Nonrenormalization theorems in extended supergravities

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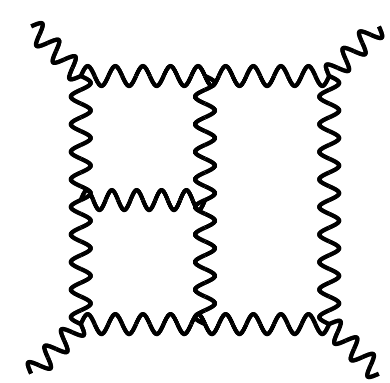
G. Bossard, P.S. Howe & K.S.S., arXiv 0901.4661, 0908.3883 & 1009.0743 [hep-th]

Ultraviolet Divergences in Gravity

- Simple power counting in gravity and supergravity theories leads to a naïve degree of divergence

$$\Delta = (D-2)L+2$$

in D spacetime dimensions. So, for D=4, L=3, one expects $\Delta = 8$. In dimensional regularization, only logarithmic divergences are seen ($\frac{1}{\epsilon}$ poles, $\epsilon = D-4$), so 8 powers of momentum would have to come out onto the external lines of such a diagram.



- Local supersymmetry implies that the pure curvature part of such a D=4, 3-loop divergence candidate must be built from the square of the Bel-Robinson tensor

Deser, Kay & K.S.S

$$\int \sqrt{-g}\,T_{\mu\nu\rho\sigma}T^{\mu\nu\rho\sigma}\,, \qquad T_{\mu\nu\rho\sigma} = R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}R_{\rho\alpha\sigma\beta} + {}^{*}R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}\,{}^{*}R_{\rho\alpha\sigma\beta}$$

- This is directly related to the α'^3 corrections in the superstring effective action, except that in the string context such contributions occur with finite coefficients. In string theory, the corresponding question is how poles might develop in $(\alpha')^{-1}$ as one takes the zero-slope limit $\alpha' \to 0$ and how this bears on the ultraviolet properties of the corresponding field theory.

Berkovits; Green, Russo & Vanhove;
cf talk by Michael Green

- The consequences of supersymmetry for the ultraviolet structure are not restricted to the requirement that counterterms be supersymmetric invariants.
- There exist more powerful "non-renormalization theorems," the most famous of which excludes infinite renormalization within D=4, N=1 supersymmetry of chiral invariants, given in N=1 superspace by integrals over half the superspace:

$$\int d^2\theta\, W(\phi(x,\theta,\bar{\theta}))\ , \quad \bar{D}\phi = 0$$

- However, maximally extended SYM and supergravity theories do not have formalisms with all supersymmetries linearly realised "off-shell" in superspace. So the power of such nonrenormalization theorems is restricted to the off-shell linearly realizable subalgebra.

- The full extent of a theory's "on-shell" supersymmetry, even though it may be non-linear, also restricts the infinities since the *leading* counterterms have to be invariant under the original unrenormalized supersymmetry transformations.
- Assuming that 1/2 supersymmetry is linearly realizable and requiring gauge and supersymmetry invariances, predictions were derived for the first divergent loop orders in maximal (N=4 ↔ 16 supercharge) SYM and (N=8 ↔ 32 sc.) SUGRA:

Howe, K.S.S & Townsend

Max. SYM first divergences, assuming half SUSY off-shell (8 supercharges)

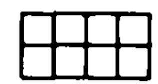
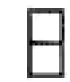
Dimension D	10	8	7	6	5	4
Loop order L	1	1	2	3	4	∞
Gen. form	$\partial^2 F^4$	F^4	$\partial^2 F^4$	$\partial^2 F^4$	F^4	finite

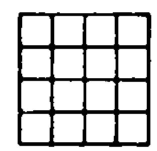
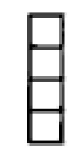
Max. SUGRA first divergences, assuming half SUSY off-shell (16 supercharges)

Dimension D	11	10	8	7	6	5	4
Loop order L	2	2	1	2	3	2	3
Gen. form	$\partial^{12} R^4$	$\partial^{10} R^4$	R^4	$\partial^4 R^4$	$\partial^6 R^4$	R^4	R^4

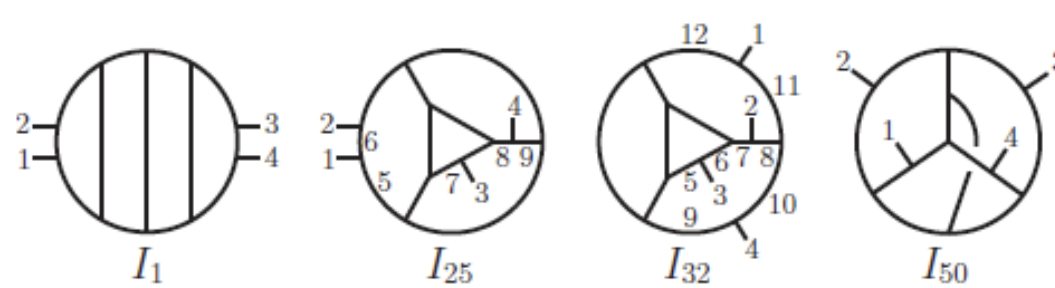
- When written in terms of the full on-shell supersymmetry, the F^4 super Yang-Mills and the R^4 supergravity candidates have similar "1/2 BPS structure". In their D=4 incarnations, they are

Howe, K.S.S. & Townsend
Kallosh

$$\Delta I_{SYM} = \int (d^4\theta d^4\bar{\theta})_{105}\, \mathrm{tr}(\phi^4)_{105}$$  **105** ϕ_{ij}  **6** of SU(4)

$$\Delta I_{SG} = \int (d^8\theta d^8\bar{\theta})_{232848} (W^4)_{232848}$$  **232848** W_{ijkl}  **70** of SU(8)

- However, it now seems that such counterterm analysis in terms of BPS degree is incomplete. The calculational front has recently progressed remarkably.



Unitarity-based calculations

Bern, Carrasco, Dixon, Johansson & Roiban

- Using unitarity and dimensional regularization, there have been significant advances in the computation of loop corrections in the maximal supersymmetric cases.
- These have led to surprising cancellations at the 3- and 4-loop orders, yielding new lowest possible orders for the super Yang-Mills and supergravity divergence onsets:

Max. SYM first divergences, current lowest possible orders.

Dimension D	10	8	7	6	5	4
Loop order L	1	1	2	3	6?	∞
BPS degree	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
Gen. form	$\partial^2 F^4$	F^4	$\partial^2 F^4$	$\partial^2 F^4$	$\partial^2 F^4$	finite

Red: known divergences

Max. supergravity first divergences, current lowest possible orders.

Dimension D	11	10	8	7	6	5	4
Loop order L	2	2	1	2	3	6?	5?
BPS degree	0	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	0	$\frac{1}{4}$
Gen. form	$\partial^{12} R^4$	$\partial^{10} R^4$	R^4	$\partial^6 R^4$	$\partial^6 R^4$	$\partial^{12} R^4$	$\partial^4 R^4$

Ectoplasm

Gates, Grisaru, Knut-Whelau, & Siegel
Berkovits and Howe
Bossard, Howe & KSS

- The construction of supersymmetric invariants is isomorphic to the construction of cohomologically nontrivial closed forms in superspace:
  $I = \int_{M_0} \sigma^* \mathcal{L}_D$ is invariant (where σ^* is a pull-back to the "body" subspace M_0) if $\mathcal{L}_D$ is a closed form in superspace, and is nonvanishing only if $\mathcal{L}_D$ is nontrivial.

- Use the BRST formalism, treating all gauge symmetries including space-time diffeomorphisms with the nilpotent BRST operator s. The invariance condition for $\mathcal{L}_D$ is $s\mathcal{L}_D + d_0\mathcal{L}_{D-1} = 0$, where d_0 is the usual bosonic exterior derivative. Since $s^2 = 0$ and s anticommutes with d_0, one obtains $s\mathcal{L}_{D-1} + d_0\mathcal{L}_{D-2} = 0$, etc.

- Solving the BRST Ward identities thus becomes a cohomological problem. Note that the supersymmetry ghost is a commuting field. One needs to study the cohomology of the nilpotent operator $\delta = s + d_0$, whose cochains $\mathcal{L}_{D-q,q}$ are (D-q) forms with ghost number q, *i.e.* (D-q) forms with q spinor indices. The spinor indices are totally symmetric since the supersymmetry ghost is commuting.

- For gauge-invariant supersymmetric integrands, this establishes an isomorphism between the cohomology of closed forms in superspace (aka "ectoplasm") and the construction of BRST-invariant counterterms.

Superspace cohomology

Bonora, Pasti & Tonin

- Flat superspace has a standard basis of invariant 1-forms

$$E^a = dx^a - \frac{i}{2} d\theta^\alpha (\Gamma^a)_{\alpha\beta}\theta^\beta$$
$$E^\alpha = d\theta^\alpha$$

dual to which are the superspace covariant derivatives (∂_a, D_α)

- There is a natural bi-grading of superspace forms into even and odd parts: $\Omega^n = \oplus_{n=p+q}\Omega^{p,q}$

- Correspondingly, the flat superspace exterior derivative splits into three parts with bi-gradings (1,0), (0,1) & (-1,2):

$$d = d_0(1,0) + d_1(0,1) + t_0(-1,2)$$

bosonic der. fermionic der. torsion

$$d_0 \leftrightarrow \partial_\mu \qquad d_1 \leftrightarrow D_\alpha$$

where for a (p,q) form in flat superspace, one has

$$(t_o\omega)_{a_2\cdots a_p\beta_1\cdots\beta_{q+2}} \sim (\Gamma^{a_1})_{(\beta_1\beta_2}\omega_{a_1\cdots a_p\beta_3\cdots\beta_{q+2})}$$

- The nilpotence of the total exterior derivative d implies the relations

$$\begin{aligned} t_0^2 &= 0 \\ t_0 d_1 + d_1 t_0 &= 0 \\ d_1^2 + t_0 d_0 + d_0 t_0 &= 0 \end{aligned}$$

- Then, since $d\mathcal{L}_D = 0$, the lowest dimension nonvanishing cochain (or "generator") $\mathcal{L}_{D-q,q}$ must satisfy $t_0 \mathcal{L}_{D-q,q} = 0$, so $\mathcal{L}_{D-q,q}$ belongs to the t_0 cohomology group $H_t^{D-q,q}$.

- Starting with the t_0 cohomology groups $H_t^{p,q}$, one then defines a spinorial exterior derivative $d_s : H_t^{p,q} \rightarrow H_t^{p,q+1}$ by $d_s[\omega] = [d_1\omega]$, where the [ ] brackets denote H_t classes.

Cederwall, Gran, Nilsson & Tsimpis
Howe & Tsimpis

- One finds that d_s is nilpotent, $d_s^2 = 0$, and so one can define spinorial cohomology groups $H_s^{p,q} = H_{d_s}(H_t^{p,q})$.

The groups $H_s^{0,q}$ give multi *pure spinors.*

- This formalism gives a way to reformulate BRST cohomology in terms of spinorial cohomology. The lowest dimension cochain, or *generator*, of a counterterm's superform will be d_s closed, *i.e.* it must be an element of $H_s^{D-q,q}$.

- Solving $d_s[\mathcal{L}_{D-q,q}] = 0$ allows one to solve for all the higher components of $\mathcal{L}_D$ in terms of $\mathcal{L}_{D-q,q}$.

Cohomological non-renormalization

- Spinorial cohomology allows one to derive non-renormalization theorems for counterterms: the cocycle structure of candidate counterterms must match that of the classical action.
    - For example, in maximal SYM, this leads to non-renormalization theorems ruling out the F^4 counterterm otherwise expected at L=4 in D=5.
    - Similar non-renormalization theorems exist in supergravity, but their study is complicated by local supersymmetry and the density character of counterterm integrands.

Duality invariance constraints

cf also Broedel & Dixon

- Maximal supergravity has a series of duality symmetries which extend the automatic GL(11-D) symmetry obtained upon dimensional reduction from D=11, e.g. E_7 in the N=8, D=4 theory, with the 70 scalars taking their values in an $E_7/SU(8)$ coset target space.
- The N=8, D=4 theory can be formulated in a manifestly E_7 covariant (but non-manifestly Lorentz covariant) formalism. Anomalies for $SU(8)$, and hence E_7, cancel.
  Bossard, Hillman & Nicolai
  Marcus
- Combining the requirement of continuous duality invariance with the spinorial cohomology requirements gives further restrictions on counterterms.

- In a curved superspace, an invariant is constructed from the top (pure "body") component in a coordinate basis: $I = \frac{1}{D!}\int d^D x\, \varepsilon^{m_D \dots m_1}\, E_{m_D}{}^{A_D} \cdots E_{m_1}{}^{A_1}\, L_{A_1 \dots A_D}(x, \theta = 0)$.

- Referring this to a preferred "flat" basis and identifying $E_M{}^A$ components with vielbeins and gravitinos, one has in D=4

$$I = \frac{1}{24}\int \big(e^a_\wedge e^b_\wedge e^c_\wedge e^d\, L_{abcd} + 4 e^a_\wedge e^b_\wedge e^c_\wedge \psi^{\underline{\alpha}} L_{abc\underline{\alpha}} + 6 e^a_\wedge e^b_\wedge \psi^{\underline{\alpha}}_\wedge \psi^{\underline{\beta}}\, L_{ab\underline{\alpha}\,\underline{\beta}}$$
$$+ 4 e^a_\wedge \psi^{\underline{\alpha}}_\wedge \psi^{\underline{\beta}}_\wedge \psi^{\underline{\gamma}} L_{a\underline{\alpha}\,\underline{\beta}\,\underline{\gamma}} + \psi^{\underline{\alpha}}_\wedge \psi^{\underline{\beta}}_\wedge \psi^{\underline{\gamma}}_\wedge \psi^{\underline{\delta}} L_{\underline{\alpha}\,\underline{\beta}\,\underline{\gamma}\,\underline{\delta}}\big)$$

  - Thus the "soul" components of the cocycle also contribute to the local supersymmetric covariantization.

- Since the gravitinos do not transform under the E_7 duality, the L_{ABCD} form components have to be separately duality invariant.

- At leading order, the $E_7/SU(8)$ coset generators of E_7 simply produce constant shifts in the 70 scalar fields. This leads to a much easier check of invariance than analysing the full spinorial cohomology problem.
- Although the pure body (4,0) component L_{abcd} of the R^4 counterterm have long been known to be shift invariant at lowest order (since all 70 scalar fields are covered by derivatives), it is harder for the fermionic soul components to be so, since they are of lower dimension.

  Howe, KSS & Townsend
- Thus, one finds that the maxi-soul (0,4) $L_{\alpha\beta\gamma\delta}$ component is not invariant under constant shifts of the 70 scalars. Hence the D=4, N=8 3-loop 1/2 BPS R^4 counterterm is not E_7 duality invariant, so it is ruled out.

  Bossard, Howe & KSS

N=5, N=6

- Similar analysis of the D=4 3-loop R^4 invariants in N=5 and N=6 supergravities shows them to be likewise ruled out by the analogous requirements of SU(5,1) and SO*(12) duality invariances.
- In N=6 supergravity, there is a 4-loop $\partial^2 R^4$ type invariant. Preliminary analysis indicates that this also is ruled out.
    - In maximal supergravity, such a $\Delta = 10$ invariant might have been expected at one loop in D=10. However, in maximal supergravity this invariant vanishes subject to the classical field equations. But in D=4, N=6 it does not vanish, so it could have been a threatening counterterm.

1/4 and 1/8 BPS counterterms in D=4

- Application of the spinorial cohomology/duality analysis to the 1/4 $\partial^4 R^4$ and 1/8 BPS $\partial^6 R^4$ candidate counterterms in D=4, N=8 supergravity is possible, but incomplete. However, in the case of the maximal D=4 theory a different type of argument based on E_7 duality invariance is possible.
  Elvang & Kiermaier (from IIA string theory)
  Bossard, Howe & KSS (purely supergravity)
  Beisert, Elvang, Freedman, Kiermaier, Morales & Stiebeger
- In fact, the *existence* of the 1/2 BPS L=1, D=8 R^4, the 1/4 BPS L=2, D=7 $\partial^4 R^4$ and the 1/8 BPS L=3, D=6 $\partial^6 R^4$ divergences together with the *uniqueness* of the corresponding D=4 counterterm structures allows one to rule out the corresponding D=4 candidates.
  Drummond, Heslop, Howe & Kerstan

- The existence of these D=8, 7 & 6 divergences indicate that the corresponding forms of the R^4, $\partial^4 R^4$ & $\partial^6 R^4$ counterterms have to be such that the purely gravitational parts of these invariants are not dressed by e^ϕ dilatonic factors – otherwise, they would violate the corresponding $SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$, $SL(5,\mathbb{R})$ & $SO(5,5)$ duality symmetries: lowest-order shift symmetries would be violated.

- Upon dimensional reduction to D=4, however, the Einstein-frame classical N=8 action is arranged to have no dilaton factors. But then the dimensional reductions of the R^4, $\partial^4 R^4$ & $\partial^6 R^4$ counterterms necessarily *do* have such dilaton factors.

- These dimensional reductions from D=8, 7 & 6 do not directly have manifest SU(8) symmetry. But they can be rendered SU(8) invariant by averaging, *ie* by integrating the dimensionally reduced counterterms over $SU(8)/(SO(3)\times SO(2))$, $SU(8)/SO(5)$ or $SU(8)/(SO(5)\times SO(5))$.
    - Terms linear in dilatons ϕ are wiped out in such averaging, but $\phi\cdot\phi$ quadratic terms survive.
    - Consequently, the dimensionally reduced SU(8) invariant 1/2, 1/4 and 1/8 BPS R^4, $\partial^4 R^4$ and $\partial^6 R^4$ N=8 counterterms all fail the test of lowest-order E_7 scalar shift symmetry.
    - Moreover, the D=4 1/2, 1/4 and 1/8 BPS counterterms are unique. So they fail the E_7 duality test and are all ruled out.

Current outlook

- All of these discussions concern BPS candidate counterterms, *ie* constrained expressions integrated over submanifolds of superspace. Non-BPS counterterms for the N=8 theory, given by full $\int d^{32}\theta$ integrals, start at $\Delta = 16$, corresponding to L=7 in D=4.
  - The first such counterterm that is manifestly E_7 invariant is $\int d^{32}\theta(\det E)$, the volume of N=8 superspace.

E_7 full superspace counterterms for L≥7:
Howe & Lindstrom
Kallosh

- Current divergence expectations for maximal supergravity:

Dimension D	11	10	8	7	6	5	4
Loop order L	2	2	1	2	3	6	7
BPS degree	0	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	0	0
Gen. form	$\partial^{12}R^4$	$\partial^{10}R^4$	R^4	$\partial^6 R^4$	$\partial^6 R^4$	$\partial^{12}R^4$	$\partial^8 R^4$

Red: known divergences

Blue: anticipated divergences