Towards a Novel Description of Flavor Dynamics in Holographic QCD

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based on 1005.2140 and ongoing work

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- In the AdS/CFT correspondence we can add quarks in the quenched approximation by adding an **open string sector** (which typically entails D-branes stretching in the holographic radial direction).

- Usually we treat this sector at low-energies with the Dirac-Born-Infeld (DBI) action. However, in certain cases this description is inappropriate (the solution develops a singularity where new dof become important).

The lack of an appropriate description impedes the discussion of fundamental questions about the flavor sector of the dual strongly coupled gauge theory, e.g.

- *the computation of the order parameter of chiral symmetry breaking,*
- *how one incorporates bare quark masses,*
- *the determination of the mesonic spectra etc.*

A setup for Holographic QCD
(the D4-D8-antiD8 system)

- The Sakai-Sugimoto model is based on the following configuration of D-branes in type IIA string theory

	0	1	2	3	4	5	6	7	8	9
N_c D4 :	x	x	x	x	x					
N_f D8 :	x	x	x	x		x	x	x	x	x
N_f anti-D8:	x	x	x	x		x	x	x	x	x

- The 4-direction is compactified with anti-periodic boundary conditions for fermions. SUSY is broken.

- Weak `t Hooft coupling: IR dynamics captured by a non-local version of the Nambu-Jona-Lasinio model ***Antonyan et al., `06***

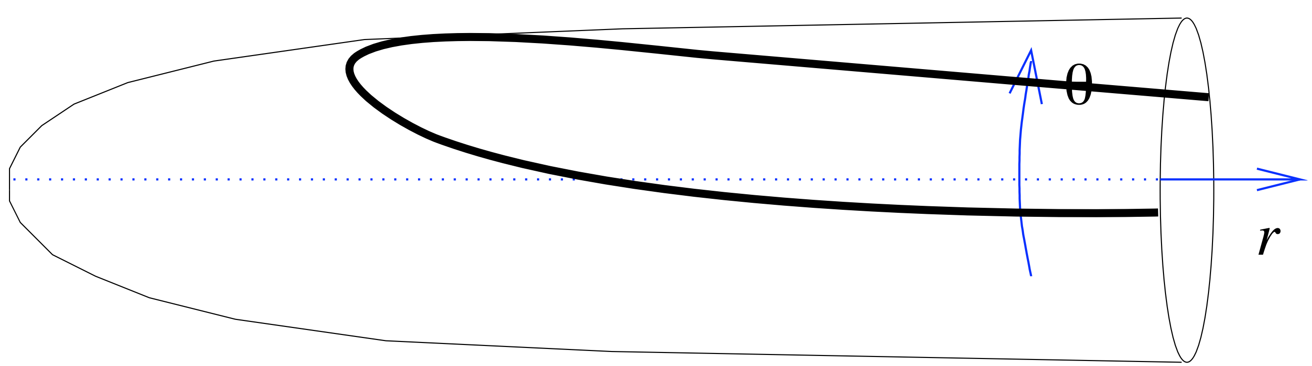
- Strong `t Hooft coupling: IR dynamics captured by QCD with $N_f$ flavors ***Sakai-Sugimoto, `04***

The D4-branes are replaced by a SUGRA solution (Wick rotated black D4-brane)

$$ds^2 = \left(\frac{u}{R}\right)^{\frac{3}{2}} \left(-dt^2 + (dx^i)^2 + f(u)(dx^4)^2\right) + \left(\frac{R}{u}\right)^{\frac{3}{2}} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)$$

$$e^{\Phi} = g_s \left(\frac{u}{R}\right)^{\frac{3}{4}} \ , \ F_4 = 3\pi N_c \ell_s^3 \varepsilon_4 \ , \ f(u) = 1 - \frac{u_{KK}^3}{u^3}$$

- This setup is controlled by three numbers: $\dfrac{u_{KK}}{\ell_s}\ ,\ \dfrac{R}{\ell_s}\ ,\ N_c$
- To avoid a conical singularity at u_{KK} : $R_4 = \dfrac{2}{3}\dfrac{R^{\frac{3}{2}}}{u_{KK}^{\frac{1}{2}}}$
- A strong coupling cutoff must be set at $u_{max} = g_s^{-4/3} R$
- The N_f D8-antiD8 pairs <u>*reconnect*</u> to N_f U-shaped D8's with a *modulus-dependent* asymptotic separation L.



- We are interested in the low-energy open string dynamics on the D8 branes

- The commonly used Dirac-Born-Infeld (DBI) action is **not** a proper effective field theory description of the dynamics in this case for the following reasons:

1. Large accelerations develop near the turning point and DBI breaks down.

2. The holographic dictionary implies that there is more than just transverse scalars and gauge fields in the low-energy description.
   A complex scalar field **T** in the bi-fundamental representation of the flavor *U(1)*x*U(1)* symmetry group **must** be present.

   The normalizable branch of **T** captures the vev of the dual meson-like operator that acts as the *order parameter for flavor chiral symmetry breaking*. The non-normalizable branch of **T** captures the *bare quark mass*.

   Fluctuations of **T** reproduce the sector of scalar mesons.

- Where does **T** come from in string theory?

- It comes from the NS- sector of the open string stretching between the asymptotic D8 and anti-D8 branches. This mode has a non-trivial wavefunction in u and becomes important (as a localized mode) in the vicinity of the turning point.

  Its effects are closely related to the geometric reconnection (motion of transverse scalars) that captures the chiral symmetry breaking. They also lead to a non-geometric smearing of the brane near the turning point.

- A proper description of this system requires an extension of the DBI action that incorporates the bi-fundamental scalar mode **T**.

- Minimal progress has been achieved in this problem.

  Technical complication: **T** is a mode that comes from a long open string that stretches a finite distance. It has non-local physics. Is it possible to treat such physics with a local effective action?

- U-shaped (hairpin-like) branes, like the D8 flavor branes in SS, appear frequently in holographic backgrounds.

- Holographic backgrounds with NSNS fluxes only are interesting cases where:

  (*a*) we can solve string theory with alpha'-exact worldsheet methods (RNS formalism),

  (*b*) we can calibrate any candidate EFT description by comparison to the exact answer,

  (*c*) we can try to import these lessons to backgrounds with RR fluxes (to obtain new information about open string dynamics in these cases and related dynamics of strongy coupled gauge theories).

The NS5-Dp-antiDp system
(an analogue of the SS model with NSNS fluxes)

- Replace the N_c D4 branes by k NS5 branes

	0	1	2	3	4	5	6	7	8	9
k NS5 :	x	x	x	x	x	x				
N_f D1 :	x					x	+			
N_f antiD1 :	x					x	+			

- The 4-direction is again compactified with anti-periodic boundary conditions for fermions. SUSY is broken.

- The D1 branes *reconnect* and form D1 hairpin branes with a modulus independent L in the (near-horizon) background of the NS5 branes

$$ds^2 = -dt^2 + (dx^i)^2 + \alpha' k d\Omega_3^2 + \alpha' k (d\rho^2 + \tanh^2\rho\, d\theta^2)\ ,\ e^{\Phi} = \frac{g_s}{\cosh\rho}$$

A Unified Effective Description of BPS, non-BPS and D-antiD branes

Based on ***Erkal, Kutasov and Lunin, '09***

- The TDBI action in flat space

$$S = -\int d^{p+1}\sigma\, V(T)\sqrt{-\det(\eta_{ab} + \partial_a X^I \partial_b X^I + F_{ab} + \partial_a T \partial_b T)}$$

$$V(T) = \frac{\tau_p}{\cosh \alpha T}\ , \ \alpha = 1, \frac{1}{\sqrt{2}} \quad \textit{bosonic, type II}$$

incorporates the tachyon T of non-BPS branes.

It has been derived in open string theory from first principles

Kutasov, VN, '03

- The <u>*real*</u> scalar field T appears in this action as an extra scalar.

The potential V(T) appears as a T-dependent contribution to the dilaton.

- Imagine an extended 11D spacetime with coordinates X^μ, T and metric

$$ds^2 = g_{\mu\nu} dX^\mu dX^\nu + dT^2$$

The **DBI** action for a p-brane in this space reads

$$S = -\int d^{p+1}\sigma \, e^{-\Phi(X,T)} \sqrt{-\det(g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + \partial_a T \partial_b T + F_{ab})}$$

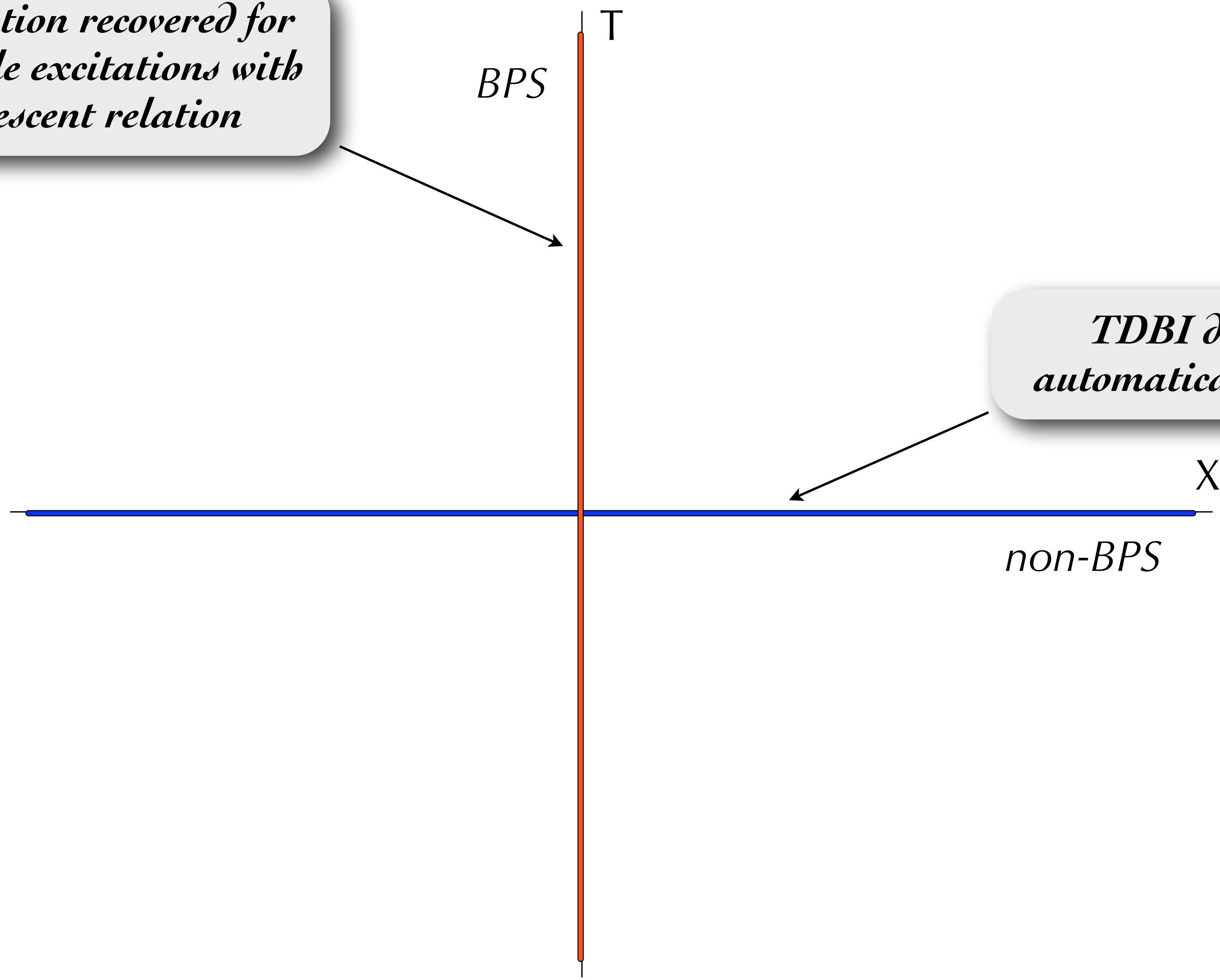
Take a dilaton with a factorized dependence

$$e^{-\Phi(X,T)} = e^{-\Phi(X)} V(T)$$

- **Observation:** BPS branes, non-BPS branes and brane-antibrane pairs arise (in flat space) as different solution curves in (X,T) space.

• **BPS and non-BPS branes**

DBI description recovered for normalizable excitations with correct descent relation



TDBI description automatically recovered

- **Brane-antiBrane pairs**

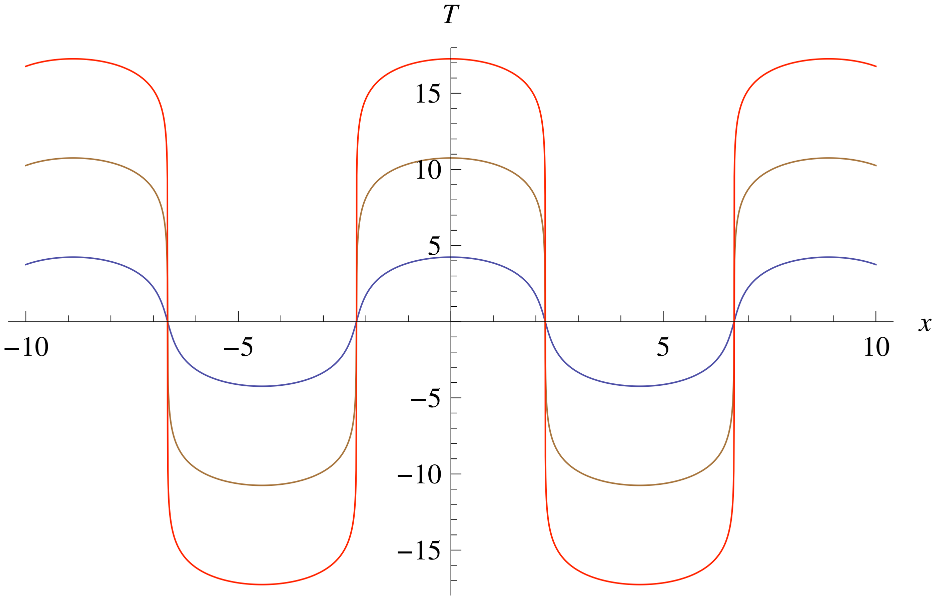
The TDBI action admits the following solution (Euclidean version of rolling tachyon solution)

$$\sinh(\alpha T) = A\cos(ax)$$

A is an arbitraty constant.

A=0 gives the non-BPS brane.

A=∞ gives an array of BPS-antiBPS branes separated by the critical distance at which the DDbar tachyon **T** is **massless** (hence the marginality of A).



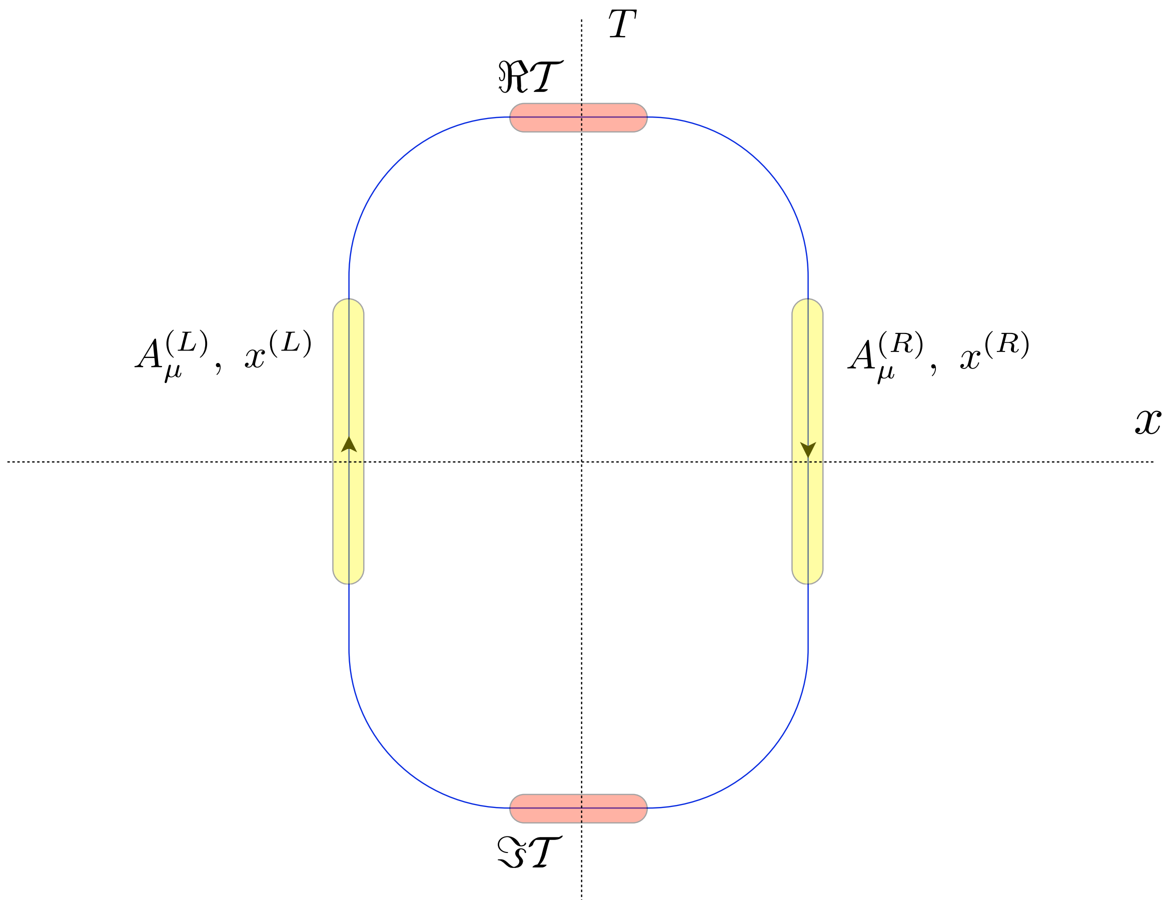
- **Parenthesis**

The TDBI action

$$S = -\int dx V(T) \sqrt{1 + \left(\frac{dT}{dx}\right)^2}$$

has a one-parameter family of periodic solutions with a modulus-*independent* period **only** when the tachyon potential V(T) is of the 1/cosh form.

- A ***closed 'paperclip'*** curve in (T,X) space reproduces the key features of the brane-antibrane system (dof, interactions of **T**, transverse scalars and U(1)xU(1) gauge field,...) & deals naturally with the non-local nature of **T**.



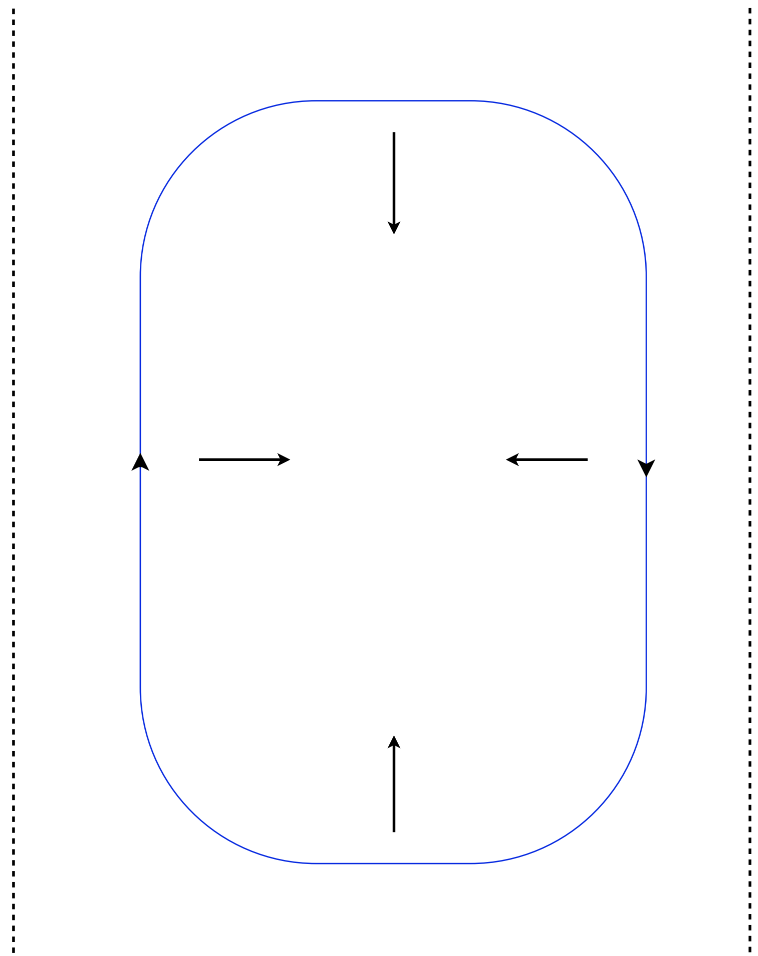
The Tachyon Paperclip

Note

There is a non-trivial transformation between the bi-fundamental tachyon **T** and the *real* tachyon T

$$\mathcal{T} \sim e^{-\alpha T}\Big|_{x=0}$$

- The TDBI action provides a new point of view on tachyon condensation in the brane-antibrane system: **a tachyon paperclip shrinks to zero size** (as a time and space dependent solution of the TDBI *real* scalar T).



Hairpin-Branes and Tachyon-Paperclips

- We will ***assume*** that the action

$$S = -\int d^{p+1}\sigma \; e^{-\Phi(X,T)} \sqrt{-\det(g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + \partial_a T \partial_b T + F_{ab})}$$

$$e^{-\Phi(X,T)} = e^{-\Phi(X)} V(T)$$

gives a sensible effective field theory description of non-BPS branes as in flat space. *We incorporate the dilaton and metric background data in the obvious way and leave the possibility open that these data affect also the tachyon potential.*

- In string theory we should be computing the disc partition function in the curved closed string background and at a suitable generalization of the rolling tachyon solution...

- To describe the dynamics of hairpin-like branes in holographic backgrounds we should be looking for a `radially condensing' tachyon-paperclip solution of the TDBI action: T=T(u,x).

  The DBI action is only a *partial* description of such branes.

  A `condensing tachyon' is necessarily present with a non-trivial profile.

- **D1 hairpins in the background of *k* NS5 branes (*our testing ground*)**

In this case we can describe explicitly these branes as boundary states in worldsheet conformal field theory for any $k>1$.

We can see explicitly how the half-winding bifundamental tachyon **T** arises in the open string spectrum and how it controls the open string theory dynamics. We would like to reproduce these results using the TDBI effective field theory.

The starting point is the TDBI action for a non-BPS **D2** brane that wraps the cigar in the (ρ,θ) directions

$$S = -\int dt\, d\rho\, d\theta\, \sinh\rho\, V(T)\sqrt{1+\frac{1}{k}(\partial_\rho T)^2+\frac{1}{k}\coth^2\rho\,(\partial_\theta T)^2}$$

Some of the main points that we find:

- The slope of the tachyon potential around T=0 is the same for all *k* and coincides with the value of the flat space result

$$\frac{1}{V(0)}\frac{d^2V}{dT^2}\bigg|_{T=0} = -\alpha^2\,,\ \alpha = \frac{1}{\sqrt{2}}$$

- We can reproduce the asymptotic behavior of the bifundamental tachyon **T** at $\rho >> \rho_0$ (correct winding and asymptotic radial dependence) with a tachyon-paperclip solution of the `rolling tachyon' form

$$T \sim a + c(k)\log\cos\theta$$

but this requires a modified (*k*-dependent) tachyon potential with large-T behavior

$$V(T) \sim e^{-\beta T}\,,\ \beta = \beta(k) > 0 \quad \left(\beta(k) = \frac{1}{\sqrt{k}}\right)$$

- There are two independent parameters controlling the leading and subleading branches of the solution

$$e^T \sim \mu_T + \ldots + M_T e^{-\rho} + \ldots$$

- In the vicinity of the turning point we find a ρ-dependent elliptical solution

$$T = A\sqrt{\rho - \rho_0}\cos\sigma + \ldots\ ,\ \theta = B\sqrt{\rho - \rho_0}\sin\sigma + \ldots$$

$$2kB^2\tanh^2\rho_0 - A^2B^2\tanh\rho_0 + 2A^2 = 0$$

- It is technically very hard to solve the highly non-linear TDBI equation everywhere and determine analytically the full condensing tachyon-paperclip solution. An approximation to T(ρ,θ=0):
  insert

$$T \sim a(\rho) + c(k)\log\cos\theta$$

  into the TDBI equation and Taylor-expand the TDBI equation around θ=0 keeping the leading order.

This manipulation leads to a simpler differential equation

$$\left(\cosh\rho\, a' - \frac{\sqrt{k}}{\sinh\rho}\right)\left(k + a'^2\right) + k\sinh\rho\, a'' = 0 \ , \ a' \equiv \frac{da}{d\rho} \qquad (*)$$

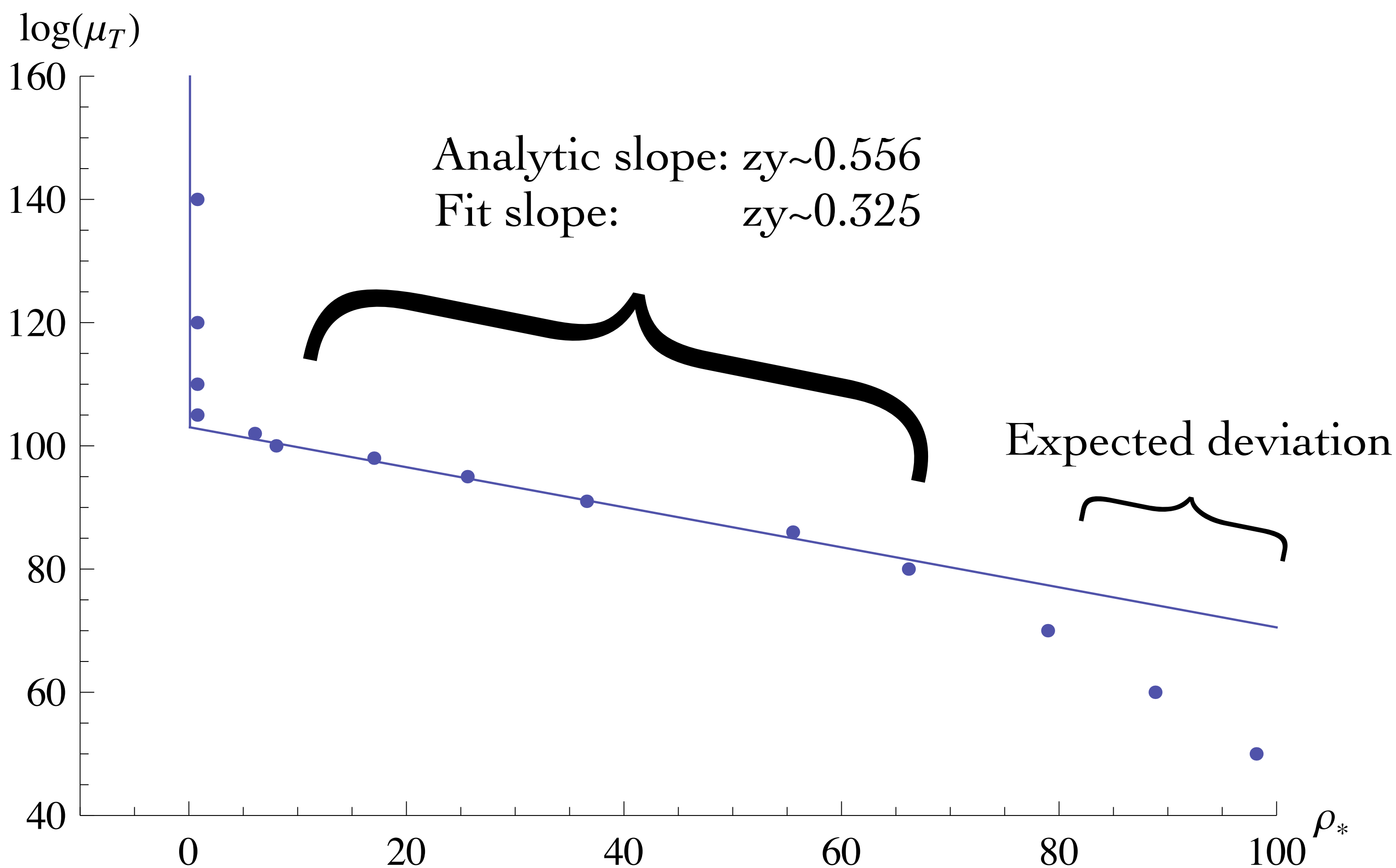
This equation has a boundary-value-dependent singularity. This singularity provides an estimate of the hairpin-brane turning point.

- Bootstrap methods in worldsheet CFT imply a specific relation between the boundary cosmological constant of the standard hairpin-brane and its turning point. In terms of the leading branch coefficient μ_T of e^T this relation translates to

$$\log\mu_T(\rho_0) = x - z\log\sinh(y\rho_0) \ , \ y = k \, , \, z = \frac{1}{2(k-1)}$$

Numerical evaluation of (*) for $k=10$.

Analytically expected: y=10, z=1/18~0.0556
From the depicted fit: y=6.5, z=0.05



D8 hairpins in the Sakai-Sugimoto model

A qualitatively similar picture is anticipated in the Sakai-Sugimoto model.

There are interesting differences, e.g. the asymptotic separation L of the hairpin branches is now modulus-dependent.

Does one need a more drastic modification of the tachyon potential V(T) to reproduce this feature?

Work is underway to determine appropriate tachyon-paperclip solutions, to put constraints on the tachyon potential, to determine the implications of this formalism for holographic QCD (details of holographic dictionary, bare quark mass dependence, mesonic spectra,...)