

# Constructing S matrix from BCFW

Hongbao Zhang

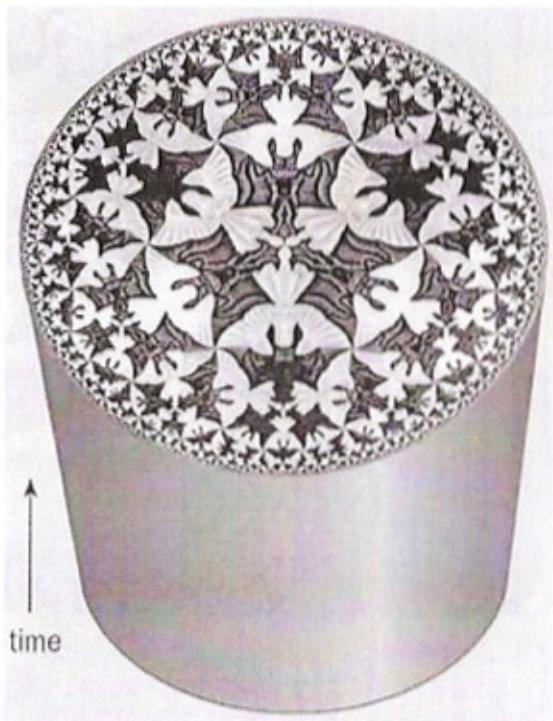
Crete Center for Theoretical Physics  
Department of Physics  
University of Crete

HZ,arXiv:1005.4462

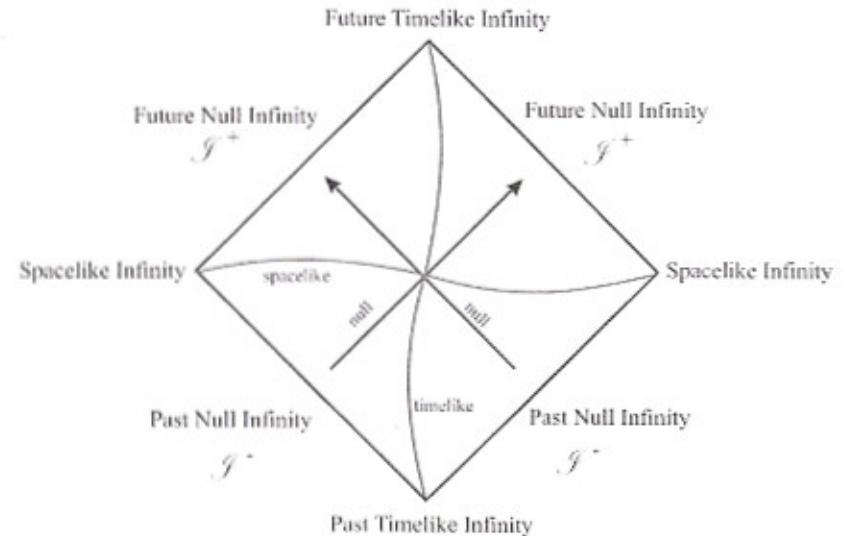
Song He and HZ,arXiv:0811.3210[JHEP1007:015(2010)]

# Putting it into a broader context

- AdS/CFT correspondence



- Grassmannian/Lagrangian duality(Arkani-Hamed, Cachazo and their company)



# Penrose's twistor prescription

- Spinor helicity
- Twistor
- Dual Spinor helicity
- Dual twistor

$$\begin{pmatrix} z^0 \\ z^1 \end{pmatrix} = \frac{i}{\sqrt{2}} \begin{pmatrix} r^0 + ir^3 & r^1 + ir^2 \\ r^1 - ir^2 & r^0 - ir^3 \end{pmatrix} \begin{pmatrix} z^0 \\ z^1 \end{pmatrix}$$

$$\omega^A = i \gamma^{AA'} \pi_{A'}$$

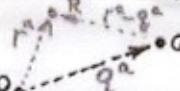
$$Z^\alpha = (\omega^A, \pi_{A'})$$

incidence:  $\omega^A = i \gamma^{AA'} \pi_{A'}$

shift of origin

$$\overset{\circ}{\omega}{}^A = \overset{\circ}{\omega}{}^A - i q^A \overset{\circ}{\pi}_{A'}$$

$$\overset{\circ}{\pi}_{A'} = \overset{\circ}{\pi}_{A'}$$



complex conjugate:  $\bar{Z}_\alpha = (\bar{\pi}_A, \bar{\omega}^{A'})$   
Null twistor:  $Z^\alpha \bar{Z}_\alpha = 0$  defines PN

Momentum-angular mom. for massless ptcle.  
 $P_\alpha P^\alpha = 0, P_0 > 0, S_\alpha = \frac{1}{2} \epsilon_{abcd} P^b M^{cd} = s P_\alpha$

$$P_{AA'} = \bar{\pi}_A \pi_{A'}, M^{AB'B'} = i(\omega^{AB}) \epsilon^{AB'} - i\epsilon^{AB} \bar{\omega}^{(A'} \pi^{B')}$$

Quantization:  $[Z^\alpha, Z^\beta] = 0, [Z^\alpha, \bar{Z}_\beta] = i \delta_\beta^\alpha$

$$S = \frac{1}{4} (Z^\alpha \bar{Z}_\alpha + \bar{Z}_\alpha Z^\alpha) = \frac{i}{2} (-2 - Z^\alpha \frac{\partial}{\partial Z_\alpha})$$

$$= \frac{i}{2} \left( \bar{\pi}^A \frac{\partial}{\partial \bar{\pi}^A} - \pi^{A'} \frac{\partial}{\partial \pi^{A'}} \right)$$

# BCFW for trees(I)

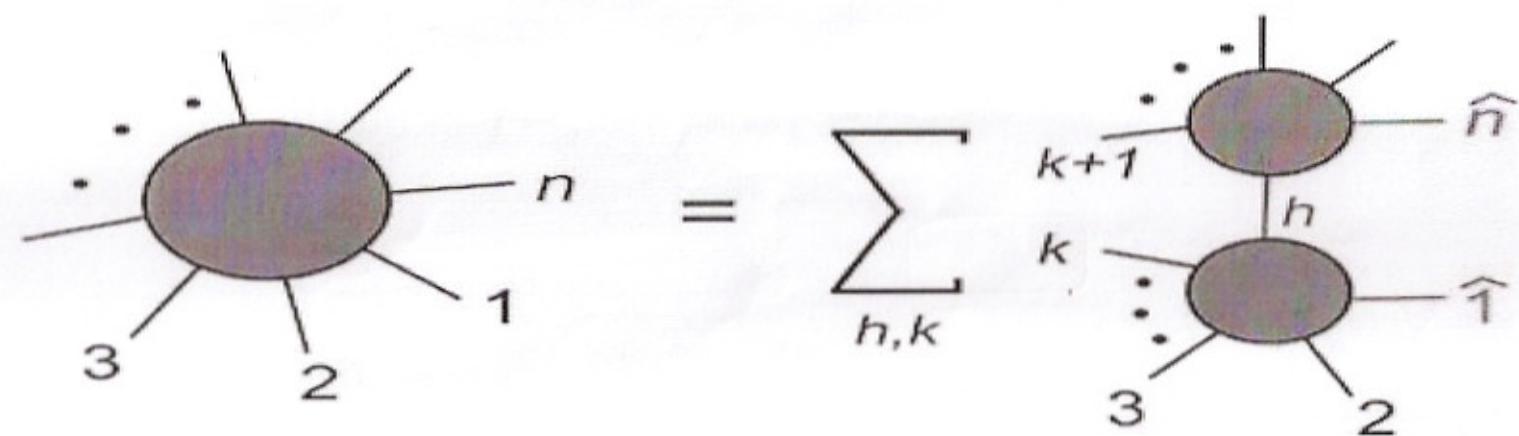
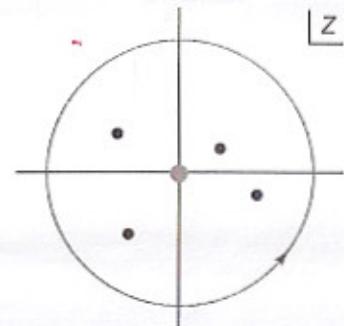
$$A(\lambda_i, \tilde{\lambda}_i \mid i=1, \dots, n)$$

$$\hat{\lambda}_1 = \lambda_1 + z\lambda_n \quad \hat{\tilde{\lambda}}_1 = \tilde{\lambda}_1 \quad \hat{\lambda}_n = \lambda_n \quad \hat{\tilde{\lambda}}_n = \tilde{\lambda}_n - z\tilde{\lambda}_1$$

$$\Rightarrow A(0) \rightarrow A(z)$$

Cauchy If  $A(\infty) = 0$  then

$$0 = \frac{1}{2\pi i} \oint dz \frac{A(z)}{z} = A(0) + \sum_k \text{Res}\left[\frac{A(z)}{z}\right] \Big|_{z=z_k}$$



## BCFW for trees(II)

- Britto,Cachazo,Feng, and Witten(2005)
- Benincasa,Boucher-Veronneau, and Cachazo(2007)
- Arkani-Hamed and Kaplan(2008)
- Cheung(2008)
- Benincasa and Cachazo(2007)
- Schuster and Toro(2008)
- He and HZ(2008)
- Feng,Huang, and Jia(2010)
- HZ(2010)

# Constructing S matrix from BCFW(I)

- 3-point scattering amplitude

determined by

$$[K] = 1 - (h_1 + h_2 + h_3) \quad \text{Poincare Symmetry}$$

$K_{abc}$ : antisymmetric

$$\begin{aligned} M_3 &= K_H \langle 1,2 \rangle^{d_3} \langle 2,3 \rangle^{d_1} \\ &\times \langle 3,1 \rangle^{d_2} + K_P [1,2]^{-d_3} [2,3]^{-d_1} \\ &\times [3,1]^{-d_2} \end{aligned}$$

where  $d_1 = h_1 - h_2 - h_3$  |  $\langle i,j \rangle = \lambda_A^i \lambda_B^j \epsilon^{AB}$

$d_2 = h_2 - h_3 - h_1$  |  $[\langle i,j \rangle] = \tilde{\lambda}_A^i \tilde{\lambda}_B^j \epsilon^{AB'}$

- 4-particle test

$$\begin{aligned} M_4^{(1,2)} &= M_4^{(1,4)} \\ \text{Diagram: } & \text{ (1)} \text{---} \text{ (2)} + \text{ (1)} \text{---} \text{ (4)} \\ &= \text{ (4)} \text{---} \text{ (2)} + \text{ (4)} \text{---} \text{ (3)} \end{aligned}$$

$$\boxed{K^{[2]} = 0}$$

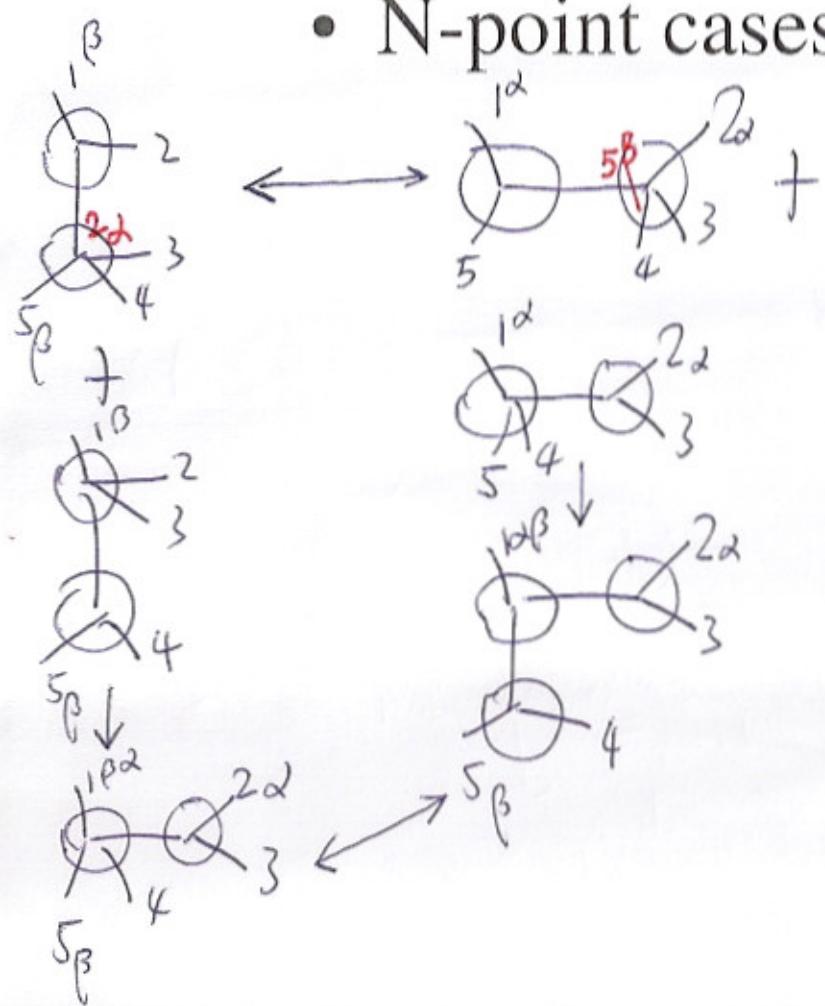
$K_{abc}$ : Jacobian

Color Stripped Amplitude

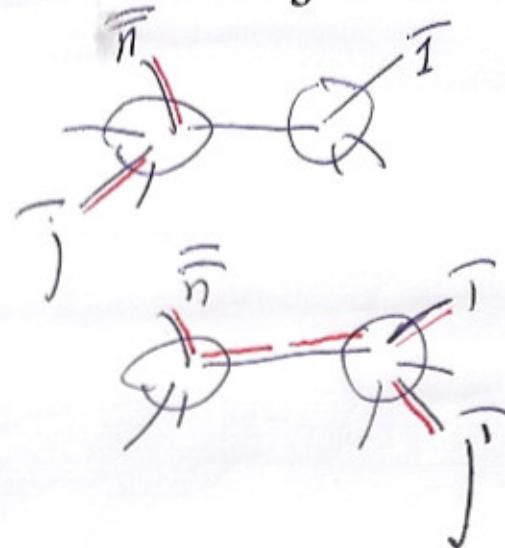
$$\begin{aligned} M(p_i, h_i; a_i) &= \sum_{\sigma \in S/C} \text{Tr}(T^{a_{\sigma(i)} \dots}) \\ &\times M(p_{\sigma(i)}, h_{\sigma(i)}) \end{aligned}$$

# Constructing S matrix from BCFW(II)

- N-point cases



- Non-adjacent cases



# Constructing S matrix from BCFW(III)

- KK relation[Kleiss and Kuijf(1989)]

$$A_n(1, \{2\}, i \{\beta\}) = (-1)^n \beta$$

$$\sum_{\sigma} A_n(1, \sigma(\{2\}, \{\beta^T\}), i)$$

- BCJ relation[Bern, Carrasco and Johansson(2008)]

$$\begin{aligned} & A_n(1, 2, \{2\}, 3, \{\beta\}) \\ &= \sum A_n(1, 2, 3, 6(\{2\}, \{\beta\})) \\ &\quad \times \prod_{k=4}^m \frac{f(3, \sigma(\{2\}, \{\beta\}), k))}{S_{2, 4, \dots, k}} \end{aligned}$$

# Future directions

- Parity identity from BCFW
- Bonus or KK and BCJ relations from Grassmannian
- Gravity

