INSTITUT DES HAUTES ÉTUDES SCIENTIFIQUES
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Quark Mass from Tachyon

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This talk is based on

O. Bergman, S.S., J. Sonnenschein, JHEP 0712 (2007) 037;
S.S., JHEP 1007 (2010) 091.

Introduction : Many models of holographic QCD

- Holographic models of QCD in string theory

- D4-branes/D6-brane model
  [Kruczenski-Mateos-Myers-Winter (2004)]

- D4-branes/D8-anti-D8-branes model
  Sakai-Sugimoto model
  [Sakai-Sugimoto (2005)]

- D4-branes model
  [Van Raamsdonk-Whyte (2010)]

- and many others

- AdS/QCD model
  5-dimensional gauge theory in anti-de Sitter space
  [Erlich-Katz-Son-Stephanov (2005)]

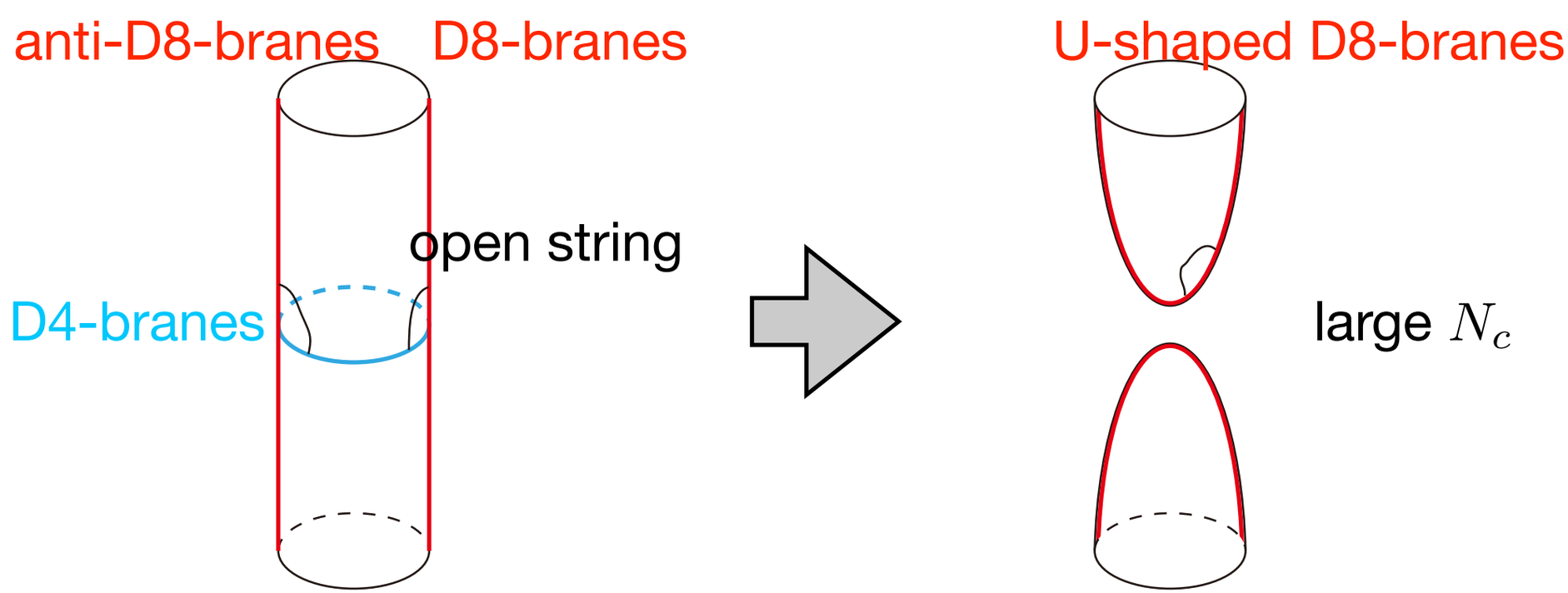
Sakai-Sugimoto model

- N_c D4-branes $\longrightarrow$ background

  N_f D8-branes and anti-D8-branes $\longrightarrow$ probe D8-branes

	0	1	2	3	x_4	U	6	7	8	9
D4	x	x	x	x	x					
D8	x	x	x	x		x	x	x	x	x

chiral symmetry $U(N_f)_R \times U(N_f)_L$ $\qquad$ $U(N_f)$

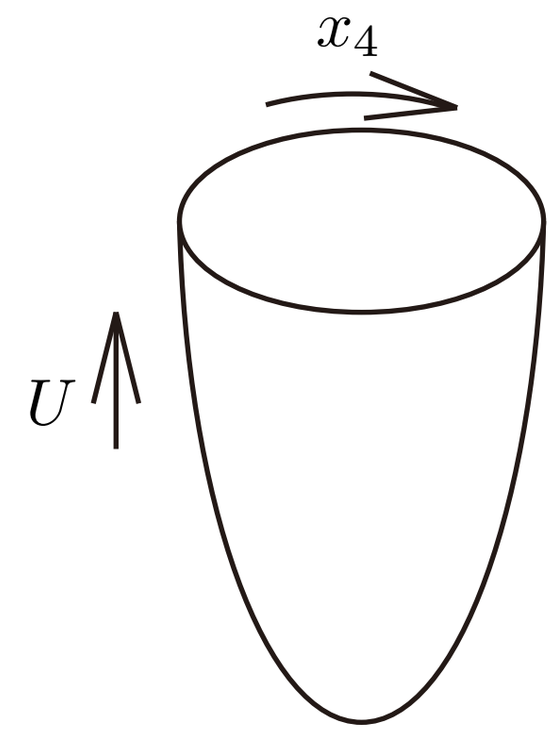


- background [Witten (1998)]

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} (\eta_{\mu\nu} dx^\mu dx^\nu + f(U)(dx_4)^2) + \left(\frac{R}{U}\right)^{\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$f(U) = 1 - \frac{U_{\rm KK}^3}{U^3} \qquad e^\phi = g_s \left(\frac{U}{R}\right)^{\frac{3}{4}}$$

$$F_4 = dC_3 = \frac{2\pi N_c}{V_4} \epsilon_4 \qquad R^3 = \pi g_s N_c l_s^3$$



- period of x_4

$$\delta x_4 = \frac{4\pi}{3} \frac{R^{\frac{3}{2}}}{U_{\rm KK}^{\frac{1}{2}}} \quad \Rightarrow \quad M_{\rm KK} = \frac{2\pi}{\delta x_4}$$

- In this background, the probe D8-branes are analysed in terms of Dirac-Born-Infeld action.

Advantages

- chiral symmetry is manifest
- good agreement with experimental data

Disadvantages

- holographic dual of massless QCD
  D4-D8 open strings :
  massless quark $\longrightarrow$ massless pion

- large N_c
  probe analysis is valid in $N_f \ll N_c$

Mass?

- The quark mass term in QCD

$$m_q \langle \bar{q} q \rangle = m_q \langle q_R^\dagger q_L + q_L^\dagger q_R \rangle$$

q_R : a fundamental of $U(N_f)_R$
q_L : a fundamental of $U(N_f)_L$

The mass and the quark bi-linear should be identified with the bi-fundamental field in the bulk.

$\longrightarrow$ the open string stretched between D8 and anti-D8-branes

(cf. in AdS/QCD [Casero, Kiritsis, Paredes (2007)])

- Pion mass
Gell-Mann-Oakes-Renner relation [Gell-Mann-Oakes-Renner (1968)]

$$m_\pi^2 \propto m_q \langle \bar{q} q \rangle$$

Quark mass from tachyon

[Bergman, S.S., Sonnenschein, JHEP 0712 (2007) 037]

- Non-compact limit ($U_{\rm KK} \to 0$) of Sakai-Sugimoto model as a toy model

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} (\eta_{\mu\nu} dx^\mu dx^\nu + (dx_4)^2) + \left(\frac{R}{U}\right)^{\frac{3}{2}} \left(dU^2 + U^2 d\Omega_4^2\right)$$

- We are now interested in the contribution of the bi-fundamental field.

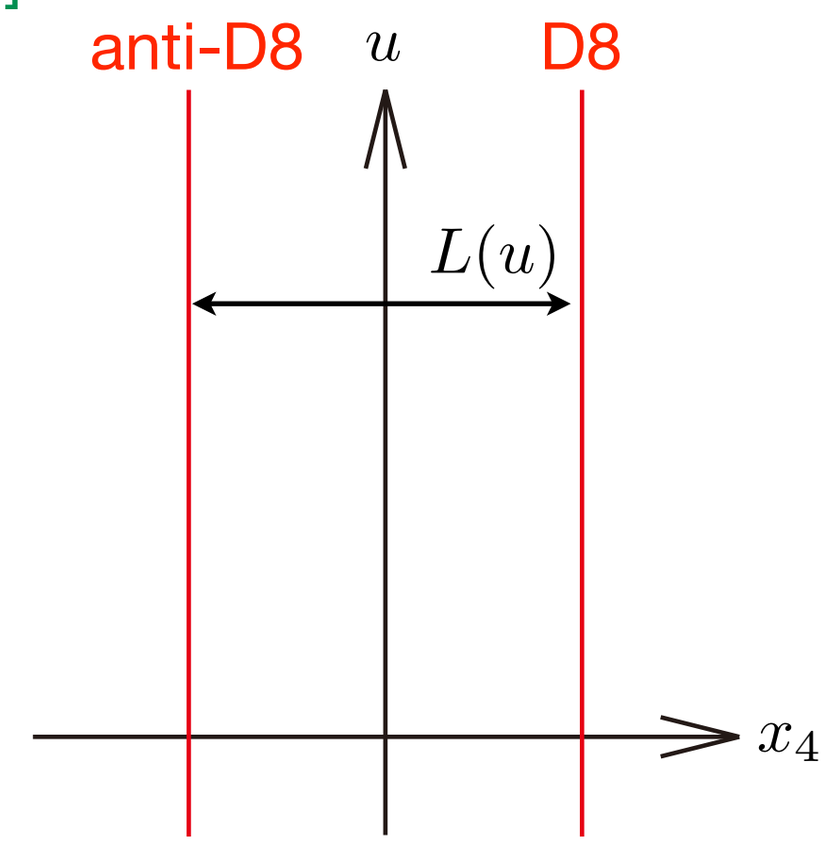
In this background, one can consider parallel D8-branes and anti-D8-branes.

Following the usual holographic dictionary,

<u>bi-fundamental field</u>		In the dual gauge theory
normalizable mode	$\Leftrightarrow$	expectation value of quark bi-linear
non-normalizable mode		quark mass

Dirac-Born-Infeld action with tachyon

- DBI action of D*p*/anti-D*p*-branes [Garousi (2005)]



- D8-brane and anti-D8-brane ($N_f = 1$)

$L(u)$: separation of D8 and anti-D8
$T(u)$: bi-fundamental "tachyon" field

$$S[T, L] = -2\mathcal{N} \int d^4x du\, V(T) u^4 \sqrt{\mathcal{D}[T, L]}$$

$$\mathcal{D} = \frac{1}{u^3} + \frac{1}{4R^2} L'^2 + \frac{2\pi\alpha'}{R^2 u^{3/2}} T'^2 + \frac{1}{2\pi\alpha' u^{3/2}} L^2 T^2$$

$$u \equiv U/R \qquad \mathcal{N} \equiv T_8 V_4 R^5 / g_s$$

- tachyon potential [Buchel-Langfelder-Walcher (2002), Leblond-Peet (2003), Lambert-Liu-Maldacena (2007)]

$$V(T) = \frac{1}{\cosh(\sqrt{\pi} T)}$$

- mass of “tachyon”

$$m_T^2 = -\frac{1}{2\alpha'} + \frac{L_{\text{proper}}^2}{(2\pi\alpha')^2} \qquad\qquad L_{\text{proper}} = u^{3/4} L$$

The tachyon is localized around $u = 0$.

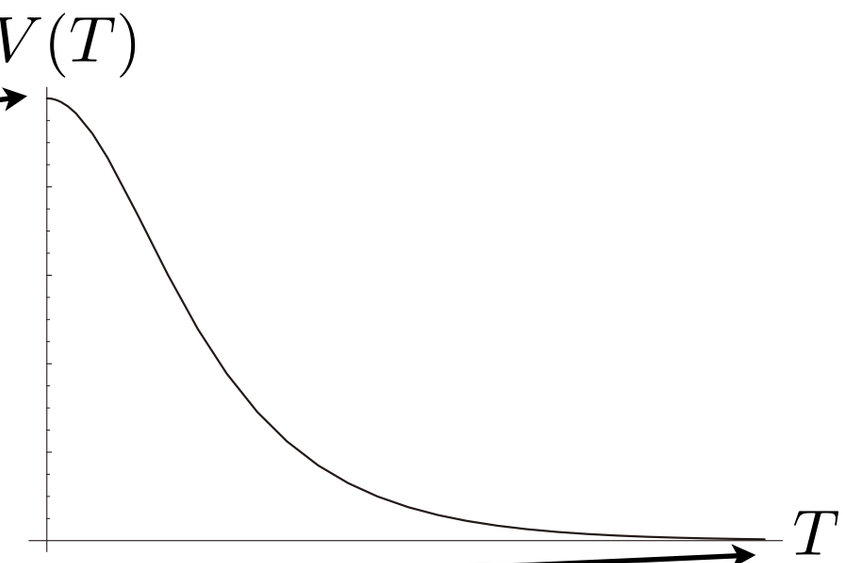
Tachyon condensation

- Let us remember tachyon condensation. [Sen (1998)]

At the top of the potential,
open string theory
D*p* and anti-D*p*

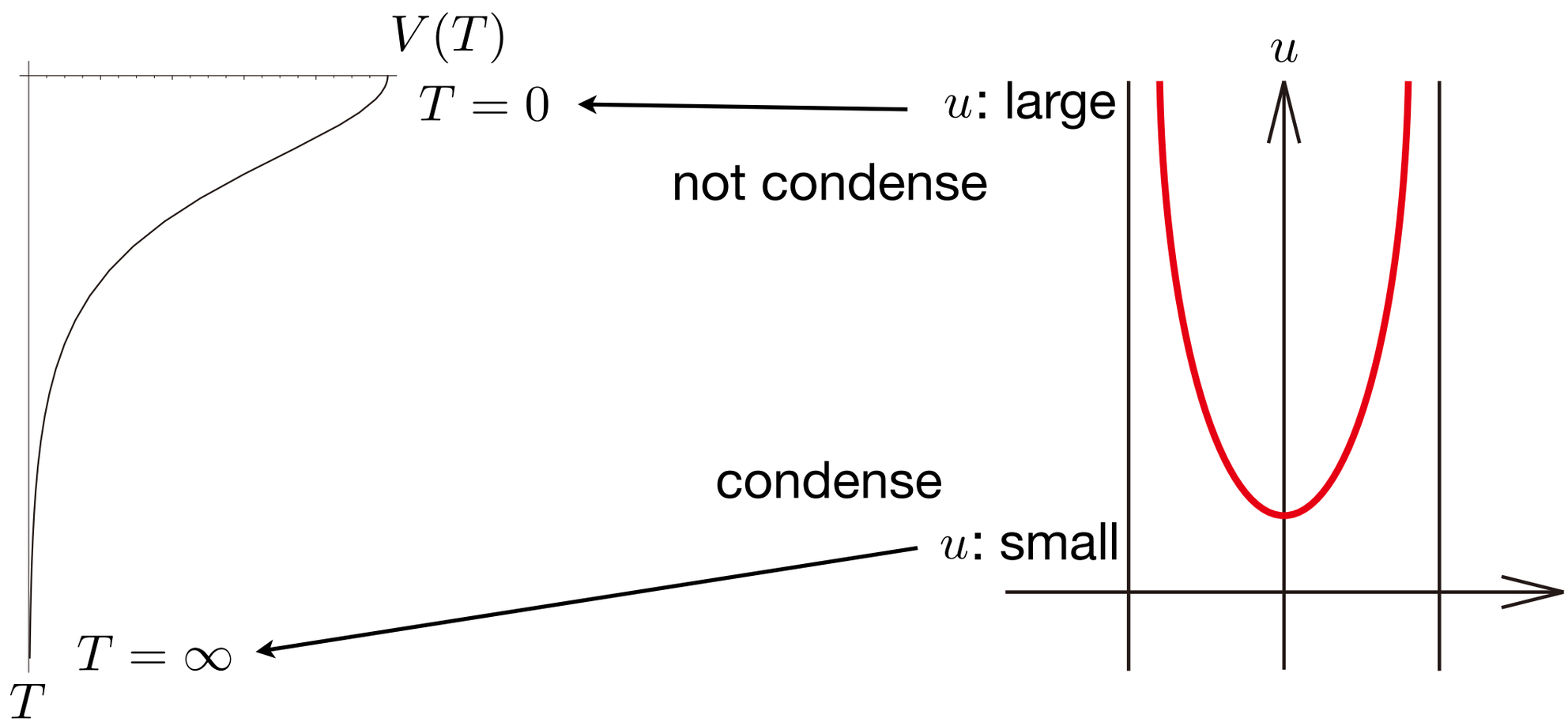
tachyon condense

At the bottom of the potential,
D*p* and anti-D*p* annihilate
closed string theory



Why U-shape?

- Tachyon is localized around $u = 0$.



Analyses of D8/anti-D8 DBI action

- The trivial solution of the equations of motion:

$$L(u) = \text{constant}, \quad T(u) = 0$$

  describes the parallel D8-brane and the anti-D8-brane.

- It is difficult to solve the full equations of motion of tachyonic DBI action. So we shall find the asymptotic solutions in the small u region (IR) and the large u region (UV).

- u: small

$$\begin{aligned} L(u) &= (u-u_0)^p[l_0 + l_1(u-u_0) + \cdots] \\ T(u) &= (u-u_0)^q[t_0 + t_1(u-u_0) + \cdots] \end{aligned}$$

$$\longrightarrow \quad q = -2, \quad 0 < p < 1, \quad t_0 = \frac{\sqrt{\pi}}{2R^2} p u_0^{3/2}$$

U-shape solution and $T(u_0) = \infty$

- u: large
  We consider the perturbation from the parallel D8 and anti-D8.

$$L(u) = L_\infty + \tilde{L}(u)\,, \quad T(u) = 0 + \tilde{T}(u)$$

The solution is

$$\begin{aligned} \tilde{L}(u) &= C_L u^{-9/2}\,, \\ \tilde{T}(u) &= u^{-2}(C_T e^{-RL_\infty u} + C'_T e^{+RL_\infty u}) \end{aligned}$$

Note that we assumed $C'_T \lesssim e^{-RL_\infty u_\infty}$.

Then we guess the following interpretations:

normalizable mode $\quad C_T \longleftrightarrow \langle \bar{q} q \rangle$

non-normalizable mode $\quad C'_T \longleftrightarrow m_q$

- Assume that the current quark mass is given by

$$m_q = \Lambda C'_T$$

- quark condensate
  (cf. $\mathcal{L}_{\mathrm{QCD}} = \cdots + m_q \bar{q} q + \cdots$)

$$\langle \bar{q} q \rangle = \left. \frac{\delta \mathcal{E}(\mathcal{C}'_{\mathcal{T}})}{\delta m_q} \right|_{m_q=0} = \frac{2\mathcal{N} L_\infty}{\Lambda R} C_T$$

Pion

- Consider the fluctuations of gauge fields

$$A^{\pm}(x^{\mu}, u) := \frac{1}{2}(A^{\mathrm{D8}} \pm A^{\overline{\mathrm{D8}}})$$

- gauge fix $\longrightarrow$ $A_u^+ = 0$

- $A_\mu^+ \longrightarrow$ vector meson

- $A_u^-, A_\mu^- \longrightarrow$ scalar meson and pseudo-scalar meson (pion)

- In terms of mode functions, the pion mass and decay constant can be evaluated.

$\longrightarrow$ GOR relation $M_0^2 \sim \dfrac{4m_q\langle\bar{q}q\rangle}{f_\pi^2}$ up to numerical factor

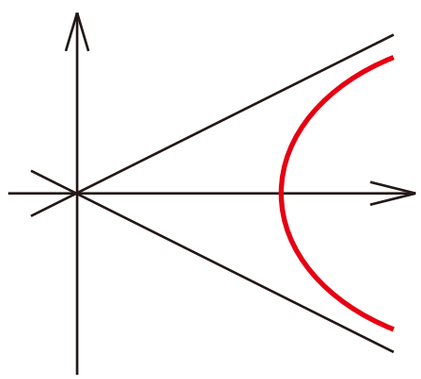
How is the quark mass generated in other HQCD models?

Van Raamsdonk-Whyte's D4-branes model

[Van Raamsdonk-Whyte (2009)]

- N_c D4-branes ----- background
  N_f D4-branes ----- probe

	0	1	2	3	x_4	r	θ	7	8	9
D4	x	x	x	x	x					
D4	x	x	x	x			x			



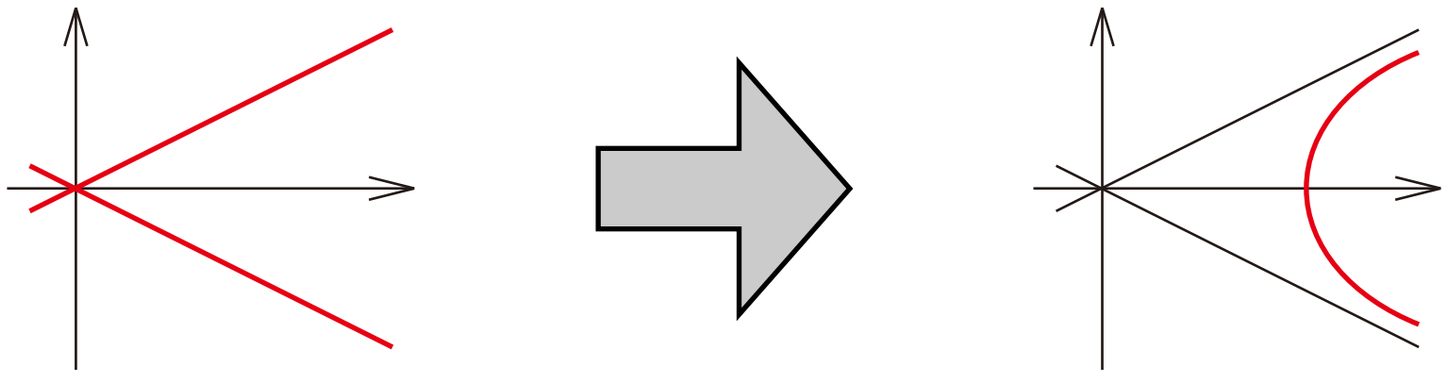
<u>Advantages</u>
The description of baryon is simpler.

<u>Disadvantages</u>
The chiral symmetry is not manifest.
The current quark mass = 0.
...

Intersecting D4-branes model

[S.S. JHEP 1007 (2010) 091]

- N_c D4-branes
  N_f intersecting D4 and anti-D4-branes $\longrightarrow$ tachyonic DBI action
  +
  Chern-Simons terms



When $T = 0$, this model is reduced to the model by Van Raamsdonk-Whyte.

- The background metric

$$ds^2 = \left(\frac{\rho}{R}\right)^{\frac{3}{2}} \left[\eta_{\mu\nu} dx^\mu dx^\nu + dx_4^2\right] + \left(\frac{R}{\rho}\right)^{\frac{3}{2}} \left[dr^2 + r^2 d\theta^2 + d\vec{x}_T^2\right]$$

$$e^\phi = g_s \left(\frac{\rho}{R}\right)^{\frac{3}{4}}, \quad \rho^2 = r^2 + \vec{x}_T^2$$

- The action

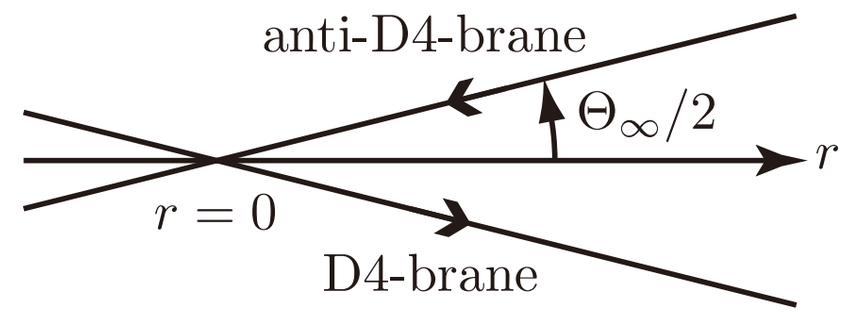
$$S = -\frac{2T_4}{g_s} \int d^4x dr\, V(T) \left(\frac{r}{R}\right)^{\frac{3}{2}} \sqrt{1 + \frac{r^2}{4}\left(\frac{d\Theta}{dr}\right)^2 + 2\pi\alpha' \left(\frac{r}{R}\right)^{\frac{3}{2}} \left(\frac{dT}{dr}\right)^2 + \frac{r^2}{2\pi\alpha'} \left(\frac{R}{r}\right)^{\frac{3}{2}} \Theta^2 T^2}$$

$$S_{\mathrm{CS}} = \int_{\mathrm{D4}} P^{(1)}[C_5] - \int_{\overline{\mathrm{D4}}} P^{(2)}[C_5]$$

$$C_5 = \frac{N_c}{16\pi^3 \alpha' R^6} \rho^3 dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4$$

- Trivial solution

$$\Theta(r) = \Theta_\infty(\text{constant})\,, \quad T(r) = 0$$

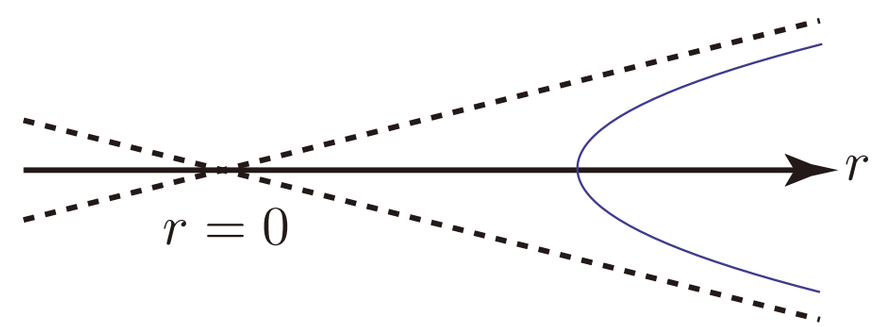


- Non-trivial solution

IR

$$\begin{aligned}\Theta(r) &= \frac{\theta_0 R^{3a}}{(2\pi\alpha')^{2a}}(r-r_0)^a + \mathcal{O}\big((r-r_0)^{a+1}\big)\,, \quad 0<a<1\\ T(r) &= \frac{(2\pi\alpha')\sqrt{\pi}a r_0^{3/2}}{2R^{3/2}}(r-r_0)^{-2} + \mathcal{O}\big((r-r_0)^{-1}\big)\,.\end{aligned}$$

UV



$$\begin{aligned}\Theta(r) &= \Theta_\infty + \frac{2\pi\alpha' C_\theta}{R^{3/2}}\frac{1}{\sqrt{r}}\,,\\ T(r) &= \frac{(2\pi\alpha')^2}{R^3\Theta_\infty^2}\frac{1}{r}\left[C_{\rm nn} I_2\left(\frac{\Theta_\infty R^{3/2}}{\pi\alpha'}\sqrt{r}\right) + C_{\rm n} K_2\left(\frac{\Theta_\infty R^{3/2}}{\pi\alpha'}\sqrt{r}\right)\right].\end{aligned}$$

- quark mass and condensate

$$C_{\rm nn} = \Lambda m_q$$

$$\langle q\bar{q}\rangle = \frac{\delta\mathcal{E}}{\delta m_q}\bigg|_{m_q=0} = \Lambda\frac{\delta\mathcal{E}}{\delta C_{\rm nn}}\bigg|_{C_{\rm nn}=0} = \frac{(2\pi\alpha')^5 T_4}{2g_s R^9}\frac{\Lambda}{\Theta_\infty^4}C_{\rm n}$$

Gauge fields

- $U(N_f = 1)$ gauge fields

$$A_i^{(\pm)} = \frac{1}{2}(A_i^{(1)} \pm A_i^{(2)})$$

- mode expansion

vector meson

$$A_\mu^{(+)}(x^\mu, r) = \sum_n a_{n\mu}^{(+)}(x^\mu)\psi_n(r) \qquad \text{(gauge } A_r^{(+)} = 0\text{)}$$

axial vector meson

$$A_\mu^{-} = A_\mu^{\perp} + A_\mu^{\parallel} \qquad (\partial_\mu A^{\perp\mu} = 0)$$

$$A_\mu^{\perp}(x^\mu, r) = \sum_n a_{n\mu}^{(-)}(x^\mu)\xi_n^{\perp}(r) \qquad A_\mu^{\parallel}(x^\mu, r) = \sum_n \partial_\mu \omega_n(x^\mu)\xi_n^{\parallel}(r)$$

pseudo-scalar meson

$$A_r^{(-)}(x^\mu, r) = \sum_n \omega_n(x^\mu)\zeta_n(r)$$

- Anti-symmetric sector

$$S[A^{(-)}] = -\int d^4x dr \left[\mathcal{C}_1 \left| F^{(-)}_{\mu\nu} \right|^2 + \mathcal{C}_2 \left| F^{(-)}_{\mu r} \right|^2 + \mathcal{C}_3 \left| A^{(-)}_{\mu} \right|^2 + \mathcal{C}_4 \left| A^{(-)}_{r} \right|^2 + \mathcal{C}_5 F^{(-)}_{\mu r} A^{(-)\mu} \right]$$

$$\mathcal{C}_1 = \frac{(2\pi\alpha')^2 T_4}{2g_s} V(T) \left(\frac{R}{r} \right)^{\frac{3}{2}} \sqrt{\mathcal{D}}\,, \quad \mathcal{C}_2 = \frac{(2\pi\alpha')^2 T_4}{g_s} V(T) \left(\frac{r}{R} \right)^{\frac{3}{2}} \frac{\mathcal{Q}}{\sqrt{\mathcal{D}}}$$

$$\mathcal{C}_3 = \frac{8\pi\alpha' T_4}{g_s} V(T) \frac{\sqrt{\mathcal{D}}}{\mathcal{Q}} T^2 + \frac{T_4}{g_s} V(T) \frac{r^4}{\mathcal{Q}\sqrt{\mathcal{D}}} \left(\frac{R}{r} \right)^{\frac{3}{2}} \Theta^2 \Theta'^2 T^4\,,$$

$$\mathcal{C}_4 = \frac{8\pi\alpha' T_4}{g_s} V(T) \left(\frac{r}{R} \right)^{3} \frac{1}{\sqrt{\mathcal{D}}} T^2\,, \quad \mathcal{C}_5 = \frac{4\pi\alpha' T_4}{g_s} V(T) \frac{r^2}{\sqrt{\mathcal{D}}} \Theta\Theta' T^2\,.$$

$$\mathcal{D} = 1 + \frac{r^2}{4}\Theta'^2 + 2\pi\alpha' \left(\frac{r}{R} \right)^{\frac{3}{2}} T'^2 + \frac{r^2}{2\pi\alpha'} \left(\frac{R}{r} \right)^{\frac{3}{2}} \Theta^2 T^2\,,$$

$$\mathcal{Q} = 1 + \frac{1}{2\pi\alpha'} R^{3/2} r^{1/2} \Theta^2 T^2$$

- eigen equations and normalisation conditions

$$-\frac{1}{2}\frac{d}{dr}\left(\mathcal{C}_2\xi_n^{\perp\prime}\right)+\frac{1}{2}\mathcal{C}_3\xi_n^{\perp}+\frac{1}{4}\frac{d\mathcal{C}_5}{dr}\xi_n^{\perp}=\left(m_n^{(-)}\right)^2\mathcal{C}_1\xi_n^{\perp}\,,$$

$$\mathcal{C}_4\zeta_n=M_n^2\left[\mathcal{C}_2\left(\zeta_n-\xi_n^{\parallel\prime}\right)+\frac{1}{2}\mathcal{C}_5\xi_n^{\parallel}\right],$$

$$\partial_r\left[\mathcal{C}_2\left(\zeta_n-\xi_n^{\parallel\prime}\right)+\frac{1}{2}\mathcal{C}_5\xi_n^{\parallel}\right]+\mathcal{C}_3\xi_n^{\parallel}+\frac{1}{2}\mathcal{C}_5\left(\zeta_n-\xi_n^{\parallel\prime}\right)=0\,,$$

$$\frac{1}{4}\delta_{mn}=\int dr\,\mathcal{C}_1\xi_m^{\perp}\xi_n^{\perp}\,,$$

$$\begin{aligned}\frac{1}{2}\delta_{mn} \quad = \quad & \int dr\Big[\mathcal{C}_2\left(\zeta_m-\xi_m^{\parallel\prime}\right)\left(\zeta_n-\xi_n^{\parallel\prime}\right)+\mathcal{C}_3\xi_m^{\parallel}\xi_n^{\parallel}\\ & +\frac{1}{2}\mathcal{C}_5\Big(\left(\zeta_m-\xi_m^{\parallel\prime}\right)\xi_n^{\parallel}+\xi_m^{\parallel}\left(\zeta_n-\xi_n^{\parallel\prime}\right)\Big)\Big]\,.\end{aligned}$$

$$\Rightarrow\quad S[A^{(-)}]=-\int d^4x\sum_n\left[\frac{1}{4}\left|f_{n\mu\nu}^{(-)}\right|^2+\frac{1}{2}\left(m_n^{(-)}\right)^2\left|a_{n\mu}^{(-)}\right|^2+\frac{1}{2}\left|\partial_\mu\omega_n\right|^2+\frac{1}{2}M_n^2\omega_n^2\right]$$

Pion

- We assume that the quark mass is small. ($C_{\rm nn}(\sim m_q) \ll 1$)

- Pion decay constant: f_π
  In QCD, it appears in the correlator of axial vector currents for a_μ^-.

$$S_{\rm eff} = \int d^4p \left[\cdots + \sum_n \left(p^2 \frac{(f_n^{(-)})^2}{p^2 + (m_n^-)^2} + f_\pi^2 \right) a_\mu^{-(n)} a^{-(n)\mu} \right]$$

We evaluate the pion decay constant by differentiating the action twice with respect to $a_\mu^{-(0)}$ and imposing $p^2 = 0$.

$$\frac{1}{2} f_\pi^2 = (\mathcal{C}_2 \xi_0^\perp \xi_0^{\perp\prime})\big|_{r=\infty}$$

If we set the boundary condition $\xi_0^\perp(\infty) = c$,

$$f_\pi^2 = \frac{(2\pi\alpha')^4 T_4^2 c^2 \beta^2}{g_s^2 R^3} \qquad \xi_0^\parallel = \alpha + \frac{\beta}{\sqrt{r}}$$

- Pion mass

$$M_0^2 = 2\int dr\, \mathcal{C}_4 \zeta_0^2$$

$$\Rightarrow \quad M_\pi^2 (= M_0^2) \approx -\frac{8\pi\alpha' g_s C_{\rm n} C_{\rm nn}}{R^6 T_4 \Theta_\infty^4 \beta^2}$$

- Gell-Mann-Oakes-Renner relation is satisfied up to a numerical factor.

$$M_\pi^2 = -8c^2 \frac{m_q \langle q\bar{q}\rangle}{f_\pi^2}$$

Conclusion and Future directions

- intersecting D4-branes model
- (tachyonic) bi-fundamental fields provide current quark mass in various models of holographic QCD.

- The solution interpolating between UV and IR?
- baryon?
- back reaction?
- more flavours?