Black Holes and Exotic States

Jan de Boer, Amsterdam

Kolymbari, September 15, 2010

Based on:

.

arXiv:0906.0011 - JdB, Sheer El-Showk, Ilies Messamah, Dieter van den Bleeken

arXiv:1004.2521 - JdB, Masaki Shigemori

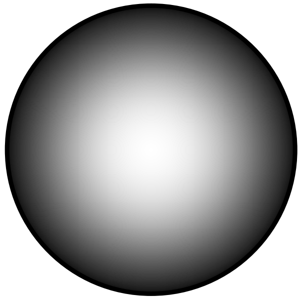
and work in progress

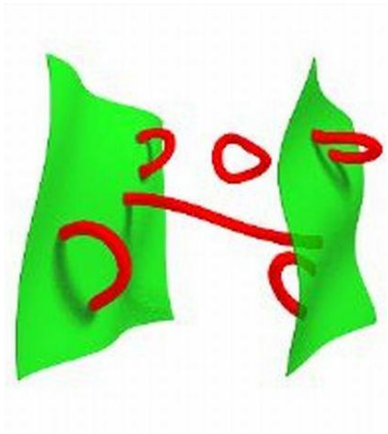
Outline

1. Higgs → Coulomb → Geometry
2. Microstates for black holes
3. A bound on the number of smooth supergravity solutions
4. Beyond Geometry
5. Conclusions

Conventional approach to black holes in string theory:

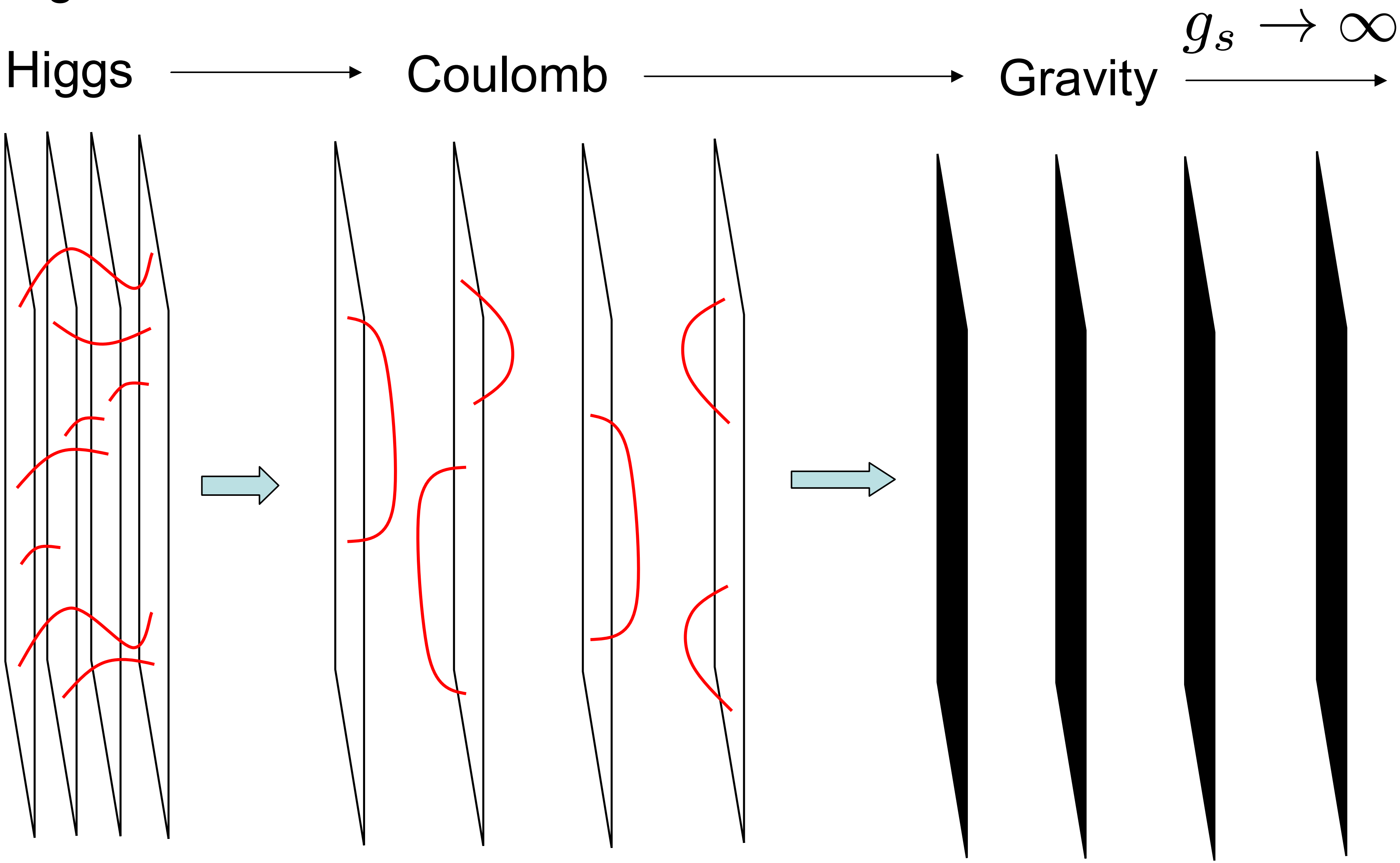
black hole





Reproduce entropy

It would be very interesting could reverse the arrows in this picture and find a geometrical description of the individual degrees of freedom.



Under suitable circumstances, the resulting geometries are smooth and source-free: they are purely geometrical.

E.g. Kaluza-Klein monopoles, D6-branes with world-volume fluxes when uplifted to M-theory, etc.

These are then usually referred to as black hole microstates, though they are better thought of as points in phase space.

A key question has always been: how many of these are there, are they typical representatives of generic microstates, and are there enough to explain the entropy of a black hole?

Example:

Recall that the F1-D0 system can puff up into a supertube, a D2-brane whose cross section can be an arbitrary curve.

Mateos, Townsend

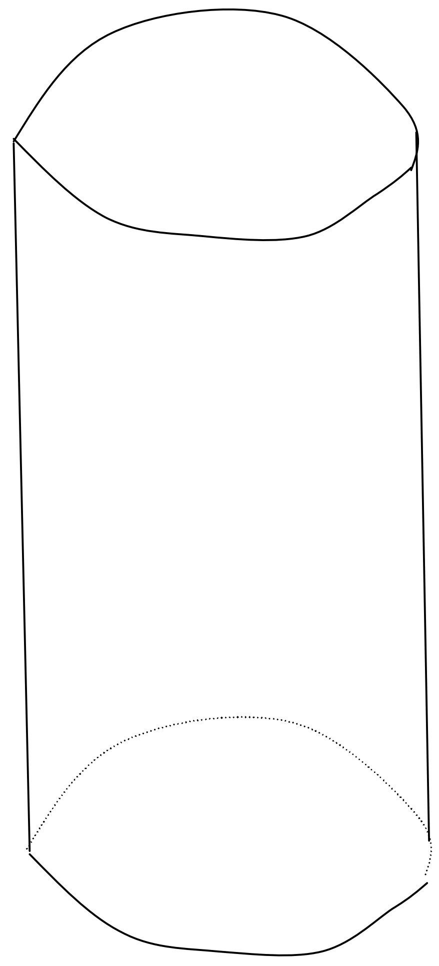
Example of Higgs-Coulomb

One can reproduce the number of F1-D0 bound states by quantizing supertubes.

Marold, Palmer

In a suitable duality frame, they can be described by smooth supergravity solutions.

Lunin, Mathur

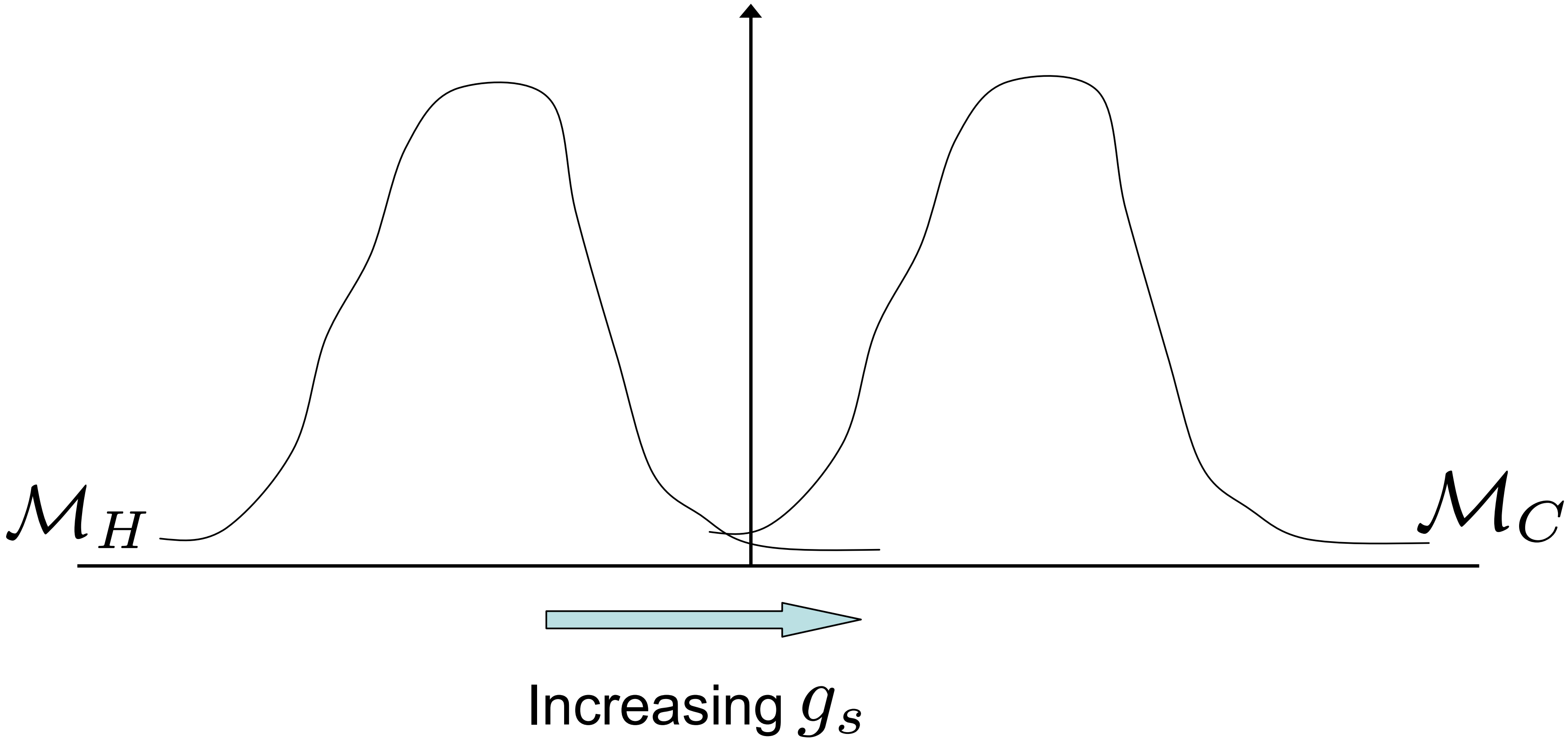


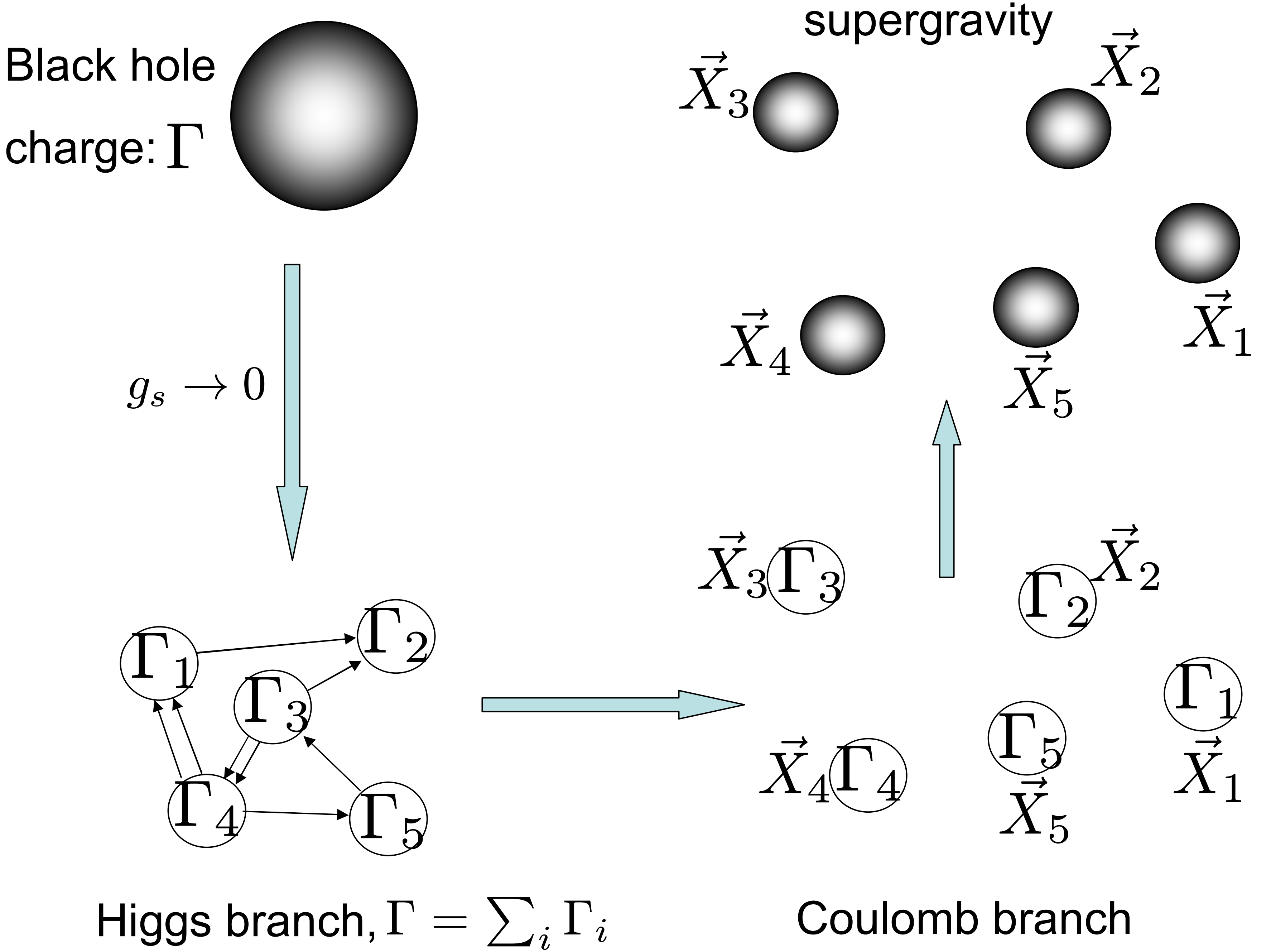
Example 2:

Wrap D0-D2-D4-D6 branes on 0,2,4,6-cycles in a CY in IIA string theory. They can form heavy BPS bound states which look like a large supersymmetric black hole in four dimensions.

Relevant gauge theory is the theory on the D-branes. After dimensional reduction over the cycles in the CY, it can be described by a 0+1 dimensional supersymmetric gauge theory. Black hole states live in the Higgs branch of such a gauge theory.

One can study such quantum mechanical systems in detail. As one increases the string coupling, states indeed move from the Higgs branch to the Coulomb branch.





Black hole
charge: Γ
supergravity
$\vec{X}_3$
$\vec{X}_2$
$\vec{X}_4$
$\vec{X}_5$
$\vec{X}_1$
$g_s \to 0$
$\vec{X}_3$ Γ_3
Γ_2 $\vec{X}_2$
Γ_1
$\vec{X}_4$ Γ_4
Γ_5
$\vec{X}_5$
$\vec{X}_1$
Γ_1
Γ_2
Γ_3
Γ_4
Γ_5
Higgs branch, $\Gamma = \sum_i \Gamma_i$
Coulomb branch

Resulting geometries describe multi-centered black hole solutions in four dimensions.

Lopes Cardoso, de Wit, Kappeli, Mohaupt; Denef; Bates, Denef; Balasubramanian, Gimon, Levi

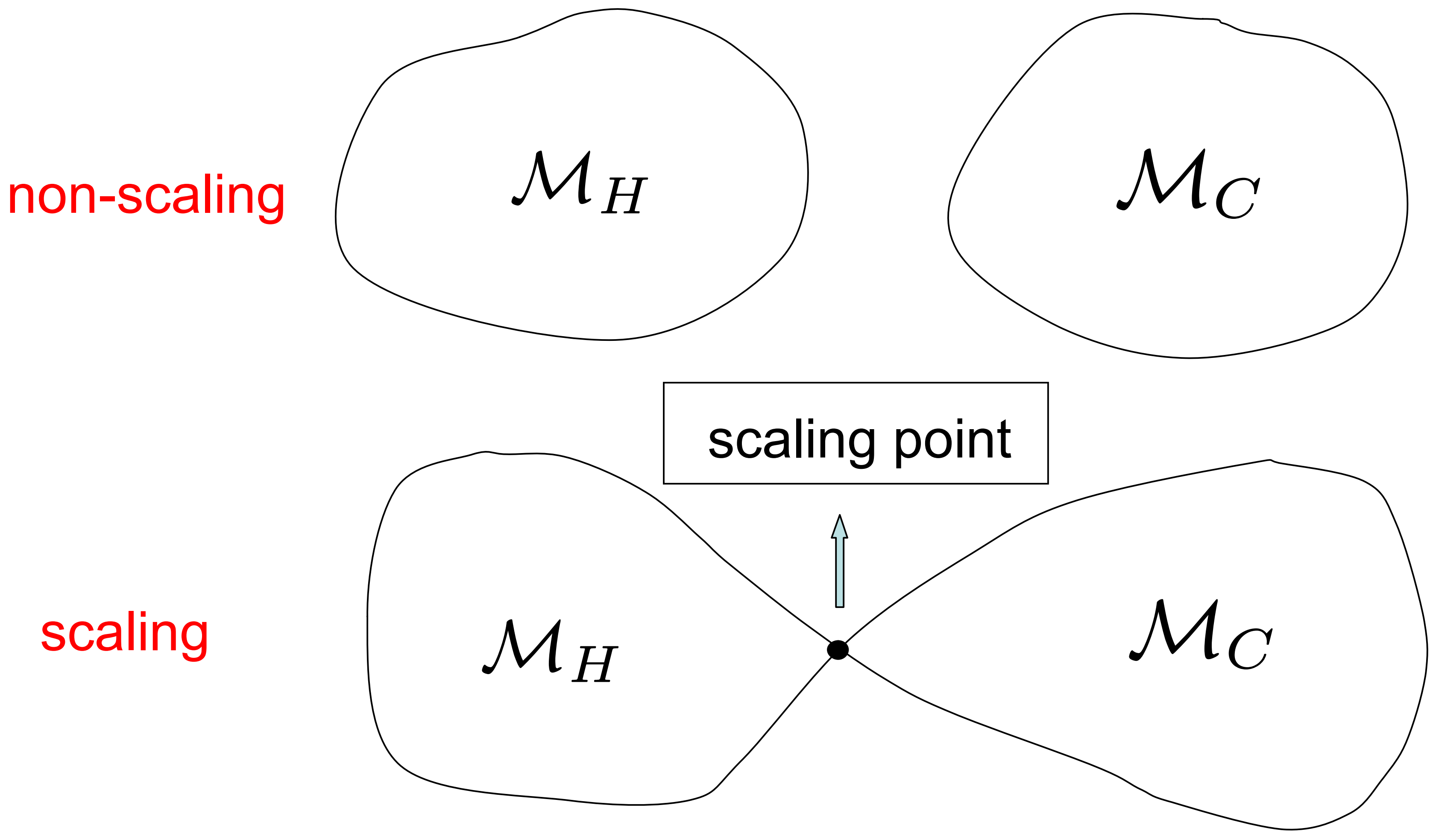
For suitable choices of the charges, these solutions become smooth after one uplifts them to five dimensions.

Space of solutions is quite complicated, described by equations of the form

$$\theta_i + \sum_{j \neq i} \frac{N_{ij}}{|\vec{x}_j - \vec{x}_i|} = 0$$

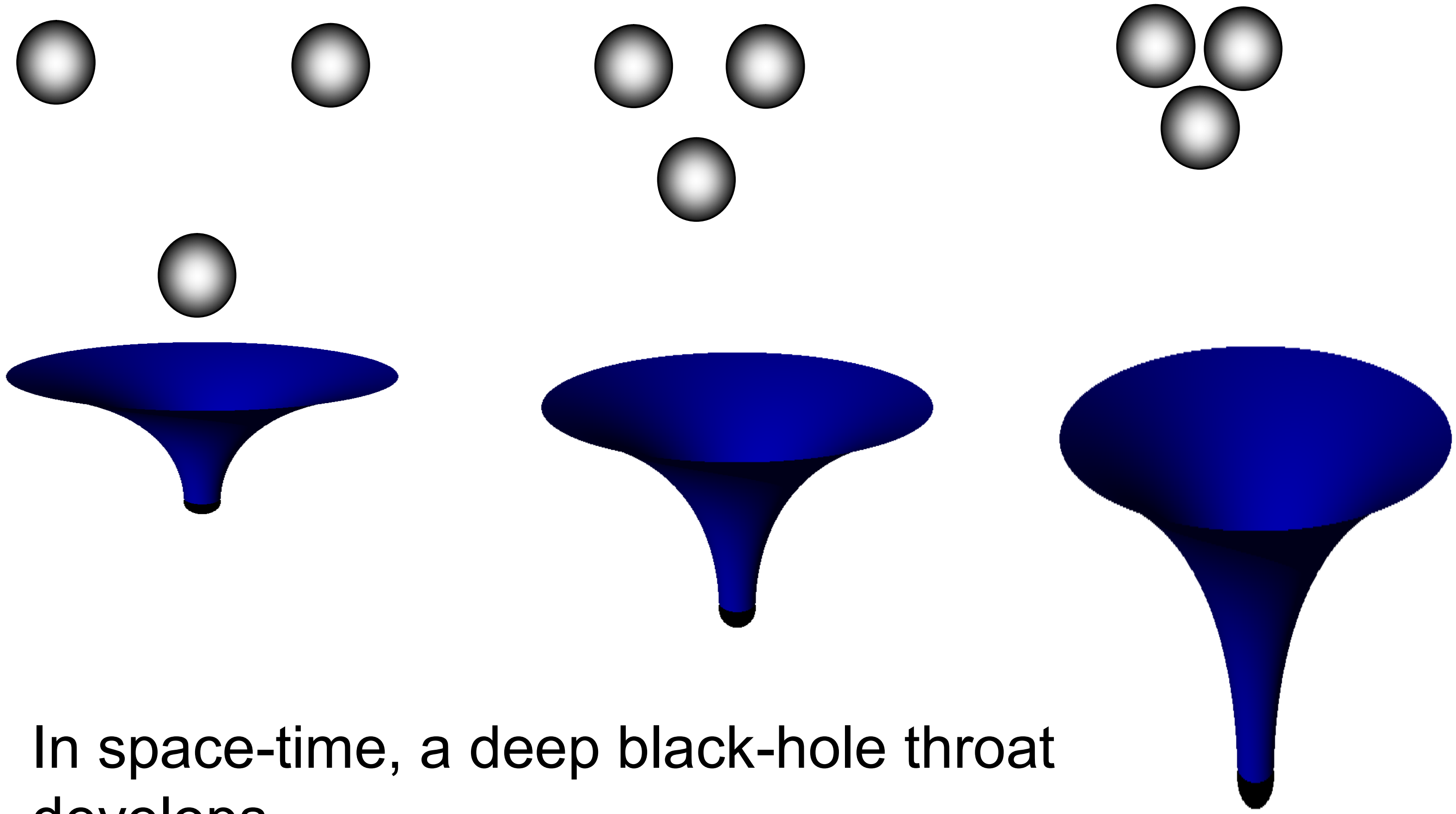
N_{ij} =#bifundamentals

Two classes of systems: scaling vs non-scaling.



Bena, Wang, Warner; Denef, Moore

Scaling solutions: solutions where the constituents can approach each other arbitrarily closely.



In space-time, a deep black-hole throat develops.

The number of states on the Higgs branch is determined by its cohomology.

On the Coulomb branch, one has to quantize the phase space of solutions. System quite similar to that of electrons in magnetic fields where the ground states correspond to the lowest Landau levels.

Theorem/conjecture: Bena, Berkooz, JdB, El-Showk, van den Bleeken

For non-scaling solutions, the number of states on the Higgs and Coulomb branches are the same.

For scaling solutions, the numbers of states are different, and the Higgs branch has more states than the Coulomb branch.

What do these smooth geometries actually represent? Do they really represent microstates of the black hole?

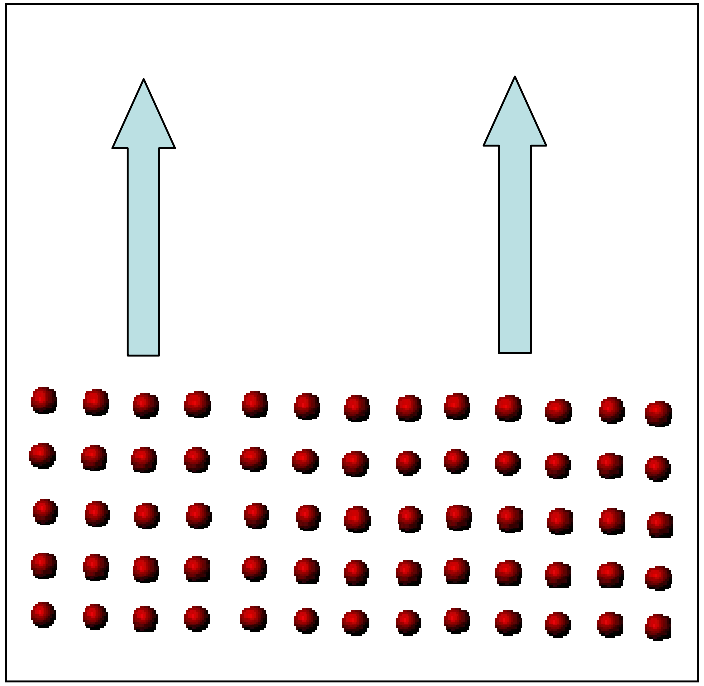
Need a cleaner setup: embed in AdS/CFT. The dual CFT has a Hilbert space $\mathcal{H} = \oplus_\Gamma \mathcal{H}_\Gamma$ and the states corresponding to the black hole span $\mathcal{H}_{\rm BH} \subset \mathcal{H}_\Gamma$. Do the smooth geometries contribute to $\mathcal{H}_{\rm BH}$ or to other states in $\mathcal{H}_\Gamma$?

Non-scaling solutions can be taken apart by changing moduli, while the black hole is still there. These do not contribute to the space of states of the black hole $\mathcal{H}_{\rm BH}$. They are best thought of as “hair”.

Scaling solutions on the other hand can not be disentangled from the black hole. In addition, they resemble a black hole (outside the horizon) arbitrarily accurately. They therefore do contribute to the space of states of the black hole.

This is rather peculiar: a macroscopic, classical thermodynamic description of a set of microstates coexists with a macroscopic, classical description of one of its microstates!

Analogy: gas of non-interacting atoms. Spread these evenly in the bottom half of a box and give them all equal upward velocity.



This will look like a classical wave going up and down. It is also one of the configurations that contributes to the partition sum of the gas and coexists with it.

This is a rather atypical setup because the atoms do not interact. In general interactions will destroy this picture. In the black hole case the system is BPS which is somewhat similar to non-interacting.

All this suggests that the smooth scaling solutions are highly atypical microstates which have a macroscopic description and coexist with the black hole.

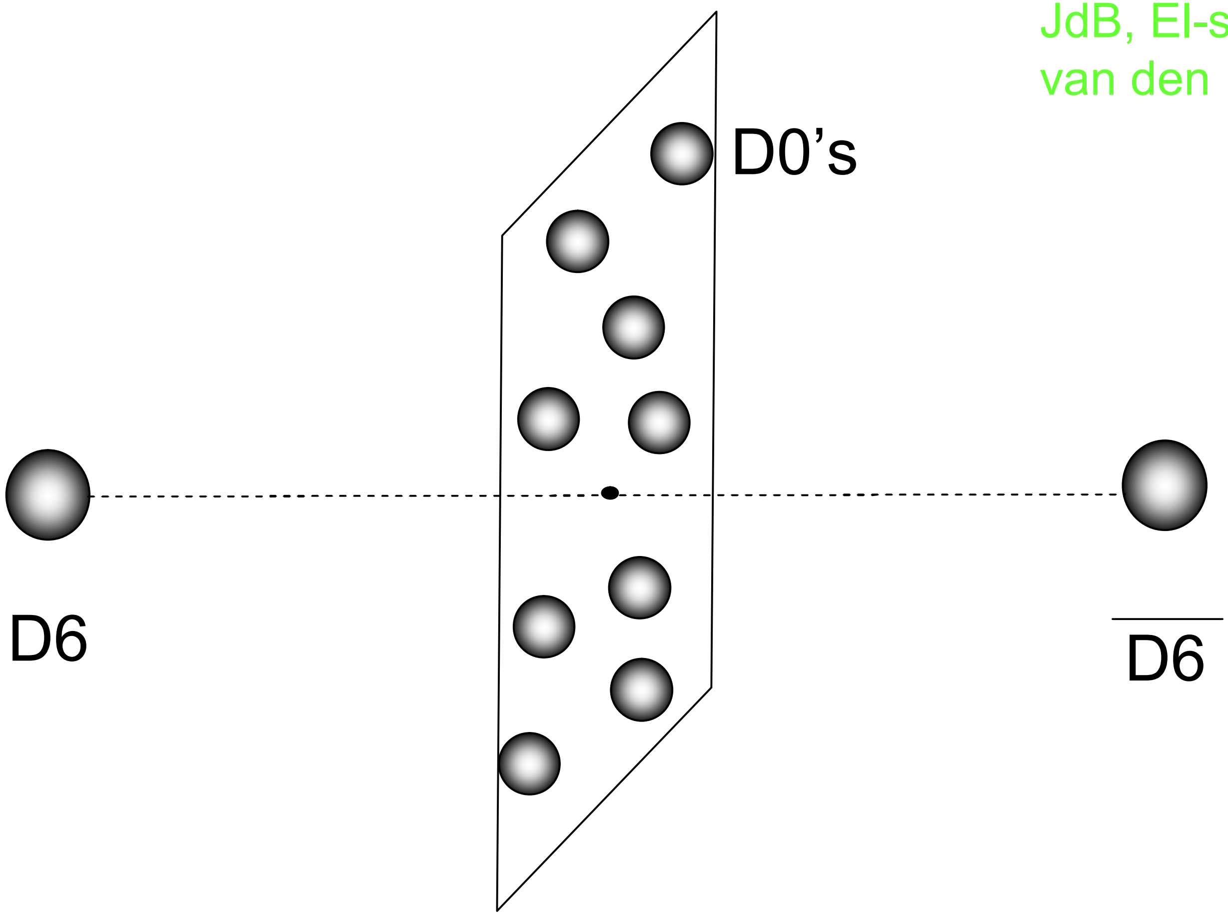
But are they indeed atypical??

Are there sufficiently many smooth supergravity solutions to account for the black hole entropy?

A priori not, we lost some states along the way, and this is not a prediction of AdS/CFT.

Largest set we have been able to find:

JdB, El-showk, Messamah, van den Bleeken



In terms of standard 2d CFT quantum numbers we find the following number of states:

$$\left(\tfrac{3}{16}\zeta(3)L_0^2\right)^{1/3} \qquad L_0 \leq c/6$$

$$\left(\tfrac{3}{2}c\zeta(3)(L_0 - \tfrac{c}{12})\right)^{1/3} \qquad L_0 \geq c/6$$

This is less than the black hole entropy, which scales as

$$S \sim 2\pi \left(\tfrac{c}{6}L_0\right)^{1/2}$$

Perhaps we are simply missing many solutions?

Try to find upper bound: count the number of states in a gas of BPS supergravitons. Idea is that all smooth BPS solutions are obtained by taking a superposition of free BPS supergravitons and letting the system backreact. Because of the BPS bound, the energy of the system cannot become be lowered.

After all, classical solutions can be thought of as coherent superpositions of gravitons…

One can explicitly find such an upperbound. It is important to take the “stringy exclusion” principle in account: the fact that the spins of primaries in a level k SU(2) WZW cannot exceed k/2.

Now we find precisely the same result as before:

$$S \sim \left(\tfrac{3}{16}\zeta(3)L_0^2\right)^{1/3} \qquad L_0 \leq c/6$$

$$S \sim \left(\tfrac{3}{2}c\zeta(3)(L_0 - \tfrac{c}{12})\right)^{1/3} \quad L_0 \geq c/6$$

Strongly suggests supergravity is not sufficient to account for the entropy.

Stringy exclusion principle is visible in classical supergravity (and not so stringy).

In particular, this suggests that all attempts to quantize gravity on its own are futile and will never lead to a consistent unitary theory with black holes.

This is of course perfectly fine: string theory was invented to yield a consistent quantum theory of gravity, so it would have been somewhat disappointing if we could get away with gravity alone.

This statement is also supported by the N=4 case, where one can show that multicentered configurations can never contribute to the index.

Dabholkar, Guica, Murthy, Nampuri

Caveat:

Aminneborg, Begtsson, Brill, Holst, Peldan

In d=3 there are many solutions which are identical to a black hole outside the horizon but have structure behind it. There may even be enough solutions of this type to account for the black hole entropy (Maloney). Not clear whether these solutions should be viewed as pure states though.

Such solutions cannot obviously be made by throwing gravitons in global AdS.

There may be other solutions with a different topology which cannot be viewed as “small” deformations of AdS.

The above counting was in d=5. Can repeat arguments in d=6.

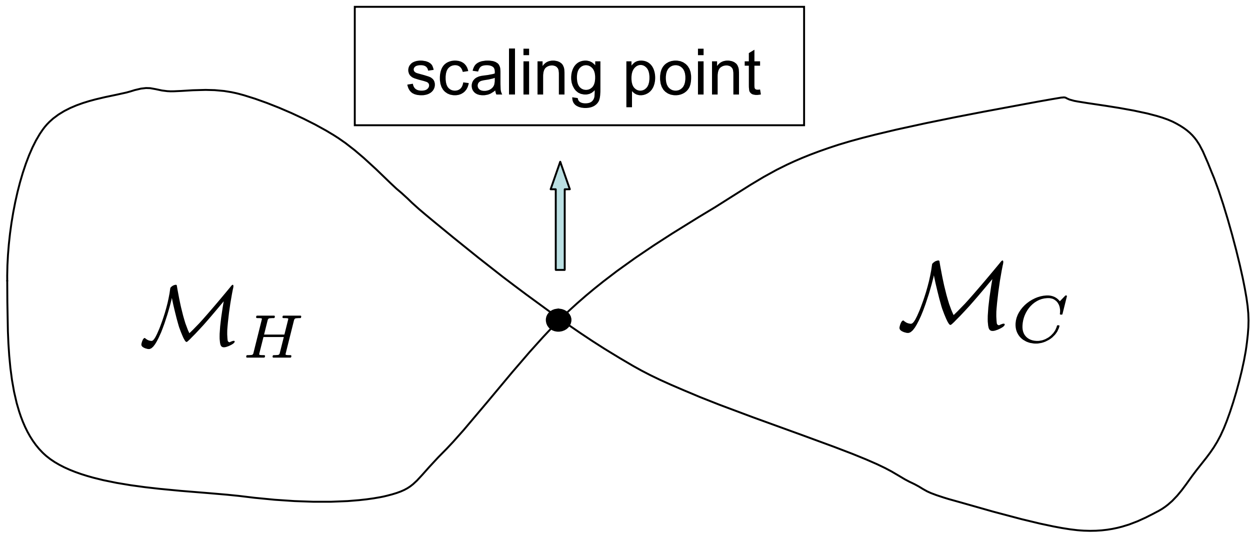
$$S \sim \left(L_0^3\right)^{1/4} \qquad L_0 \lesssim c$$

$$S \sim \left(c^2 L_0\right)^{1/4} \qquad L_0 \gtrsim c$$

In other dimensions find similar results and never recover growth a la Cardy.

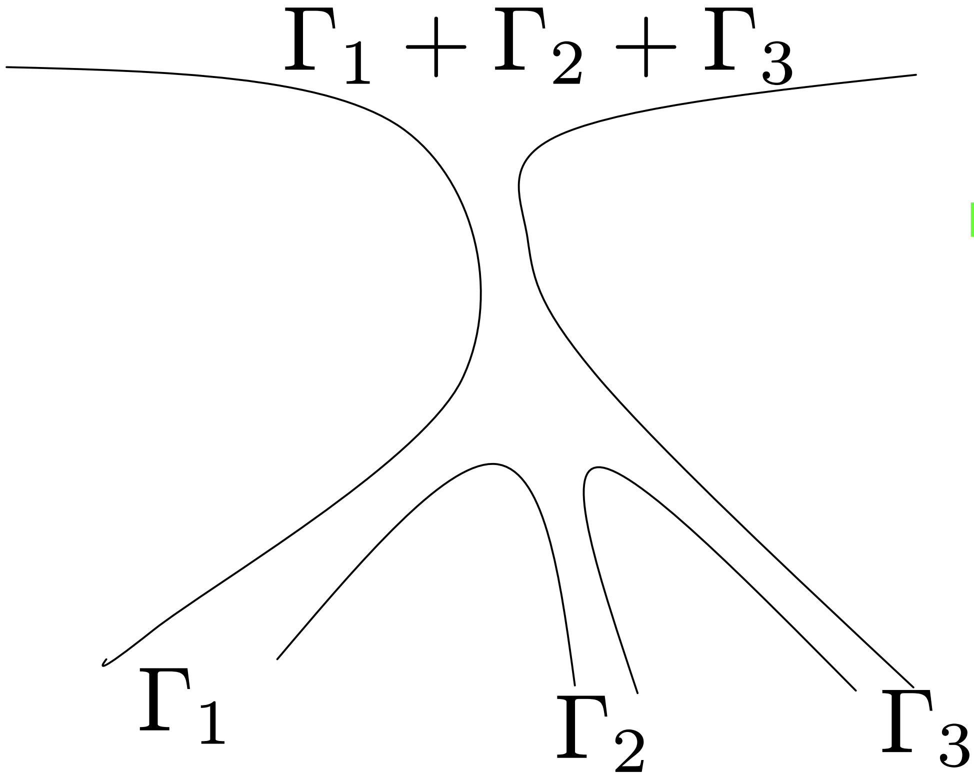
Can we say something about the missing states? They got lost when we passed from the Higgs to the Coulomb branch whenever there are scaling solutions.

Recall cartoon of moduli space:



The missing states on the Higgs branch all sit in the middle cohomology and have no spacetime angular momentum. If anything, they must sit at the scaling point.

Interestingly, the scaling point has zero symplectic volume but in a sense still represents a large class of solutions. These correspond to tree-like AdS2 solutions.



Maldacena, Strominger

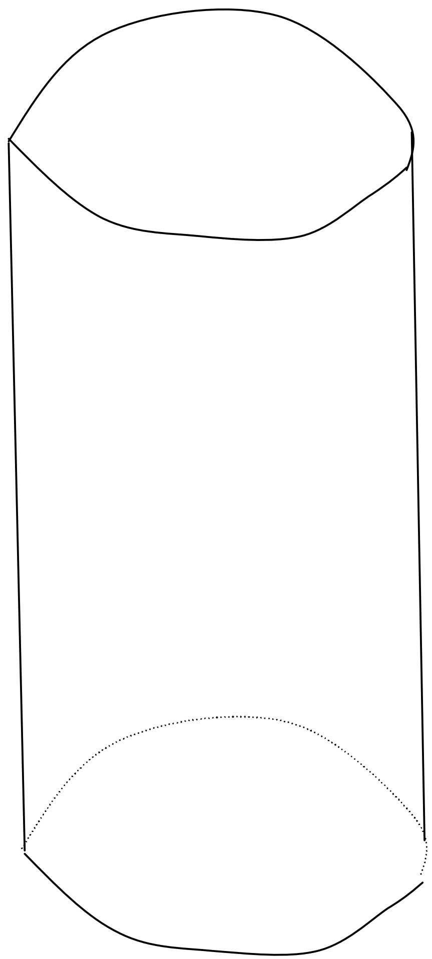
One could try to quantize these solutions, but one would need to give them small velocities (adiabatic approximation). Superficially leads to a continuous spectrum of non-BPS states, hard to see how many BPS states can arise from the bottom of this continuum.

Thus, staying purely in supergravity, it seems very difficult to get a complete description of the microscopic degrees of freedom of large black holes.

Is it possible to go beyond supergravity without invoking all of closed string field theory?

Beyond supergravity?

Recall that the F1-D0 system can puff up into a supertube, a D2-brane whose cross section can be an arbitrary curve.



If we T-dualize the F1-D0 system, many other systems can be shown to puff up into supertubes.

In IIA on T^6, with D4 branes wrapping the 6789 and D4 branes wrapping the 4589 directions, the resulting supertube is made of an extended object which one gets by T-dualizing an NS5-brane in two transversal directions: a 5^2_2-brane.

These exotic objects also appear when U-dualizing conventional branes in three dimensions. Their tensions can involve strange powers of g_s such as g_s^{-3}, and strange powers of the radii. Elitzur, Giveon, Kutasov, Rabinovici; Obers, Pioline

Above reasoning suggests such exotic branes may play an important role in understanding microstates.

JdB, Shigemori

What do such exotic branes correspond to?

In three-dimensions, they are point-particles. Their charges are given by monodromies of the scalars of 3d supergravity. Those scalars take values in $SO(16)\backslash E_8(\mathbb{R})/E_8(\mathbb{Z})$

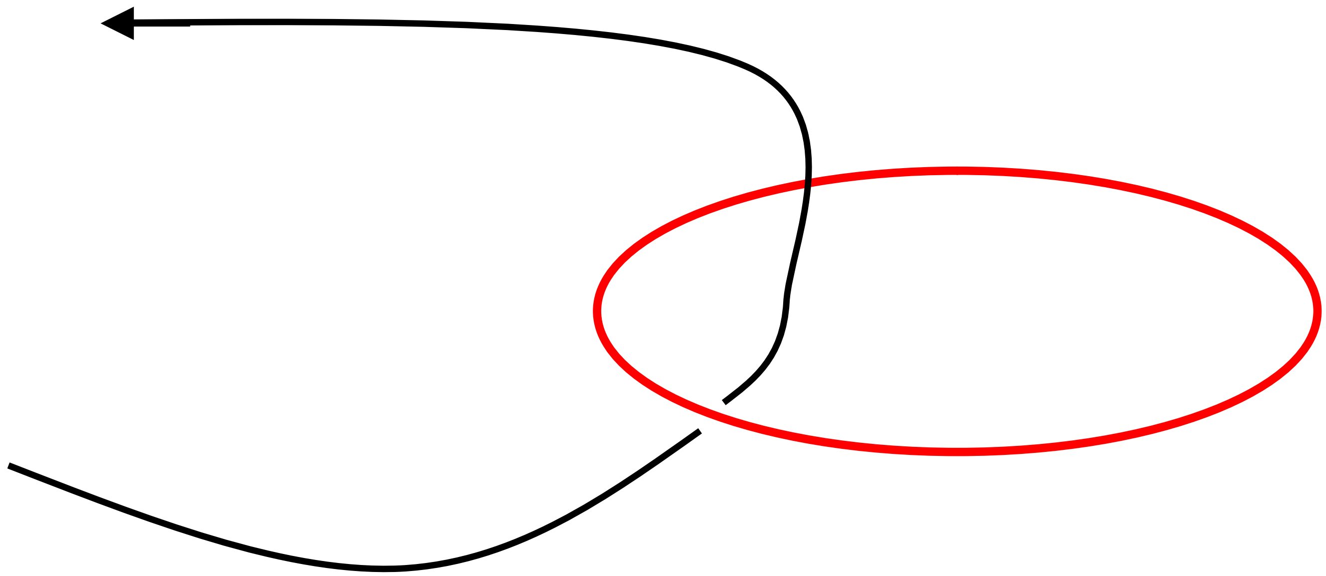
Charges are not additive: they take values in $E_8(\mathbb{Z})$.

From a higher dimensional point of view, one undergoes a U-duality as one encircles these exotic branes, just as in non-geometric backgrounds.

Not much is known about these solutions. Supersymmetric solutions of 3d maximally supersymmetric supergravity have not been classified.

It is difficult to find explicit solutions of wiggly, supersymmetric non-geometric objects which are not simply U-duals of known things.

Can explicitly construct the metric for the 5_2^2 supertube.



As one moves through the loop, one picks up a non-geometric twist. Not visible at infinity.

Do non-geometric solutions possible carry enough entropy?

If one only includes T-folds, one can describe solutions using a truncation of string field theory which contains both the massless modes of strings as well as their winding modes.

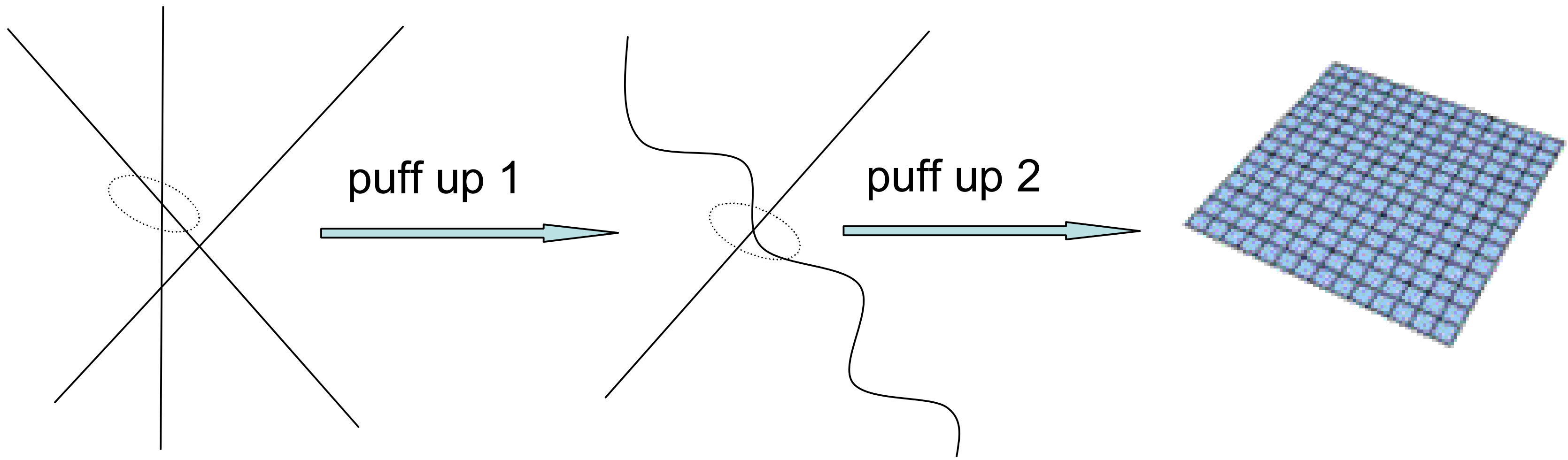
Hohm, Hull, Zwiebach

Such a theory has roughly twice as many fields as supergravity and it is hard to see how that would evade the counting arguments made before.

To really get much more entropy would need a theory which contains many more degrees of freedom. Perhaps including the whole tower of U-dual images of fundamental strings will do the job……

There is hope: large supersymmetric black holes involve intersections of at least three branes.

Can therefore puff up twice:



Each supertube-like puff up introduces an arbitrary curve. Final object may contain arbitrary functions of two coordinates. Consistent with a supersymmetry analysis: many possible solutions.

Bena, JdB, Shigemori, Warner

OUTLOOK:

Described progress towards understanding which microscopic degrees of freedom of black holes may be visible in gravity and which ones may not. More work needs to be done. Black holes remain elusive.

It appears that supersymmetric black holes cannot be described in terms of gravitational degrees of freedom only, but perhaps non-geometric solutions of gravity may allow one to improve the situation.