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# Flows Between 2-Dim SCFT and Y-M Instantons

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- Introduction
- 6-Dim  $\mathcal{N}=1$  SUGRA  
& Solution
- Conclusions

## I Introduction

5-BRANE — 1-BRANE system in Type II B  
 theories compactified on  $T^4$  or  $K3$   
 are well understood.

In  $Q_1$  &  $Q_5$  large limit the  
 near horizon geometry is  $AdS_3 \times S^3 \times T^4$   
 or  $K3$

and the dual SCFT is ~~that~~  $\mathcal{N} = (4,4)$   
 $\mathcal{N} = (4,4)$  and is in the moduli space  
 of a symmetric product theory of  
 $Q_1 Q_5$  copies of  $T^4$  or  $K3$

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Detailed checks exist

In the Holographic context

Elliptic genus (d'Brer)

gravity side =  $(4,4)$  SCFT

in the limits where AdS/CFT holds

→ I-5 Brane System in Heterotic or Type I like theories is much less understood.

Near horizon geometry  $\sim \text{AdS}_3 \times S^3 \times \dots$

Dual SCFT is  $\mathcal{N} = (4,0)$

These theories should have some non-standard features

e.g.  $C_L + C_R$  at the subleading orders in  $Q_1$  &  $Q_5$ . (Due to gravitational Chern Simons term)

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From the gravity side a good starting point will be

3-Dim gauged SUGRA and study  
-th fixed pts that admit  $AdS_3$  solutions

→ There is a classification of 2-derivative

3-Dim gauged SUGRA

(de Wit, Hennig, Samtleben)

In particular for  $N=4$ , Scalar manifold is product of 2-Quaternionic spaces. The R-Sym.  $SO(4) \sim SU(2)_+ \otimes SU(2)_-$  where  $SU(2)_+$  &  $SU(2)_-$  act on the two spaces

Gauging of isometries can be done subject to an algebraic constraint?

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→ For a single Quaternionic case we considered cosets

$$SO(4, 1 + \dim G)$$

where  $G$  is a semi-simple group.

One can consistently gauge

$$SU(2) \times G$$

→ This theory can be obtained a consistent  $SU(2)$  reduction from  $N=1$  6-dim theory ~~with~~ with vector multiplets in group  $G$  and 1-tensor multiplet.

(GAVA, KARNDUMRI, KSN)

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→ Study critical pts preserving  
 $N=4$  SUSY, and found  
flows between them

↓ Lifting this solution to 6-Dim  
One ends up with an instanton  
solution.

→ generalize to Multiinstanton  
solutions

(Duff, Liu & Pope)

(Liam, Liu, Orriat & Pope)

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## 6-Dim. $N=1$ SUGRA

(eg. obtained by compactifying heterotic or Type I theory compactified on K3)

- Gravity multiplet (which contains a self dual 3-form field strength)
  - 1 tensor multiplet (anti-self dual 3-form)
  - Vector multiplets in some gauge group  $G$
  - Hyper multiplets
- Combining self dual & anti-self dual 3-Forms
- $F_3$  (unconstrained)

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$$dF_{(3)} = v \text{Tr} F_{(2)} \wedge F_{(2)}$$

$$d^*(e^{2\theta} F_{(3)}) = \tilde{v} \text{Tr} F_{(2)} \wedge F_{(2)}$$

$\theta$  is -the scalar in -the Tensor multiplet

$v, \tilde{v}$  are certain constants that can be obtained from 10 - dim theory and depends on - the compactification.

For simplicity we set  $\tilde{v} = 0$  &  $v = 1$  in the following .

$$\Rightarrow F_{(3)} = dB_{(2)} + \omega_{(3)} \text{YM}$$

YM - Chern Simons

$$\text{where } d\omega_3 = \text{Tr. } F_{(2)} \wedge F_{(2)}$$

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We are looking for a Dyonic 1-brane solution which carries  $Q_1$  electric charge &  $Q_5$  magnetic charge

$$Q_5 \sim \int_{S^3} F_{(3)} = \int_{S^3} dB_{(2)} + \int_{S^3} \omega_{(3)} \gamma_M$$

At  $r \rightarrow \infty$  YM vacuum  $\Rightarrow \bar{F}_{(2)} = 0$

$$\Rightarrow A = g^{-1} \partial g \quad g \in G$$

$g : S^3 \rightarrow G$  (for simplicity  
we take  $G = SU(2)$ )

Winding # =  $n$

defines  $|n\rangle$  vacuum

$$\int_{S^3} \omega_{(3)} \gamma_M \sim n$$

$\Rightarrow |n\rangle_{YM}$  vacuum contributes  $n$  units of  $Q_5$  charge.

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~~Now~~ The Near Horizon limit  
 of the Dyonic I-brane Solution  
 (w 5-brane - I-brane Bound  
 states from 10-Dim point of view)

will be

$$\text{AdS}_3 \times S^3$$

where radius of  $\text{AdS}_3 \sim Q, Q_5$

$\Rightarrow$  Trace anomaly in the 2-dim  
 dual Conformal Field theory

$$c_L + c_R \sim Q, Q_5$$

$\Rightarrow$  Changing the winding number  $|n\rangle$   
 will change the central charge.

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Hence Instantons that tunnel between different winding numbers  $|n\rangle$  should give rise to a flow between different CFT's.

Comment

$\rightarrow$  if  $\tilde{v} \neq 0$  then  $Q$

$$Q_1 = \int_{S^3} e^{2\phi} \times F_3 \quad \text{will also}$$

pick a contribution from the winding number of YM-vacua  $|n\rangle$ .

Thus instantons will also change  $Q$ ,

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SOLUTION

(Duff, Liu &amp; Pope)

(Lima, Liu, Ovrut &amp; Pope)

Looking for a solution of the form

$$ds^2 = e^{2f(y)}(dx^4)^2 + e^{-2f(y)} ds_4^2$$

where  $f(y)$  depends on transverse directions

$$F_{(3)} = dx^0 \wedge dx^1 \wedge V_{(0,1)}(y) + G_{(0,3)}(y)$$

$y$ -M- Instantons will sit in the  
4-Transverse directions labelled by  $y$

ie  $F_{(2)} \wedge F_{(2)}$  is  $(0,4)$  Form

$$dF_{(3)} = \text{Tr } F_{(2)} \wedge F_{(2)}$$

$$\Rightarrow dV_{(0,1)} = 0 \implies V = d\lambda$$

$$\text{AND } dG_{(0,3)} = \text{Tr } F_{(2)} \wedge F_{(2)}$$

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$${}^*F_{(3)} = e^{-4f} \hat{\star} d\Lambda + e^{4f} dx^0 dx'^1 \hat{\star} G_{(0,3)}$$

$$d(e^{20} {}^*F_3) = 0 \Rightarrow$$

$$\boxed{d(e^{20-4f} \hat{\star} d\Lambda) = 0}$$

$$d(e^{20+4f} \hat{\star} G_{(0,3)}) = 0$$

$$\Rightarrow \hat{\star} G_{(0,3)} = e^{-20-4f} d\tilde{\Lambda}(y)$$

$$\Rightarrow \boxed{d(e^{-20-4f} \hat{\star} d\tilde{\Lambda}) = \text{Tr } F_{(2)} \wedge F_{(2)}}$$

SUSY variation

$$\delta \psi_M = D_M \epsilon + \frac{1}{24} e^\theta F_{(3)} \Gamma_M \epsilon$$

$$\delta \chi = \frac{1}{2} \not{\partial} \theta \epsilon - \frac{1}{12} e^\theta F_{(3)} \epsilon$$

$$\delta \gamma = F_{(2)} \epsilon$$

For  $F_{(2)}$  self dual in Transverse directions  
 $\epsilon$  is chiral in Transverse directions

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$\Rightarrow \epsilon$  has definite chirality in  $(x^0, x')$  directions.

$\delta \psi_M = 0$  &  $\delta \chi = 0$  can be solved by

$$ds_4^2 = \sum_i dy_i^2 \quad (\text{flat})$$

$$\epsilon = e^{f/2} \cdot (\text{constant spinor})$$

$d\Lambda = \frac{1}{2} d(e^{-\theta+2f})$
$d\tilde{\Lambda} = \frac{1}{2} d(e^{\theta+2f})$

Plugging this in - the 2 Laplacian eqs,  
for  $\Lambda$  &  $\tilde{\Lambda}$  gives

$$\square e^{\theta-2f} = 0$$

$$\square e^{-\theta-2f} = \text{Tr } F_{(2)} \Lambda F_{(2)}$$

where  $\square$  is the Laplacian in flat  $\mathbb{R}^4$   
along  $y_i$  directions.

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$$e^{\theta - 2f} = \frac{Q_1}{r^2}$$

$$r^2 = \sum_i y_i \bar{y}_i$$

$$e^{-\theta - 2f} = \frac{1}{\square} \operatorname{Tr} F_{(2)} \wedge F_{(2)}$$

For t'Hooft instanton solutions  
(in singular gauge)

$$A_{jk} \approx \bar{\sigma}_{ikj} \partial_j \ln \rho$$

$$\rho = 1 + \sum_{a=1}^n \frac{\gamma_a^2}{(\gamma - \gamma_a)^2} \quad n = \text{instanton number}$$

$$\operatorname{Tr} F_{(2)} \wedge F_{(2)} = \square \square \ln \tilde{\rho}$$

$$e^{-\theta - 2f} = \frac{Q_5}{r^2} + \square \ln \tilde{\rho}$$

$$\text{where } \tilde{\rho} = \rho \prod_{a=1}^n (\gamma - \gamma_a)^2$$

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## Asymptotic behaviour of the solution

$$e^{-4f} \rightarrow \begin{cases} \frac{16 Q_1 Q_5}{r^4} & r \rightarrow 0 \\ \frac{16 Q_1 (Q_5 + n)}{r^4} & r \rightarrow \infty \end{cases}$$

$$\frac{1}{8\pi^2} \int_{S^3} F_{(3)} \rightarrow \begin{cases} Q_5 & r \rightarrow 0 \\ Q_5 + n & r \rightarrow \infty \end{cases}$$

$$\frac{1}{8\pi^2} \int_{S^3} e^{2\phi} * F_{(3)} \rightarrow \begin{cases} Q_1 & r \rightarrow 0 \\ Q_1 & r \rightarrow \infty \end{cases}$$

(This is because ~~also~~ we have  
set  $\tilde{v} = 0$ )

The metric in the two limits  $r \rightarrow 0$   $r \rightarrow \infty$   
becomes that of  $AdS_3 \times S^3$

$$ds^2 = \frac{r^2}{L^2} (dx^m)^2 + L^2 \left( \frac{dr^2}{r^2} + \frac{ds^2}{S^3} \right)$$

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where  $L \approx \begin{cases} (Q_1 Q_5)^{1/4} & r \rightarrow 0 \\ (Q_1 (Q_5 + n))^{1/4} & r \rightarrow \infty \end{cases}$

$$\Rightarrow \frac{c|_{r=0}}{c|_{r=\infty}} = \frac{Q_1 Q_5}{Q_1 (Q_5 + n)} < 1$$

$r = \infty$  is the UV point

and  $r = 0$  is the IR point

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To understand the operator in the Boundary CFT that is responsible for the flow we expand the solution near the UV point and from the leading deviation from  $\text{AdS}_3$

$$\sim \frac{1}{r^2}$$

→ Operator has  $\Delta = 2$

→ Flow is a VEV Flow

$\Delta = 2$  means that the operator is marginal at U.V. point BUT NOT EXACTLY MARGINAL

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## Conclusions

- 1) 4-dm Instantons generate flows between  $(4,0)$  SCFT's that are relevant for 5-brane - 1-brane systems in Type I / Heterotic like theories
- 2) Deformations are due to  $\Delta = 2$  operators that are not exactly Marginal.
- 3) Need to understand the Instanton Moduli Space in terms of SCFT.
- 4) what are these  $(4,0)$  SCFTs??