Higher derivative gravities and AdS/CFT

Andrei Parnachev

C.N.Yang ITP, Stony Brook University, USA

Introduction and Summary

CFT duals of theories with Einstein-Hilbert gravity in AdS_5 necessarily have $a = c$. Higher derivative gravities lift this restriction. Will consider Lovelock gravities with $\Lambda < 0$, which give rise to CFTs.

In these CFTs $\langle T_{ab} T_{cd} T_{ef} \rangle$ satisfy a constraint which follows from the superconformal Ward identity. Moreover, acasual propagation at finite T is equivalent to the violation of the energy flux positivity.

One can use it as a starting point to show that energy flux positivity = ghost free CFT

Outline

- Lovelock gravities, black holes and graviton propagation. Acasual propagation in CFT at $T > 0$.
- Energy flux in CFTs.
- Computing energy flux in CFTs dual to Lovelock gravities. Holographic Weyl anomaly and $\langle TTT \rangle$ 3-point function.
- Conformal field theory. Energy flux positivity = no ghosts in CFTs.
- Conclusions.

Lovelock gravities

Gauss-Bonnet gravity Lagrangian:

$$\mathcal{L} = R + \frac{6}{L^2} + \lambda L^2 (R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd})$$

We will be interested in the $\lambda \sim 1$ regime.

More generally, we can add terms $\mathcal{O}(R^k)$ which are Euler densities in $2k$ dimensions:

$$\lambda_k L^{2k-2} \delta^{a_1 \ldots b_k}_{c_1 \ldots d_k} R^{c_1 d_1}{}_{a_1 b_1} \ldots R^{c_k d_k}{}_{a_k b_k}$$

They become non-trivial for gravity theories in AdS_D with $D > 2k$.

Lovelock gravities

Special properties of Lovelock gravities

- Equations of motion don't contain 3rd order derivatives g'''
- Metric and Palatini formulations are equivalent
- No ghosts around flat space
- Exact black hole solutions can be found

The last property allows one to study dual CFTs at finite temperature.

Small fluctuations

Consider propagation of gravitons in the black hole background. (D=5 Gauss-Bonnet: Brigante, Liu, Myers, Shenker, Yaida)

$$ds^2 = -\frac{f(r)}{\alpha}dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}\left(\sum dx_i^2 + 2\phi(t,r,z)dx_1 dx\right.$$

Fourier transform:
$\phi(t,r,z) = \int dwdqexp(-iwt + iqz)$
After substitutions and coordinate transformations, get Schrodinger equation with $\hbar \to 1/\tilde{q} = T/q$:

$$-\frac{1}{\tilde{q}^2}\partial_y^2\Psi(y) + V(y)\Psi(y) = \frac{w^2}{q^2}\Psi(y)$$

Small fluctuations

Spectrum = states in finite T CFT.
In the $\tilde{q} \gg 1$ regime, there are stable states with $\partial w/\partial q > 1$ in some region of the parameter space.
Causality places constraints on Lovelock couplings:

$$\sum_k [(d-2)(d-3) + 2d(k-1)]\lambda_k \alpha^{k-1} < 0$$

where α defines the AdS radius $L_{AdS}^2 = L^2/\alpha$ and satisfies $\sum_k \lambda_k \alpha^k = 0$.

This effect is absent at $T = 0$; appears as $\mathcal{O}(T/q)$ correction from the tails of black hole metric.

Energy flux in CFT

Define $\varepsilon(\hat{n}) = \lim_{r\to\infty} r^2 \int dt T^0_i \hat{n}_i$

Conjecture (Hofman, Maldacena) $\langle\varepsilon(\hat{n})\rangle \geq 0$.

Consider a state created by $\epsilon^{ij}T_{ij}$

$$\langle\varepsilon(\hat{n})\rangle \sim 1 + t_2\left(\frac{\epsilon_{ij}\epsilon^{il}\hat{n}^j\hat{n}_l}{\epsilon_{ij}\epsilon^{ij}} - \frac{1}{d-1}\right) + t_4\left(\frac{(\epsilon^{ij}\hat{n}^i\hat{n}_j)^2}{\epsilon_{ij}\epsilon^{ij}} - \frac{2}{d^2-1}\right.$$

t_2 and t_4 are determined by the 2 and 3-point functions of T_{ab}.

Fact: (Osborn, Petkou) $\langle TT\rangle$ and $\langle TTT\rangle$ in CFTs are completely determined by 3 parameters a, b, c.

Computation of $\langle TTT \rangle$ in Lovelock gravity duals

We compute $\langle TTT \rangle$ by computing holographic Weyl anomaly in $d = 6$ CFT. Under Weyl rescaling, $g \to e^{2\sigma} g$, action of CFT in curved space is not invariant: $\delta_\sigma S = \sqrt{\det g} A_W \sigma$

$$A_W = (\text{Euler density}) + (\text{B} - \text{type terms}) + \nabla_i J^i$$

Fact: (Bastianelli, Frolov, Tseytlin) B-type terms are related to $2, \ldots, d/2 + 1$ point functions of T_{ab}. In $d = 6$ B-type anomaly is $b_1 I_1 + b_2 I_2 + b_3 I_3$ where $I_i \sim R^3$. Can construct the linear map

$$(b_1, b_2, b_3) \leftrightarrow (n_1, n_2, n_3) \leftrightarrow (a, b, c)$$

Computation of b_1, b_2, b_3

Holographic Weyl anomaly (Henningson, Skenderis)

$$ds^2 = L_{AdS}^2 \left(\frac{dt^2}{4\rho^2} + \frac{g_{ij}}{\rho} dx^i dx^j \right)$$

$g_{ij} = g_{ij}^{(0)} + \rho g_{ij}^{(1)} + \rho^2 g_{ij}^{(2)} + \ldots$ Procedure: solve Lovelock EOMs, find $g_{ij}^{(1)}$ and $g_{ij}^{(2)}$ as functions of $g_{ij}^{(0)}$, substitute into the Lovelock action and extract $1/\rho$ term. Did this for cubic Lovelock R_{abcd}^2, R_{abcd}^3.

Results: $t_4 = 0$, constraints on Lovelock parameters from flux positivity are the same as those from causality at finite T. Also holds in GB in any dimensions: Myers et al, Camanho&Edelstein

Comments

- Superconformal Ward identity implies $t_4 = 0$. This is the case in Lovelock theory. Similar to $a = c$ in Einstein-Hilbert gravity (which was required by $\mathcal{N} = 4$ SUSY)
- The exact matching between the causality and flux positivity constraints in Lovelock theories hints at their special role in AdS/CFT.
- Both causality and flux positivity constraints involve UV physics. We will see below that the most singular term in the TT OPE is responsible for this.

CFT

Consider the TT OPE (Osborn, Petkou)

$$T_{ab}(x)T_{cd}(0) \sim \frac{C_T \mathcal{I}_{ab,cd}(x)}{x^{2d}} + \hat{A}_{abcd\alpha\beta}(x)T_{\alpha\beta}(0) + \ldots$$

Take an expectation value of both sides and Fourier transform to arrive at two-point function. Zero temperature result comes from the first term:

$$G_{ab,cd}(k) \sim k^4 \log k^2$$

(index structure is completely fixed by Ward identities)

At finite temperature $\langle T_{\alpha\beta}\rangle = C\mathrm{diag}[3,1,1,1]T^4$

CFT

$TT \sim T$ term in the OPE gives $\mathcal{O}(T^4/q^4)$ correction, denoted by $G_{ab,cd}(k)_T$. But this correction contains a pole! For example,

$$G_{12,12}(k)_T = C_t \frac{w^2 + q^2}{w^2 - q^2}$$

where $C_t = -5(7a+2b-c)/(14a-2b-5c)$ is precisely the quantity which must stay positive for the energy flux to remain positive!

Negative residue = ghost state.

CFT

$$1+t_2\left(\frac{\epsilon_{ij}\epsilon^{il}\hat{n}^j\hat{n}_l}{\epsilon_{ij}\epsilon^{ij}}-\frac{1}{3}\right)+t_4\left(\frac{(\epsilon^{ij}\hat{n}^i\hat{n}_j)^2}{\epsilon_{ij}\epsilon^{ij}}-\frac{2}{15}\right) \geq 0$$

implies 3 inequalities: [for ϵ^{ij} being tensor, vector or scalar with respect to residual SO(2)]
$C_t, C_v, C_s \geq 0$.
Was matched to causality of 3 independent channels in $G_{ab,cd}(k)_T$ in gravity (Buchel, Myers; Hofman)

Using TT OPE, the poles in 3 independent channels in $G_{ab,cd}(k)_T$ are shown to be equal to C_t, C_v, C_s.
Energy flux positivity = absence of ghosts

Comments

- True for any state that leads to $\langle T_{ab} \rangle \neq 0$
- In theories whose operator content consists of T_{ab} nothing can spoil the correspondence. Presumably the case for Lovelock duals. Tachyons are also ghosts (would be nice to verify directly).
- Lightlike poles from the OPE. Generally expect thinly spaced quasinormal poles: $w = q - inT$, which form a continuum in the $q/T \to \infty$ limit. Presumably CFT computation observes the lower edge of this continuum. Less singular terms in the OPE are down by powers of T/q.

More comments

Correpondence can be possibly spoiled by relevant and marginal operators whose VEV violates Lorentz invariance.

- Scalars give rise to $T^{\Delta} q^{2(2-\frac{\Delta}{2})}$ and do not contribute to the poles.
- Vector: $V^0 = 0$
- The only possible complication may come from the conserved traceless symmetric spin-2 operator, other than T_{ab}. Does the theory decouple into the sum of noninteracting CFTs?

Conclusions/Summary

- Exact correspondence between the positivity of energy flux and causality in Lovelock duals.
- 3-point functions satisfy constraint which follows from supersymmetry.
- CFT result: this is no surprise; these tachyons must also be ghosts.