Thermal Brane/anti-Brane Blackfold

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10xx.xxxx (to appear) with G. Grignani, T. Harmark, A. Marini, M. Orselli

0912.2352 (JHEP), 0910.1601 (JHEP) & 0902.0427 (**PRL)** + to appear

(with R. Emparan, T. Harmark, V. Niarchos)

Plan

- Introduction + motivation
- Short review of blackfold approach
- EOM and action
- F-string on D3-brane at finite T
- CM revisited: spike and throat
- Brane/anti-brane separation at finite T
- Comparison of phases + phase transition
- Conclusion & outlook

Black holes and branes in string theory

◄ black holes/branes (SUSY and non-SUSY) + play a major role in string theory

- microscopic entropy counting, stringy effects,..
- AdS/CFT: dual to thermal states in finite temperature field
- AdS/CMT: many recent applications

► exact analytic solutions exist for very symmetric cases:
but very rich landscape of possible black holes
(illustrated already by considering pure gravity in higher dimensions)

Recently: effective fluid-dynamical approach has been developed to construct new (approximate analytic) black hole solutions in higher dimensions

⇨ blackfolds

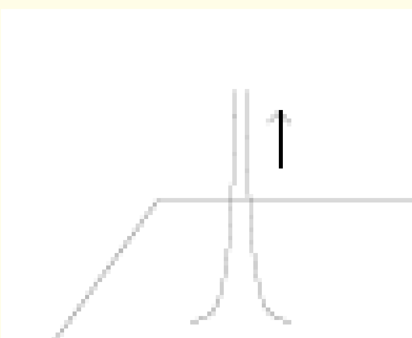
- has revealed many new black objects with novel horizons in gravity (extension to charged blackfolds of supergravity to appear)

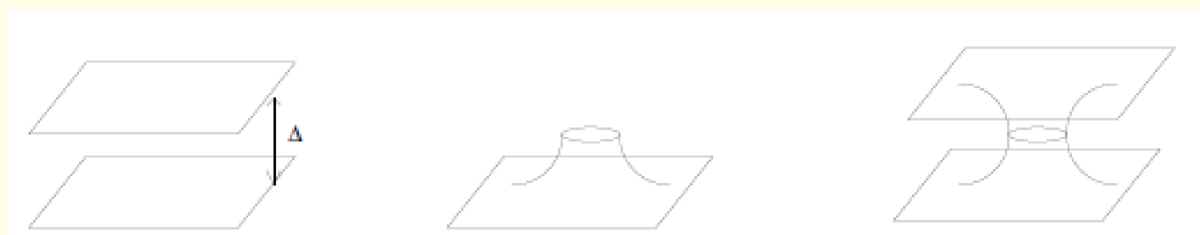
Thermalized version of Callan/Maldacena

This talk: apply these techniques to find and study thermal (non-extremal) version of interesting ST configurations:

⇒ Callan/Maldacena spike + brane/anti-brane config.

DBI solution for D3-brane in flat space in which brane has localized spike
= fundamental string ending on D3-brane
by gluing: brane/anti-brane configurations connected by a string





Motivations

► interesting to see if CM picture continues to hold when the system is heated up: non-extremal fundamental string spike ?

• CM (and related work) inspired construction of Wilson loops from D3-branes with electric flux (Drukker,Fiol + others)
- relevant for mutiple wound WL or WL in higher representations (non-trivial checks of AdS/CFT)

► interesting to examine whether D-brane configurations exist in the finite temperature geometry dual to gauge theory loop linking periodic Euclidean time

• multiple wound Polyakov loops examined by Hartnoll,Kumar
- solution for D5-brane case (antisymmetric rep)
- no solution for D3-brane case (symmetric rep)

(see also: Grignani, Semenoff)

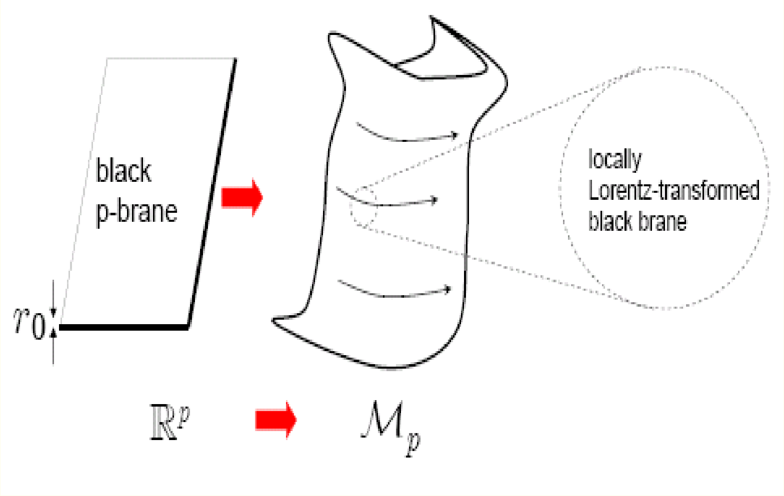
◄ thermal CM (this talk) may be first step to revisit the D3-brane case

Lightening review of blackfold approach

▶ basic idea:

take black branes (possibly charged, intersections/bound states) and curve them (e.g. into black holes with compact horizon topologies)

blackfold limit (=test-brane or probe approximation)



size-scale of the brane

length scale of the worldvolume $\mathcal{W}_{p+1}$

Blackfold equations

▶ equilibrium conditions:

Emparan,Harmark,Niarchos,NO

blackfold equations

intrinsic
(Euler equations of fluid
+ charge current conservation)

extrinsic
(generalized geodesic eqn. for
brane embedding)

$$D_a T^{ab} = 0$$

$$D_a J^{a a_1 .. a_p} = 0$$

fluid stress tensor
of the brane
(position dependent)

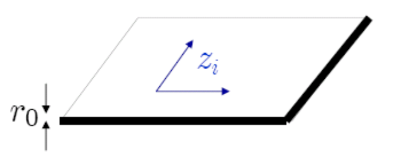
second fundamental
form

- gives novel stationary black holes + allows study of time evolution
- possible to include backreation in perturbative expansion
  (incorporate higher-derivative corrections)

Stationary blackfolds and action principle

blackfold equations determine brane collective coordinates

(depending on wv coords σ^α)



positions in directions
transverse to worldvolume

horizon thickness

velocity

charge

For stationary configurations: can solve blackfold equations explicitly for thickness, velocity + charge

$\rightarrow$ only need to solve extrinsic equations for the embedding

▶ extrinsic equations can be integrated to action $\sim$ Gibbs free energy:

varying G $\Rightarrow$ 1st law of thermodynamics

1st law of thermo $\Leftrightarrow$ blackfold equations for stationary configurations

Blackfold dynamics generalizes DBI dynamics

► consider e.g. DBI action for D3-brane with non-zero E-field:
EOM can be written as

$$T^{ab}_{\text{DBI}} K_{ab}{}^{\rho} = \frac{1}{4!} J^{abcd} F^{\rho}{}_{\lambda abcd}$$

background potential

DBI energy-momentum tensor

D3-brane current

$$D_a J^{abcd} = 0$$

◄ heat up system + assume (for simplicity) flat background:

- describes thermal version of Callan/Maldacena spike or brane/anti-brane configuration.
- can be solved exactly and analyzed

Grignani,Harmark,Marini,NO,Orselli (to appear)

Setup for thermal CM

- 10D flat background metric

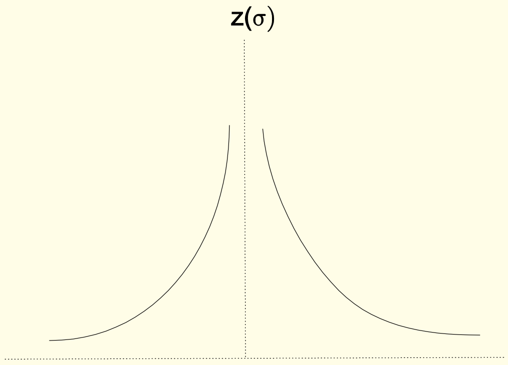
$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + \sum_{i=1}^{6} dx_i^2$$

► embedding of 3-brane

$$t = \tau\ , \quad r = \sigma_1 \equiv \sigma\ , \quad x_1 = z(\sigma)\ , \quad \theta = \sigma_2\ , \quad \phi = \sigma_3$$

- induced metric

$$\gamma_{ab} d\sigma^a d\sigma^b = -d\tau^2 + \left(1 + z'(\sigma)^2\right) d\sigma^2 + \sigma^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$



- k F-strings ending on N coincident infinitely extended D3-branes
- or stretching between two parallel systems

$$z(\sigma) \to 0 \quad \text{for} \quad \sigma \to \infty$$

$$z'(\sigma) \to -\infty \quad \text{for} \quad \sigma \to \sigma_0$$

brane/anti-brane: attach mirror solution at σ_o

brane separation $\Delta = 2\, z(\sigma_o)$

Action for black D3-F1 system

► action

$$I = \beta \mathcal{F} = \beta(M - TS)$$

$$M = \int dV_{(3)} T_{00}$$

$$S = \int dV_{(3)}\; s$$

• need to impose T=const. on blackfold & charges N,k are conserved

$$\mathcal{F}(T,N,k) = \frac{2T_{\mathrm{D3}}^2}{\pi T^4} \int_{\sigma_0}^{\infty} d\sigma \sqrt{1 + z'(\sigma)^2}\, F(\sigma)$$

$$F(\sigma) = \sigma^2 \frac{1 + 4\sinh^2\alpha(\sigma)}{\cosh^4\alpha(\sigma)}$$

with

$$\cosh^2\alpha = \frac{3}{2}\,\frac{\cos\frac{\delta}{3} + \sqrt{3}\sin\frac{\delta}{3}}{\cos\delta} \qquad \cos\delta(\sigma) \equiv \bar{T}^4 \sqrt{1 + \frac{\kappa^2}{\sigma^4}}$$

definitions

$$\bar{T} \equiv \frac{T}{T_{\mathrm{bnd}}} \qquad T_{\mathrm{bnd}}(N) \equiv \left(\frac{4\sqrt{3}T_{\mathrm{D3}}}{9\pi^2 N}\right)^{\frac{1}{4}} \qquad \kappa \equiv \frac{k T_{\mathrm{F1}}}{4\pi N T_{\mathrm{D3}}}$$

◄ relation to DBI: $\displaystyle \lim_{T\to 0} \mathcal{F} = N H_{\mathrm{DBI}}$

Analytic solution

► EOM can be integrated exactly

$$\boxed{z'(\sigma) = -\left(\frac{F(\sigma)^2}{F(\sigma_0)^2} - 1\right)^{-\frac{1}{2}}}$$

• focus on branch connected to extremal (other one connected to neutral)

⇨ reproduces CM throat and spike in zero temperature limit

$$-z'(\sigma) = \sqrt{\frac{\kappa^2 + \sigma_0^4}{\sigma^4 - \sigma_0^4}}\left[1 + \mathcal{O}(\bar{T}^4)\right]$$

$$\sim \frac{\kappa}{\sigma^2} \quad (\text{for } \sigma_0 = 0)$$

◄ validity of the probe approximation:

$$r_{\mathrm{C}}(\sigma) << \sigma$$

$$r_{\mathrm{C}}(\sigma) << L_{\mathrm{curv}} = |K^{-1}|$$

charge radius of the brane:

$$r_{\mathrm{C}}^4 \sim \left(1 + \frac{\kappa^2}{\sigma^4}\right)\frac{N}{T_{\mathrm{D3}}}$$

$$\sigma_0^3 \gg \sqrt{k} g_s l_s^3$$

Phases at zero T

σ_o = throat size

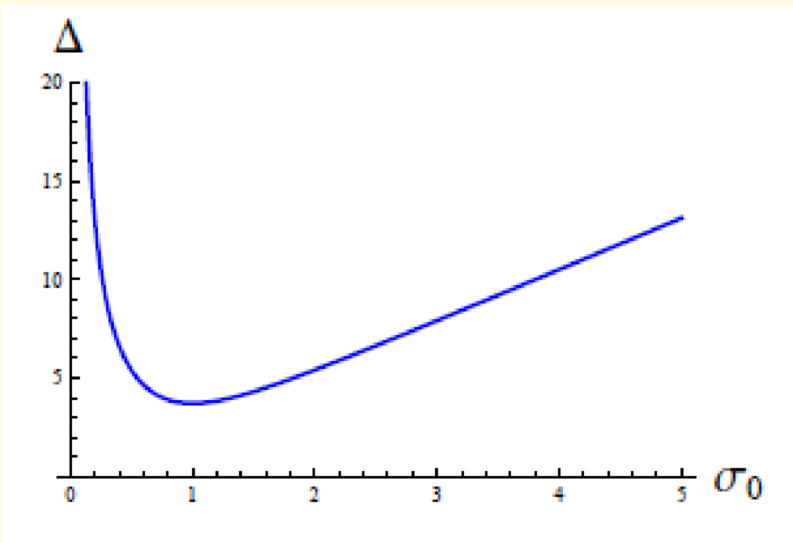
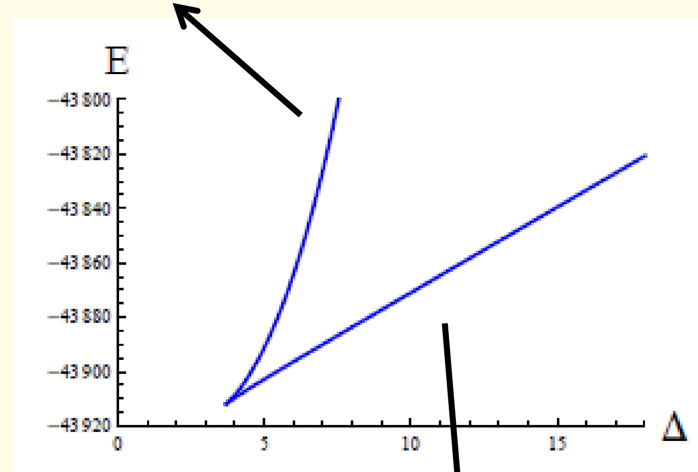


Figure 1: Δ for $\kappa = 1$ in the Callan-Maldacena case

thick throat



thin throat

$$\Delta_{\text{min}} \sim \sqrt{\kappa} \;\; \text{at} \;\; \sigma_0 = \sqrt{\kappa}$$

Phases at finite T

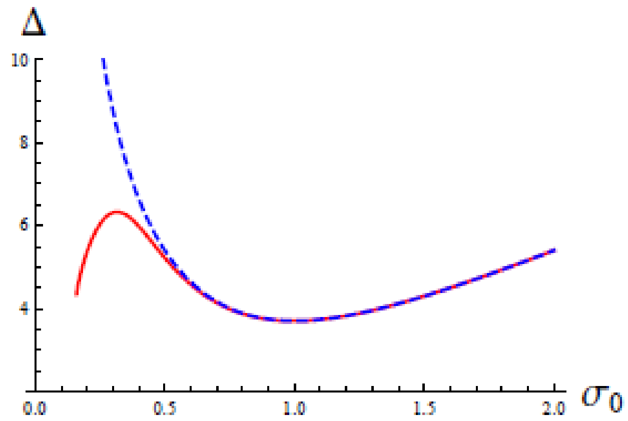


Figure 2: Δ for $\kappa = 1$, $\bar{T} = 0.4$, blue dashed line: Callan-Maldacena, red line: non extremal case.

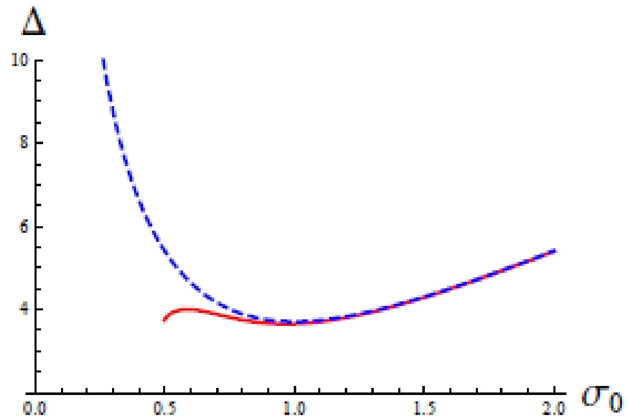


Figure 3: Δ for $\kappa = 1$, $\bar{T} = 0.7$, blue dashed line: Callan-Maldacena, red line: non extremal case.

lower bound on σ

$$\sigma \geq \sigma_{\text{min}} \equiv \sqrt{\kappa} \left(\bar{T}^{-8} - 1 \right)^{-\frac{1}{4}}$$

new behavior at finite T: brane separation Δ cannot become arbitrary big on thin throat branch

Small and large T limit

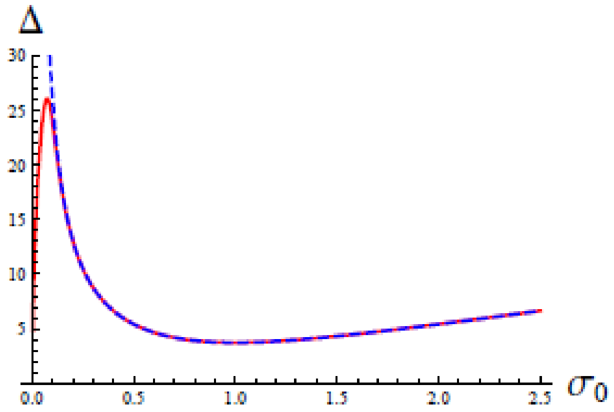


Figure 4: Δ for $\kappa = 1$, $\bar{T} = 0.05$, blue dashed line: Callan-Maldacena, red line: non extremal case.

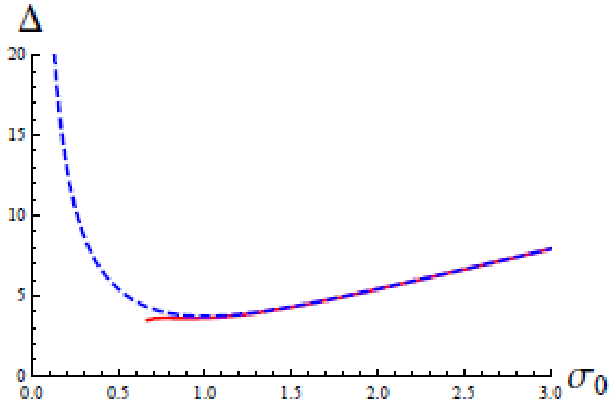


Figure 5: Δ for $\kappa = 1$, $\bar{T} = 0.8$, blue dashed line: Callan-Maldacena, red line: non extremal case.

small T:
overlaps more and more with CM but still maximum of Δ before reaching minimum radius

large T:
only thick throat branch behavior

Free energy

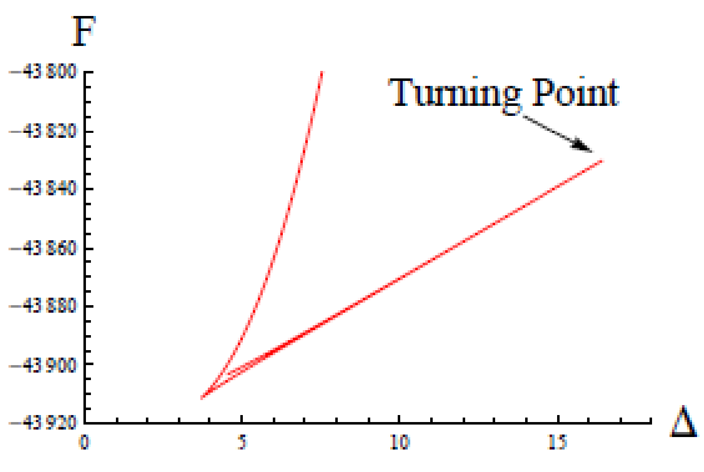


Figure 6: $\delta\mathcal{F}(\bar{T},\Delta(\hat{\sigma}_0);\hat{\sigma}_0)$.

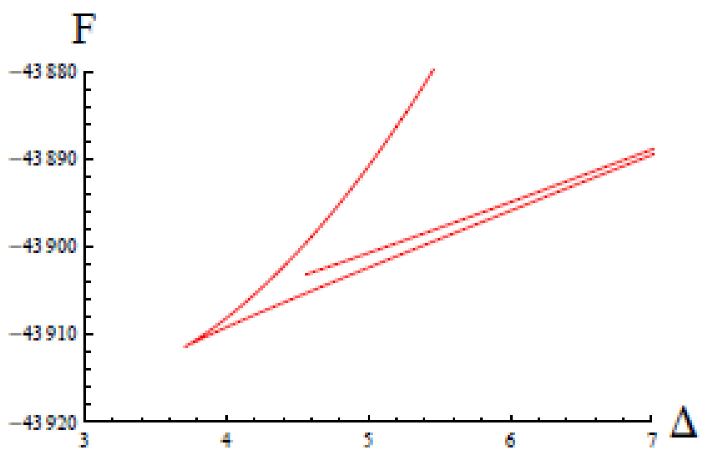


Figure 7: $\delta\mathcal{F}(\bar{T},\Delta(\hat{\sigma}_0);\hat{\sigma}_0)$.

for $\Delta < \Delta_{max}$

free energy minimized on (thin throat) branch with $d\,\Delta/d\,\sigma_o < 0$

for $\Delta > \Delta_{max}$

only available branch is thick throat branch with $d\,\Delta/d\,\sigma_o > 0$

0^{th} order phase transition at critical temperature

Temperature dependence of extrema

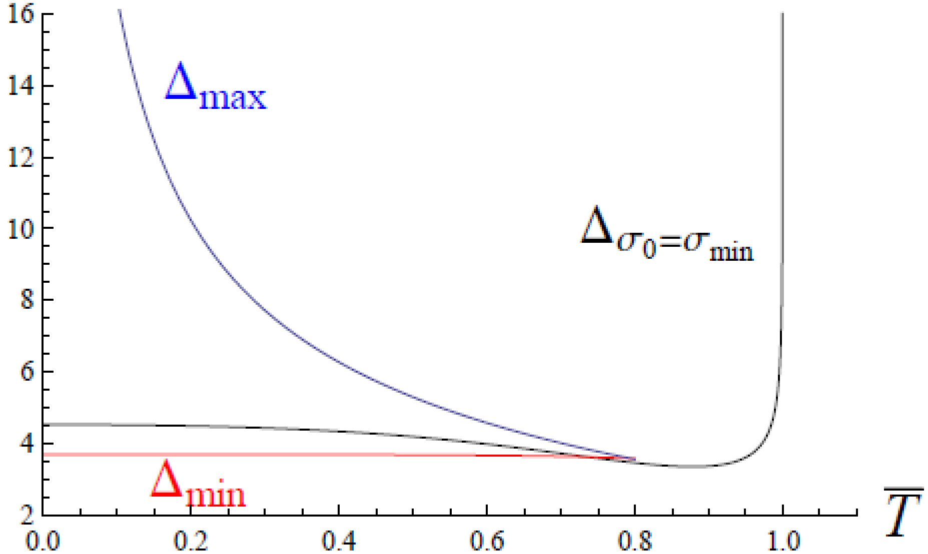


Figure 8: $\Delta_{\max}$ (blue curve), $\Delta_{\min}$ (red curve), Δ at $\sigma_{\min}$ (black curve) as a function the temperature $\bar{T}$.

Summary & Outlook

- reviewed the blackfold approach to construct (novel) black objects

- applied to non-extremal CM solution
  - thermal brane/anti-brane
  - interesting new physics when heated up:
    0^{th} order phase transition (connection with tachyon condensation ?)
  - spike ? can show that non-extremal F1 string can be matched onto
    the solution by examining mass density of the solution
  - easy to construct solution with N1 branes and N2 anti-branes

- can use results + approach to revisit the Polyakov-Maldacena loop
  at strong coupling

- powerful tool to examine other extremal and non-extremal configurations
  in string theory
  - blackfolds with multiple charge — Emparan,Harmark,Niarchos,NO (to appear)
  - blackfolds in AdS — Caldareli,Emparan,Rodriguez/Armas,NO (to appear)

- further elucidate relation of blackfold action with DBI