Massive Type IIA and Strong Coupling

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Outline

- Motivations
- Massive Type IIA cannot be Strongly Coupled
- Examples based on Membranes Theories
- Generalizations
- Conclusions and Open Problems

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based on

O. Aharony, D. Jafferis, A. Tomasiello, A. Z. arXiv:1007.2451

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- We can use AdS/CFT when available.
  Understanding different large N limits of gauge theories; type IIA $\rightarrow$ M theory/ strings $\rightarrow$ membranes
- Interesting solutions for the $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence.

Strongly coupled type IIA

We all known that the strong coupling of IIA is M theory

$$\boxed{ds^2_{IIA} \qquad \to \qquad e^{-2\phi/3} ds^2_{IIA} + e^{4\phi/3}(dx_{11} + A)^2}$$

$$D0 \text{ branes} \qquad \to \qquad \text{KK modes}$$

$$F_0 \qquad \to \qquad ?$$

- Duality does not work for massive IIA ($F_0 \neq 0$): no free "massive parameters in 11dim, D0 have tadpoles.
- Is this an imperfection in our understanding?

A general bound on the dilaton

The 00 component of Einstein equations in massive type IIA reads

$$e^{-2\phi}\left[\left(R_{00}+2\nabla_0\nabla_0\phi-\frac{1}{4}H_0{}^{PQ}H_{0PQ}\right)\right]=\frac{1}{4}\Big(\sum_{k=2,4}F^2_{0,k-1}+\sum_{k=0,2,4}F^2_{\perp,k}\Big)$$

$$F_k = e^0\wedge F_{0,k-1}+F_{\perp,k}$$

$$F_0=\frac{n_0}{2\pi l_s} \qquad \longrightarrow \qquad e^{-2\phi}R^2\sim 1/l_s^2$$

flux quantization condition R local curvature

- no weakly curved strongly coupled solutions in massive type IIA
- but strongly coupled IIA = M theory ($F_0=0\,,\;\; F_k^2\sim n_k^2 R^{2k}$)

Observations

whenever the coupling is large in massive IIA the curvature in some direction must grow, opening room for new physics, T duality to other weakly coupled descriptions [Hull...]

- no sign of strong coupling in all massive type IIA *AdS* vacua
- UV completion of Sakai-Sugimoto model still unknown...
- we can use AdS/CFT which provides a non-perturbative definitions of some IIA backgrounds

AdS_4 solutions in IIA: maximal supersymmetry

Familar duality between $\mathcal{N} = 6$ theories [Aharony-Bergman-Jafferis-Maldacena]

$\boxed{AdS_4 \times \mathbb{CP}^3 \text{ in IIA}}$ $\boxed{U(N)_k \times U(N)_{-k} \text{ CS}}$

$$k \equiv \int_{\mathbb{CP}^1} F_2 \qquad (L/l_s)^4 \sim N/k$$
$$N \equiv \int_{\mathbb{CP}^3} F_6 \qquad g_s^4 \sim N/k^5$$

- For $N \gg k^5$ large dilaton and small curvature: M theory on $AdS_4 \times S^7/\mathbb{Z}_k$.
- New perspective for the large N limit:

Type IIA :	t'Hooft limit	$N \to \infty, \ \ x = N/k$ fixed
		$g_s = x^{5/4}/N, \quad L^2 = \sqrt{x}$
M theory :	large N	$N \to \infty, \ \ k$ fixed

AdS_4 solutions in massive type IIA

Other dualities in massive type IIA with $\mathcal{N} = 1, 2, 3$ supersymmetry

$$\boxed{U(N_1)_{k_1} \times U(N_2)_{-k_2}} \qquad \boxed{ds^2_{\mathcal{N}=1,2,3} = ds^2_{\text{warped AdS}_4} + ds^2_{\mathbb{CP}^3,\mathcal{N}=1,2,3}}$$

$\mathcal{N} = 1 \quad \rightarrow \quad SO(5)$ symmetry
$\mathcal{N} = 2 \quad \rightarrow \quad SO(4)$ symmetry
$\mathcal{N} = 3 \quad \rightarrow \quad SO(3)$ symmetry (sugra known only at perturbative level)

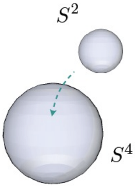
Parameters [Aharony-Bergmann-Jafferis; Gaiotto-Tomasiello]:

$n_0 = F_0$	$k_1 - k_2$
$n_2 = \int_{\mathbb{CP}^1} F_2$	k_2
$n_4 = \int_{\mathbb{CP}^2} F_4$	$N_2 - N_1$
$n_6 = \int_{\mathbb{CP}^3} F_6$	N_2

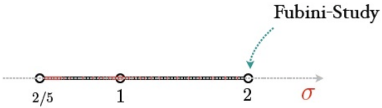
What happens to CS theories for large N with fixed levels?

$\mathcal{N}=1$ solutions in massive type IIA

Solution is a product of AdS_4 with an S^2 bundle over S^4 ($SO(5)$ symmetry):
[A. Tomasiello]



$$ds^2_{\mathbb{CP}^3,\mathcal{N}=1} = L^2\left(\frac{1}{8}(dx^i + \epsilon^{ijk}A^j x^k)^2 + \frac{1}{2\sigma} ds^2_{S^4}\right)$$



Parameters are: $L, \;\; g_s, \;\; \sigma \in [\frac{2}{5}, 2], \;\; b \;\;$ (B-field zero mode on $\mathbb{CP}^1$)

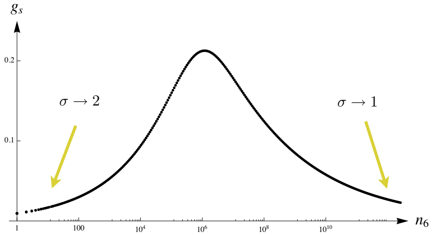
Related to quantized fluxes: $n_0, \;\; n_2, \;\; n_4, \;\; n_6$

$$\boxed{\int e^{-B} F_k \equiv n_k (2\pi l_s)^{k-1}}$$

A phase transition

for simplicity:

$$n_4 = 0\,, \quad n_2 \equiv k\,, \quad n_6 \equiv N$$



$$\boxed{N \ll \frac{k^3}{n_0^2}}$$

$\sigma \longrightarrow 2$ (Fubini-Study on $\mathbb{CP}^3$)

$$l \sim \frac{N^{1/4}}{k^{1/4}}\,, \quad g_s \sim \frac{N^{1/4}}{k^{5/4}}$$ ($\mathcal{N} = 6!$)

$$\boxed{N \gg \frac{k^3}{n_0^2}}$$

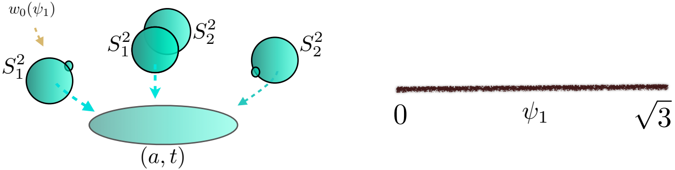
$\sigma \longrightarrow 1$ (Nearly-Kahler metric on $\mathbb{CP}^3$)

$$l \sim \frac{N^{1/6}}{n_0^{1/6}}\,, \quad g_s \sim \frac{1}{N^{1/6} n_0^{5/6}}$$

$\mathcal{N} = 2$ solutions in massive type IIA

Solution is a warped product of AdS_4 with an $SO(4)$ invariant metric:

$$ds^2_{\mathbb{CP}^3,\mathcal{N}=2} = \frac{w_1(t)}{4}\mathrm{d}s^2_{S^2_1} + \frac{w_2(t)}{4}\mathrm{d}s^2_{S^2_2} + \frac{1}{8}\epsilon^2(t)\mathrm{d}t^2 + \frac{1}{64}\Gamma^2(t)(\mathrm{d}a + A_2 - A_1)^2$$



Parameters are: $L,\ g_s,\ \phi_1 \in [0,\sqrt{3}],\ b$ (B-field zero mode on $\mathbb{CP}^1$)

Related to quantized fluxes: $n_0,\ n_2,\ n_4,\ n_6$

$\mathcal{N} = 2$ solutions in massive type IIA

This time solution is only know numerically. Reduced to 3 1-order differential equations:

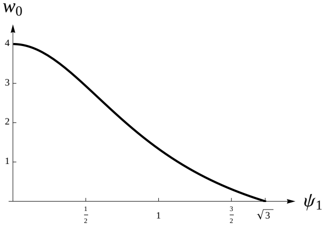
$$\psi' = \frac{\sin(4\psi)}{\sin(4t)} \frac{C_{t,\psi}(w_1 + w_2) + 2\cos^2(2t) w_1 w_2}{C_{t,\psi}(w_1 + w_2)\cos^2(2\psi) + 2w_1 w_2}$$

$$w_1' = \frac{4w_1}{\sin(4t)} \frac{C_{t,\psi}(w_1 w_2 - 2\, w_2 - 2\sin^2(2\psi) w_1)}{C_{t,\psi}(w_1 + w_2)\cos^2(2\psi) + 2w_1 w_2}$$

$$w_2' = \frac{4w_2}{\sin(4t)} \frac{C_{t,\psi}(w_1 w_2 - 2\, w_1 - 2\sin^2(2\psi) w_2)}{C_{t,\psi}(w_1 + w_2)\cos^2(2\psi) + 2w_1 w_2}$$

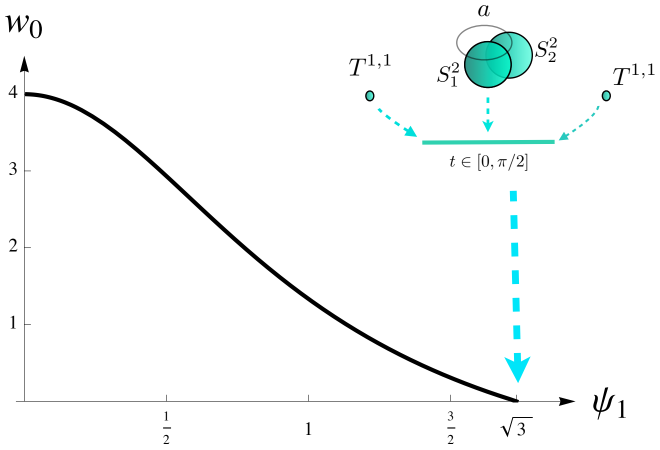
$$C_{t,\psi} \equiv \cos^2(2t)\cos^2(2\psi) - 1$$

- All other quantities known analitically
- Regular metric for all $\psi_1 \in [0, \sqrt{3}]$
- Conifold singularity at $\psi_1 = \sqrt{3}$



$\mathcal{N} = 2$ solutions in massive type IIA

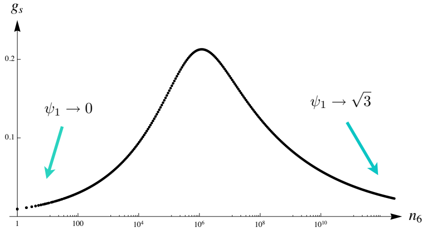
A better picture is:



A phase transition again

for simplicity:

$$n_4 = 0\,,\quad n_2 \equiv k\;,\quad n_6 \equiv N$$



$$\boxed{N \ll \frac{k^3}{n_0^2}}$$

$\psi_1 \longrightarrow 0$ $(\mathcal{N} = 6!)$

$$l \sim \frac{N^{1/4}}{k^{1/4}}\,,\quad g_s \sim \frac{N^{1/4}}{k^{5/4}}$$

$$\boxed{N \gg \frac{k^3}{n_0^2}}$$

$\psi_1 \longrightarrow \sqrt{3}$ (conifold point)

$$l \sim \frac{N^{1/6}}{n_0^{1/6}}\,,\quad g_s \sim \frac{1}{N^{1/6} n_0^{5/6}}$$

Light states and monopoles

Our original motivation for studying $\mathcal{N} = 2$ examples is the existence of light monopole states!

- In $\mathcal{N} = 6$ CS $U(N_1) \times U(N_2)$ with $k_1 = k_2 = k$:

BPS states A^k, B^k (D0/KK states)

$$\boxed{\Delta = k/2}$$

- In $\mathcal{N} = 2$ CS $U(N_1) \times U(N_2)$ with $k_1 \neq k_2$:

BPS states $A^{k_1 k_2}, B^{k_1 k_2}$ (D0/D2)

$$\boxed{\Delta = \frac{k_1 k_2}{2} + (k_2 - k_1)^2 - (k_2 - k_1)(N_1 - N_2)}$$

$$\left(\delta(\Delta) = -\frac{1}{2} \sum_{\text{fermions}} |q| R\right)$$

Light states survives for large N, k fixed

D0/D2 bound states

A supersymmetric D2 can wrap the non trivial two-cycle $S^2 = S^2_1 + S^2_2$:

$$m_{D2}L = n_0 L \frac{1}{(2\pi)^2 g_s l_s^3} \int \sqrt{\det(g + \mathcal{F} - B)} = \left(\frac{n_2^2}{2} - n_0 n_4 \right) F(\psi_1)$$

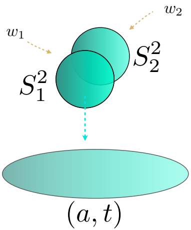
- tadpole cancelled by worldvolume flux
- bound state of n_0 D2 and n_2 D0
- numerically: function $F(\psi_1)$ is remarkably constant
- in the sugra limit it perfectly matches the monopole dimension
- $m_{D2}L \sim L^3/g_s \sim N^{2/3}$ for large N, but $S^2 \to 0$

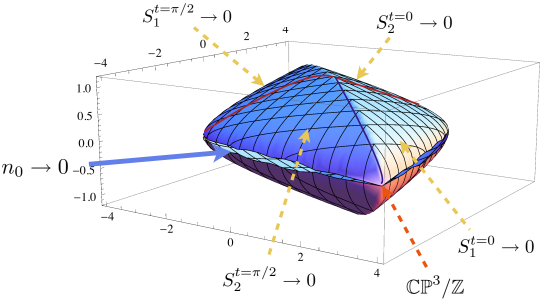
$$\boxed{m_{D2}L = \left(\frac{n_2^2}{2} - n_0 n_4 \right)}$$

Other $\mathcal{N}=2$ solutions in massive type IIA

More general warped AdS_4 solutions with an $SO(4)$ invariant metric:

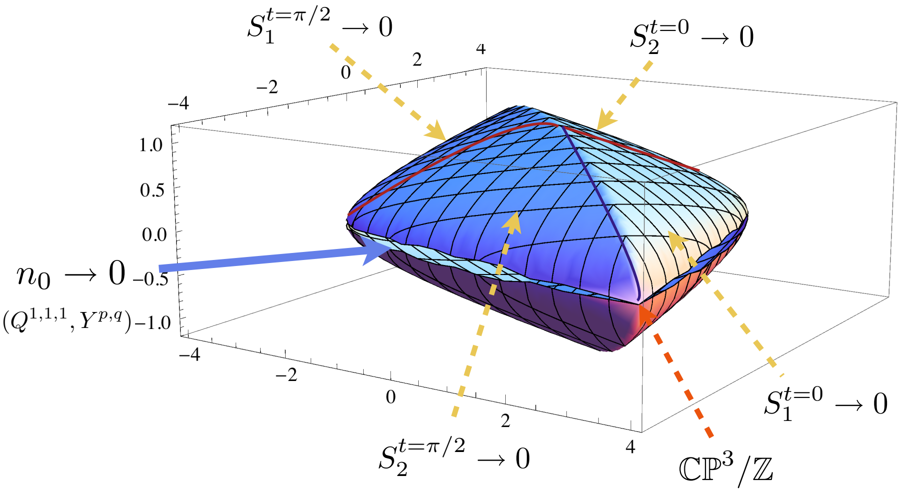
$$ds^2_{\mathcal{N}=2} = \frac{w_1(t)}{4}\mathrm{d}s^2_{S^2_1} + \frac{w_2(t)}{4}\mathrm{d}s^2_{S^2_2} + \frac{1}{8}\,\epsilon^2(t)\mathrm{d}t^2 + \frac{1}{64}\Gamma^2(t)(\mathrm{d}a + A_2 - A_1)^2$$





Various degenerations on faces. Sasaki-Einsten manifold for $n_0 = 0$, including $Q^{1,1,1}$. More general quiver CS theories. [A. Tomasiello, A.Z.]

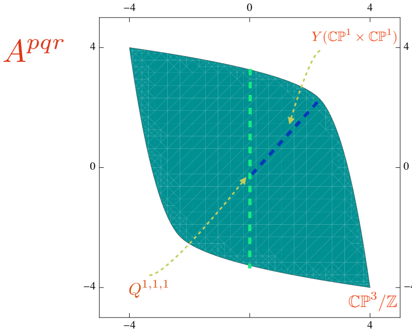
Moduli space for $\mathcal{N} = 2$ solutions

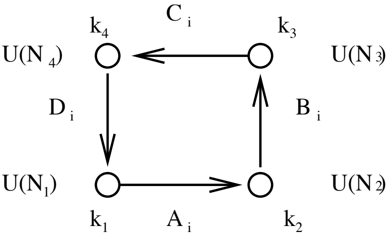


Conclusions

- Region of strong curvature in massive type IIA still need to be understood
- Conifold singularities seem to appear frequently in supergravity solutions; new light states, conifold transitions?
- What string description for large N limit with fixed fluxes?
- Moduli space for generalized geometries are still to be understood

Lift to M theory and Sasaki-Einstein ($n_0 = 0$)





$$W = \lambda \epsilon_{ij} \epsilon_{pq} A_i B_p C_j D_q$$

- solutions with $\psi_1 = 0$ can be lifted to M theory and become toric Sasaki-Einstein manifolds
- metric already known but not much studied [waldram-gauntlett-martelli-sparks; pope-liu]
- (part of the) A^{prk} admit a dual CS quiver description

Quantization of fluxes

$$\boxed{\int e^{-B} F_k \equiv n_k (2\pi l_s)^{k-1}}$$

$$\begin{pmatrix} \frac{1}{lg_s} f_0(\sigma) \\ \frac{l}{g_s} f_2(\sigma) \\ \frac{l^3}{g_s} f_4(\sigma) \\ \frac{l^5}{g_s} f_6(\sigma) \end{pmatrix} = \begin{pmatrix} n_0^b \\ n_2^b \\ n_4^b \\ n_6^b \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ b & 1 & 0 & 0 \\ \frac{1}{2} b^2 & b & 1 & 0 \\ \frac{1}{6} b^3 & \frac{1}{2} b^2 & b & 1 \end{pmatrix} \begin{pmatrix} n_0 \\ n_2 \\ n_4 \\ n_6 \end{pmatrix} \qquad l = L/(2\pi l_s)$$

One can invert these equations eliminating b:

$$\begin{aligned} \frac{n_2^3 + 3n_0^2 n_6 - 3n_0 n_2 n_4}{n_0^3} &= \frac{f_2^3 + 3f_0^2 f_6 - 3f_0 f_2 f_4}{f_o^3} L^6 \\ n_0^3 (n_2^3 + 3n_0^2 n_6 - 3n_0 n_2 n_4) &= f_0^3 (f_2^3 + 3f_0^2 f_6 - 3f_0 f_2 f_4) \frac{1}{g_s^6} \\ \frac{(n_2^2 - 2n_0 n_4)^3}{(n_2^3 + 3n_0^2 n_6 - 3n_0 n_2 n_4)^2} &= \frac{(f_2^2 - 2f_0 f_4)^3}{f_2^3 + 3f_0^2 f_6 - 3f_0 f_2 f_6)^2} \equiv \rho(\sigma) \end{aligned}$$

Probes

For $n_0 = 0$, $D0$ are monopoles, $D4$ are baryons.

- **Monopoles:**

  $D0/D2$ bound state $\quad (n_{\rm D2} n_2 + n_{\rm D0} n_0) \int \mathcal{A} = 0 \quad$ (no tadpole)

  $\left(m_{\rm D2} L \sim \frac{L^3}{g_s} \, , \;\; m_{\rm D0} L \sim \frac{L}{g_s} \right)$

  $$\text{Phase I}: \quad k^2 \sqrt{1 + \frac{N}{k^3}} \qquad (D0 \text{ dominates})$$

  $$\text{Phase II}: \quad N^{2/3} \sqrt{1 + \left(\frac{N}{k^3}\right)^{-2/3}} \qquad (D2 \text{ dominates})$$

- **Baryons:**

  $D4$ branes $\quad$ Phase I / II: $\quad N$

Digression on monopoles

CS theories on $\mathbb{R} \times S^2$ may contain monopole operators

$$\boxed{\int_{S^2} \mathrm{Tr}(F_i) = 2\pi n_i}$$

Real scalar field in $\mathcal{N} = 2$ theories (A_μ, σ):

$$F_i \equiv \sigma_i = \mathrm{diag}(w_i^1, \ldots, w_i^{N_i})|_{S^2} \qquad \longrightarrow \qquad n_i = \sum_a w_i^a$$

- This configuration gives rise to monopole operators

$$T_{\mathrm{mon}} : \qquad (k_i w_i^1, \ldots, k_i w_i^{N_i}) \qquad \text{irrep of } U(N_1) \times U(N_2)$$

  to be dressed with fundamental fields $\mathrm{T}\mathcal{O}(\mathrm{A}, \mathrm{B})$

- no fundamental fields charges under diagonal $U(1)$ $\quad \longrightarrow \quad \boxed{\sum_i k_i n_i = 0}$

Digression on monopoles

For the monopole to be BPS

$$\begin{aligned} A\,A^\dagger - B^\dagger B &= \tfrac{k_1}{2\pi}\sigma_1 \\ B\,B^\dagger - A^\dagger A &= -\tfrac{k_2}{2\pi}\sigma_2 \\ \sigma_1 A_i &= A_i\sigma_2 \\ \sigma_2 B_i &= B_i\sigma_1 \end{aligned} \qquad \rightarrow \qquad \textstyle\sum_i k_i n_i = 0$$

$$\begin{aligned} \mathcal{N}=6:\ \sigma_1 &= w_1 = (1,0,\ldots,0) & \rightarrow \quad & \text{BPS states } A^k, B^k \\ \sigma_2 &= w_2 = (1,0,\ldots,0) & & \\ \mathcal{N}=2:\ \sigma_1 &= w_1 = (k_1, .._{k_2}.., k_1, 0, \ldots) & \rightarrow \quad & \text{BPS states } A^{k_1 k_2}, B^{k_1 k_2} \\ \sigma_2 &= w_2 = (k_2, .._{k_1}.., k_2, 0, \ldots) & & \end{aligned}$$

$$U(N_1)\times U(N_2) \rightarrow U(k_1)\times U(k_2) \qquad \Delta_{1-\text{loop}} = -\frac{1}{2}\sum_{\text{fermions}} |q|R$$

contribution to Δ due to rectangular matrices

charge fermions: $A^{(i)}\psi - \psi A^{(j)}$

Digression on monopoles

Correction to gauge/global/R-symmetry charges due to fermionic zero modes:

- two adjoint fermions with R-charge $+1$
- four bi-fundamental fermions with charge $-1/2$
- square or rectangular matrices with coupling q given by $A^{(i)}\psi - \psi A^{(j)}$

$$\begin{aligned}\Delta &= -\frac{1}{2}\sum_{\text{fermions}} |q|R \\ &= -\frac{1}{2}\left[2\times 1\left(n_1(N_1-n_1)+n_2(N_2-n_2)\right) - 4\times\frac{1}{2}\left(n_1(N_2-n_2)+n_2(N_1-n_1)\right)\right] \\ &= (n_1-n_2)^2 - (N_1-N_2)(n_1-n_2)\end{aligned}$$

In more general $\mathcal{N}=2$ CS theories corrections also to gauge charge.