Small hairy black holes in $\mathrm{AdS}_5 \times \mathrm{S}^5$

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Main Idea:

- We will construct a new class of black hole solutions with "scalar hair" i.e. with a nontrivial profile for a scalar field.
- We will do this in the context of the AdS/CFT correspondence, our black holes will be asymptotically $AdS_5 \times S^5$.
- Our solutions are exact (i.e. in analytic form), in the limit where the black holes are small.

Motivations

- **Phase diagram of large N gauge theories**

  • Black Holes in AdS correspond to saddle points of dual gauge theory at finite temperature/chemical potential.

  • Hairy black holes $\rightarrow$ **new phases** of strongly coupled gauge theory.

- **Relation to “Hairy Black Branes”**

  • Superconductivity in AdS/CFT etc.

- **Entropy of Supersymmetric Black Holes in AdS$_5$**

  • Gutowski-Reall black hole and entropy of 1/16 BPS states in $\mathcal{N} = 4$ SYM.

Introduction

Black Hole instabilities

Charged black holes may be unstable in the presence of charged scalar fields:

- **Classical Picture:** charged particles can have negative energy if close enough to horizon $\rightarrow$ "charge analogue" of Penrose process, pair production.
- **Superradiance:** reflection coefficient > 1 if $\omega < e\mu$.
- **In AdS superradiance amplified** $\rightarrow$ "black hole bomb".
- **Gubser's observation:** charged black branes in Poincare AdS may be unstable to the condensation of charged scalar.

Hairy black holes in AdS

- Planar AdS: no expansion parameter, (mostly) numerical work.
- Global AdS: we have new parameter that we can tune, the size of black hole in AdS units. It will allow us to perform analytic computations.

Small hairy black holes in global AdS

The Setup

- AdS/CFT: IIB on AdS$_5 \times$S^5 is dual to $\mathcal{N} = 4$ SYM on $S^3 \times \mathrm{time}$.
- We will be working in global AdS: bulk is solid cylinder with "confining" gravitational potential.
- We focus in the regime $N \to \infty$ and $\lambda = g^2_{YM} N \gg 1$, where IIB supergravity is reliable.
- We will analyze the scalar field condensation for charged AdS black holes.

A consistent truncation

- We do not want to work with the full 10-dimensional IIB supergravity theory.
- We consider the reduction of IIB supergravity on AdS$_5\times$S^5 down to AdS$_5$.
- $SO(6)$ isometry of $S^5 \to U(1)^3$ gauge fields in AdS$_5$.
- 3 R-charges Q_1, Q_2, Q_3 corresponding to angular momentum on S^5.
- To simplify, we take $Q_1 = Q_2 = Q_3 = Q$.
- Under this reduction we get scalar fields which are charged under the $U(1)$ (are dual to chiral primary operators in $\mathcal{N} = 4$).

A consistent truncation

- In the end we get a 5-d theory with bosonic fields

$$g_{\mu\nu}, \quad A_\mu, \quad \phi$$

and action of the form

$$S = \int \sqrt{g}(R + 12 + F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 + \Delta(\Delta - 4)|\phi|^2 + \text{interactions} + \text{fermions}) \tag{1}$$

- The field ϕ is dual to the operator $\mathcal{O}_\phi = \mathrm{tr}X^2 + \mathrm{tr}Y^2 + \mathrm{tr}Z^2$ in the gauge theory. It has conformal dimension $\Delta = 2$ and charge $e = 2$.
- This action is a consistent truncation of 10d IIB supergravity.
- We will be studying charged black holes (and their instabilities) within this truncation.

Reissner Nordstrom black holes in AdS

- The RNAdS black hole is

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2$$

with $f(r) = r^2 + 1 - \frac{(R^2+\mu^2+1)R^2}{r^2} + \frac{\mu^2 R^4}{r^4}$ and the gauge field

$$A = \mu\left(1 - \frac{R^2}{r^2}\right)dt$$

- The black hole is parametrized by the horizon radius R and the chemical potential μ (or equivalently its mass and charge).
- The requirement for regularity is

$$\mu^2 \leq (1 + 2R^2)$$

which is saturated for extremal black holes.

Reissner Nordstrom black holes in AdS

- Classical black holes have mass and charge which scale like N^2. We work in $N \to \infty$, so we introduce the rescaled

$$m = \frac{M}{N^2}, \qquad q = \frac{Q}{N^2}$$

- Rewriting the regularity condition in terms of the mass m and charge q of the black hole we have

$$m \geq 3q + 3q^2 - 6q^3 + \mathcal{O}(q^4)$$

- Notice that the BPS bound reads

$$m \geq 3q$$

- Comparing the two we see that there are no supersymmetric RNAdS black holes. The extremality bound is reached before the BPS bound as we lower the mass, for fixed charge.

Instability of global RNAdS black holes

- In the presence of a charged scalar field the RNAdS black hole may become unstable near extremality (i.e. at low temperature).
- At that point the scalar field condenses.
- To determine the onset of the instability we need to compute the spectrum of quasinormal modes (QN modes) of the scalar field in the background of the black hole.

Instability of global RNAdS black holes

- Quasinormal modes are solutions of the (linearized) equation of motion for the scalar field, with ingoing boundary conditions at the horizon of the black hole and normalizable conditions at infinity.
- These modes have the general form

$$\phi \sim e^{-i\omega t} f(r, \Omega)$$

- Because of the two boundary conditions only discrete (complex) frequencies ω are allowed.
- In general $\mathrm{Im}\,\omega < 0$ so these modes are exponentially decaying in time (the black hole is stable under perturbations).
- The onset of the instability is when a QN mode becomes unstable (ω has positive imaginary part).

Instability for small RNAdS black holes

- In general it is not possible to compute the QN spectrum analytically.
- Consider the limit $R_{horizon}/R_{AdS} \to 0$ i.e. a very small RNAdS black hole.
- In this limit the wave equation simplifies in two different regimes.
- In the "far regime" $r >> R_{horizon}$ the geometry in the bulk is approximately the same as that of empty AdS.
- Then we also have the "near regime" where the solution can be computed for $r \ll 1$
- By matching the solution in the overlap regime we can determine the QN spectrum analytically in an expansion in $R_{horizon}/R_{AdS}$.

Instability for small RNAdS black holes

- At leading order we find that the s-wave QN mode of a charged scalar field becomes unstable when

$$\Delta - \mu e < 0$$

- When this quantity is negative the mode becomes dynamically unstable and undergoes Boes-Einstein condensation.
- In our case we have $\mu \approx 1$ for small RNAdS black hole, and $\Delta = 2$, $e = 2$.
- What is the endpoint of the condensation?

Solitons in AdS

- Notice that the wavelength of the unstable mode is much larger than the black hole horizon.
- To leading order, we can ignore the (small) black hole at the center of AdS and simply consider empty AdS held at chemical potential $\mu \approx 1$ which triggers the Bose-Einstein condensation of the scalar field.
- The endpoint of the condensation will be a spherically symmetric, nontrivial configuration of the scalar field, i.e. a soliton. (a coherent state superposition of the excitations of the scalar field).

Equations of motion for soliton

We have to solve

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - 6g_{\mu\nu} = -\frac{3}{2}T^{EM}_{\mu\nu} + \frac{3}{8}T^{mat}_{\mu\nu} \tag{2}$$

where

$$\begin{aligned} T^{EM}_{\mu\nu} =& F_{\mu}{}^{\sigma}F_{\sigma\nu} - \frac{1}{4}g_{\mu\nu}F^{\alpha\sigma}F_{\sigma\alpha} \\ T^{mat}_{\mu\nu} =& \frac{1}{2}\left[D_\mu\phi\,(D_\nu\phi)^* + D_\nu\phi\,(D_\mu\phi)^*\right] - \frac{1}{2}g_{\mu\nu}|D_\sigma\phi|^2 + 2\phi\phi^* g_{\mu\nu} \\ & - \frac{1}{4(4+\phi\phi^*)}\left[\partial_\mu(\phi\phi^*)\partial_\nu(\phi\phi^*) - \frac{1}{2}g_{\mu\nu}[\partial_\sigma(\phi\phi^*)]^2\right] \end{aligned} \tag{3}$$

the Maxwell equation

$$\nabla_\sigma F_\mu{}^\sigma = \frac{i}{4}\left[\phi(D_\mu\phi)^* - \phi^* D_\mu\phi\right] \tag{4}$$

and the scalar equation

$$D_\mu D^\mu\phi + \phi\left[\frac{[\partial_\sigma(\phi\phi^*)]^2}{4(4+\phi\phi^*)^2} - \frac{\nabla^2(\phi\phi^*)}{2(4+\phi\phi^*)} + 4\right] = 0. \tag{5}$$

Soliton in perturbation theory

- We expand the fields in a small parameter ε which characterizes the condensate of the scalar field

$$\begin{aligned} g_{\mu\nu} &= g^{(0)}_{\mu\nu} + \varepsilon\, g^{(1)}_{\mu\nu} + \varepsilon^2\, g^{(2)}_{\mu\nu} + \ldots \\ A_0 &= 1 + \varepsilon\, A^{(1)}_0 + \varepsilon^2\, A^{(2)}_0 + \ldots \\ \phi &= \varepsilon\, \phi^{(1)} + \varepsilon^2\, \phi^{(2)} \ldots \end{aligned} \tag{6}$$

  where $g^{(0)}_{\mu\nu}$ is the metric of empty AdS$_5$.

- It is straightforward to determine the unknown functions by solving the EOMS perturbatively in ε.

- For example, we find

$$\phi^{(1)} = \frac{1}{1+r^2}, \qquad \phi^{(2)} = 0, \qquad \phi^{(3)} = \frac{1}{8(1+r^2)^3}, \ldots$$

Soliton in perturbation theory

- For the mass and charge we find

$$
\begin{aligned}
m =& \frac{3}{4}\left(\frac{\varepsilon^2}{4} + \frac{\varepsilon^4}{192} + \frac{\varepsilon^6}{1920} + \frac{169\varepsilon^8}{2211840} + \mathcal{O}\left(\varepsilon^{10}\right)\right) \\
q =& \frac{1}{2}\left(\frac{\varepsilon^2}{8} + \frac{\varepsilon^4}{384} + \frac{\varepsilon^6}{3840} + \frac{169\varepsilon^8}{4423680} + \mathcal{O}\left(\varepsilon^{10}\right)\right)
\end{aligned}
\tag{7}
$$

- The soliton is supersymmetric i.e. $m = 3q$.
- We found a supersymmetric condensate of the unstable scalar mode.
- It is an 1/8 BPS configuration.

Hairy black hole in perturbation theory

Small hairy black holes in global AdS

- For small $R_{horizon}$ the hairy black hole $\approx$ small RNAdS black hole surrounded by a cloud of the supersymmetric soliton.
- The hairy configuration can be analytically computed in a double expansion in the field amplitude ε and the horizon radius $R_{horizon}$.
- One indeed finds hairy black hole solutions provided that

$$3q \leq m \leq 3q + q^2 + \mathcal{O}(q^4)$$

At the lower bound they become pure supersymmetric soliton (constructed) and at the upper bound usual RNAdS black holes (without condensate).

- The explicit form (not presented here) of the hairy black hole solutions can be determined analytically order by order in perturbation theory.

Thermodynamics of hairy black holes

- For m and q where both RNAdS and hairy black hole exist, hairy BH dominates entropically.

- For small q, hairy black hole $\approx$ noninteracting mix of the supersymmetric soliton and a standard RNAdS black hole:

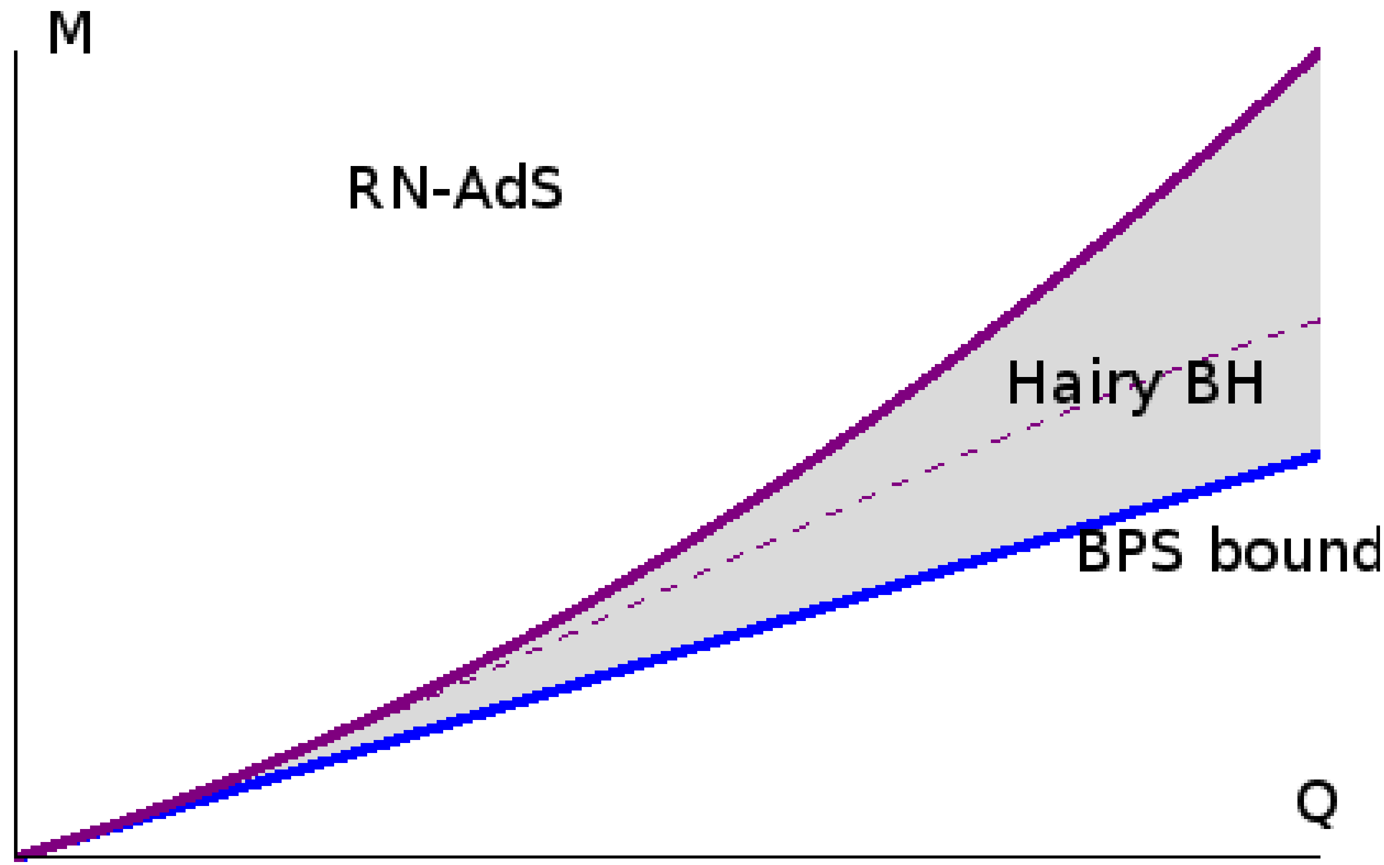
  imagine splitting the total charge q as

  $$q = q_{soliton} + q_{\mathrm{RNAdS}}$$

  The entropy of the soliton is zero and $m_{soliton} = 3q_{soliton}$. Then we compute the entropy of the RNAdS black hole with mass $m_{BH} = m - m_{soliton}$ and charge $q - q_{soliton}$, as a function of $q_{soliton}$, and maximize with respect to it. We find the entropy of the corresponding hairy black hole.

Phase diagram at small charge

Small hairy black holes
in global AdS



- Microcanonical phase diagram.
- States with entropy of order N^2 down to the BPS bound.

The phase diagram at large charge

What happens at finite q?

- Our previous results were robust and the phase diagram cleanly derived in an expansion around $q = 0$.

- However we would like to understand what happens for finite q, where unfortunately we do not have analytic control.

- This is also important in order to study the $q \to \infty$ limit, or the planar limit to make contact with the hairy brane story.

- For general q we expect the same qualitative behavior: for fixed charge, as we lower the mass we start with the RNAdS black hole, then it becomes unstable and we enter a hairy black hole phase, which extends all the way down to the BPS bound $m = 3q$ reducing to a pure-soliton configuration.

- In the following we will focus on the lower endpoint of the phase diagram, that is, on solitonic configurations saturating the BPS bound (easier to find, have to solve SUSY equations).

Spherically symmetric supersymmetric configurations

- We look for spherically symmetric configurations in the bulk saturating the BPS bound $m = 3q$, this time keeping q finite.
- 1/8 BPS solutions have been partly classified. For the spherically symmetric case with all 3 R-charges equal, the supersymmetric solutions have the form

$$ds^2 = -\frac{1+\rho^2 h^3}{h^2}\, dt^2 + \frac{h}{1+\rho^2 h^3}\, d\rho^2 + \rho^2\, h\, d\Omega_3^2 \tag{8}$$

$$A = h^{-1} dt, \qquad \phi = 2\sqrt{(h+\rho h'/2)^2 - 1}$$

The entire solution is then determined by the single function $h(\rho)$ which has to satisfy the following ordinary differential equation

$$(1+\rho^2\, h^3)\,(3\, h' + \rho\, h'') = \rho\left[4 - (2\, h + \rho\, h')^2\right] h^2 \tag{9}$$

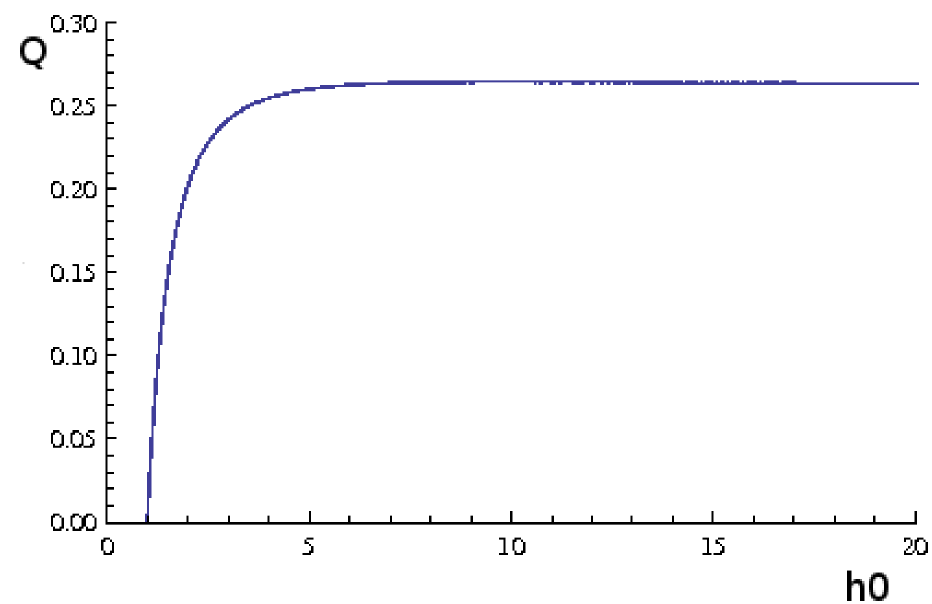
which unfortunately cannot be solved analytically, so we look for numerical solutions.

Numerical Results for smooth solitons

The phase diagram at large charge

- We demand smoothness at the center of AdS. We fix the "initial conditions" at the center by specifying $h_0 = h(0)$ and then solve the equation outwards. The solution asymptotes to AdS.

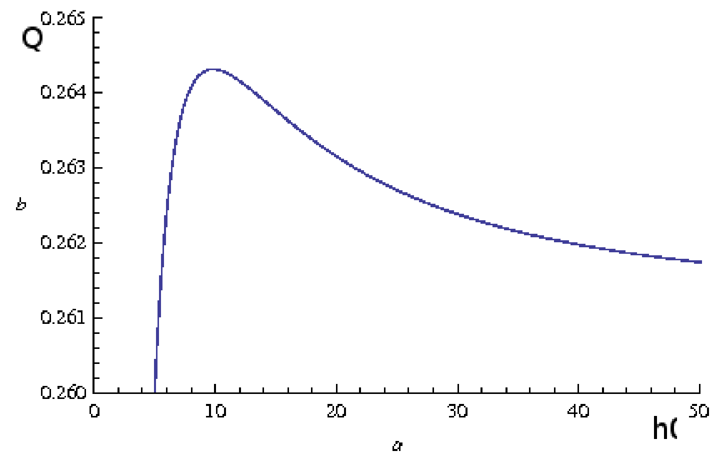
- Below we plot the charge q of the soliton as a function of h_0. The mass is fixed by the BPS bound $m = 3q$.

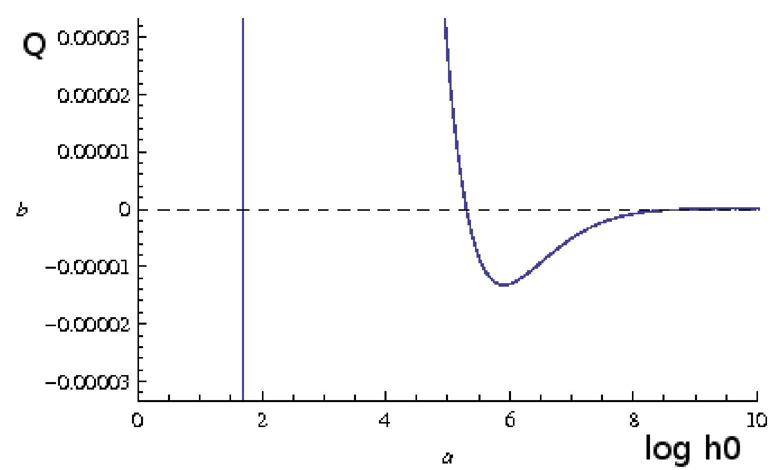


- We notice the existence of a maximum charge q_c for 1/8 BPS smooth solitons.

Oscillatory behavior near the critical charge

- If look for solutions around q_c more carefully we find that $q(h_0)$ actually oscillates around q_c. This means that there are many BPS solutions with the same value of q!





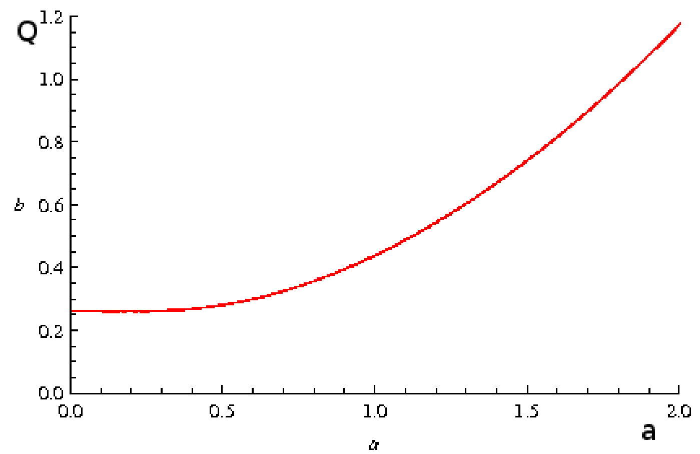
Oscillatory behavior near the critical charge

- All solutions supersymmetric so (presumably) stable.
- The oscillatory behavior is related to the presence of a special critical (singular) solution which acts as an attractor point in the space of solutions of the differential equation that we are trying to solve.
- The oscillations around the critical value q_c can be determined analytically.
- Solutions with the same q have different values of $\langle \mathcal{O}_\phi \rangle$.

What happens for $q > q_c$?

The phase diagram at large charge

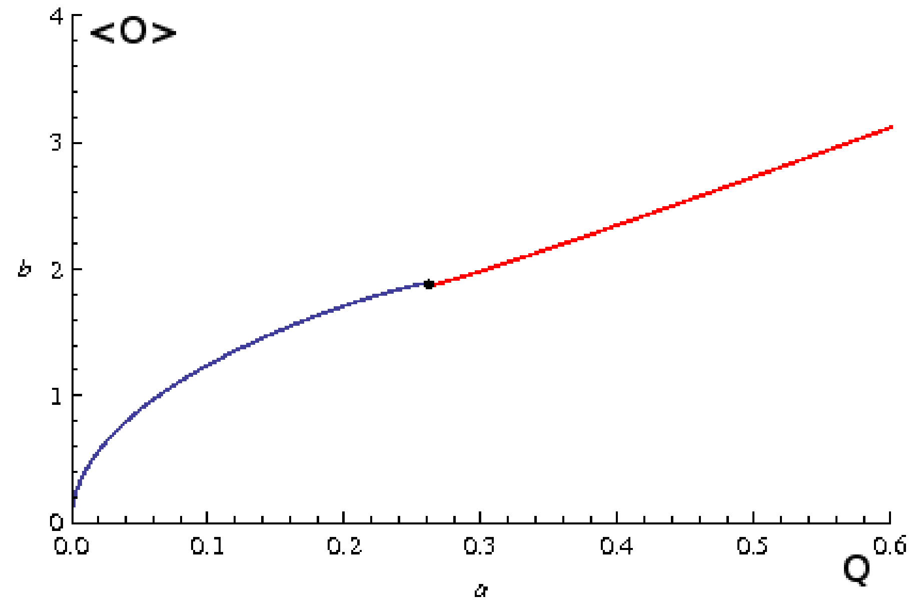
- There are no smooth spherically symmetric 1/8 BPS solutions.
- However there is a 1-parameter family of supersymmetric solutions with an $\frac{a}{\rho}$ kind of singularity.
- The charge of these solutions as a function of a looks like

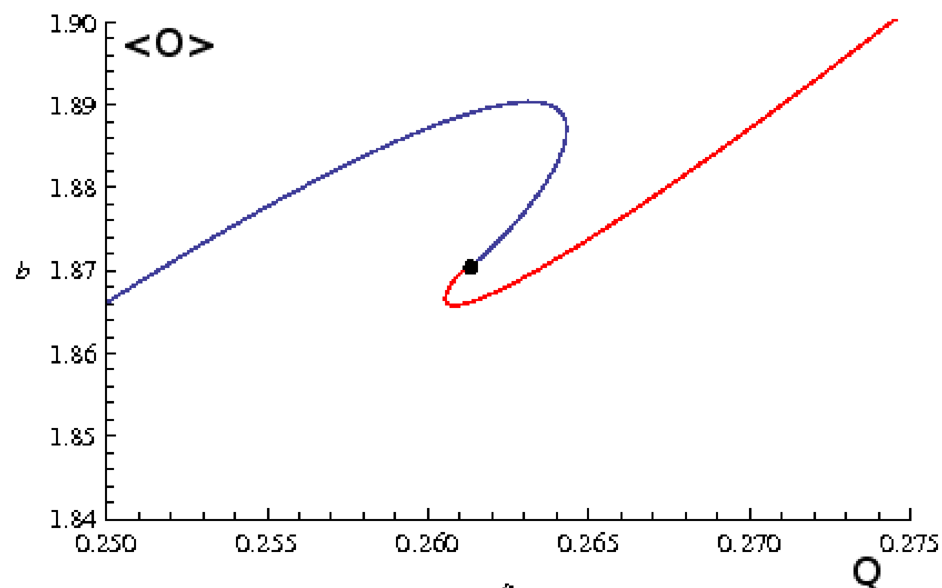


- It seems plausible that this is the BPS limit of hairy black holes for $q > q_c$.

Spherically symmetric supersymmetric solutions

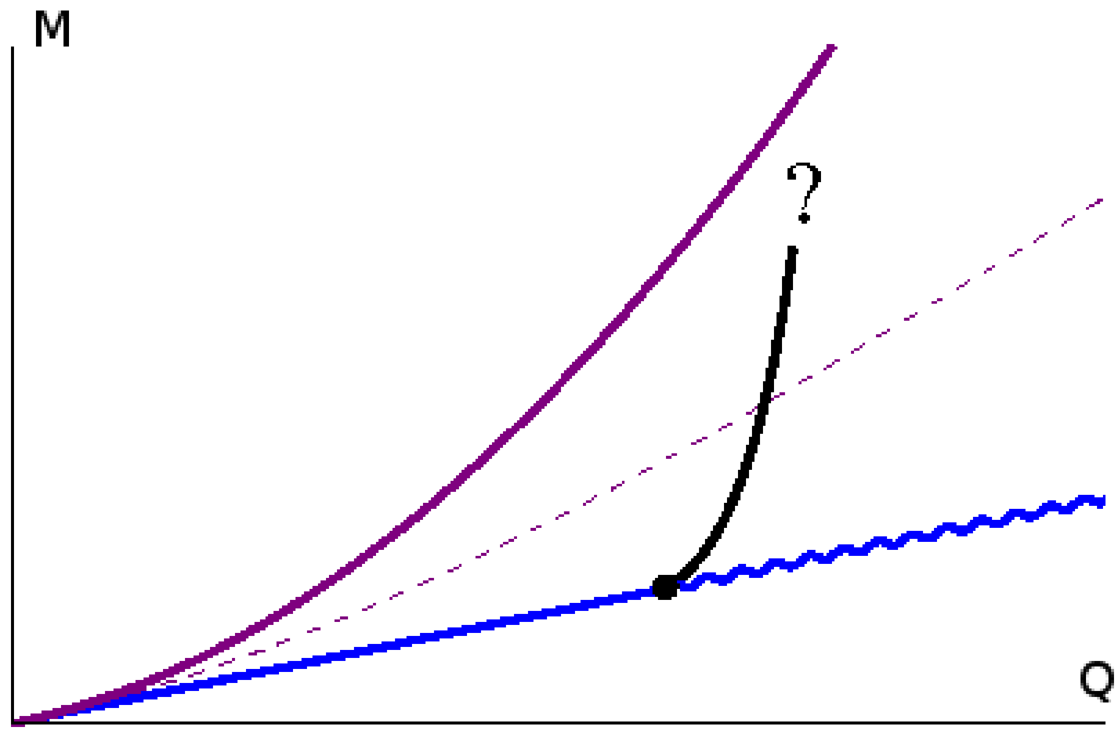
The phase diagram at large charge





The conjectured phase diagram for all values of Q

The phase diagram at
large charge



Including rotation in AdS_5

- It would be interesting to explore the phase diagram of hairy black holes inluding angular momentum in AdS_5.
- In this case there are BPS black holes, the Gutowski Reall black hole.
- Puzzle: there are extremal AdS BHs for all values of the 5 parameters $(J_1, J_2; Q_1, Q_2, Q_3)$. Only a 4-parameter sub-family is BPS.
- This seems peculiar from boundary counting of 1/16 BPS operators (at weak coupling).
- Could there be new hairy BPS black holes extending the Gutowski-Reall family?
- Work in progress...

Summary and further directions

- We found analytic expressions for small hairy black holes in AdS.
- We presented a conjecture for the microcanonical phase diagram of the $\mathcal{N} = 4$ on $S^3 \times R$, as a function of the R-charge.
- What is the nature of the singular supersymmetric solutions? Are they singular in 10d?
- Is there condensation of other chiral primaries?
- It would be interesting to explore the similar story for rotating hairy black holes and to understand the entropy puzzles of the Gutowski-Reall black hole.
- What is the gauge theory interpretation of the “hairy” phase? Can we find an analogue at weak 't Hooft coupling?