

Non-Geometric String Theory Backgrounds

↙ from Gauged Supergravities to Interacting Chiral Bosons ↘

Nikolaos Prezas

*Albert Einstein Center for Fundamental Physics
Institute for Theoretical Physics
University of Bern*

based on

- "Conformal Chiral Boson Models on Twisted Doubled Tori and Non-Geometric String Backgrounds", with S. Avramis and J. P. Derendinger,
Nucl. Phys. **B827** (2010) 281; arXiv:0910.0431
- "Worldsheet Theories for Non-Geometric String Backgrounds",
with G. Dall'Agata, **JHEP 0808** (2008) 088; arXiv:0712.1026
- "Gauged Supergravities from Twisted Doubled Tori and Non-Geometric String Backgrounds", with G. Dall'Agata, H. Samtleben and M. Trigiante,
Nucl. Phys. **B799** (2008) 80; arXiv:0712.1026

Motivation and overview

Fluxes, twists and supergravity gaugings

Non-geometric backgrounds and twisted doubled tori

Interacting chiral boson models: Lorentz invariance

Interacting chiral boson models: one-loop effective action

Summary and open problems

Standard lore about string theory

- ▶ Consistent **unification** of gauge and gravitational interactions
- However, it requires (typically) more dimensions than four!
- How can we make contact to four-dimensional physics ?

Compactification approach

- ▶ Assumption: spacetime of the form $\mathcal{M}_4 \times Y_6$
- Classify possible **internal spaces** Y_6
- Derive **effective theory** in 4 dimensions

Traditional compactification scheme (80s)

- ▶ Non-trivial metric on Y_6 + SUSY → **Calabi–Yau**
(or manifolds with exceptional holonomy)
- However: huge **vacuum degeneracy**
- **Moduli**: scalars without potential (fluctuations of internal fields)
- Problems: long range forces + loss of predictive power
- **Moduli stabilization**

Flux compactifications (90s)

- ▶ Enlarge landscape: **fluxes** of antisymmetric tensor fields
- ▶ Generate **potential** for "moduli" fields
- Special holonomy → manifolds with ***G*-structure**
- Scalar potential + extended SUSY: **gauged supergravity**
- Fluxes \leftrightarrow gauging parameters

Scherk–Schwarz reductions - twisted tori - geometric fluxes

- ▶ Twisted reduction on a torus: **twisted torus**
[Scherk, Schwarz '79; Kaloper, Myers '99]
- Induces scalar **potential** and **non-abelian** gauge symmetry
- Consistent truncation on a (local) group manifold

Are there more general backgrounds/fluxes ?

- ▶ **Non-geometric** backgrounds: T-folds
[Flournoy, Wecht, Williams '02; Hellerman, McGreevy, Williams '02; Dabholkar, Hull '02;
Kachru, Schulz, Tripathy, Trivedi '02]
- Background data: patched with **T-dualities**
- Associated **non-geometric fluxes** (from duality covariance)
[Shelton, Taylor, Wecht '05; Dabholkar, Hull '05; Aldazabal, Camara, Font, Ibanez '06; Shelton,
Taylor, Wecht '06]

Are there more general effective theories ?

- ▶ General framework: **embedding tensor**
[de Wit, Samtleben, Trigiante '05]
- Yields all supergravity gaugings for $\mathcal{N} \geq 1$

Non-geometric backgrounds and doubled geometries

- ▶ Inherently stringy constructions: **winding modes**
- **Doubling** of coordinates
[Witten '88; Duff '90; Tseytlin '90, '91; Kugo, Zwiebach '92; Maharana, Schwarz '92; Siegel '93]
- **Doubled torus**: geometry for non-geometric string backgrounds
[Hull '04, '06, '07]
- **Twisted doubled torus (TDT)**: electric $\mathcal{N} = 4$ gaugings
[Hull, Reid-Edwards '07; Dall'Agata, Prezas, Samtleben, Trigiante '07]

Worldsheet approach: chiral bosons

- ▶ Momentum and winding modes on equal footing: **chiral bosons**
[Siegel '84; Floreanini, Jackiw '87]
- Interacting models: conditions from **2d Lorentz invariance**
[Tseytlin '90, '91]
- **TDT as target space:** Lorentz invariant!
[Dall'Agata, Prezas '08]
- Conditions for **conformal invariance**
[Avramis, Derendinger, Prezas '09]
- Computation of the one-loop effective action
- **Agreement with corresponding gauged supergravity**

Motivation and overview

Fluxes, twists and supergravity gaugings

Non-geometric backgrounds and twisted doubled tori

Interacting chiral boson models: Lorentz invariance

Interacting chiral boson models: one-loop effective action

Summary and open problems

Compactifications without fluxes

- ▶ Consider a toroidal reduction of string/M-theory
- Abelian gauge theory without scalar potential
- Existence of global duality symmetries

[Cremmer, Scherk, Ferrara '79; Cremmer, Julia '79]

Flux compactifications

- ▶ Our setup: heterotic strings on \mathbb{T}^6 (ignoring the gauge sector)
- ▶ Let us assume NS-NS 3-form flux in the internal torus:

$$\langle H_{ijk}(x, y) \rangle = h_{ijk}$$

- Decompose 2-form, metric tensor and dilaton:

$$B_{MN}(x, y) \rightarrow B_{\mu\nu}(x), B_{\mu i}(x), B_{ij}(x)$$

$$G_{MN}(x, y) \rightarrow G_{\mu\nu}(x), G_{\mu i}(x), G_{ij}(x)$$

$$\phi(x, y) \rightarrow \phi(x)$$

- 3-form kinetic term yields **potential** for metric moduli

$$\int_{\mathcal{M}_4 \times \mathbb{T}^6} H \wedge *H = \int_{\mathcal{M}_4} d^4x \sqrt{-g_4} (\textcolor{red}{h_{ijk} h_{mln}} G^{im}(x) G^{jl}(x) G^{kn}(x) + \dots)$$

and **non-abelian** gauge interactions

$$\int_{\mathcal{M}_4} d^4x \sqrt{-g_4} (\dots + \partial_\mu B_\nu^i G^{\mu j} G^{\nu k} \textcolor{red}{h_{ijk}} + \dots)$$

- Generators: Z_i for $G_{\mu i}$, X^i for $B_{\mu i}$. **Gauge algebra:**

$$[X^i, X^j] = 0$$

$$[Z_i, X^j] = 0$$

$$[Z_i, Z_j] = h_{ijk} X^k$$

Scherk-Schwarz reductions (a.k.a. twisted tori, geometric fluxes)

- ▶ Can we enrich the previous gauge algebra ?
- ▶ Consider reduction ansatz

$$\begin{aligned} ds_{(10)}^2 &= G_{\mu\nu}(x) dx^\mu dx^\nu \\ &+ (\delta_{ab} + G_{ab}(x))(\eta^a + A_\mu^a(x) dx^\mu)(\eta^b + A_\nu^b(x) dx^\nu) \end{aligned}$$

with $\eta^a(y)$ being left-invariant 1-forms satisfying

$$d\eta^a = -\frac{1}{2}\tau_{bc}{}^a \eta^b \wedge \eta^c$$

- Similar expansion for other fields
- $\tau_{bc}{}^a$ are structure constants of Lie algebra \mathfrak{g} (Jacobi identity)
- ▶ **Consistent truncation** to singlets under \mathcal{G}_L

[Duff, Pope '85; Hull, Reid-Edwards '05]

- **Geometric flux**: spin connection condensate

$$\langle \omega_{bc}{}^a(x, y) \rangle = \frac{1}{2} \tau_{bc}{}^a$$

- Compactification on a **twisted torus**: local group manifold \mathcal{G}/Γ
- Equivalent to a **twisted reduction** on a torus

[Scherk, Schwarz '79]

$$\eta^a(y) = U_j^a(y) dy^i$$

- Introduces y -dependance that is eliminated if

$$U_b^{-1i}(y) U_c^{-1j}(y) \partial_{[i} U_{j]}^a = \tau_{bc}{}^a$$

- Combining with NS-NS flux yields gauge algebra:

[Kaloper, Myers '99]

$$[X^a, X^b] = 0$$

$$[X^a, Z_b] = \tau_{bc}{}^a X^c$$

$$[Z_a, Z_b] = \tau_{ab}{}^c Z_c + h_{abc} X^c$$

- Part of isometry group \mathcal{G}_R appears as gauge symmetry

- Introduce $\mathbb{X}_A = \{Z_a, X^a\}$, gauge algebra:

$$[\mathbb{X}_A, \mathbb{X}_B] = \mathcal{T}_{AB}{}^C \mathbb{X}_C$$

- Fluxes \Rightarrow gauge algebra structure constants $\mathcal{T}_{AB}{}^C$
- Jacobi for $\mathcal{T}_{AB}{}^C$: Bianchi for fluxes and Jacobi for $\tau_{bc}{}^a$
- ▶ Pack scalar fields in $\frac{O(6,6)}{O(6) \times O(6)}$ matrix:

$$M_{AB} = \begin{pmatrix} G_{ab} - B_{ac} G^{cd} B_{db} & B_{ac} G^{cb} \\ -G^{bc} B_{ca} & G^{ab} \end{pmatrix}$$

- Scalar potential:

$$V \sim \mathcal{T}_{DA}{}^C \mathcal{T}_{CB}{}^D M^{AB} + \frac{1}{3} \mathcal{T}_{CE}{}^A \mathcal{T}_{DF}{}^B M_{AB} M^{CD} M^{EF}$$

- ▶ What type of theory is this ?
- 4 dimensions, 16 supersymmetries, gravity, non-abelian gauge symmetry, scalar potential $\Rightarrow \mathcal{N} = 4, D = 4$ gauged supergravity

General gauge algebras

[Shelton, Taylor, Wecht '05; Dabholkar, Hull '05; Aldazabal, Camara, Font, Ibanez '06]

- ▶ Fill in the gaps in Kaloper–Myers gauge algebra:

$$\begin{aligned}[Z_a, Z_b] &= \tau_{ab}{}^c Z_c + h_{abc} X^c \\ [X^a, Z_b] &= \tau_{bc}{}^a X^c - Q_b{}^{ac} Z_c \\ [X^a, X^b] &= Q_c{}^{ab} X^c + R^{abc} Z_c\end{aligned}$$

- Physical and geometric fluxes: **non-semisimple** gauge groups
- $Q_c{}^{ab}$: non-abelian gauge symmetry from B-field gauge vectors!
- ▶ What is the origin of the new "fluxes" Q and R ? **T-duality**
- ▶ Organizing principle: $\mathcal{N} = 4, D = 4$ gauged supergravity

Gauged supergravity perspective

- ▶ General gauging: adjoint of gauge group → fundamental of duality group

[de Wit, Samtleben, Trigiante '05]

$$\mathbb{X}_A = \theta_A^m t_m$$

- Embedding tensor θ_M^a : subject to conditions \Rightarrow full theory
- In particular: gauge algebra \Rightarrow all possible fluxes!

- ▶ Focus on $\mathcal{N} = 4, D = 4$ gauged supergravity
- $\mathcal{N} = 4$ **graviton** multiplet: graviton, axio-dilaton, 6 vectors
- $\mathcal{N} = 4$ **vector** multiplet: vector, 6 scalars
- **Duality group** of ungauged theory: $SL(2) \times O(6, 6+n)$
- Scalars

$$\tau \in \frac{SL(2)}{U(1)}, \quad M_{AB} \in \frac{O(6, 6+n)}{O(6) \times O(6+n)}$$

- Embedding tensor (subject to quadratic constraints)

[Schön, Weidner '06]

$$f_{\alpha ABC}, \xi_{\alpha A}$$

- Heterotic reduction ($n = 0$): T-duality group $O(d, d), d = 6$

- Electric gaugings

$$f_{-ABC} = 0, \quad \xi_{\alpha A} = 0$$

- For electric gaugings: embedding tensor \equiv structure constants

$$f_{+ABC} := f_{ABC} = \mathcal{T}_{AB}{}^D \eta_{CD} = \mathcal{T}_{ABC}$$

- $O(d, d)$ metric η_{AB}

$$\eta_{AB} = \begin{pmatrix} 0 & \mathbb{1}_d \\ \mathbb{1}_d & 0 \end{pmatrix}$$

- $\mathcal{N} = 4$ gauge algebra

$$[\mathbb{X}_A, \mathbb{X}_B] = \mathcal{T}_{AB}{}^C \mathbb{X}_C$$

- η_{AB} invariant metric on gauge algebra
- $\mathcal{N} = 4$ scalar potential

$$V(M) \sim \frac{1}{2} \left(\frac{1}{3} M^{AA'} M^{BB'} M^{CC'} + \left(\frac{2}{3} \eta^{AA'} - M^{AA'} \right) \eta^{BB'} \eta^{CC'} \right) \mathcal{T}_{ABC} \mathcal{T}_{A'B'C'}$$

- Consider geometric \mathbb{T}^6 symmetry: $GL(6) \subset O(6, 6)$
- ▶ Decompose fundamental rep $\mathbf{12} \rightarrow \mathbf{6} \oplus \bar{\mathbf{6}}$

$$\mathbb{X}_A = \{Z_a, X^a\}$$

$$f_{ABC} = (f_{abc}, f_{ab}{}^c, f_a{}^{bc}, f^{abc})$$

- $\mathcal{N} = 4$ electric gaugings: include all possible (NS-NS) fluxes

[Derendinger, Petropoulos, Prezas '07]

$$h_{abc}, \tau_{ab}{}^c, Q_a{}^{bc}, R^{abc}$$

- There is a similar set of S-dual fluxes: f_{-ABC}
- $\xi_{\alpha I}$: reduction with $SO(1, 1)$ duality twist
 [Derendinger, Petropoulos, Prezas '07]
- ▶ Similar story: $\mathcal{N} = 8$ gauged supergravity (type II/M-theory)

Motivation and overview

Fluxes, twists and supergravity gaugings

Non-geometric backgrounds and twisted doubled tori

Interacting chiral boson models: Lorentz invariance

Interacting chiral boson models: one-loop effective action

Summary and open problems

Duality related backgrounds

- ▶ h, τ, Q, R are related by T-duality
- ▶ **Toy model** (not a solution but can be promoted to one)
[Kachru, Schulz, Tripathy, Trivedi '02; Lowe, Nastase, Ramgoolam '03]
- 3-torus with N units of **NS-NS flux** h_{xyz}

$$ds^2 = dx^2 + dy^2 + dz^2, \quad H = Ndx \wedge dy \wedge dz, \quad B = Nx dy \wedge dz$$

- $x \sim x + 1$: gauge transformation on B
- ▶ T-duality along $z \Rightarrow$ twisted torus (**geometric flux** τ_{xy}^z)

$$ds^2 = dx^2 + dy^2 + (dz + Nx dy)^2, \quad H = 0$$

$$\tau_{y,z}(x) = Nx + i$$

- $x \sim x + 1$: **$SL(2; \mathbb{Z})$** modular transformation on fiber $\mathbb{T}_{y,z}^2$
- Bianchi II or nilmanifold

- T-duality along $y \Rightarrow$ T-fold (**Q-flux** Q_x^{zy})

$$ds^2 = dx^2 + \frac{1}{1+N^2x^2}(dz^2 + dy^2), \quad B = \frac{Nx}{1+N^2x^2}dy \wedge dz$$

- $x \sim x + 1$: T-duality group $O(2, 2; \mathbb{Z})$ action on $M_{AB} \equiv \{G_{ab}, B_{ab}\}$

$$M_{AB}(x+1) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -N & 1 & 0 \\ N & 0 & 0 & 1 \end{pmatrix} M_{AB}(x)$$

- There is a **local** geometric description
- Globally well-defined up to **T-duality**
- In this sense: **non-geometric**

- ▶ T-duality along x : cannot be done using Buscher rules
- At the level of the effective theory: R-flux R^{xyz}
- In this case there is no local geometric description
- Explicit dependence on dual coordinate \tilde{x}
- ▶ *Other non-geometric string constructions:* S-folds, U-folds, mirror-folds, asymmetric orbifolds, CFT constructions

Doubling and doubled tori

[Witten '88; Duff '90; Tseytin '90, '91; Kugo, Zwiebach '92; Maharana, Schwarz '92]

- ▶ y^i conjugate to **momenta**, \tilde{y}^i conjugate to **windings**
- Double coordinates $Y^I = \{y^i, \tilde{y}^i\} \Rightarrow$ **doubled torus** \mathbb{T}^{2n}

$$dS^2 = 2dy^i d\tilde{y}_i, \quad O(n, n; \mathbb{Z}) \in GL(2n; \mathbb{Z})$$

- Realize T-duality group $O(n, n; \mathbb{Z})$ geometrically
- ▶ Geometry for non-geometric string backgrounds

[Hull '04, '06, '07]

- ▶ **Scherk–Schwarz**: torus \rightarrow twisted torus \rightarrow geometric fluxes
- ▶ Natural idea: why not **double everything** and then **twist**
- Generalization of Scherk–Schwarz on the **doubled torus!**

Twisted doubled tori (TDT)

[Hull, Reid-Edwards '07; Dall'Agata, Prezas, Samtleben, Trigiante '07]

- ▶ Consider ordinary compactification on \mathbb{T}^d
- Full gauge group $U(1)^{2d}$: isometry of the **doubled torus** \mathbb{T}^{2d}
- Suggests: generic gauging \mathcal{G} from **twisting the doubled torus!**
- Generalization of Scherk–Schwarz via a **double field theory**
- ▶ Gauge algebra and G_{ab} potential in Scherk-Schwarz reduction (**twisted torus**) of pure gravity:

$$[Z_a, Z_b] = \tau_{ab}^k Z_c$$

$$V \sim 2 \tau_{bc}^a \tau_{ad}^b g^{cd} + \tau_{bc}^a \tau_{ef}^d g_{ad} g^{be} g^{cf}$$

- Gauge algebra $\mathbb{X}_A = \{Z_a, X^a\}$ and $M_{AB} \equiv \{G_{ab}, B_{ab}\}$ potential for electric $\mathcal{N} = 4$ gaugings (**twisted doubled torus**)

$$[\mathbb{X}_A, \mathbb{X}_B] = \mathcal{T}_{AB}^{C} \mathbb{X}_C$$

$$V \sim 3 \mathcal{T}_{DA}^{C} \mathcal{T}_{CB}^{D} M^{AB} + \mathcal{T}_{CE}^{A} \mathcal{T}_{DF}^{B} M_{AB} M^{CD} M^{EF}$$

- ▶ General gauging: Scherk–Schwarz – but doubled!
- ▶ Geometric flux on doubled torus \Leftrightarrow all fluxes
- ▶ Twisted doubled torus (TDT): local group manifold \mathcal{G}

$$E^A = E_I^A(Y) dY^I$$

$$dE^A = -\frac{1}{2} \mathcal{T}_{BC}^A E^B \wedge E^C$$

$$\mathcal{T}_{BC}^A = \{ h_{abc}, \tau_{bc}{}^a, Q_a{}^{bc}, R^{abc} \}$$

- ▶ Geometric interpretation of embedding tensor
 - ▶ Gauging \Leftrightarrow Fluxes \Leftrightarrow TDT \implies spacetime data (partially)
- [Dall'Agata, Prezas, Samtleben, Trigiante '07]
- ▶ Is it really the underlying geometry or just bookkeeping tool ?

Motivation and overview

Fluxes, twists and supergravity gaugings

Non-geometric backgrounds and twisted doubled tori

Interacting chiral boson models: Lorentz invariance

Interacting chiral boson models: one-loop effective action

Summary and open problems

Chiral bosons

- ▶ Twisted doubled torus as string target space ?
- Treat momentum and winding independently
- ▶ Chiral boson theory

[Siegel '84; Floreanini, Jackiw '87]

- Floreanini–Jackiw (FJ) approach ($\partial_{\pm} = \partial_0 \pm \partial_1$)

$$S_+ = \frac{1}{2} \int d^2\sigma \partial_1 \phi_+ \partial_- \phi_+$$

$$\partial_1(\partial_- \phi_+) = 0 \Rightarrow \partial_- \phi_+ = 0$$

- Lorentz invariant on-shell

$$\delta_L S_+ = \frac{1}{2} \int d^2\sigma (\partial_- \phi_+)^2$$

- Similarly for antichiral boson $S_- = -\frac{1}{2} \int d^2\sigma \partial_1 \phi_- \partial_+ \phi_-$

- ▶ Introducing usual and **dual** fields

$$\phi = \frac{1}{\sqrt{2}}(\phi_+ + \phi_-), \quad \chi = \frac{1}{\sqrt{2}}(\phi_+ - \phi_-)$$

$$S = S_+ + S_- = \frac{1}{2} \int d^2\sigma \left(\partial_0 \phi \partial_1 \chi + \partial_1 \phi \partial_0 \chi - (\partial_1 \phi)^2 - (\partial_1 \chi)^2 \right)$$

- Symmetric under $\phi \leftrightarrow \chi$
- Double integration by parts, set $p = \partial_1 \chi$: **first order**

$$S = \frac{1}{2} \int d^2\sigma \left(2\partial_0 \phi p - (\partial_1 \phi)^2 - p^2 \right)$$

- Non-local **duality** relation $p \equiv \partial_1 \chi = \partial_0 \phi$
- **Second order** action of free boson ϕ

$$S = \frac{1}{2} \int d^2\sigma \left((\partial_0 \phi)^2 - (\partial_1 \phi)^2 \right)$$

- T-duality invariant formulation
- Start with compact boson at radius R

$$S = \frac{R^2}{2} \int d^2\sigma \left((\partial_0\phi)^2 - (\partial_1\phi)^2 \right)$$

- First order

$$S = \frac{1}{2} \int d^2\sigma \left(-\frac{p^2}{R^2} + 2p\partial_0\phi - (R\partial_1\phi)^2 \right)$$

- Introduce dual field $\partial_1\chi = p$, integrate by parts

$$S = \frac{1}{2} \int d^2\sigma \left(\partial_1\chi\partial_0\phi + \partial_0\chi\partial_1\phi - (R^{-1}\partial_1\chi)^2 - (R\partial_1\phi)^2 \right)$$

- Manifest T-duality invariance: $\phi \leftrightarrow \chi, R \leftrightarrow R^{-1}$
- Dual boson at radius R^{-1}

$$S = \frac{R^{-2}}{2} \int d^2\sigma \left((\partial_0\chi)^2 - (\partial_1\chi)^2 \right)$$

Interacting chiral bosons

- ▶ Consider the following action ($\mathbb{Y}^I = \{y^i, \tilde{y}^i\}$)

[Tseytlin '90, '91]

$$S = \frac{1}{2} \int d^2\sigma \left(-H_{IJ}(\mathbb{Y}) \partial_1 \mathbb{Y}^I \partial_1 \mathbb{Y}^J + (\eta_{IJ}(\mathbb{Y}) + C_{IJ}(\mathbb{Y})) \partial_0 \mathbb{Y}^I \partial_1 \mathbb{Y}^J \right)$$

- H_{IJ}, η_{IJ} : symmetric, C_{IJ} antisymmetric
- Interacting generalization of Floreanini-Jackiw
- ▶ Free theory

$$H_{IJ} = \begin{pmatrix} \mathbb{1}_d & 0 \\ 0 & \mathbb{1}_d \end{pmatrix}, \quad \eta_{IJ} = \begin{pmatrix} 0 & \mathbb{1}_d \\ \mathbb{1}_d & 0 \end{pmatrix}, \quad C_{IJ} = \text{const.}$$

- Yields $d + d$ copies of FJ for $y_\pm^i = y^i \pm \tilde{y}^i$: **double torus**
- Integrate out \tilde{y}^i (or y^i): original (or dual) torus vacuum

- Lorentz invariance condition

$$\eta_{IJ} \left(\partial_0 \mathbb{Y}^I \partial_0 \mathbb{Y}^J + \partial_1 \mathbb{Y}^I \partial_1 \mathbb{Y}^J \right) - 2H_{IJ} \partial_0 \mathbb{Y}^I \partial_1 \mathbb{Y}^J = 0$$

- Rearrange to

$$(\eta - H\eta^{-1}H)_{IJ} \partial_1 \mathbb{Y}^I \partial_1 \mathbb{Y}^J + \eta^{IJ} V_I V_J = 0 \text{ with}$$

$$V_I \equiv \eta_{IJ} \partial_0 \mathbb{Y}^J - H_{IJ} \partial_1 \mathbb{Y}^J$$

- Sufficient

$$(\eta - H\eta^{-1}H)_{IJ} = 0 \quad \text{and} \quad \eta^{IJ} V_I V_J = 0$$

- Notice: equation of motion

$$\begin{aligned} 2\partial_1 V_I &+ \partial_I H_{JK} \partial_1 \mathbb{Y}^J \partial_1 \mathbb{Y}^K \\ &- G_{IJK} \partial_0 \mathbb{Y}^J \partial_1 \mathbb{Y}^K - 2\eta_{JL} \Gamma_{IK}^L (\eta) \partial_0 \mathbb{Y}^J \partial_1 \mathbb{Y}^K = 0 \end{aligned}$$

- ▶ Lorentz invariant models

- ▶ Standard sigma models

[Tseytlin '90, '91]

$$H_{IJ} = \begin{pmatrix} G_{ij} - B_{ik}G^{kl}B_{lj} & B_{ik}G^{kj} \\ -G^{ik}B_{kj} & G^{ij} \end{pmatrix}, \eta_{IJ} = \begin{pmatrix} 0 & \mathbb{1}_d \\ \mathbb{1}_d & 0 \end{pmatrix}, C_{IJ} = \text{const.}$$

- Metric $G_{ij} = G_{ij}(y)$, B-field $B_{ij} = B_{ij}(y)$
- Integrating out \tilde{y}^i

$$S = \int d^2\sigma (\sqrt{\gamma}\gamma^{\mu\nu}G_{ij} + \epsilon^{\mu\nu}B_{ij})\partial_\mu y^i\partial_\nu y^j$$

- ▶ Interacting non-abelian chiral scalars

[Depireux, Gates, Park '89; Gates, Siegel '88; Tseytlin '90, '91]

- ▶ $\eta = \pm H$: quantum Lorentz anomaly

[Dall'Agata, Prezas '08]

► Twisted doubled tori

[Dall'Agata, Prezas '08]

- Gauge algebra \rightarrow group representative \rightarrow vielbein

$$dE^A = -\frac{1}{2} \mathcal{T}_{BC}^A E^B \wedge E^C, \quad E^A = E_I^A(Y) dY^I$$

- Construct η_{IJ} and H_{IJ} as

$$\eta_{IJ} = \eta_{AB} E_I^A E_J^B; \quad \eta_{AB} = \begin{pmatrix} 0 & \mathbb{1}_d \\ \mathbb{1}_d & 0 \end{pmatrix}$$

$$H_{IJ} = H_{AB} E_I^A E_J^B; \quad \eta_{AB} = H_{AC} \eta^{CD} H_{DB}$$

- $H_{AB} \in \frac{O(d,d)}{O(d) \times O(d)}$ (cf. $\mathcal{N} = 4$ moduli M_{AB})
- First Lorentz invariance condition is satisfied by construction
- Add generalized three-form flux

$$G_{ABC} = 3\partial_{[A} C_{BC]} = \mathcal{T}_{ABC}$$

- Equation of motion reads

$$(D_I V_J - \frac{1}{2} T_{IJ}^{K} V_K) \partial_1 \mathbb{Y}^I = 0$$

- Second Lorentz invariance condition is satisfied
- Compact gaugings

$$T_{AB}^{D} H_{DC} = -T_{AC}^{D} H_{DB}$$

- Choose $G_{ABC} = -T_{ABC} \Rightarrow V^I = 0$
- Electric $\mathcal{N} = 4$ gauging and moduli matrix $M_{AB} \Leftrightarrow$ Lorentz invariant chiral boson model
- Related (equivalent ?) second order constrained formulation

[Albertsson, Kimura, Reid-Edwards '08, Hull, Reid-Edwards '09, Reid-Edwards '09]

- ▶ Application: recover spacetime background fields

[Dall'Agata, Prezas '08]

- Impossible to integrate out dual coordinates for R -flux
- No local geometric description

Motivation and overview

Fluxes, twists and supergravity gaugings

Non-geometric backgrounds and twisted doubled tori

Interacting chiral boson models: Lorentz invariance

Interacting chiral boson models: one-loop effective action

Summary and open problems

One-loop effective action

- ▶ Can the TDT models be promoted to conformal field theories ?
- ▶ Starting point

$$S = \frac{1}{2} \int d^2\sigma \left(-H_{IJ}(\mathbb{Y}) \partial_1 \mathbb{Y}^I \partial_1 \mathbb{Y}^J + (\eta_{IJ}(\mathbb{Y}) + C_{IJ}(\mathbb{Y})) \partial_0 \mathbb{Y}^I \partial_1 \mathbb{Y}^J \right)$$

- Earlier work: H_{IJ} and η_{IJ} depend on base coordinates X^m

[Berman, Copland, Thompson '07; Berman, Copland '07]

- Background field method $\mathbb{Y}^I = \mathbb{Y}_{cl}^I + \pi^I(\xi^I)$

[Alvarez-Gaume, Freedman, Mukhi '81; Braaten, Curtright, Zachos '85; Mukhi '86; Howe, Papadopoulos, Stelle '88]

- Covariant expansion, recursive algorithm

[Mukhi '86]

$$S_n = \frac{1}{n!} \mathcal{D}^n S \equiv \frac{1}{n!} \left(\int d^2\sigma \xi^I(\sigma) \frac{D}{D\mathbb{Y}^I(\sigma)} \right)^n S$$

► η -covariant expansion

$$S_1 = \int d^2\sigma \left(\frac{1}{2} \eta_{IJ} (\partial_0 \mathbb{Y}^I D_1 \xi^J + D_0 \xi^I \partial_1 \mathbb{Y}^J) - H_{IJ} \partial_1 \mathbb{Y}^I D_1 \xi^J - \frac{1}{2} D_K H_{IJ} \xi^K \partial_1 \mathbb{Y}^I \partial_1 \mathbb{Y}^J + \frac{1}{2} G_{IJK} \xi^K \partial_0 \mathbb{Y}^I \partial_1 \mathbb{Y}^J \right)$$

$$\begin{aligned} S_2 = & \frac{1}{2} \int d^2\sigma \left(-H_{IJ} D_1 \xi^I D_1 \xi^J + \eta_{IJ} D_0 \xi^I D_1 \xi^J \right. \\ & + \left(\frac{1}{2} D_J G_{IKL} + R_{KIJL} \right) \xi^I \xi^J \partial_0 \mathbb{Y}^K \partial_1 \mathbb{Y}^L \\ & - \frac{1}{2} (D_I D_J H_{KL} + H_{KM} R^M{}_{IJL} + H_{LM} R^M{}_{IJK}) \xi^I \xi^J \partial_1 \mathbb{Y}^K \partial_1 \mathbb{Y}^L \\ & \left. + \frac{1}{2} G_{IJK} \xi^K (\partial_0 \mathbb{Y}^I D_1 \xi^J + D_0 \xi^I \partial_1 \mathbb{Y}^J) - 2 D_K H_{IJ} \xi^K D_1 \xi^I \partial_1 \mathbb{Y}^J \right) \end{aligned}$$

- One-loop effective action $S_{\text{eff}} = S_{\text{cl}} + \Gamma$

$$\exp(i\Gamma[\mathbb{Y}]) = \int \mathcal{D}\xi \exp(iS_2[\mathbb{Y}; \xi])$$

- Propagators of tangent space fields ξ^A

[Tseytlin '90, '91; Berman, Copland, Thompson '07]

$$\langle \xi^A(\sigma) \xi^B(\sigma') \rangle = H^{AB} \Delta(\sigma - \sigma') + \eta^{AB} \bar{\Delta}(\sigma - \sigma')$$

$$\begin{aligned} \Delta(\sigma - \sigma') &= \frac{1}{2} (\Delta_+(\sigma - \sigma') + \Delta_-(\sigma - \sigma')) = -\frac{1}{4\pi} \ln(\sigma - \sigma')^2 \\ \bar{\Delta}(\sigma - \sigma') &= \frac{1}{2} (\Delta_+(\sigma - \sigma') - \Delta_-(\sigma - \sigma')) = -\frac{1}{2\pi} \operatorname{arctanh} \frac{\sigma^1 - \sigma'^1}{\sigma^0 - \sigma'^0} \end{aligned}$$

- Regularize: $\ln(\sigma - \sigma')^2 \rightarrow \ln((\sigma - \sigma')^2 + \mu^2)$, set
 $\frac{\sigma^1 - \sigma'^1}{\sigma^0 - \sigma'^0} = \tanh \delta$
- Limiting expressions

$$\Delta(0) \rightarrow -\frac{1}{2\pi} \ln \mu , \quad \bar{\Delta}(0) \rightarrow -\frac{1}{2\pi} \delta$$

- Breakdown of **scale invariance** and **Lorentz invariance**

► Weyl anomaly and global Lorentz anomaly

$$W = \frac{1}{2} \int d^2\sigma \left(W_{IJ}^{00} \partial_0 \mathbb{Y}^I \partial_0 \mathbb{Y}^J + W_{IJ}^{01} \partial_0 \mathbb{Y}^I \partial_1 \mathbb{Y}^J + W_{IJ}^{11} \partial_1 \mathbb{Y}^I \partial_1 \mathbb{Y}^J \right)$$

$$L = \frac{1}{2} \int d^2\sigma \left(L_{IJ}^{00} \partial_0 \mathbb{Y}^I \partial_0 \mathbb{Y}^J + L_{IJ}^{01} \partial_0 \mathbb{Y}^I \partial_1 \mathbb{Y}^J + L_{IJ}^{11} \partial_1 \mathbb{Y}^I \partial_1 \mathbb{Y}^J \right)$$

- Vanishing on-shell: equations of motion and/or constraints
- Weyl anomaly coefficients:

$$W_{IJ}^{00} = \frac{1}{4} (H^{A[C} H^{D]B} - \eta^{A[C} \eta^{D]B}) Q_{AB,I}^0 Q_{CD,J}^0$$

$$W_{IJ}^{01} = H^{AB} S_{AB,IJ}^{01} + \frac{1}{2} (H^{A[C} H^{D]B} - \eta^{A[C} \eta^{D]B}) Q_{AB,I}^0 Q_{CD,J}^1$$

$$- \frac{1}{4} (H^{A[C} \eta^{D]B} + \eta^{A[C} H^{D]B}) (Q_{AB,I}^0 P_{CD,J}^1 + P_{AB,I}^1 Q_{CD,J}^0)$$

$$W_{IJ}^{11} = H^{AB} S_{AB,IJ}^{11} + \frac{1}{4} (H^{A[C} H^{D]B} - \eta^{A[C} \eta^{D]B}) Q_{AB,I}^1 Q_{CD,J}^1$$

$$- \frac{1}{4} (H^{A[C} \eta^{D]B} + \eta^{A[C} H^{D]B}) (Q_{AB,I}^1 P_{CD,J}^1 + P_{AB,I}^1 Q_{CD,J}^1)$$

$$- \frac{1}{4} (H^{A[C} H^{D]B} + 3\eta^{A[C} \eta^{D]B}) P_{AB,I}^1 P_{CD,J}^1$$

- Lorentz anomaly coefficients

$$L_{IJ}^{00} = 0$$

$$L_{IJ}^{01} = \eta^{AB} S_{AB,IJ}^{01} - \frac{1}{2} \eta^{A[C} \eta^{D]B} (Q_{AB,I}^0 \mathcal{P}_{CD,J}^1 + \mathcal{P}_{AB,I}^1 Q_{CD,J}^0)$$

$$\begin{aligned} L_{IJ}^{11} &= \eta^{AB} S_{AB,IJ}^{11} - \frac{1}{2} (H^{A[C} \eta^{D]B} + \eta^{A[C} H^{D]B}) \mathcal{P}_{AB,I}^1 \mathcal{P}_{CD,J}^1 \\ &\quad - \frac{1}{2} \eta^{A[C} \eta^{D]B} (Q_{AB,I}^1 \mathcal{P}_{CD,J}^1 + \mathcal{P}_{AB,I}^1 Q_{CD,J}^1) \end{aligned}$$

- Basic building blocks:

$$\begin{aligned} S_{AB,IJ}^{11} &= -\frac{1}{2} D_A D_B H_{IJ} - \frac{1}{2} (H_{IK} R^K{}_{ABJ} + H_{JK} R^K{}_{ABI}) - H_{CD} \Omega_I{}^C{}_A \Omega_J{}^D{}_B \\ &\quad - D_A H_{CI} \Omega_J{}^C{}_B - D_A H_{CJ} \Omega_I{}^C{}_B \end{aligned}$$

$$S_{AB,IJ}^{01} = R_{IABJ} + \frac{1}{2} D_B G_{AIJ} + \eta_{CD} \Omega_I{}^C{}_A \Omega_J{}^D{}_B + \frac{1}{2} (G_{IDA} \Omega_J{}^D{}_B - G_{JDA} \Omega_I{}^D{}_B)$$

$$Q_{AB,I}^1 = -2 \Omega_I{}^C{}_A H_{CB} - 2 D_A H_{BI}$$

$$Q_{AB,I}^0 = -\frac{1}{2} G_{IAB} + \Omega_I{}^C{}_A \eta_{CB}$$

$$\mathcal{P}_{AB,I}^1 = \frac{1}{2} G_{IAB} + \Omega_I{}^C{}_A \eta_{CB}$$

Specialize to twisted doubled tori

- ▶ Use general flux $G_{ABC} = \lambda T_{ABC}$
- ▶ **Lorentz anomaly** ($T_{AB\hat{C}} = T_{AB}{}^D H_{DC}$, $T_{AB\hat{I}} = T_{AB}{}^C H_{CD} E_I^D$)

$$L_{IJ}^{01} = \frac{\lambda^2 - 1}{4} T_{IAB} T_J{}^{AB}$$

$$L_{IJ}^{11} = -\frac{(\lambda - 1)^2}{4} T_{I\hat{A}B} T_J{}^{AB} - \frac{\lambda - 1}{4} (T_{\hat{I}AB} T_J{}^{AB} + T_{JAB} T_I{}^{AB})$$

- Vanish for all gaugings when $\lambda = 1$
- Compact gaugings

$$L_{IJ}^{11} = -\frac{\lambda^2 - 1}{4} T_{I\hat{A}\hat{B}} T_J{}^{AB} ,$$

- Vanish for $\lambda = \pm 1$ as well
- **Classical Lorentz invariance, once established, survives in the quantum theory**

► Weyl anomaly

$$\begin{aligned}
 W_{IJ}^{00} &= \frac{(\lambda - 1)^2}{16} (\mathcal{T}_{IAB} \mathcal{T}_J{}^{AB} - \mathcal{T}_{IAB} \mathcal{T}_J{}^{\hat{A}\hat{B}}) \\
 W_{IJ}^{01} &= \frac{\lambda - 1}{4} \left(\mathcal{T}_{IAB} \mathcal{T}_J{}^{AB} - \mathcal{T}_{IAB} \mathcal{T}_J{}^{\hat{A}\hat{B}} + (\lambda + 1) \mathcal{T}_{IAB} \mathcal{T}_J{}^{\hat{A}B} \right) \\
 W_{IJ}^{11} &= \frac{(\lambda + 1)(8 - 3(\lambda + 1))}{16} \mathcal{T}_{IAB} \mathcal{T}_J{}^{AB} + \frac{(\lambda + 1)(8 - (\lambda + 1)) - 16}{16} \mathcal{T}_{IAB} \mathcal{T}_J{}^{\hat{A}\hat{B}} \\
 &\quad - \frac{1}{4} (\mathcal{T}_{IAB} \mathcal{T}_J{}^{AB} - \mathcal{T}_{IAB} \mathcal{T}_J{}^{\hat{A}\hat{B}}) - \frac{\lambda - 1}{4} (\mathcal{T}_{IAB} \mathcal{T}_J{}^{\hat{A}B} + \mathcal{T}_{JAB} \mathcal{T}_I{}^{\hat{A}B})
 \end{aligned}$$

- General gaugings with $\lambda = 1$: $W_{IJ}^{00} = W_{IJ}^{01} = 0$
- Condition for (one-loop) conformal invariance**

$$W_{AB}^{11} = \frac{1}{4} (\eta_{AE} \eta_{BE'} - H_{AE} H_{BE'}) (\eta^{CC'} \eta^{DD'} - H^{CC'} H^{DD'}) \mathcal{T}_{CD}{}^E \mathcal{T}_{C'D'}{}^{E'} = 0$$

- Compact gaugings: $W_{IJ}^{00} = W_{IJ}^{01} = 0$ for $\lambda = \pm 1$ and

$$W_{IJ}^{11} = -\frac{\lambda^2 - 1}{4} \mathcal{T}_{IAB} \mathcal{T}_J{}^{AB}$$

- Conformal models for $\lambda = \pm 1$**

Correspondence with $\mathcal{N} = 4$ electric gaugings

- ▶ Introduce projectors (recall $\eta = H\eta^{-1}H$)

$$P_{\pm}^{ABCD}(H) = \frac{1}{2}(\eta^{A(C}\eta^{D)B} \pm H^{A(C}H^{D)B})$$

- Define

$$Z_{AB}(H) = \frac{1}{2}(\eta^{C[C'}\eta^{D']D} - H^{C[C'}H^{D']D})\mathcal{T}_{CDA}\mathcal{T}_{C'D'B}$$

- Weyl anomaly

$$W_{AB} = P_{-AB}{}^{CD}(H)Z_{CD}(H)$$

- $\mathcal{N} = 4$ gauged supergravity potential (electric gaugings)

$$V(M) = \frac{1}{2} \left(\frac{1}{3} M^{AA'} M^{BB'} M^{CC'} + \left(\frac{2}{3} \eta^{AA'} - M^{AA'} \right) \eta^{BB'} \eta^{CC'} \right) \mathcal{T}_{ABC} \mathcal{T}_{A'B'C'}$$

- Incorporate constraint $M_{AB} \in \frac{O(d,d)}{O(d) \times O(d)}$

$$\hat{V} = V + \Lambda_A{}^B (M^{AC} \eta_{CB} - \eta^{AC} M_{CB})$$

- Condition for extremum: set to zero

$$\begin{aligned}\frac{\partial \hat{V}}{\partial M^{AB}} &= -Z_{AB}(M) + \frac{1}{2}(\eta_{AC}\eta_{BD} + M_{AB}M_{CD})\Lambda^{CD} \\ &= -Z_{AB}(M) + P_{+ABCD}(M)\Lambda^{CD}\end{aligned}$$

- Identify $H_{AB} = M_{AB}$ and act with $P_-(H)$

$$P_-^{ABCD}(H) Z_{CD}(H) = 0$$

- $\mathcal{N} = 4$ electric vacuum \Rightarrow conformal chiral boson model

- ▶ Assume now conformal invariance condition is satisfied. Then

$$Z_{AB}(H) = P_{+ABCD}(H) Z^{CD}(H)$$

- Thus

$$\left. \frac{\partial \hat{V}}{\partial M^{AB}} \right|_{M=H} = P_{+ABCD}(H) \left(\Lambda^{CD} - Z^{CD}(H) \right)$$

- $\mathcal{N} = 4$ vacuum for $\Lambda^{CD} = Z^{CD}(H)$
- ▶ $\mathcal{N} = 4$ electric vacua \Leftrightarrow conformal chiral boson models
- ▶ SUSY: TDT is even-dim group manifold $\Rightarrow \mathcal{N} = 2$ SCFT

Compact gaugings and chiral WZW models

- ▶ **Compact gaugings** \Rightarrow **conformal chiral boson models**
- ▶ Compact gauging conditions for $H_{AB} = \delta_{AB}$

$$h_{abc} = Q_a{}^{bc}, \quad R^{abc} = \tau_{bc}{}^a, \quad \tau_{ab}{}^c = -\tau_{ac}{}^b, \quad Q_c{}^{ab} = -Q_a{}^{cb}$$

- Change basis $\{Z_a, X^a\}$ to $\{T_a^\pm = \frac{1}{2}(Z_a \pm X^a)\}$
- Gauge algebra is a sum

$$[T_a^\pm, T_b^\pm] = f_{ab}^{\pm c} T_c^\pm, \quad [T_a^+, T_b^-] = 0$$

$$f_{ab}^{\pm c} = \tau_{ab}{}^c \pm Q_a{}^{bc}$$

- Gauge group $\mathcal{G}_L \times \mathcal{G}_R \in O(d) \times O(d)$

- ▶ Corresponding chiral boson model (chiral basis $y_{L,R}^i = y^i \pm \tilde{y}^i$)

$$S = \frac{1}{4} \int d^2\sigma \left((G_{ij}^L + \lambda B_{ij}^L) \partial_- y_L^i \partial_1 y_L^j - (G_{ij}^R + \lambda B_{ij}^R) \partial_+ y_R^i \partial_1 y_R^j \right)$$

- $G_{IJ}^{L,R}$ and $dB^{L,R}$ invariant metric and 3-form of $\mathcal{G}_{L,R}$
- Chiral and antichiral WZW based on \mathcal{G}_L and \mathcal{G}_R

[Sonnenchein '88; Tseytlin '91]

- Kac–Moody symmetry \Rightarrow all order conformal invariance!
- When $\mathcal{G}_L = \mathcal{G}_R$: ordinary WZW model
- In line with previous observations for $\mathcal{G}_L = \mathcal{G}_R = SU(2)$

[Dabholkar, Hull 05; Dall'Agata, Prezas '08]

- WZW model is realized in terms of non-geometric fluxes
- Notion of non-geometricity needs refinement

Motivation and overview

Fluxes, twists and supergravity gaugings

Non-geometric backgrounds and twisted doubled tori

Interacting chiral boson models: Lorentz invariance

Interacting chiral boson models: one-loop effective action

Summary and open problems

Summary

- ▶ **Supergravity gaugings:** non-geometric string backgrounds
 - Embedding tensor \Rightarrow gauge algebra \Rightarrow all fluxes!
 - New interpretation: geometric flux on **doubled torus**
 - **Twisted doubled tori:** underlying geometries
- ▶ **Interacting chiral bosons:** doubled geometry sigma models
 - Twisted doubled tori \Rightarrow Lorentz invariant chiral bosons
 - Conformal invariance condition \Leftrightarrow Vacua of $\mathcal{N} = 4$ potential
 - Compact gaugings are always conformal: **chiral WZW models**
- ▶ *Twisted doubled tori are actual physical backgrounds*
- ▶ *Scherk–Schwarz on doubled geometries \rightarrow general supergravity gaugings*

Open problems

- ▶ *Straightforward generalizations*
- Include dilaton (gaugings with $SO(1, 1)$ duality twist)
- Include non-compact spacetime dimensions
- Include worldsheet supersymmetry
- Full equations of motion of $\mathcal{N} = 4$ gauged supergravity
- ▶ Other classes of Lorentz invariant chiral boson models ?
- ▶ *Conformal field theory aspects*
- Origin of conformal invariance condition
- Deformations of (chiral, gauged) WZW models ?
- [Tseytlin '93]
- Relation to affine Virasoro constructions

► Relation to other worldsheet models

[Albertsson, Kimura, Reid-Edwards '08; Hull, Reid-Edwards '09; Reid-Edwards '09; Halmagyi '09]

● Non-commutativity of Q-flux and non-associativity of R-flux ?

[Mathai, Rosenberg '04; Bouwknegt, Hannabuss, Mathai '04; Ellwood, Hashimoto '06]

► Connection with double field theory

[Hohm, Hull, Zwiebach '10]

● Equations of motion: beta functionals

● Explicit Scherk–Schwarz on twisted doubled torus: electric
 $\mathcal{N} = 4$ gaugings

► Exotic constructions

● Incorporate $SL(2, \mathbb{Z})$ S-duality, double the doubled torus!

● Account for all $\mathcal{N} = 4$ gaugings

● E_7 invariant membrane theory!

● Account for all $\mathcal{N} = 8$ gaugings