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Crete Conference on Gauge theories and the Structure of Spacetime

Negative refraction from holography

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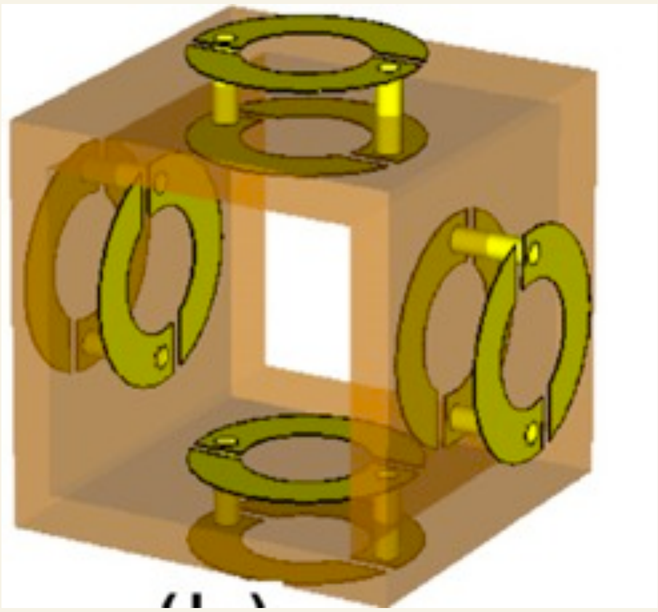
In 1968 Veselago predicted the possibility of materials with a negative index of refraction

Around 2000 the prediction was confirmed: samples of NR materials were built using arrays of conducting rings and wires (*metamaterials*)



Typically NR requires a strong magnetic activity, a property observed in holographic duals of quantum critical theories [Hartnoll]

This gives an opportunity to study the phenomenon in an exactly computable model and hopefully to understand some of its general features

We find qualitative agreement with properties of experimentally realized materials

Negative refraction

Conventional description

$$\mathbf{D} = \epsilon \mathbf{E} \qquad \mathbf{B} = \mu \mathbf{H}$$

$$\epsilon = \epsilon' + i\epsilon''$$

$$\mu = \mu' + i\mu''$$

Causality of response functions and unitarity

$$\epsilon'' > 0 \quad \mu'' > 0$$

$$\epsilon(\omega)\mu(\omega) \equiv n^2(\omega) = \frac{k^2}{\omega^2}$$

$$\mathbf{S} = \mathbf{E} \wedge \mathbf{H} = \hat{k}\frac{n}{\mu}\mathbf{E}^2$$

for transverse monochromatic wave

n' phase velocity

$\mathrm{Re}\left(\frac{n}{\mu}\right)$ power flow

Condition for NR $\quad n' < 0 \quad \mathrm{Re}\left(\frac{n}{\mu}\right) > 0$

$$\boxed{n_{DL} \equiv |\mu|\epsilon' + |\epsilon|\mu' < 0}$$

Depine-Lakhtakia '03

$\epsilon' < 0\,,\mu' < 0$ doubly-negative materials

sufficient but not necessary

Alternative description to take spatial dispersion into account : use only D, E, B [Agranovich, Gartstein]

$$D_i = \epsilon_{ij}(\omega, k) E_j$$

Longitudinal and transverse parts

$$\epsilon_{ij} = \epsilon_T(\omega, k) \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) + \epsilon_L(\omega, k) \frac{k_i k_j}{k^2}$$

dispersion relations

$$\epsilon_T(\omega, k) = \frac{k^2}{\omega^2} \qquad \epsilon_L(\omega, k) = 0$$

Expanding in k

$$\epsilon_T(\omega, k) = \epsilon(\omega) + \frac{k^2}{\omega^2}\left(1 - \frac{1}{\mu(\omega)}\right) + \mathcal{O}(k^4)$$

$$\frac{k^2}{\omega^2} = \epsilon(\omega)\mu(\omega) \quad \text{is recovered}$$

However this is a priori not the same μ !!

$$1 - \frac{1}{\mu} = \omega^2 \lim_{k\to 0} \frac{\epsilon_T - \epsilon_L}{k^2} \qquad \text{[Landau]}$$

Ours is an effective μ that reproduces the correct dispersion relations

It is not a response function, $\mu'' > 0$ not required

$$\boxed{\epsilon_T'' > 0}$$

Linear response

EM current induced by an external field

$$J_i(x,t) = \int dx'\, dt'\, G_{ij}(x-x', t-t') A_j(x', t')$$

where G is the retarded current correlator

$$G_{ij}(x,t) = -i\theta(t)\, \langle [J_i(x,t), J_j(0,0)] \rangle$$

dressed photon propagator

$$D_T(\omega,k) = \frac{4\pi}{\omega^2 \epsilon_T(\omega,k) - k^2}$$

$$\epsilon_T(\omega,k) = 1 - \frac{4\pi q^2}{\omega^2} G_T(\omega,k)$$

Holographic setup

Charged black hole in AdS

5d Einstein-Maxwell $S = \frac{1}{2e^2} \int d^5x \sqrt{-g} \left(R + 6 - \frac{1}{2} F_{mn} F^{mn} \right)$

metric $$ds^2 = \frac{(2-a)^2}{16 b^2 u} (d\mathbf{x}^2 - f(u) dt^2) + \frac{du^2}{4u^2 f}$$

$$A_t = \Sigma(1-u) \qquad f(u) = (1-u)(1+u-au^2)$$

$$T = \frac{2-a}{4\pi b} \qquad \Sigma = \frac{1}{2b}\sqrt{\frac{3}{2}a} \qquad \rho = \frac{l}{2e^2 b^2}\Sigma$$

temperature | chemical potential | charge density

To compute the current correlators we consider fluctuations of $A_x = \Sigma B_x$ and h_{tx}

The equations decouple in the new variables

$$\Phi_\pm(u) = \frac{h'_{tx}(u)}{u} - 3aB_x + \frac{1}{u} C_\pm(k^2) B_x$$

$$\Phi''(u) + g(u)\Phi'(u) + V_\pm(\omega^2, k^2, u)\Phi(u) = 0$$

at u=1 infalling b.c. $\quad \Phi_\pm(u) \sim (1-u)^{-i\omega/4\pi T}$

$$B'_x(u)(B^0, \cancel{h^0_{xt}}) \sim B^0 k^2 \ln u + \text{reg.}$$

only interested in current correlators

Correlators are obtained with the SS prescription from the boundary action

$$S_{bdy} = -\frac{3al}{32e^2b^4}\int \frac{d^4k}{(2\pi)^4} f(u) B_x(-k,u) B'_x(k,u) + S_{ct}$$

$$S_{ct} = (C + D\ln u)\int d^4x F_{ij}F^{ij}$$

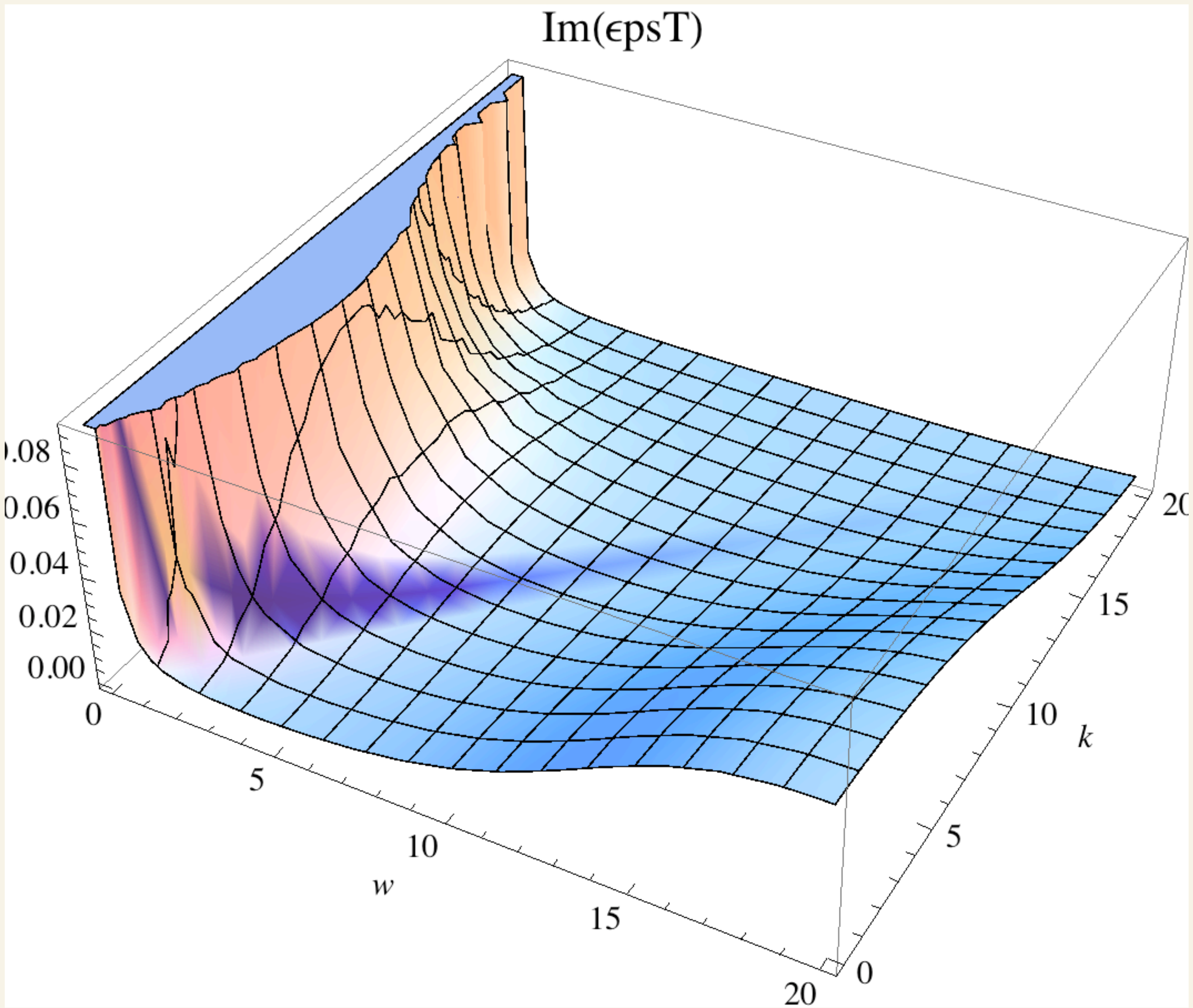
The constant C is an ambiguity in the renormalization prescription

$$\delta G_T = C(\omega^2 - k^2)$$

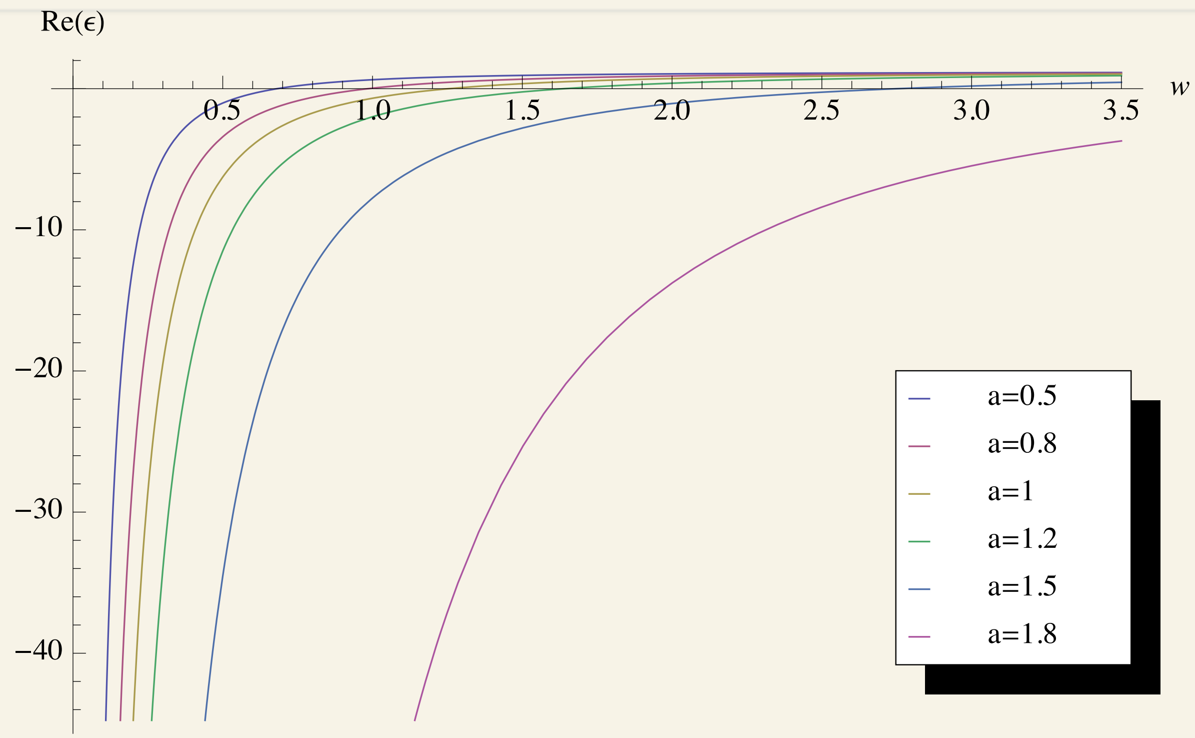
$$\delta\epsilon_T = -4\pi C$$

We fix it by requiring $\epsilon_T(\omega, k) \to 1$ for $\omega \to \infty$

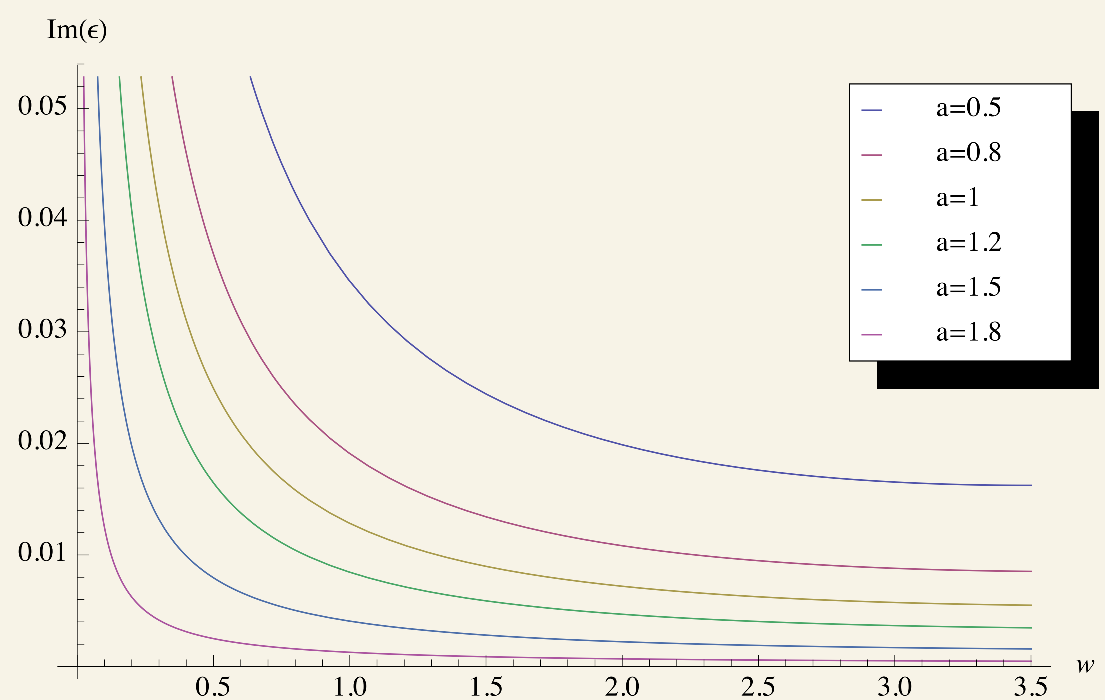
We solve the equations numerically in ω , order by order in k^2



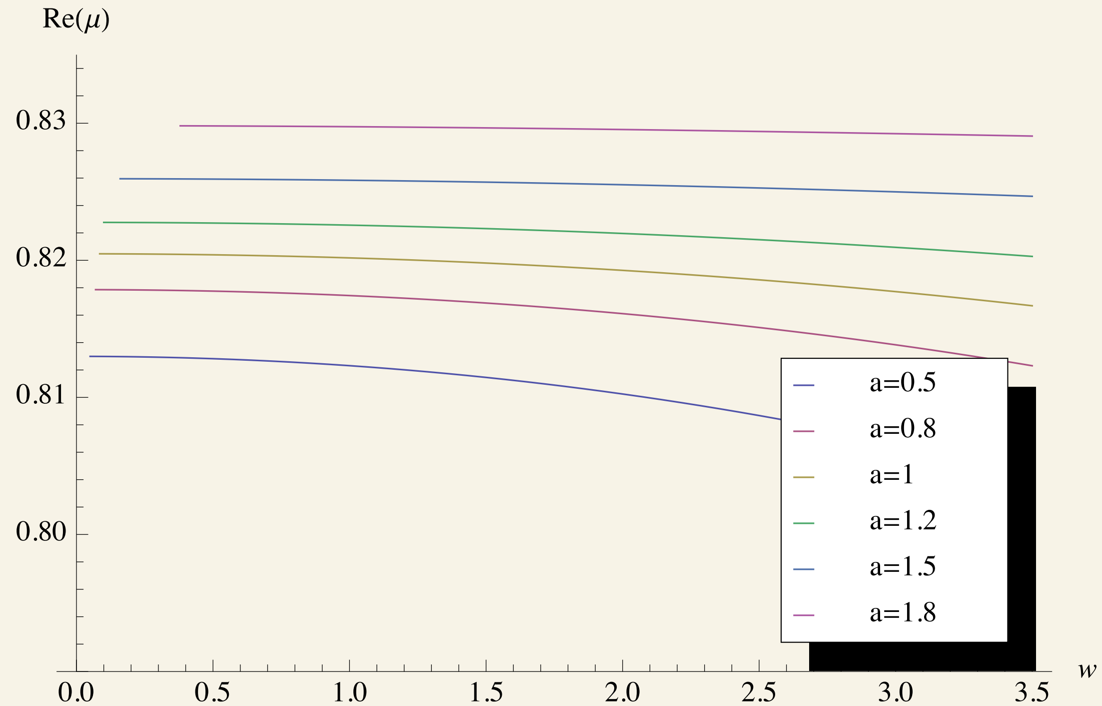
$\epsilon'(\omega)$



$\epsilon''(\omega)$

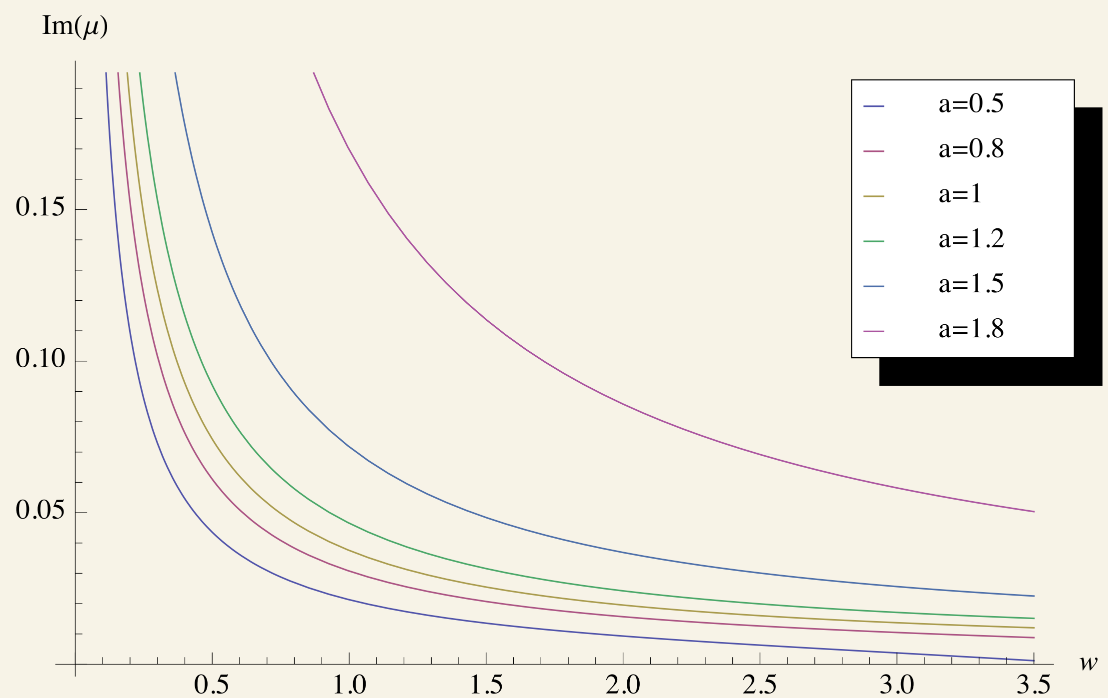


$\mu'(\omega)$



Single-neg material

$\mu''(\omega)$



...goes negative

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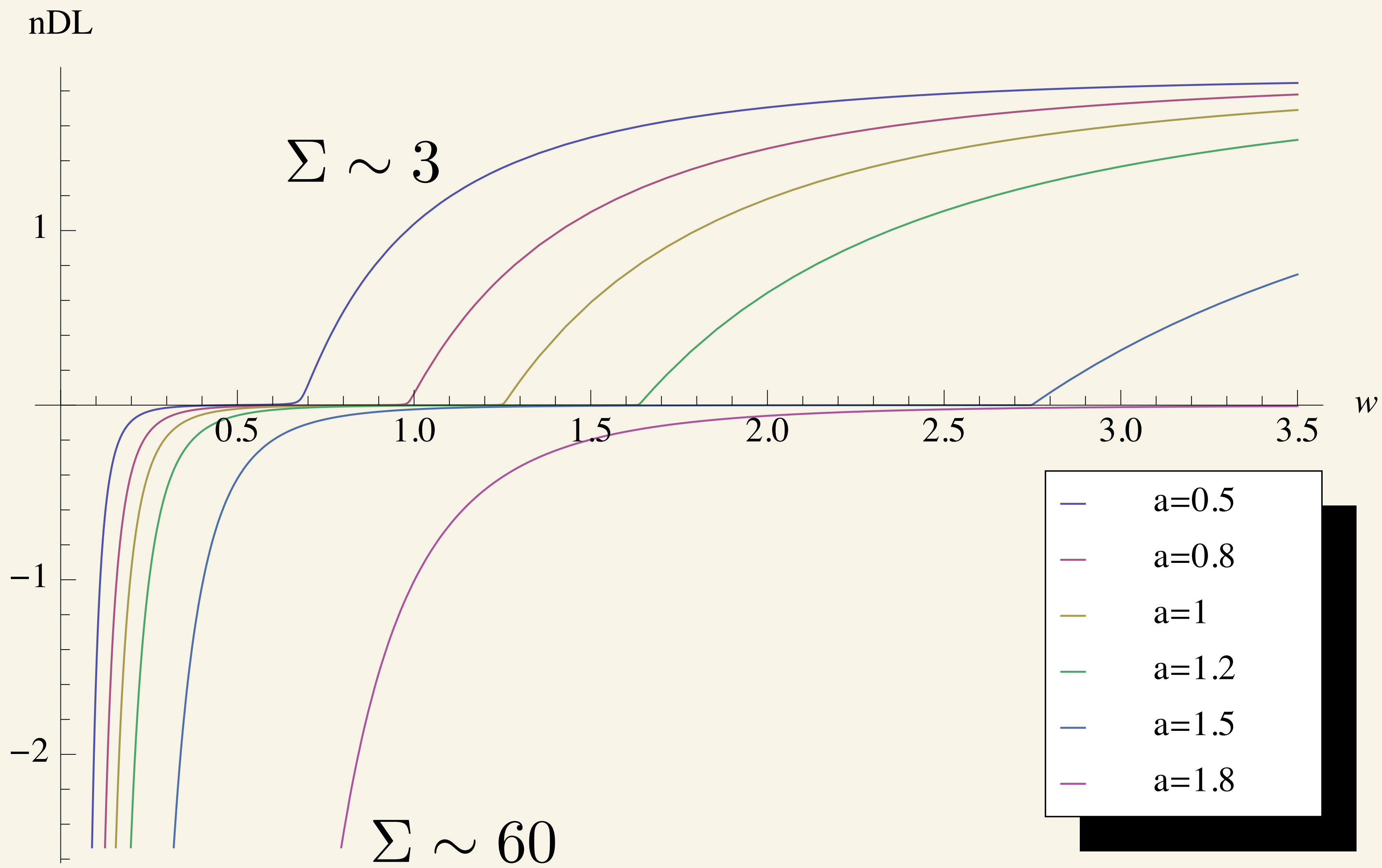

nDL
1
−1
−2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
w
$\Sigma \sim 3$
$\Sigma \sim 60$
a=0.5
a=0.8
a=1
a=1.2
a=1.5
a=1.8

We can check the numerics against analytic results obtained in the hydrodynamic limit $\omega, k << T, \Sigma$

Matsuo et al. '08

$$G_T = \frac{il\omega}{e^2}\left(\frac{3a}{2b^2}\frac{1}{2i(1+a)\omega - bk^2} - \frac{(2-a)^2}{8b(1+a)^2}\right)$$

Low-frequency behavior follows from relativistic hydro

Hartnoll, Kovtun, Müller, Sachdev '07

$$J^\mu = \rho u^\mu + \nu^\mu \qquad J_T = \rho v_T$$

In the absence of external magnetic field v_T decouples

$$\omega(\varepsilon + P)\, v_T - ik^2\eta\, v_T = 0$$

hydro correlator $$G_T^{hy} = \frac{iB\omega}{i\omega - Dk^2}$$

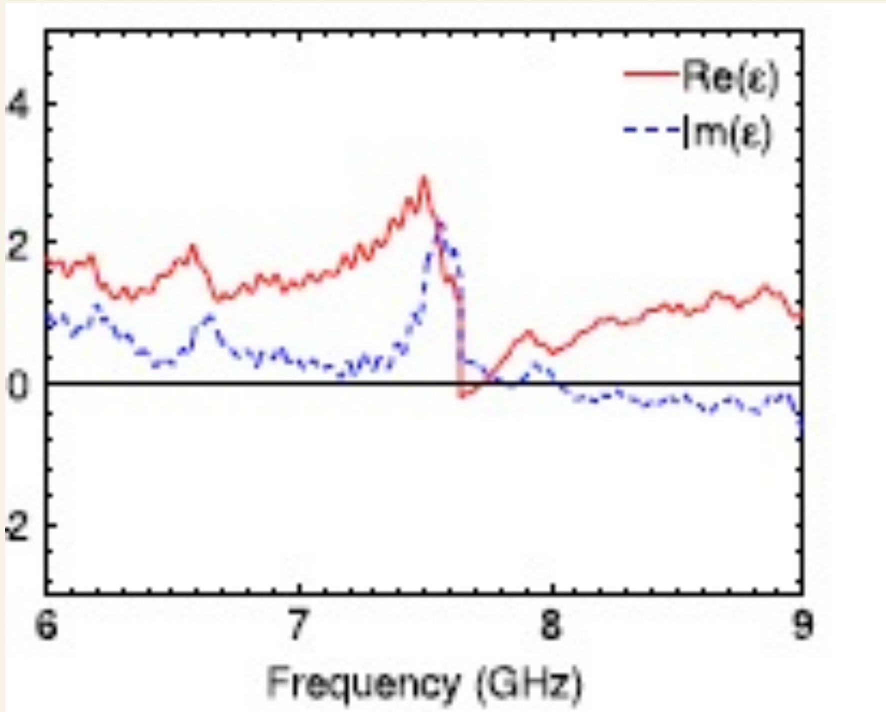
with $$B = \frac{\rho^2}{\varepsilon + P} \qquad D = \frac{\eta}{\varepsilon + P}$$

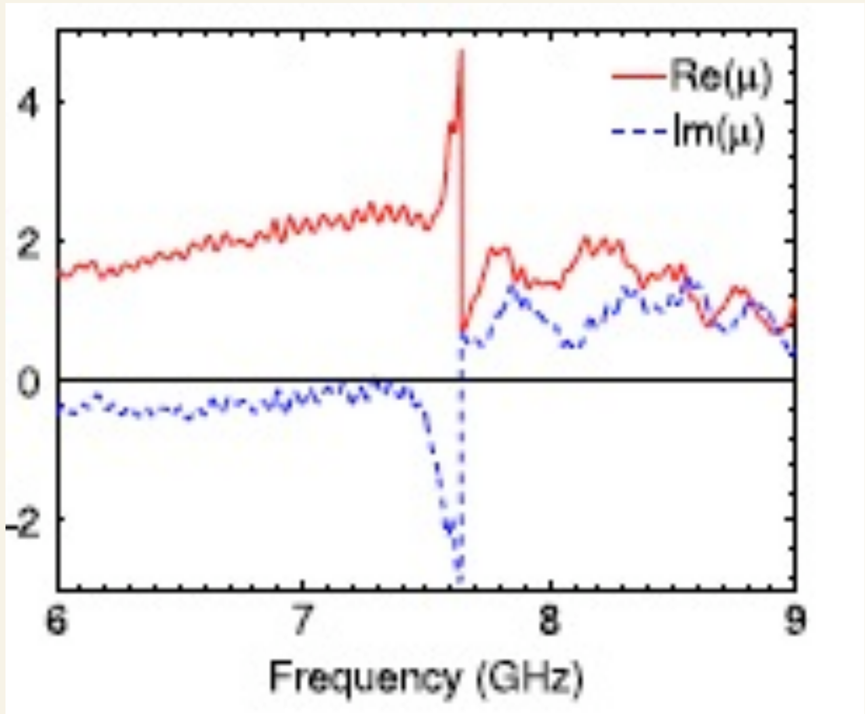
in agreement with the explicit computation

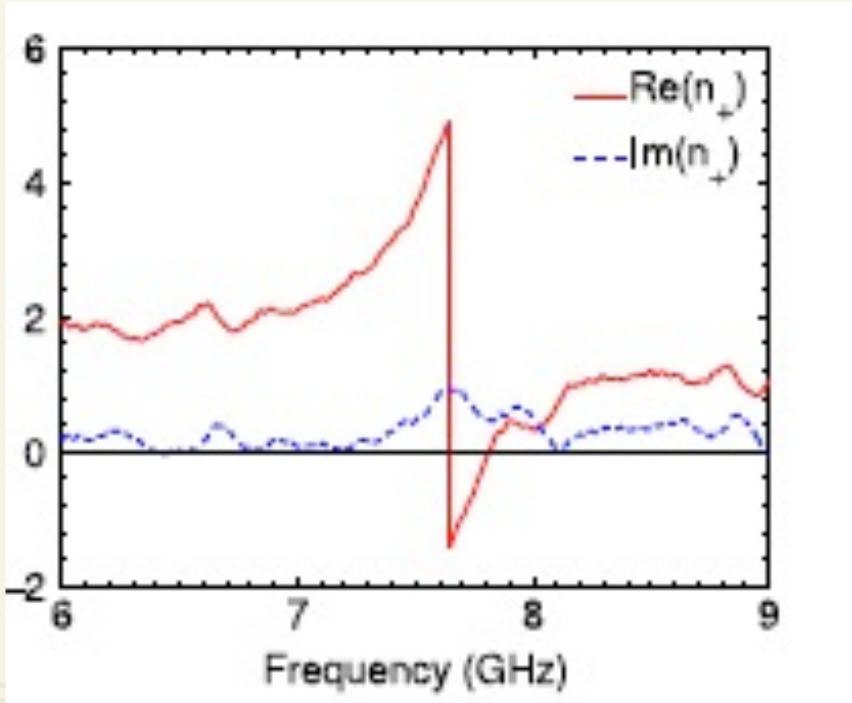
From this expression one finds

$$n_{DL} < 0 \quad \text{for} \quad \omega < 4\pi q^2 B$$

Experimental data







Review on Chiral Metamaterials, J. Opt. A (2009)

Future directions

- Longitudinal modes
- Doubly-neg materials
- Inhomogeneity, anisotropy
- Non-relativistic backgrounds
- 2+1 dimensions
- Superconductors (no neg refr in the probe approx) [Gao, Zhang, 1008.0720]