace{c_\phi^2 \Delta - m_\phi^2}_{\text{IR: z=1}}$$

FERMI, MAGIC
→ $M>10^{11}$GeV
for photon

- **UV: z=3** , renormalizable nonlinear theory

RG flow

- **IR: z=1** , familiar Lorentz invariant theory

Note: we need a mechanism or symmetry to make “limits of speed” of different species to be essentially the same.
c.f. Iengo, Russo, and Serone (2009)

$\delta c = 0$ is an IR fixed point but approach seems slow.

Perhaps, embedding into an unified theory is necessary.

Scale-invariant cosmological perturbations from Horava-Lifshitz gravity without inflation

arXiv:0904.2190 [hep-th]

c.f. Basic mechanism is common for “Primordial magnetic field from non-inflationary cosmic expansion in Horava-Lifshitz gravity”, arXiv:0909.2149 [astro-th.CO] with S.Maeda and T.Shiromizu.

Usual story with z=1

- $\omega^2 \gg H^2$: oscillate

  $\omega^2 \ll H^2$: freeze

  oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

- Scaling law

  $t \to b\, t$ ($E \to b^{-1}E$)

  $x \to b\, x$

  $\phi \to b^{-1}\, \phi$

  $\Rightarrow \quad \delta\phi \propto E \sim H$

  Scale-invariance requires almost const. H, i.e. inflation.

UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$

  OK for $a \sim t^p$ with $p > 1/3$

- Scaling law

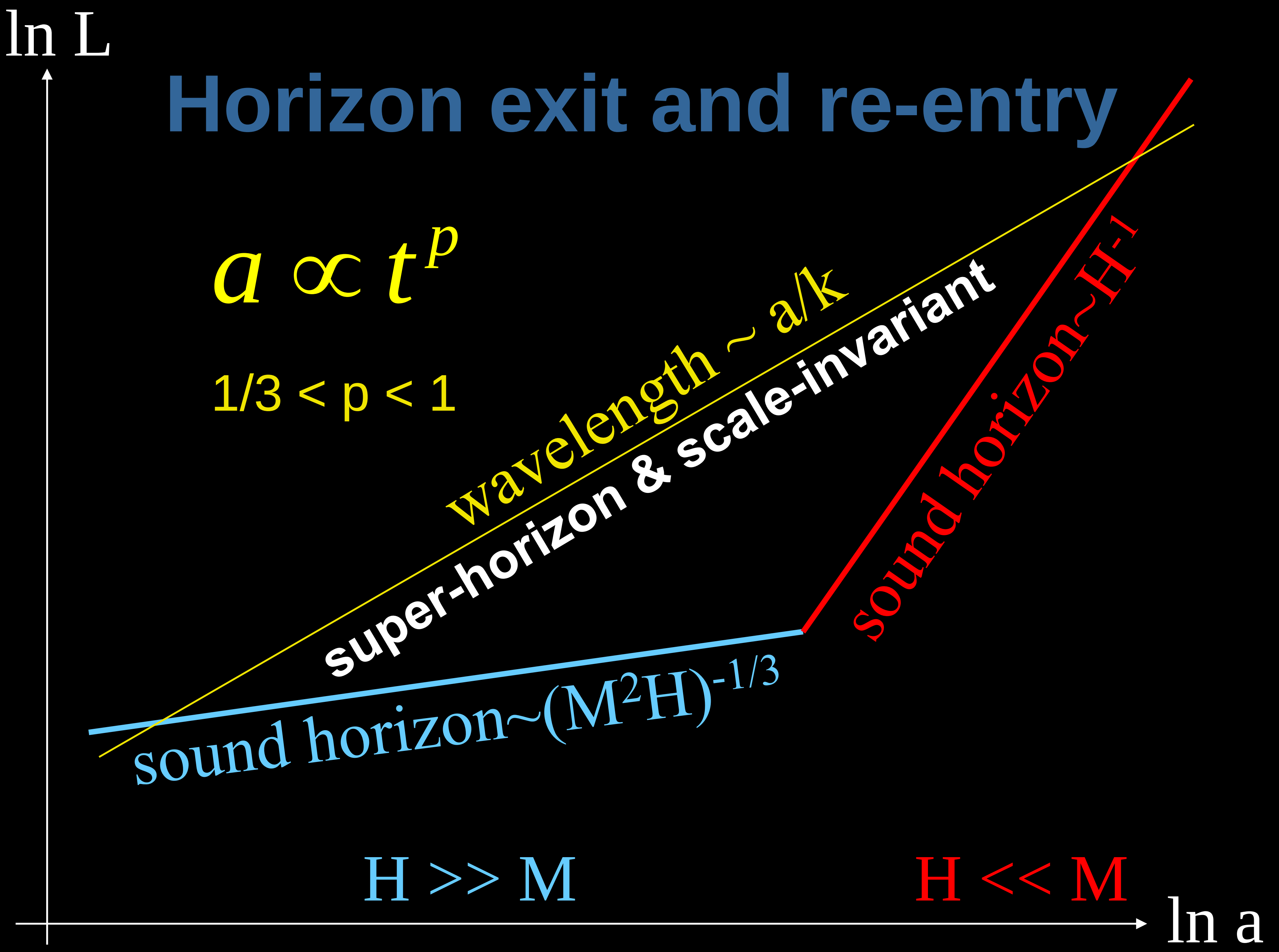
  $t \to b^3 t$ ($E \to b^{-3}E$)

  $x \to b\,x$

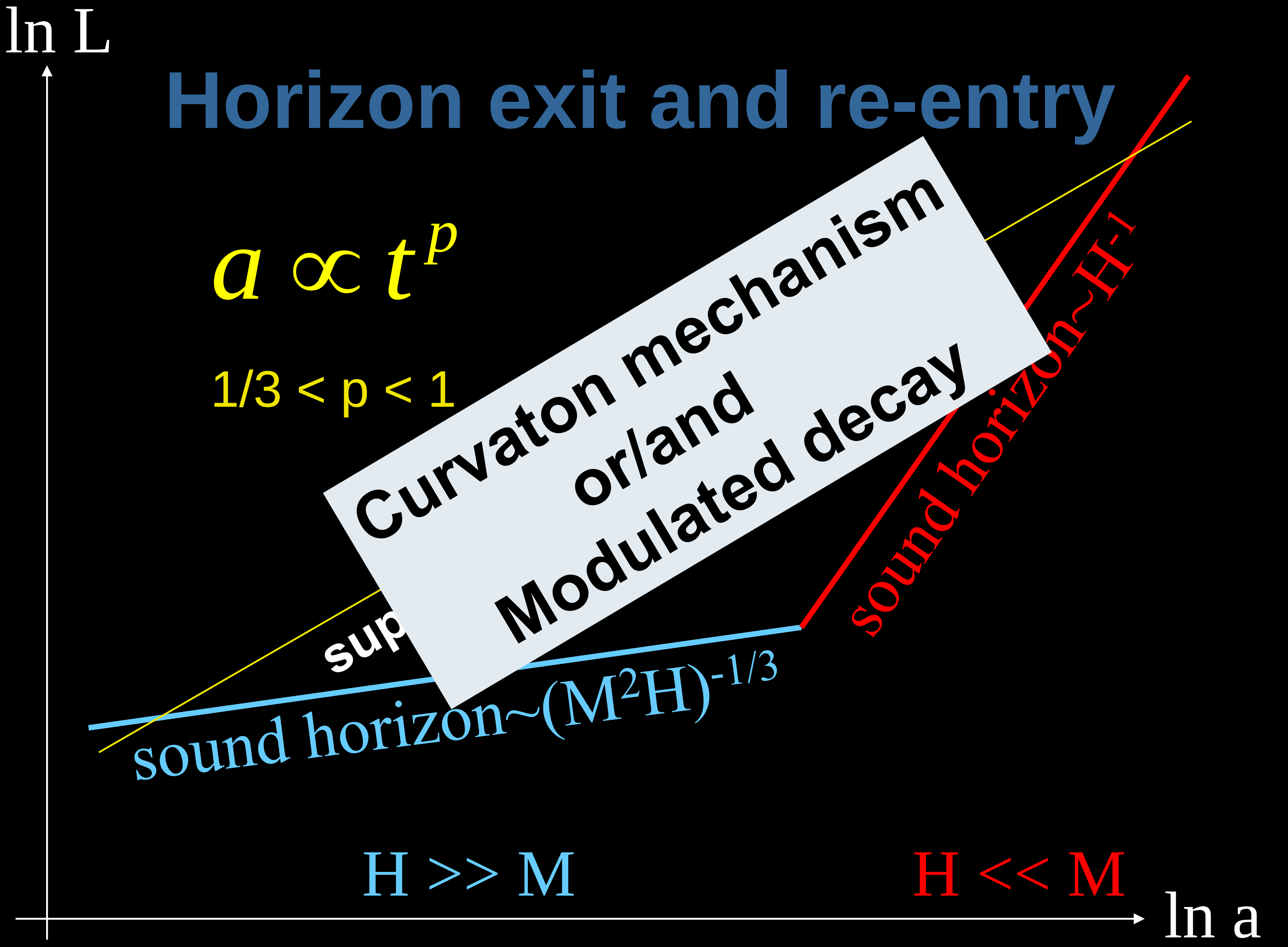
  $\phi \to b^0 \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$

  Scale-invariant fluctuations!

Horizon exit and re-entry



Horizon exit and re-entry



GOING BACK TO HORAVA‘S IDEA

Horava-Lifshitz gravity

Horava (2009)

- Basic quantities:
  lapse $N(t)$, shift $N^i(t,x)$, 3d spatial metric $g_{ij}(t,x)$
- ADM metric (emergent in the IR)
  $ds^2 = -N^2dt^2 + g_{ij}\,(dx^i + N^i dt)(dx^j + N^j dt)$
- Foliation-preserving deffeomorphism
  $t \rightarrow t'(t), \quad x^i \rightarrow x'^i(t,x^j)$
- Anisotropic scaling with z=3 in UV
  $t \rightarrow b^z\, t, \quad x^i \rightarrow b\, x^i$
- Ingredients in the action

$$Ndt \qquad \sqrt{g}\,d^3x \qquad g_{ij} \qquad D_i \qquad R_{ij}$$

$$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - D_i N_j - D_j N_i\right)$$

(C_{ijkl} = 0 in 3d)

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g}\, d^3x \left(K_{ij}K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g}\, d^3x \Big[\quad D_i R_{jk} D^i R^{jk} \qquad D_i R D^i R$$

$$R_i^j R_j^k R_k^i \qquad R R_i^j R_j^i \qquad R^3 \qquad \Big]$$

c.f. $D_i R_{jk} D^j R^{ki}$ is written in terms of other terms

Relevant deformations (with parity)

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g}\, d^3x \left[\qquad R_i^j R_j^i \qquad\qquad R^2 \qquad \right]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g}\, d^3x \left[\qquad\qquad R \qquad\qquad \right]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g}\, d^3x \left[\qquad\qquad 1 \qquad\qquad \right]$$

IR action with z=1

- **UV: z=3** , power-counting renormalizability

  ⬇ RG flow

- **IR: z=1** , seems to recover GR iff λ → 1

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g}\, d^3x \Big(\overbrace{K_{ij}K^{ij} - \lambda K^2}^{\text{kinetic term}} + \underbrace{R - 2\Lambda}_{\text{IR potential}} \Big)$$

note:

Renormalizability has not been proved.

RG flow has not yet been investigated.

Projectability condition

- Infinitesimal tr. $\delta t = f(t)$, $\delta x^i = \zeta^i(t,x^j)$

$$\delta g_{ij} = \partial_i \zeta^k g_{jk} + \partial_j \zeta^k g_{ik} + \zeta^k \partial_k g_{ij} + f \dot{g}_{ij}$$

$$\delta N_i = \partial_i \zeta^j N_j + \zeta^j \partial_j N_i + \dot{\zeta}^j g_{ij} + \dot{f} N_i + f \dot{N}_i$$

$$\delta N = \zeta^i \partial_i N + \dot{f} N + f \dot{N}$$

- Space-independent N cannot be transformed to space-dependent N.
- N is gauge d.o.f. associated with the space-independent time reparametrization.
- It is natural to restrict N to be space-independent.
- Consequently, Hamiltonian constraint is an equation integrated over a whole space.

Note

- Imposing local Hamiltonian constraint would result in theoretical inconsistencies and phenomenological obstacles.
- "Strong coupling in Horava gravity"
  by C.Charmousis, et.al., arXiv:0905.2579
  "A trouble with Horava-Lifshitz gravity"
  by M.Li and Y.Pang, arXiv:0905.2751
  "A dynamical inconsistency of Horava gravity"
  by M.Henneaux, et.al., arXiv:0912.0399
- Those problems disappear once we notice that there is no local Hamiltonian constraint.
  (c.f. section 5 of arXiv:0905.3563)

Dark matter as integration constant in Horava-Lifshitz gravity

arXiv:0905.3563 [hep-th]

See also arXiv:0906.5069 [hep-th]
Caustic avoidance in Horava-Lifshitz gravity

Structure of GR

- 4D diffeomorphism $\rightarrow$
  4 constraints = 1 Hamiltonian + 3 momentum
  @ each time @ each point
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

FRW spacetime in GR

- $ds^2 = - dt^2 + a^2(t) (dx^2 + dy^2 + dz^2)$
- Approximates overall behavior of our patch of the universe inside the Hubble horizon.
- Hamiltonian constraint
  → Friedmann eq

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} \rho_i$$

  E.o.m. of matter
  → conservation eq.

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$

- Dynamical eq
  is not independent
  but follows from the above n+1 eqs.

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} P_i$$

Structure of HL gravity

- Foliation-preserving diffeomorphism
  = 3D spatial diffeomorphism
  + space-independent time reparametrization
- 3 local constraints + 1 global constraint
  = 3 momentum @ each time @ each point
  + 1 Hamiltonian @ each time integrated
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

FRW spacetime in HL gravity

- Approximates overall behavior of our patch of the universe inside the Hubble horizon.
- No “local” Hamiltonian constraint
  E.o.m. of matter
  → conservation eq.

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$

- Dynamical eq
  can be integrated to give
  Friedmann eq with
  “dark matter as
  integration constant”

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} P_i$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \left(\sum_{i=1}^{n} \rho_i + \frac{C}{a^3} \right)$$

IR limit of HL gravity

$$\frac{1}{16\pi G_N}\int N dt\sqrt{g}\, d^3x\left(K_{ij}K^{ij}-\lambda K^2+R-2\Lambda\right)$$

- Looks like GR iff λ = 1. So, we assume that λ = 1 is an IR fixed point of RG flow.
- Global Hamiltonian constraint

$$\int d^3x\sqrt{g}(G^{(4)}_{\mu\nu}+\Lambda g^{(4)}_{\mu\nu}-8\pi G_N T_{\mu\nu})n^\mu n^\nu=0$$

$$n_\mu dx^\mu=-Ndt,\quad n^\mu\partial_\mu=\frac{1}{N}(\partial_t-N^i\partial_i)$$

- Momentum constraint & dynamical eq

$$(G^{(4)}_{i\mu}+\Lambda g^{(4)}_{i\mu}-8\pi G_N T_{i\mu})n^\mu=0$$

$$G^{(4)}_{ij}+\Lambda g^{(4)}_{ij}-8\pi G_N T_{ij}=0$$

Dark matter as integration constant

- Def. $T^{HL}{}_{\mu\nu}$ $\quad G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu} \right)$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad\qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K\rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

Dark matter as integration constant

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GR is NOT recovered but GR+CDM is!

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Dark matter as integration constant

- Def. $T^{HL}_{\mu\nu}$: $G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu}\right)$

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- Global Hamiltonian constraint

$$\int d^3x \sqrt{g}\rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

Indeed, one can prove that there is no exactly static "CDM" star: "CDM" accretes! [K.Izumi and S.Mukohyama]

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K\rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

Micro to Macro

- Overall behavior of smooth $T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu$ is like pressureless dust.
- Microscopic lumps (sequences of caustics & bounces) of ρ^{HL} can collide and bounce. (cf. early universe bounce [Calcagni 2009, Brandenberger 2009]) If asymptotically free, would-be caustics does not gravitate too much.
- Group of microscopic lumps with collisions and bounces → When coarse-grained, can it mimic a cluster of particles with velocity dispersion?
- Dispersion relation of matter fields defined in the rest frame of "dark matter"
  → Any astrophysical implications?

Summary so far

- The z=3 scaling solves horizon problem and leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p>1/3$.
- The lack of local Hamiltonian constraint may explain "dark matter" without dark matter. GR is NOT recovered: constraint algebra is smaller than GR since the time slicing and the "dark matter" rest frame are synchronized in the theory level.

The rest of this talk

- Comments on scalar graviton
- Non-Gaussianity

Propagating d.o.f.

- Minkowski + perturbation
  $N = 1$, $N^i = 0$, $g_{ij} = \delta_{ij} + h_{ij}$
- Residual guage freedom =
  time-independent spatial diffeo.
- Momentum constraint
  $$\partial_t \partial_i H_{ij} = 0 \qquad\qquad H_{ij} \equiv h_{ij} - \lambda h \delta_{ij}$$
- Fix the residual guage freedom by setting
  $\partial_i H_{ij} = 0$ at some fixed time surface.
- Decompose H_{ij} into trace and traceless parts
  TT part : 2 d.o.f. (usual tensor graviton)
  Trace part : 1 d.o.f. (scalar graviton)

Scalar graviton and λ → 1

$$h_{ij} = \tilde{H}_{ij} + \frac{1-\lambda}{2(1-3\lambda)} H\delta_{ij} - \frac{\partial_i \partial_j}{2\partial^2} H$$

- In the limit λ → 1, the scalar graviton H becomes pure gauge. So, it decouples.
- However, its kinetic term will vanish

$$I_{kin} \sim \int dt d^3x \left[(\partial_t \tilde{H}_{ij})^2 + \frac{\lambda - 1}{2(3\lambda - 1)} (\partial_t H)^2 \right]$$

  and H gets strongly self-coupled.
- This is not a problem in renormalizable theories if there is "Vainshtein effect", i.e. decoupling of the strongly-coupled sector from the rest of the world.

Linear instability of scalar graviton

Appendix C of arXiv:0911.1814 with K.Izumi

- Sign of (time) kinetic term $(\lambda-1)/(3\lambda-1) > 0$.
- The dispersion relation in flat background
  $\omega^2 = k^2 \times [c_s^2 + O(k^2/M^2)]$ with $c_s^2 = -(\lambda-1)/(3\lambda-1)<0$
  → IR instability in linear level
  (Wang&Maartens; Blas,et.al.; Koyama&Arroja 2009)
- Slower than Jeans instability of "DM as integration const" if
  $t_J \sim (G_N\rho)^{-1/2} < t_L \sim L/|c_s|$.
- Tamed by Hubble friction or/and $O(k^2/M^2)$ terms if
  $H^{-1} < t_L$ or/and $L < 1/(|c_s|M)$.
- Thus, the linear instability does not show up if
  $|c_s| < \mathrm{Max}\,[|\Phi|^{1/2}, HL, 1/(ML)]$. $(\Phi \sim -G_N\rho L^2)$
  L>0.01mm (Shorter scales → similar to spacetime foam)
- Phenomenological constraint on properties of RG flow.

Non-Gaussianity

work in progress

(~ 1 week old)

Bispectrum of z=3 scalar

Leading 3-point interactions with shift symmetry

$$L_1 = -\frac{\alpha_1}{M^5}(\Delta\phi)^3 \qquad L_2 = -\frac{\alpha_2}{M^5}\Delta\phi D^i D^j \phi D_i D_j \phi$$

$$L_3 = -\frac{\alpha_3}{M^5} D^i D^j \phi D_j D_k \phi D^k D_i \phi \qquad L_4 = \frac{\alpha_4}{M^5} D^i \Delta\phi D_i \phi \Delta\phi$$

$$L_5 = \frac{\alpha_5}{M^5} D^i D^j D^k \phi D_i D_j \phi D_k \phi \qquad L_6 = -\frac{\alpha_6}{M^5} \Delta^2 \phi D^i \phi D_i \phi$$

$$L_7 = -\frac{\alpha_7}{M^5} D_i D_j \Delta\phi D^i \phi D^j \phi$$

Corresponding $H_I dt$ has scaling dim 0 !

Order estimate

Power spectrum

$$P_\phi \propto \langle 0|\phi\phi|0\rangle \propto M^2 \times \left(\frac{H}{M}\right)^{2\times 0} = M^2$$

Bispectrum

$$B_\phi \propto \langle 0|\phi\phi\phi|0\rangle_c \propto i\int dt_1 \langle [H_I(t_1), \phi\phi\phi]\rangle_c \propto \alpha \times M^3 \times \left(\frac{H}{M}\right)^{0+3\times 0} = \alpha M^3$$

After conversion to curvature perturbation

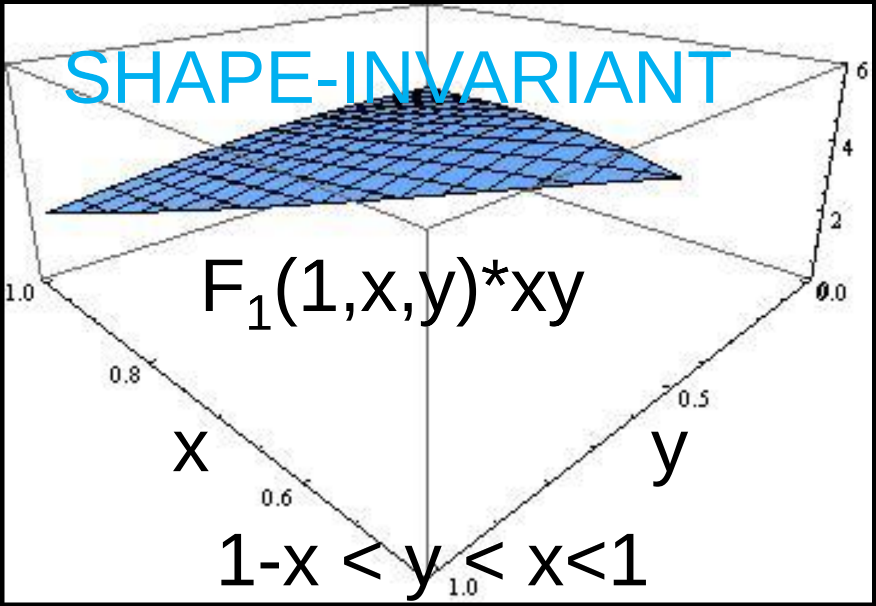
$$B_\varsigma \sim \alpha \times (P_\varsigma)^{3/2}$$

$$f_{NL} \sim \frac{B_\zeta}{(P_\zeta)^2} \sim \alpha (P_\zeta)^{-1/2} \sim 10^5 \times \alpha$$

Totally independent of background evolution!

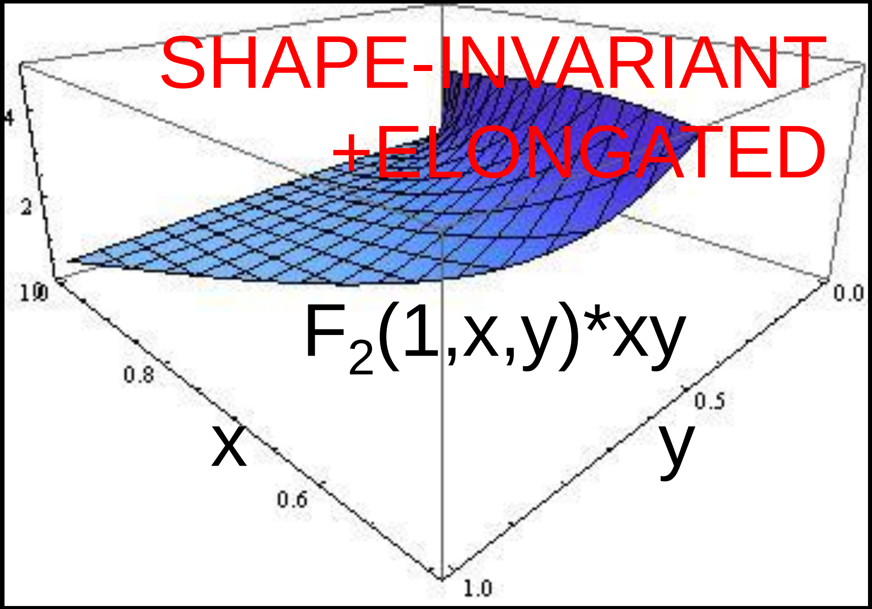
$\Rightarrow$ Strong constraint on α, perhaps requiring asymptotic freedom of the theory.

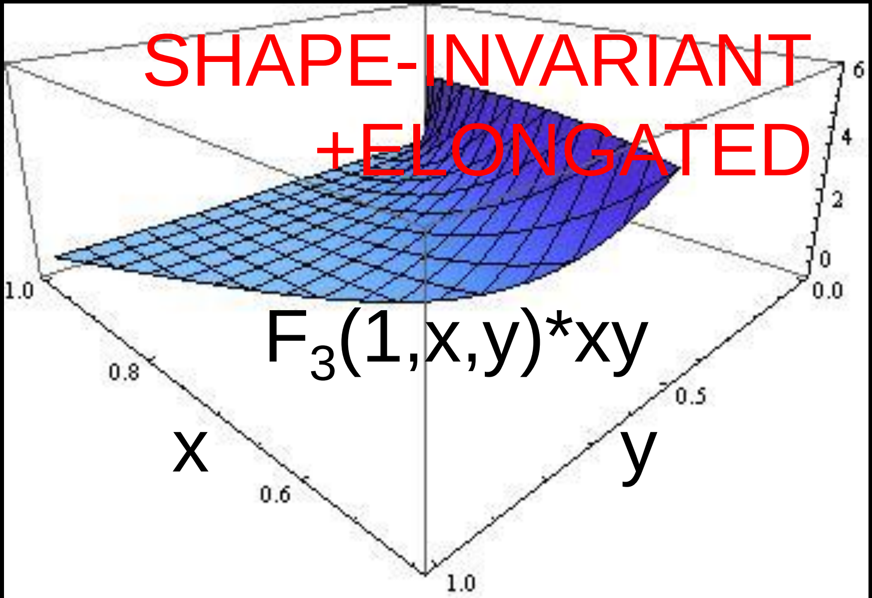
Shape of bispectrum

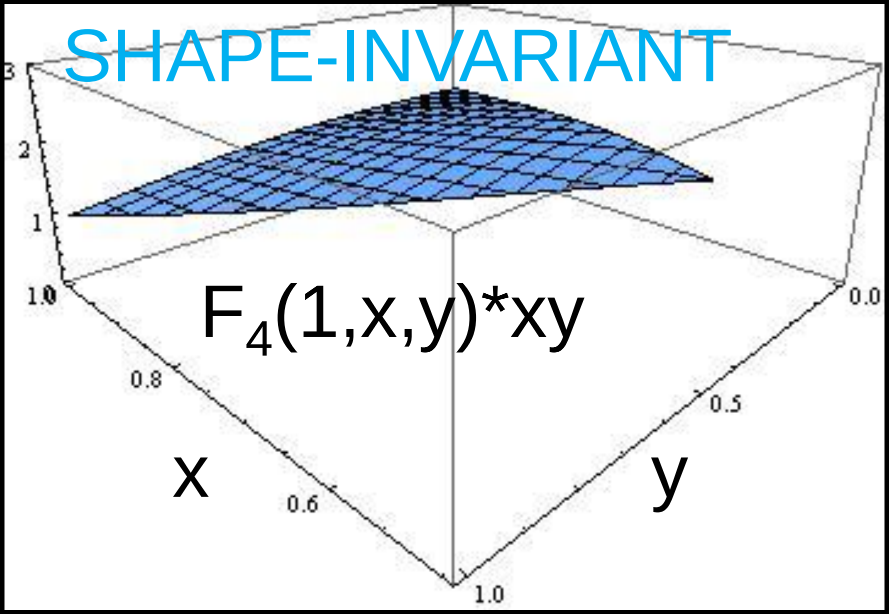


$$B_\phi(k_1,k_2,k_3) = \frac{M^3}{4}\sum_{i=1}^{7}\alpha_i F_i(k_1,k_2,k_3)$$

Totally independent of background evolution!






SHAPE-INVARIANT
F4(1,x,y)*xy
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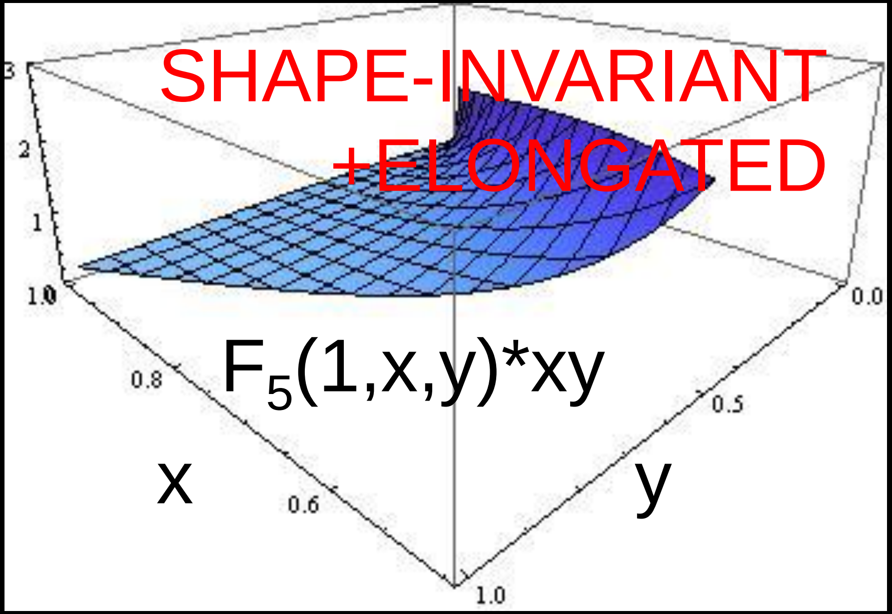
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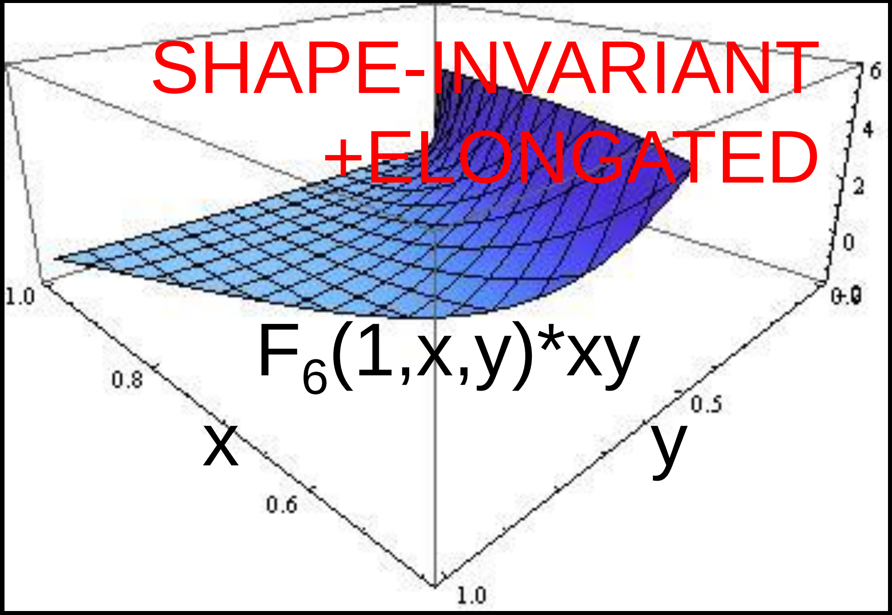
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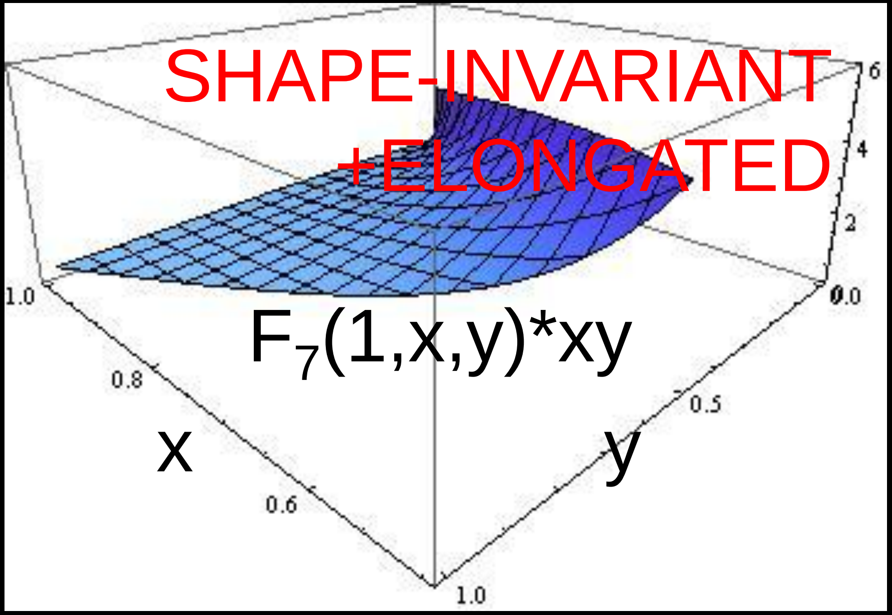
SHAPE-INVARIANT
+ELONGATED
F7(1,x,y)*xy
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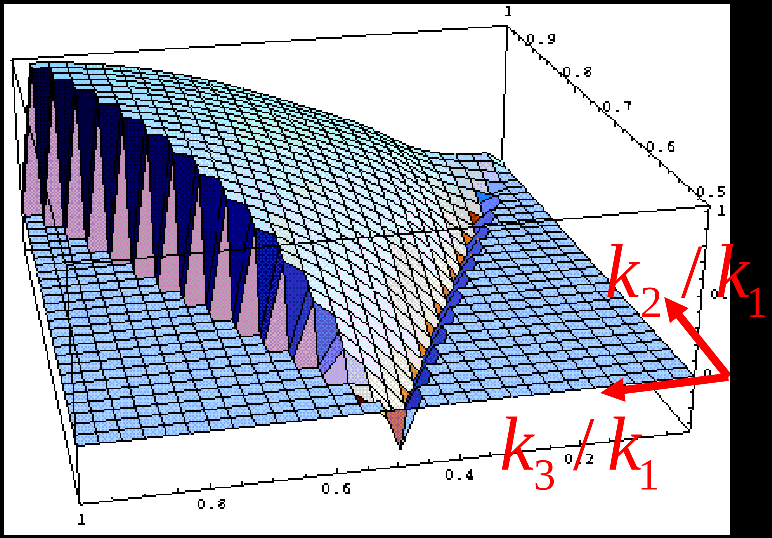
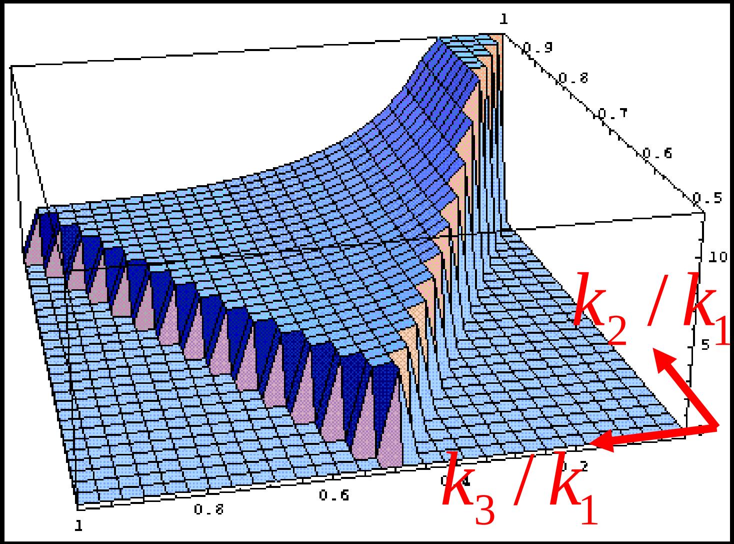
Other models



Equilateral

DBI inflation

ghost inflation



Squeezed

multi-field slow-roll

curvaton, etc.

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- While there are many fundamental issues to be addressed, it is interesting to investigate cosmological implications.
- The z=3 scaling solves horizon problem and leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p > 1/3$.
- HL gravity does NOT recover GR at low-E but can instead mimic GR+CDM: “dark matter as an integral constant”. Constraint algebra is smaller than GR since the time slicing and the “dark matter” rest frame are synchronized.
- Large non-Gaussianity is expected, perhaps requiring asymptotic freedom, if cosmological perturbation is produced during the z=3 regime. The shape (elongated-triangle + shape-invariant) of bispectrum is distinguishable from other models.

Future works

- Renormalizability beyond power-counting
- RG flow: is λ = 1 an IR fixed point ? Does it satisfy the stability condition for the scalar graviton? ($|c_s| < \mathrm{Max}\,[|\Phi|^{1/2}, HL, 1/(ML)]$ for $L>0.01$mm)
- Embedding into an unified theory : can we get a common “limit of speed” ?
- Is there Vainshtein effect? [with K.Izumi]
- Micro & macro behavior of “CDM”
- Adiabatic initial condition for “CDM” from the z=3 scaling
- Spectral tilt from anomalous dimension
- …

Backup slides

Black holes with N=N(t)?

- Schwarzschild BH in PG coordinate

$$ds^2 = -dt_P^2 + \left(dr \pm \sqrt{\frac{2m}{r}} dt_P \right)^2 + r^2 d\Omega^2$$

exact sol
for λ = 1

- Gaussian normal coordinate

$$ds^2 = -dt_G^2 + \cdots$$

approx sol
for λ = 1

Lemaitre reference frame
Doran coordinate

A free scalar field (I)

$$I = \frac{1}{2}\int dt d^3\vec{x} a^3 N \sqrt{q}\left[\frac{1}{N^2}\left(\partial_t \Phi - N^i \partial_i \Phi\right)^2 + \Phi \mathcal{O} \Phi\right]$$

$$\mathrm{O} = \underbrace{\frac{\Delta^3}{M^4}}_{\text{UV: z=3}} - \frac{\kappa\Delta^2}{M^2} + \underbrace{\Delta - m_\phi^2}_{\text{IR: z=1}}$$

FRW background with H >> M

$$I_{UV} = \frac{1}{2}\int d\eta d^3\vec{x}\left[a^2(\partial_\eta \delta\Phi)^2 + \frac{1}{M^4 a^2}\delta\Phi(\delta^{ij}\partial_i\partial_j)^3\delta\Phi\right]$$

$$(\delta\Phi_1, \delta\Phi_2)_{KG} = -i\int d\vec{x}^3 a^2\left(\delta\Phi_1 \partial_\eta \delta\Phi_2^* - \delta\Phi_2^* \partial_\eta \delta\Phi_1\right)$$

A free scalar field (II)

Normalized mode function

$$\phi_{\vec{k}} = \frac{e^{i\vec{k}\cdot\vec{x}}}{(2\pi)^3} \times 2^{-1/2} k^{-3/2} M \exp\left(-i \frac{k^3}{M^2} \int \frac{d\eta}{a^2}\right)$$

for $a \propto t^p, \quad p > 1/3$

$$\int^{\eta_\infty} \frac{d\eta}{a^2} = \int^{t_\infty} \frac{dt}{a^3}$$ converges

$\phi_{\vec{k}}$ initially oscillates and freezes @ $\omega^2 \sim H^2$

Power spectrum

$$\mathcal{P}_{\delta\Phi}^{1/2} = \sqrt{\frac{k^3}{2\pi^2}} \left|(2\pi)^3 \phi_{\vec{k}}\right| = \frac{M}{2\pi}$$

independent of H and scale-invariant!

General case

- General solution to the momentum constraint and dynamical eq.

$$G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} + O(\lambda - 1)$$
$$+ \,(\text{higher curvature corrections})$$
$$= 8\pi G_N \left(T_{\mu\nu} + \rho^{HL} n_\mu n_\nu \right)$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

- Bianchi identity → (non-)conservation eq → initial condition of "dark matter"

$$+ \,(\text{higher curvature corrections})$$

Four versions of HL gravity

- There are at least four versions of the theory: w/wo detailed balance & w/wo projectability.
- Only the version without the detailed balance condition with the projectability condition has a potential to be theoretically consistent and cosmologically viable.
- Horava's original proposal was with the projectability condition and with/without the detailed balance condition.
- There is an attempt to extend the non-projectable theory by introducing $a_i = (\ln N)_{,i}$ [Blas, Pujolas and Sibiryakov 2009].

“On the extra mode and inconsistency of Horava gravity”, by Blas, Pujolas and Sibiryakov, arXiv:0906.3046

- This paper has three statements about the projectable version: (i) Formation of caustics without taking into account backreaction of higher curvature terms to geometry; (ii) Relation to ghost condensate without taking into account difference in symmetries; (iii) Low strong-coupling scale of their low-E EFT away from $\lambda=1$. This does not imply breakdown of the underlining UV theory. (See “note added” in arXiv:0906.5069.)
- Contrary to (iii), we know that the scalar graviton gets strongly coupled only at $\lambda=1$. This is not a problem if there is “Vainstein effect” and if the theory is renormalizable.

Stellar center is dynamical in Horava-Lifshitz gravity

arXiv:0911.1814 [hep-th]
with K.Izumi

Black holes and stars

- Schwarzschild geometry in PG coordinate (N=1) is locally an exact solution with $\lambda = 1$.
- Kerr geometry in Doran coordinate ($N=1, N^i=0$) is locally an approximate solution with $\lambda = 1$.
- Those solutions are "black" for low-E probes but not "black" for high-E probes. Visible singularity?
- Extrinsic curvature diverges at the center of those solutions → UV effects such as deviation of λ from 1 → Do UV effects resolve BH singularity?
- To answer this question, we probably need to evolve a regular initial data towards BH formation.
- As a first step, let us consider stellar solutions.

Basic setup

Painlevé-Gullstrand coordinate

$$N = 1 \qquad N^i \partial_i = \beta(x)\partial_x$$

$$g_{ij}dx^i dx^j = dx^2 + r^2(x)d\Omega_2^2$$

Matter sector

$$T_{\mu\nu} = \rho(x)u_\mu u_\nu + P(x)\left[g^{(4)}_{\mu\nu} + u_\mu u_\nu\right]$$

$$u^\mu = \frac{\xi^\mu}{\sqrt{1-\beta^2}} \qquad \xi^\mu = \left(\frac{\partial}{\partial t}\right)^\mu$$

- The energy density ρ is a piecewise-continuous non-negative function of the pressure P.
- The central pressure P_c is positive.

No static star solution

- Momentum conservation equation

$$P'(1-\beta^2)+(\rho+P)(1-\beta^2)'=0$$

- Global-staticity → $1-\beta^2 > 0$ everywhere.
- Regularity of K^x_x → β' is finite → P' is also finite → $\beta(x)$ and $P(x)$ are continuous → $\rho(x)+P(x)$ is piecewise-continuous.
- $P_c>0$ & P continuous & ρ non-negative → $\rho+P>0$ in a neighborhood of the center.
- Define x_0 as the minimal value for which at least one of $(\rho+P)|_{x=x_0}$, $\lim_{x\to x_0-0}(\rho+P)$ and $\lim_{x\to x_0+0}(\rho+P)$ is non-positive.

$$\ln(1-\beta_0^2) - \ln(1-\beta_c^2) = -\int_{P_c}^{P_0} \frac{dP}{\rho(P)+P}$$

- L.h.s. is non-positive ← β_c=0 & r_c'=1 ← regularity of R & $K^\theta{}_\theta$
- R.h.s. is positive ← P_0 is non-positive ← ρ is non-negative & at least one of $(\rho+P)|_{x=x_0}$, $\lim_{x\to x_0-0}(\rho+P)$ and $\lim_{x\to x_0+0}(\rho+P)$ is non-positive & P(x) is continuous
- Contradiction! → no spherically-symmetric globally-static solutions → stellar center is dynamical
- The proof is insensitive to the structure of higher-derivative terms → valid for any z

$$\ln(1-\beta_0^2) - \ln(1-\beta_c^2) = -\int_{P_c}^{P_0} \frac{dP}{\rho(P)+P}$$

- L.h.s. is non-positive ← β_c=0 & r_c'=1 ← regularity of R & K^θ_θ
- R.h.s. is positive ← P_0 is non-positive ← ρ is non-negative & at least one of $(\rho+P)|_{x=x_0}$, $\lim_{x\to x_0-0}(\rho+P)$ and $\lim_{x\to x_0+0}(\rho+P)$ is non-positive & P(x) is continuous

The proof supports “DM as integration constant”: “DM” accreates toward a star and makes stellar center dynamical

- Contradiction! → no spherically-symmetric globally-static solutions → stellar center is dynamical
- The proof is insensitive to the structure of higher-derivative terms → valid for any z