PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

Holographic C-theorems

with A. Sinha

(arXiv:1006.1263 & in progress)

Motivation:

"role of higher curvature interactions on AdS/CFT calculations"

Overview:

1. Introductory remarks on c-theorem and CFT's
2. Holographic c-theorem I: Einstein gravity
3. Holographic c-theorem II: Quasi-topological gravity
4. Holographic c-theorem III: Higher curvature theories
5. a_d^*, Entanglement entropy and Beyond
6. Concluding remarks

Zamolodchikov c-theorem (1986):

- renormalization-group (RG) flows can seen as one-parameter motion

$$\frac{d}{dt} \equiv -\beta^i(g)\,\frac{\partial}{\partial g^i}$$

in the space of (renormalized) coupling constants $\{g^i,\ i = 1, 2, 3, \cdots\}$ with beta-functions as "velocities"

- for unitary, renormalizable QFT's in two dimensions, there exists a positive-definite real function of the coupling constants $c(g)$:

1. monotonically decreasing along flows: $\frac{d}{dt}c(g) \le 0$

2. "stationary" at fixed points $g^i = (g^*)^i$:

$$\beta^i(g^*) = 0 \longleftrightarrow \frac{\partial}{\partial g^i}c(g) = 0$$

3. at fixed points, it equals central charge of corresponding CFT

$$c(g^*) = c$$

C-theorems in higher dimensions??

d=2: $$\langle T_\mu{}^\mu \rangle = -\frac{c}{12} R$$

d=4: $$\langle T_\mu{}^\mu \rangle = \frac{c}{16\pi^2} I_4 - \frac{a}{16\pi^2} E_4 - \frac{a'}{16\pi^2} \nabla^2 R$$

$$I_4 = C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \quad \text{and} \quad E_4 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

- in 4 dimensions, have three central charges: $c,\ a,\ a'$
- do any of these obey a similar "c-theorem" under RG flows?

✗ <u>a'-theorem</u>: a' is scheme dependent (not globally defined)

✗ <u>c -theorem</u>: there are numerous counter-examples

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<u>a -theorem</u>: proposed by Cardy (1988)

- numerous nontrivial examples, eg, perturbative fixed points (Jack & Osborn), SUSY gauge theories (Anselmi et al)

(Anselmi, Freedman, Grisaru & Johansen, hep-th/9708042)

SUSY example:

- SU(N_c) supersymmetric QCD with N_f flavors of massless quarks

with $3/2 \leq N_f/N_c \leq 3$

- in UV, asymptotically free:

$$a_{UV} = \frac{1}{48}\left(9\,N_c^2 - 9 + 2\,N_f\,N_c\right)$$
$$c_{UV} = \frac{1}{24}\left(3\,N_c^2 - 3 + 2\,N_f\,N_c\right)$$

- in IR, flows to nontrivial conformal fixed point:

$$a_{IR} = \frac{3}{16}\left(2\,N_c^2 - 1 - 3\,\frac{N_c^4}{N_f^2}\right)$$
$$c_{IR} = \frac{1}{16}\left(7\,N_c^2 - 2 - 9\,\frac{N_c^4}{N_f^2}\right)$$

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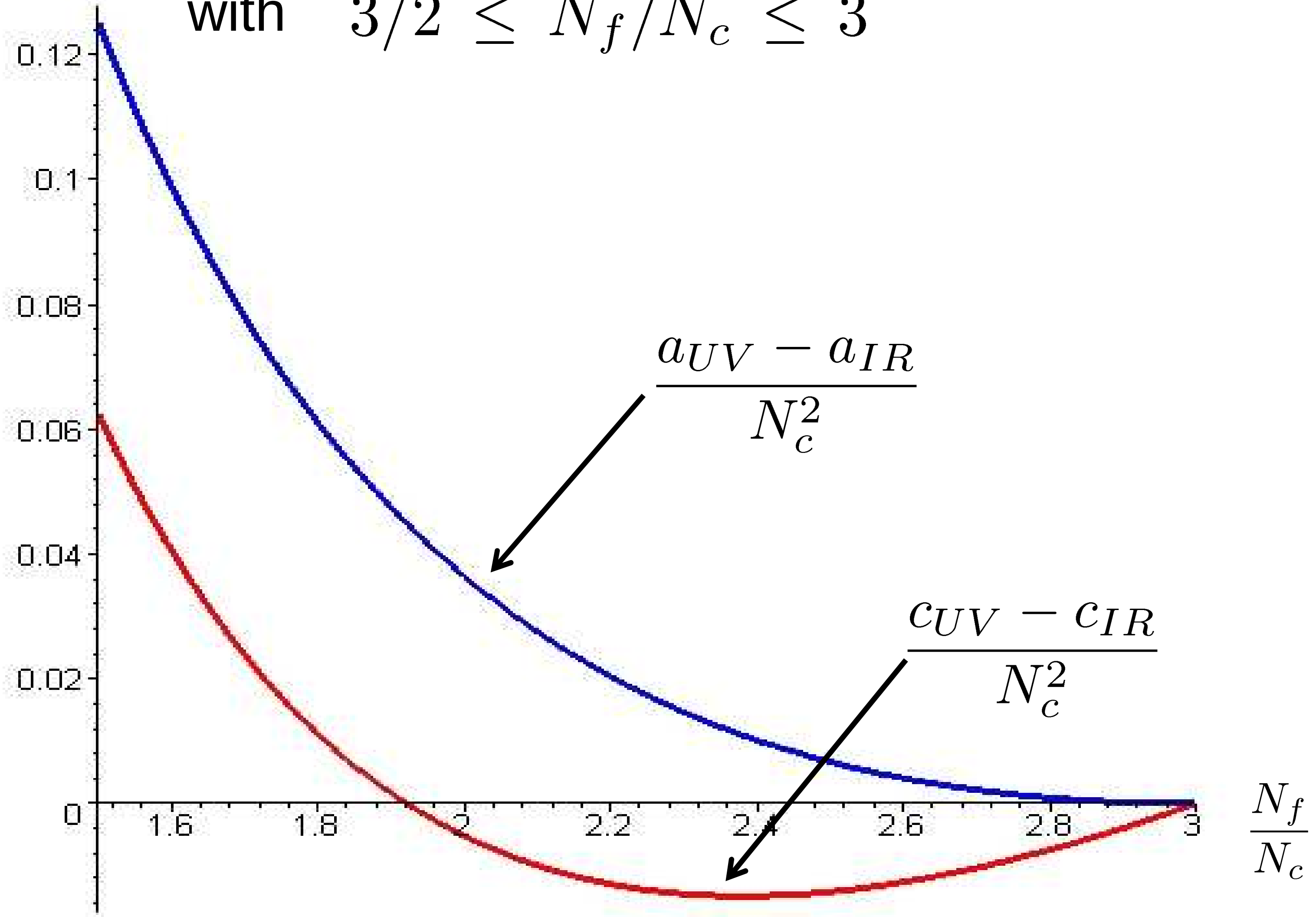

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- holographic field theories with gravity dual $(a = c)$
- no completely general proof
- counterexample proposed: Shapere & Tachikawa, 0809.3238

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Counterexample to a-theorem:

- flow between two N = 2 superconformal gauge theories

UV: gauge group SU(N_c+1) with N_f=2N_c fundamental hyper's
IR: gauge group SU(N_c) with N_f=2N_c fundamental hyper's (m=0)

$$a_{UV} - a_{IR} = \frac{1}{72}\left(19\,N_c - 7\,N_c^2 + 15\right) \qquad \left(\leq 0 \quad \text{for } N_c \geq 4\right)$$

- loophole: accidental U(1) symmetry appears in the IR limit

- Possibilities?:

i) no theorem exists

ii) "C-theorem" exists but C(g*) ≠ a — **universal**

Today's Talk

iii) "C-theorem" exists and C(g*) = a, but theorem needs fine-print for d>2 — **not universal**

iv) something more interesting??

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(Freedman, Gubser, Pilch & Warner, hep-th/9904017)
(Girardello, Petrini, Porrati and Zaffaroni, hep-th/9810126)

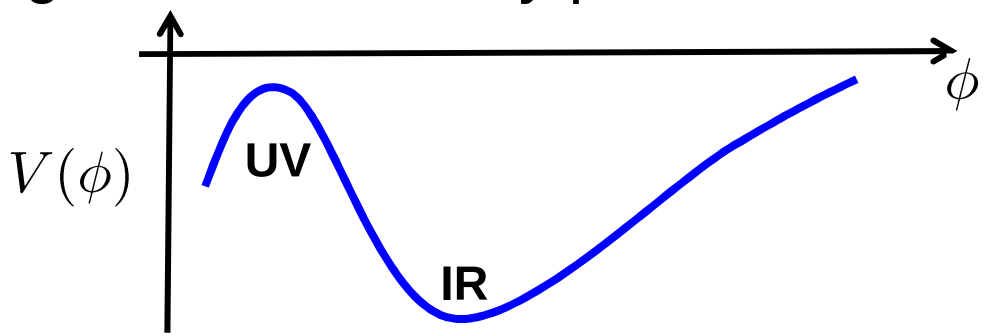
Holographic RG flows:

$$I = \frac{1}{2\ell_P^3}\int d^5x\,\sqrt{-g}\,(R+\mathcal{L}_{\rm matter})$$

- assume stationary points: matter fields fixed and $\mathcal{L}_{\rm matter} = \frac{12}{L^2}\alpha_i^2$

(eg, scalar field: $\mathcal{L}_{\rm matter} = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$)

- consider metric: $ds^2 = e^{2A(r)}(-dt^2+dx_1^2+dx_2^2+dx_3^2)+dr^2$
- at stationary points, AdS$_5$ vacuum: $A(r) = r/\tilde{L}$ with $\tilde{L} = L/\alpha_i$
- RG flows are solutions starting at one stationary point and ending at another



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Holographic RG flows:

- for general flow solutions, define: $a(r) \equiv \frac{\pi^2}{\ell_P^3 A'(r)^3}$

$$a'(r) = -\frac{3\pi^2}{\ell_P^3 A'(r)^4} A''(r) = -\frac{\pi^2}{\ell_P^3 A'(r)^4} \left(T^t{}_t - T^r{}_r \right) \geq 0$$

Einstein equations **null energy condition**

- at stationary points, $a(r) \to a^* = \pi^2 \tilde{L}^3 / \ell_P^3$ and hence

$$\boxed{a^*_{UV} \geq a^*_{IR}}$$

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$$\boxed{a_{UV} \geq a_{IR}}$$

- using holographic trace anomaly: $a^* = a$

(e.g., Henningson & Skenderis)

→ supports Cardy's conjecture

- for Einstein gravity, central charges equal $(a = c)$: $c_{UV} \geq c_{IR}$

(Freedman, Gubser, Pilch & Warner, hep-th/9904017)

Holographic RG flows:

$$I = \frac{1}{2\ell_P^{d-1}} \int d^{d+1}x \sqrt{-g}\, (R + \mathcal{L}_{\rm matter})$$

- same story is readily extended to (d+1) dimensions
- defining: $a_d(r) \equiv \dfrac{\pi^{d/2}}{\Gamma\left(d/2\right) \left(\ell_P A'(r)\right)^{d-1}}$

$$a'(r) \quad = \quad -\underbrace{\frac{(d-1)\pi^{d/2}}{\Gamma\left(d/2\right)\ell_P^{d-1} A'(r)^d} A''(r) = -}_{\text{Einstein equations}}\underbrace{\frac{\pi^{d/2}}{\Gamma\left(d/2\right)\ell_P^{d-1} A'(r)^d}\left({T^t}_t - {T^r}_r\right) \geq 0}_{\text{null energy condition}}$$

- at stationary points, $a(r) \to a^* = \pi^{d/2}/\Gamma(d/2)\, (\tilde{L}/\ell_P)^{d-1}$ and so

$$\boxed{a^*_{UV} \geq a^*_{IR}}$$

- using holographic trace anomaly: $a^* \propto$ central charges

(for even d! what about odd d?) (e.g., Henningson & Skenderis)

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Improved Holographic RG Flows:

- add higher curvature interactions to bulk gravity action

$\longrightarrow$ provides holographic field theories with, eg, $a \neq c$
so that we can clearly distinguish evidence of a-theorem

(Nojiri & Odintsov; Blau, Narain & Gava)

$\longrightarrow$ more generally broadens class of dual CFT's

Improved Holographic RG Flows:

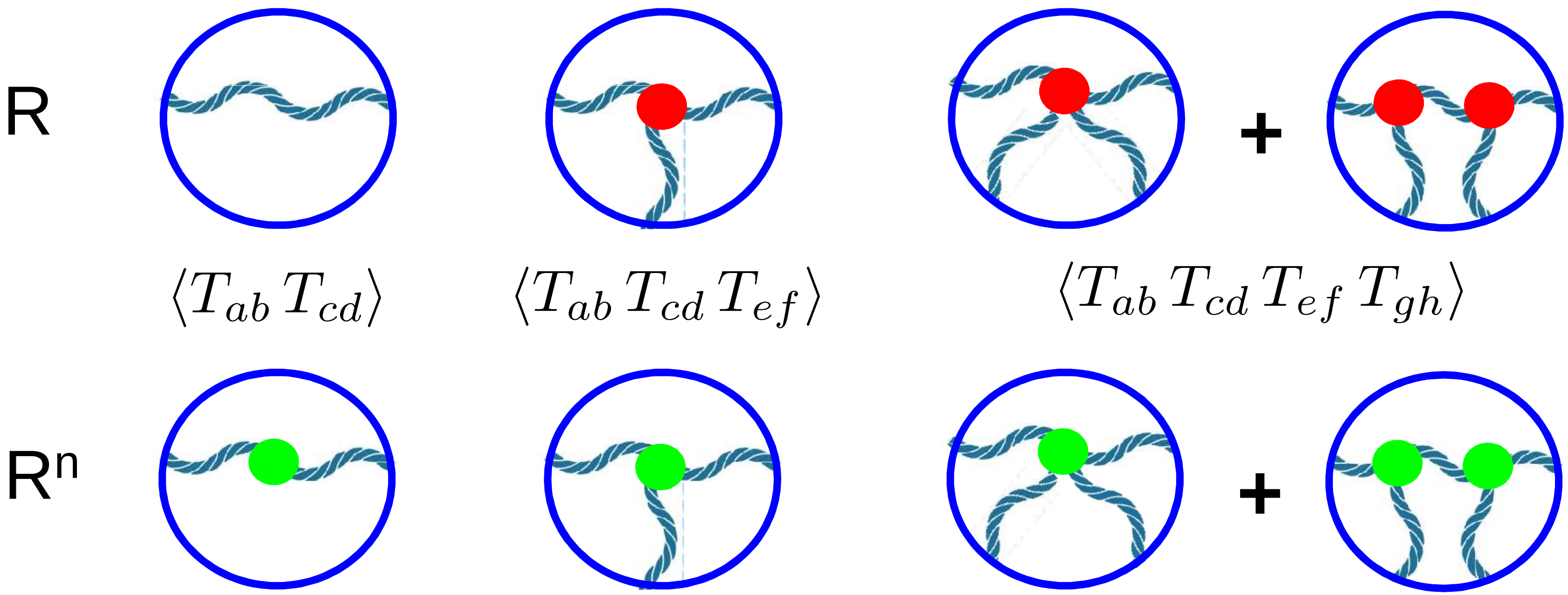
- engineering the gauge theory: gauge groups, field content, . . .

versus

- engineering the CFT: parameters in n-point correlators, . . .

graviton, h_{ab} $\longleftrightarrow$ stress tensor, T_{ab}

- gravitational action naturally connected to correlators of T_{ab}



- adding higher curvature terms changes both parameter values and also form of n-point functions in dual CFT's

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Higher Curvature Terms in Derivative Expansion

- in strings, sugra action corrected by higher curvature terms

α' corrections: $\alpha'/L^2 \simeq 1/\sqrt{\lambda}$

string loops: $g_s \simeq \lambda/N_c$

- perturbing sugra theory with higher curvature terms provides insight into finite $N_c,\ \lambda$ corrections in gauge theory
- here I want to go beyond perturbative framework to study RG flows (i.e., want to consider finite values of new couplings)
- if we go to finite parameters where one of the higher curvature terms is important, expect all are important
- ultimately one needs to fully develop string theory for interesting holographic background's

Higher Curvature Terms without Derivative Expansion

- **instead** consider "toy models" with finite R^n interactions (where we can maintain control of calculations)
- with AdS/CFT, higher curvature couplings become dials to adjust parameters characterizing the dual CFT
- note that any one R^n interaction implicitly determines an infinite number of couplings in T_{ab} correlators

What about the swampland?

- constrain gravitational couplings with consistency tests (positive fluxes; causality; unitarity) and keep fingers crossed!
- seems an effective approach with Lovelock gravity

(eg, Brigante, Liu, Myers, Shenker & Yaida)

(see also Parnachev's talk)

Quasi-Topological gravity:

(Myers & Robinsion, 1003.5357)

$$I=\frac{1}{2\ell_P^3}\int d^5x\,\sqrt{-g}\left[\frac{12}{L^2}+R+L^2\frac{\lambda}{2}\,\chi_4+L^4\frac{7\mu}{4}\,\mathcal{Z}_5\right]$$

with $$\chi_4 \;=\; R^{abcd}R_{abcd}-4R_{ab}R^{ab}+R^2$$

$$\begin{aligned}\mathcal{Z}_5 \;=\;& R_a{}^c{}_b{}^d R_d{}^e{}_c{}^f R_e{}^a{}_f{}^b+\frac{1}{56}\big(21R_{abcd}R^{abcd}R-72R_{abcd}R^{abc}{}_eR^{de}\\ &+120R_{abcd}R^{ac}R^{bd}+144R_a{}^bR_b{}^cR_c{}^a-132R_a{}^bR_b{}^aR+15R^3\big)\end{aligned}$$

- three dimensionless couplings, $L/\ell_P\,,\ \lambda\,,\ \mu\,,$ allow us to explore dual CFT's with most general three-point function $\langle T_{ab}\,T_{cd}\,T_{ef}\rangle$

"maintain control of calculations"

- analytic black hole solutions
- linearized eom in AdS_5 are second order (in fact, Einstein eq's!)
- can be extended to higher dimensions (D≥7)
- gravitational couplings constrained – see Parnachev's talk

(Myers, Paulos & Sinha, 1004.2055)

Quasi-Topological gravity:

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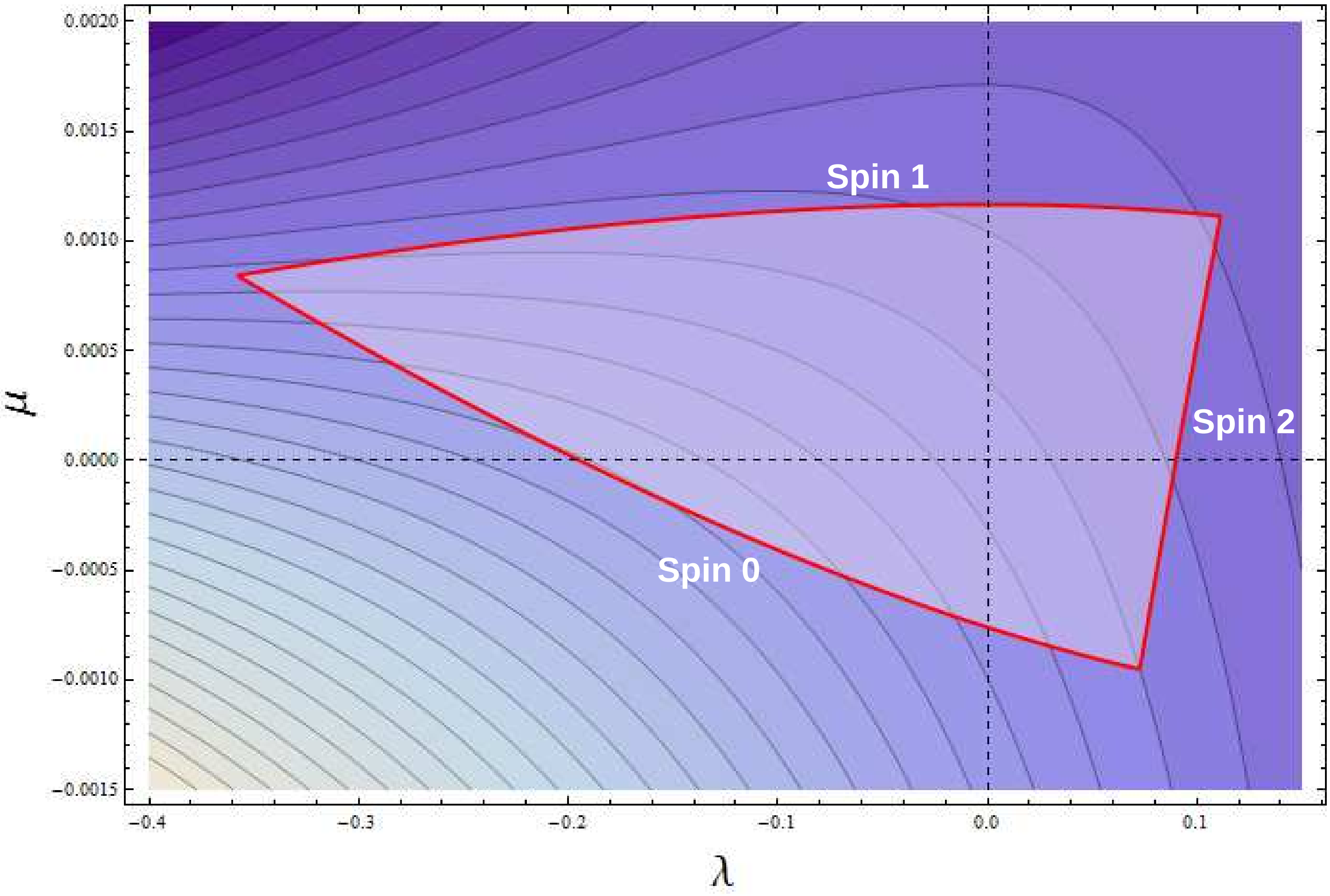

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with
$$\chi_4 = R^{abcd} R_{abcd} - 4 R_{ab} R^{ab} + R^2$$

$$\mathcal{Z}_5 = R_a{}^c{}_b{}^d R_d{}^e{}_c{}^f R_e{}^a{}_f{}^b + \frac{1}{56}\big(21 R_{abcd} R^{abcd} R - 72 R_{abcd} R^{abc}{}_e R^{de} + 120 R_{abcd} R^{ac} R^{bd} + 144 R_a{}^b R_b{}^c R_c{}^a - 132 R_a{}^b R_b{}^a R + 15 R^3\big)$$

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anticipate RG flows

with
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$$\left.+120R_{abcd}R^{ac}R^{bd} + 144R_a{}^bR_b{}^cR_c{}^a - 132R_a{}^bR_b{}^aR + 15R^3\right)$$

- so calculate!
- curvature in AdS$_5$ vacuum: $\dfrac{1}{\tilde{L}^2} = \dfrac{f_\infty}{L^2}$,

where $\quad \alpha^2 - f_\infty + \lambda f_\infty^2 + \mu f_\infty^3 = 0$

- holographic trace anomaly:

(Myers, Paulos & Sinha, 1004.2055)

$$a = \pi^2\frac{\tilde{L}^3}{\ell_P^3}\left(1 - 6\lambda f_\infty + 9\mu f_\infty^2\right)\,, \qquad c = \pi^2\frac{\tilde{L}^3}{\ell_P^3}\left(1 - 2\lambda f_\infty - 3\mu f_\infty^2\right)$$

RG flows in Quasi-Topological gravity:

- consider metric: $ds^2 = e^{2A(r)}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + dr^2$

$\longrightarrow$ AdS$_5$ vacua: $A(r) = r/\tilde{L}$

- natural to define "flow functions":

$$a(r) \equiv \frac{\pi^2}{\ell_P^3 A'(r)^3}\left(1 - 6\lambda L^2 A'(r)^2 + 9\mu L^4 A'(r)^4\right)$$

$$c(r) \equiv \frac{\pi^2}{\ell_P^3 A'(r)^3}\left(1 - 2\lambda L^2 A'(r)^2 - 3\mu L^4 A'(r)^4\right)$$

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- in general flows:

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assume null energy condition

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✓

- can try to be more creative in defining c(r) but we were unable to find a expression where flow is guaranteed to be monotonic

- our toy model seems to provide support for Cardy's “a-theorem” in four dimensions

Higher Dimensions: $D = d + 1 \ (d \geq 6)$

- straightforward to reverse engineer "a-theorem" flows
- eq's of motion:

$$T^t{}_t - T^r{}_r = (d-1)\, A''(r)\, \left(1 - 2\lambda L^2 A'(r)^2 - 3\mu L^4 A'(r)^4\right)$$

- expression with natural flow:

$$a_d(r) \equiv \frac{\pi^{d/2}}{\Gamma(d/2)\,(\ell_P A'(r))^{d-1}} \left(1 - \frac{2(d-1)}{d-3}\lambda L^2 A'(r)^2 - \frac{3(d-1)}{d-5}\mu L^4 A'(r)^4\right)$$

$$\Longrightarrow \quad a_d'(r) = -\frac{\pi^{d/2}}{\Gamma(d/2)\ell_P^{d-1} A'(r)^d} \left(T^t{}_t - T^r{}_r\right) \geq 0$$

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- flow between stationary points (where $a_d^* \equiv a_d(r)|_{AdS}$)

$$(a_d^*)_{UV} \geq (a_d^*)_{IR}$$

What is a_d^* ??

What is a_d^* ??

$$a_d^* = \frac{\pi^{d/2}\tilde{L}^{d-1}}{\Gamma(d/2)\ell_P^{d-1}}\left(1 - \frac{2(d-1)}{d-3}\lambda f_\infty - \frac{3(d-1)}{d-5}\mu f_\infty^2\right)$$

where AdS curvature: $\frac{1}{\tilde{L}^2} = \frac{f_\infty}{L^2}\,, \quad \alpha^2 - f_\infty + \lambda f_\infty^2 + \mu f_\infty^3 = 0$

- a_d^* is NOT C_T , coefficient of leading singularity in

$$\langle\, T_{ab}(x)\, T_{cd}(0)\,\rangle = \frac{C_T}{x^{2d}}\,\mathcal{I}_{ab,cd}(x)$$

- a_d^* is NOT C_S , coefficient in entropy density: $s = C_S\, T^{d-1}$

What is a_d^* ??

$$a_d^* = \frac{\pi^{d/2}\tilde{L}^{d-1}}{\Gamma(d/2)\ell_P^{d-1}}\left(1 - \frac{2(d-1)}{d-3}\lambda f_\infty - \frac{3(d-1)}{d-5}\mu f_\infty^2\right)$$

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- trace anomaly for CFT's with even d:

$$\langle\, T_\mu{}^\mu \,\rangle = \sum B_i(\text{Weyl invariant})_i - 2(-)^{d/2} A\,(\text{Euler density})_d$$

- verify that we have precisely reproduced central charge

$$\boxed{a_d^* = A}$$

(Henningson & Skenderis; Nojiri & Odintsov; Blau, Narain & Gava; Imbimbo, Schwimmer, Theisen & Yankielowicz)

→ agrees with Cardy's proposal (1988)

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(Henningson & Skenderis; Nojiri & Odintsov; Blau, Narain & Gava; Imbimbo, Schwimmer, Theisen & Yankielowicz)

What is a_d^* for odd d?? **(Later!)**

RG flows in Quasi-Topological gravity:

Comment:

- "c-theorem" still assume null energy condition

$\longrightarrow$ construct a toy model with reasonable physical properties

creative gravity

status quo matter

$$V(\phi^*) = -\frac{12}{L^2}\alpha^2$$

- natural to consider more general models:

$$I = \frac{1}{2\ell_P^3}\int d^5x\,\sqrt{-g}\,\Big[-V(\phi)\,+R+\frac{L^2}{2}\lambda(\phi)\,\chi_4+\frac{7L^4}{4}\mu(\phi)\,\mathcal{Z}_5$$
$$+L^2\gamma(\phi)\,R^{ab}\partial_a\phi\,\partial_b\phi+L^4\gamma'(\phi)R^2\nabla^2\phi+\cdots\Big]$$

What are the rules??

Motivation:

"role of higher curvature interactions on AdS/CFT calculations"

Overview:

1. Introductory remarks on c-theorem and CFT's

2. Holographic c-theorem I: Einstein gravity

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5. a_d^*, Entanglement entropy and Beyond

6. Concluding remarks

More Improved Holographic RG Flows:

- quasi-topological gravity obeys c-theorem in very nontrivial way
- how robust really is this result??
- new toy models: start with arbitrary curvature-cubed action

$$I=\frac{1}{2\ell_P^{d-1}}\int \mathrm{d}^{d+1}x\,\sqrt{-g}\,\left[\frac{d(d-1)}{L^2}\alpha^2+R+L^2\widetilde{\mathcal{X}}+L^4\widetilde{\mathcal{Z}}\right]$$

where
$$\begin{aligned}\widetilde{\mathcal{X}} &= b_1 R_{abcd}R^{abcd}+b_2 R_{ab}R^{ab}+b_3R^2\ ,\\ \widetilde{\mathcal{Z}} &= c_1\,R_a{}^c{}_b{}^d R_c{}^e{}_d{}^f R_e{}^a{}_f{}^b+c_2\,R_{ab}{}^{cd}R_{cd}{}^{ef}R_{ef}{}^{ab}+c_3\,R_{abcd}R^{abc}{}_eR^{de}\\ &\quad +c_4\,R_{abcd}R^{abcd}R+c_5\,R_{abcd}R^{ac}R^{bd}+c_6\,R_a{}^bR_b{}^cR_c{}^a\\ &\quad +c_7\,R_a{}^bR_b{}^aR+\,c_8\,R^3\,.\end{aligned}$$

- AdS vacua: $\dfrac{1}{\tilde{L}^2}=\dfrac{f_\infty}{L^2}$, where $\quad \alpha^2-f_\infty+\lambda f_\infty^2+\mu f_\infty^3=0$

$$\begin{aligned}\lambda &= \frac{d-3}{d-1}\left(2\,b_1+d\,b_2+d(d+1)\,b_3\right)\\ \mu &= -\frac{d-5}{d-1}\left((d-1)\,c_1+4\,c_2+2d\,c_3+2d(d+1)\,c_4+d^2\,c_5+d^2\,c_6+d^2(d+1)\,c_7+d^2(d+1)^2c_8\right)\end{aligned}$$

More Improved Holographic RG Flows:

- is it reasonable to expect any theory to obey a c-theorem? **NO**
- how do we constrain theory to be physically reasonable?
- recall one of the nice properties of quasi-top. gravity was that linearized graviton equations in AdS were 2nd order
- greatly facilitates calculations but deeper physical significance
- analogy with higher derivative scalar field eq. (in flat space)

$$\left(\nabla^2+\frac{a}{M^2}(\nabla^2)^2\right)\phi=0 \longrightarrow \frac{1}{q^2(1-a\,q^2/M^2)}=\frac{1}{q^2}-\underset{\uparrow\ \text{ghost}}{\frac{1}{q^2-M^2/a}}$$

- graviton ghosts will be generic with 4th order equations

$\longrightarrow$ couple to additional non-unitary tensor operator in dual CFT

More Improved Holographic RG Flows:

$$I = \frac{1}{2\ell_P^{d-1}} \int \mathrm{d}^{d+1}x\,\sqrt{-g}\,\left[\frac{d(d-1)}{L^2}\alpha^2 + R + L^2\widetilde{\mathcal{X}} + L^4\widetilde{\mathcal{Z}}\right]$$

where
$$\begin{aligned}
\widetilde{\mathcal{X}} &= b_1 R_{abcd}R^{abcd} + b_2 R_{ab}R^{ab} + b_3 R^2\ , \\
\widetilde{\mathcal{Z}} &= c_1\, R_a{}^c{}_b{}^d R_c{}^e{}_d{}^f R_e{}^a{}_f{}^b + c_2\, R_{ab}{}^{cd} R_{cd}{}^{ef} R_{ef}{}^{ab} + c_3\, R_{abcd}R^{abc}{}_e R^{de} \\
&\quad + c_4\, R_{abcd}R^{abcd}R + c_5\, R_{abcd}R^{ac}R^{bd} + c_6\, R_a{}^b R_b{}^c R_c{}^a \\
&\quad + c_7\, R_a{}^b R_b{}^a R + c_8\, R^3\,.
\end{aligned}$$

• demand that linearized graviton equations in AdS were 2nd order

R^2 interaction is GB

$$b_1 = b_3\,, \qquad b_2 + 4b_1 = 0$$

$$\begin{aligned}
3c_1 - 24c_2 - 4(d+1)c_3 - 4d(d+1)c_4 \\
-(2d-1)c_5 - 3d\,c_6 - d(d+1)c_7 = 0
\end{aligned}$$

$$3c_1 - 12c_2 - 2d\,c_3 - 2(d^2+d-4)c_4 + 2c_5 + 4d\,c_7 + 6d(d+1)c_8 = 0$$

Work in progress!

More Improved Holographic RG Flows:

- as before, try reverse engineer "c-theorem" flows by examining eom: $T^t{}_t - T^r{}_r = \cdots$ ← contains 4-derivative terms!

- need an extra constraint to reduce this expression to 2nd order

$$4c_2 + (d+1)c_3 + 4d\,c_4 + d\,c_5 + \frac{d^2+1}{2}c_6 + d(d+1)c_7 + 4d^2 c_8 = 0$$

- with extra constraint, eq's of motion yield:

$$T^t{}_t - T^r{}_r = (d-1)\,A''(r)\,\left(1 - 2\lambda L^2 A'(r)^2 - 3\mu L^4 A'(r)^4\right)$$

- expression with natural flow:

$$a_d(r) \equiv \frac{\pi^{d/2}}{\Gamma(d/2)\,(\ell_P A'(r))^{d-1}}\left(1 - \frac{2(d-1)}{d-3}\lambda L^2 A'(r)^2 - \frac{3(d-5)}{d-5}\mu L^4 A'(r)^4\right)$$

$$\Rightarrow \quad a'_d(r) = -\frac{\pi^{d/2}}{\Gamma(d/2)\ell_P^{d-1}A'(r)^d}\left(T^t{}_t - T^r{}_r\right) \geq 0$$

assume null energy condition

More Improved Holographic RG Flows:

- as before, reverse engineer "c-theorem" flows
- with extra constraint, flow eq's of motion yield:

$$T^t{}_t - T^r{}_r = (d-1)\, A''(r)\, \left(1 - 2\lambda L^2 A'(r)^2 - 3\mu L^4 A'(r)^4\right)$$

- expression with natural flow:

$$a_d(r) \equiv \frac{\pi^{d/2}}{\Gamma(d/2)\,(\ell_P A'(r))^{d-1}} \left(1 - \frac{2(d-1)}{d-3}\lambda L^2 A'(r)^2 - \frac{3(d-1)}{d-5}\mu L^4 A'(r)^4\right)$$

$$\Longrightarrow \quad a_d'(r) = -\frac{\pi^{d/2}}{\Gamma(d/2)\ell_P^{d-1} A'(r)^d}\left(T^t{}_t - T^r{}_r\right) \geq 0$$

- flow between stationary points (where $a_d^* \equiv a_d(r)|_{AdS}$)

$$(a_d^*)_{UV} \geq (a_d^*)_{IR}$$

What is a_d^* ??

What is a_d^* ??

$$a_d^* = \frac{\pi^{d/2} L^{d-1}}{\Gamma(d/2)\ell_P^{d-1} f_\infty^{(d-1)/2}} \left(1 - \frac{2(d-1)}{d-3}\lambda f_\infty - \frac{3(d-1)}{d-5}\mu f_\infty^2\right)$$

where AdS curvature: $\frac{1}{\tilde{L}^2} = \frac{f_\infty}{L^2}\,, \quad \alpha^2 - f_\infty + \lambda f_\infty^2 + \mu f_\infty^3 = 0$

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$$\boxed{a_d^* = A}$$

(Henningson & Skenderis; Nojiri & Odintsov; Blau, Narain & Gava; Imbimbo, Schwimmer, Theisen & Yankielowicz)

$\longrightarrow$ again, agrees with Cardy's proposal (1988)

More Improved Holographic RG Flows:

- need an extra constraint to reduce flow eq.'s to 2nd order

$$4c_2 + (d+1)c_3 + 4dc_4 + dc_5 + \frac{d^2+1}{2}c_6 + d(d+1)c_7 + 4d^2 c_8 = 0$$

- why an extra constraint?
  - original constraints ensure boundary CFT is unitary at fixed pts
  - away from fixed pts, nonunitary operators "pollute" boundary theory and c-theorem is lost in general
  - additional constraint ensures boundary theory is unitary along flows, as well as at fixed points

 **Work in progress!** 

What is a_d^* ??

$$a_d^* = \frac{\pi^{d/2} L^{d-1}}{\Gamma(d/2)\ell_P^{d-1} f_\infty^{(d-1)/2}} \left(1 - \frac{2(d-1)}{d-3}\lambda f_\infty - \frac{3(d-1)}{d-5}\mu f_\infty^2\right)$$

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(Henningson & Skenderis; Nojiri & Odintsov; Blau, Narain & Gava; Imbimbo, Schwimmer, Theisen & Yankielowicz)

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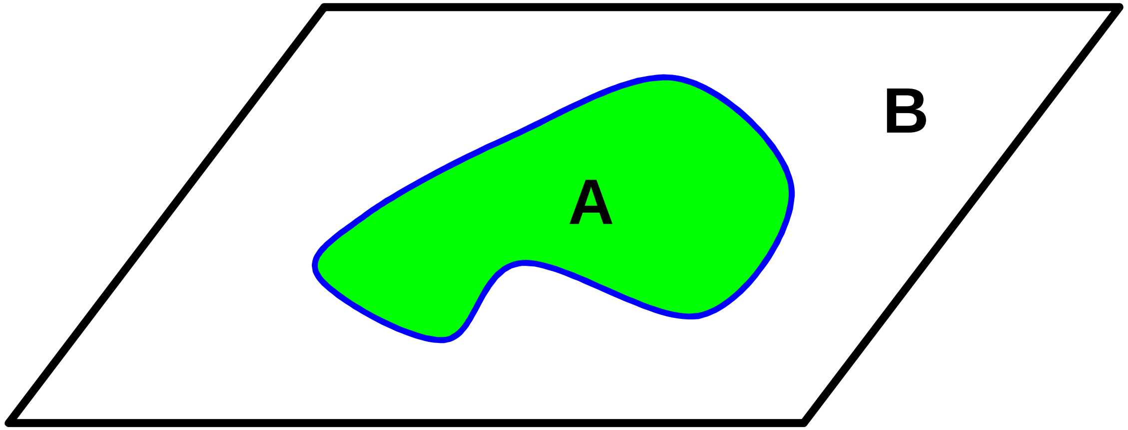
6. Concluding remarks

(see Kulaxizi's talk)

a_d^* and Entanglement Entropy

- introduce a(n arbitrary) boundary dividing the system in two
- integrate out degrees of freedom in outside region
- remaining dof are described by a density matrix ρ_A

$\longrightarrow$ entanglement entropy: $S = -Tr\left[\rho_A\, \log \rho_A\right]$



- full result sensitive to UV physics: $S \,=\, c_0 \dfrac{A_{boundary}}{\delta^{d-2}} \,+\, \cdots$
- universal information appears in subleading terms:

$$S \,=\, \cdots \,+\, c_d \log\left(R/\delta\right) \,+\, \cdots \qquad \text{for even d}$$

a_d^* and Entanglement Entropy

- in 1003.5357, studied black hole thermodynamics for quasi-topological gravity with various horizons: R^{d-1}, S^{d-1}, H^{d-1}
- allows for the following observation:
- place CFT on hyperbolic hyperplane (ie, R X H^{d-1})

$\longrightarrow$ ground-state energy density is now negative

- heat system up until energy density is precisely zero, $\rho_E = 0$

$\longrightarrow$ entropy density: $$s = (4\pi)^{d/2}\,\Gamma\left(d/2\right)\, a_d^*\, T^{d-1}$$
$$= \frac{2\pi}{\pi^{d/2}}\Gamma\left(d/2\right)\, \frac{a_d^*}{\tilde{L}^{d-1}}$$

Why entanglement entropy?

a_d^* and Entanglement Entropy

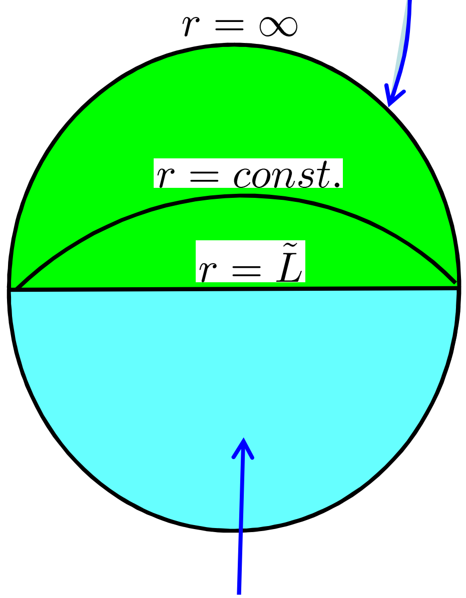
- CFT on hyperbolic hyperplane H^{d-1} at finite T tuned to $\rho_E = 0$

  $\longrightarrow$ bulk spacetime is pure AdS_{d+1}

- so why is there entropy at all??

$$ds^2 = \frac{dr^2}{\left(\frac{r^2}{\tilde{L}^2} - 1\right)} - \left(\frac{r^2}{\tilde{L}^2} - 1\right) dt^2 + r^2\, d\Sigma_2^{d-1}$$

t= const. slice of AdS



second asymptotic region

entanglement entropy: hyperbolic foliation divides boundary into two halves

- can make precise connection between horizon entropy of “bh” and entanglement in boundary CFT

  instead go back to “standard” definition

a_d^* and Entanglement Entropy

(see Kulaxizi's talk)

- place boundary CFT on S^{d-1} X R, divide sphere in half on an equator and calculate entanglement entropy

EE Calculation:

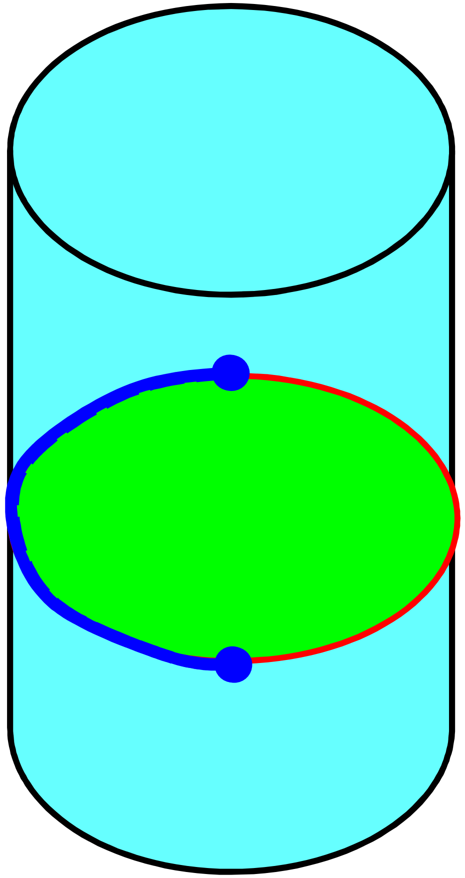
1) construct n-fold cover of (euclidean) geometry

2) calculate partition function Z_n on this geometry

3) analytically continue Z_n to real n

4) calculate entropy:

$$S = -\lim_{n\to 1}\left(n\frac{\partial}{\partial n}-1\right)\log Z_n$$



- would like to interchange steps 2) & 3)

a_d^* **and Entanglement Entropy**

(see Kulaxizi's talk)

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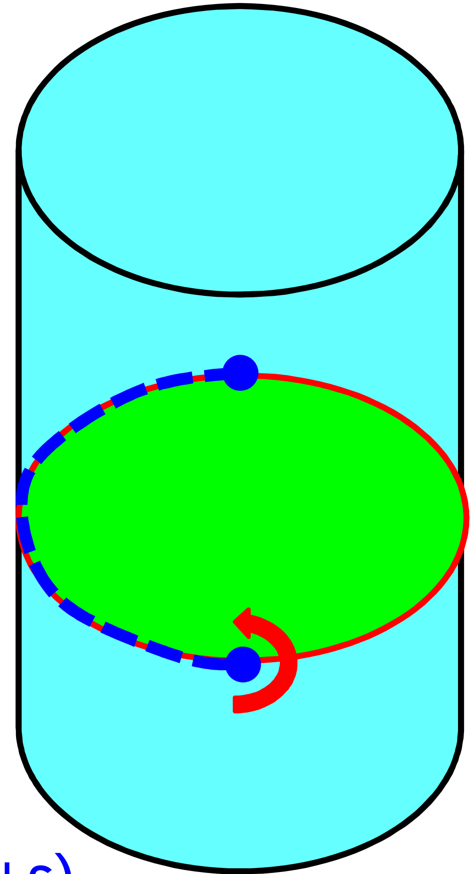

- would like to interchange steps 2) & 3)

→ work on geometry with conical defect (n=1+ε)

- this interchange requires angle is a symmetry, but not case here

a_d^* **and Entanglement Entropy**

(see Kulaxizi's talk)

- place boundary CFT on S^{d-1} X R, divide sphere in half on an equator and calculate entanglement entropy

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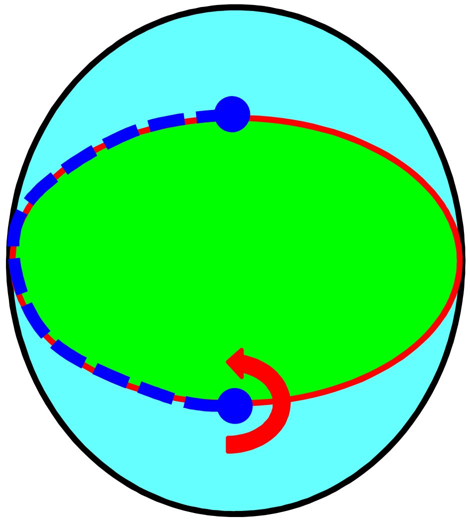


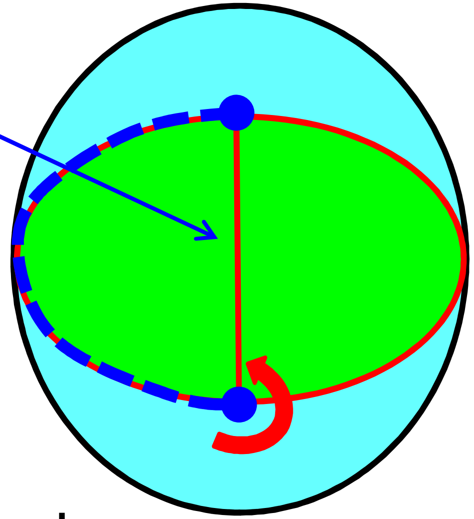
- use conformal symmetry to "compactify":

$$S^{d-1} \text{ X R} \longrightarrow S^d$$

- now have desired symmetry and so can calculate as above

a_d^* and Entanglement Entropy

$$S = -\lim_{n\to 1}\left(n\frac{\partial}{\partial n}-1\right)\log Z_n$$



- AdS/CFT translates to gravity calculation:

$$Z_n = \exp\left[-I_{gravity,n}\right]$$ on AdS$_{d+1}$ with S^d boundary

- note we consider completely general covariant gravity action:

$$I = \int d^{d+1}x\sqrt{-g}\,\mathcal{L}(g^{ab}, R^{ab}{}_{cd}, \nabla_e R^{ab}{}_{cd}, \ldots, matter) + boundary\ term$$

- note conical singularity extends through AdS bulk
- regulate singularity and determine response to small deficit

(e.g., Fursaev & Solodukhin, hep-th/9501127)

$$S = -2\pi\ \frac{\partial\mathcal{L}}{\partial R^{ab}{}_{cd}}\varepsilon^{ab}\,\varepsilon_{cd}\Big|_{AdS} \times \int_{defect} d^{d-1}x\,\sqrt{h}$$

a_d^* and Entanglement Entropy

- we had "expected" $S \propto A$ from trace anomaly (for even d)

$$\langle\, T_\mu{}^\mu \,\rangle = \sum B_i (\text{Weyl invariant})_i - 2(-)^{d/2} A \,(\text{Euler density})_d$$

(Imbimbo, Schwimmer, Theisen & Yankielowicz)

- short cut for holographic type-A trace anomaly:

  given any covariant gravitational action:

$$I = \int d^{d+1}x \sqrt{-g}\, \mathcal{L}(g^{ab}, R^{ab}{}_{cd}, \nabla_e R^{ab}{}_{cd}, \dots, matter) + boundary\ term$$

$$\longrightarrow \quad A = -\frac{\pi^{d/2}\, \tilde{L}^{d+1}}{2\Gamma\left(\frac{d}{2}+1\right)}\, \mathcal{L}\Big|_{AdS}$$

$$\boxed{a_d^* \equiv -\frac{\pi^{d/2}\, \tilde{L}^{d+1}}{2\Gamma\left(\frac{d}{2}+1\right)}\, \mathcal{L}\Big|_{AdS}}$$ for even or odd d

a_d^* and Entanglement Entropy

- consider equations of motion:

$$I = \int d^{d+1}x\sqrt{-g}\,\mathcal{L}(g^{ab}, R^{ab}{}_{cd}, \nabla_e R^{ab}{}_{cd}, \ldots, matter) + boundary\ term$$

- make variation of g^{ab} and $R^{ab}{}_{cd}$ "separately"

$$0 = -\frac{1}{2}\mathcal{L}\, g_{ab}\,\delta g^{ab} + \frac{\delta\mathcal{L}}{\delta g^{ab}}\,\delta g^{ab} + \frac{\delta\mathcal{L}}{\delta R^{ab}{}_{cd}}\,\delta R^{ab}{}_{cd}$$

with $\delta R^{ab}{}_{cd} = g^{be}(\nabla_c\delta\Gamma^a{}_{ed} - \nabla_d\delta\Gamma^a{}_{ec}) + R^a{}_{ecd}\delta g^{be}$

$$\delta\Gamma^a{}_{ed} = \frac{1}{2}g^{af}(\nabla_e\delta g_{fd} + \nabla_d\delta g_{fe} - \nabla_f\delta g_{ed})$$

- result of integration by parts hidden in $\delta\mathcal{L}/\delta g^{ab}$ and $\delta\mathcal{L}/\delta R^{ab}{}_{cd}$

a_d^* and Entanglement Entropy

- consider equations of motion:

$$I = \int d^{d+1}x\sqrt{-g}\,\mathcal{L}(g^{ab}, R^{ab}{}_{cd}, \nabla_e R^{ab}{}_{cd}, \ldots, matter) + boundary\ term$$

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- result of integration by parts hidden in $\delta\mathcal{L}/\delta g^{ab}$ and $\delta\mathcal{L}/\delta R^{ab}{}_{cd}$
- evaluate for AdS_{d+1} solution (maximally symmetric, $\nabla_a[\cdots] = 0$)

$$\delta R^{ab}{}_{cd} = -\frac{1}{\tilde{L}^2}(\delta^a{}_c g_{ed} - \delta^a{}_d g_{ec})\delta g^{be}$$

a_d^* and Entanglement Entropy

- consider equations of motion:

$$I = \int d^{d+1}x\sqrt{-g}\,\mathcal{L}(g^{ab}, R^{ab}{}_{cd}, \nabla_e R^{ab}{}_{cd}, \ldots, matter) + boundary\ term$$

- make variation of g^{ab} and $R^{ab}{}_{cd}$ "separately"

$$0 = -\frac{1}{2}\mathcal{L}\,g_{ab}\,\delta g^{ab} + \cancel{\frac{\delta\mathcal{L}}{\delta g^{ab}}\delta g^{ab}} + \frac{\delta\mathcal{L}}{\delta R^{ab}{}_{cd}}\,\delta R^{ab}{}_{cd}$$

with $\delta R^{ab}{}_{cd} = \cancel{g^{be}(\nabla_c\delta\Gamma^a{}_{ed} - \nabla_d\delta\Gamma^a{}_{ec})} + R^a{}_{ecd}\delta g^{be}$

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- evaluate for AdS_{d+1} solution (maximally symmetric, $\nabla_a[\cdots] = 0$)

$$\boxed{\left.\frac{\delta\mathcal{L}}{\delta R^{ab}{}_{cd}}\right|_{AdS}\delta^b{}_d = -\frac{\tilde{L}^2}{4}\,\mathcal{L}|_{AdS}\;\delta^a{}_b}$$

a_d^* and Entanglement Entropy

- puzzle pieces:

entanglement entropy: $S = -2\pi \, \frac{\partial \mathcal{L}}{\partial R^{ab}{}_{cd}} \varepsilon^{ab} \, \varepsilon_{cd} \Big|_{AdS} \times \int_{defect} d^{d-1}x \, \sqrt{h}$

trace anomaly: $$a_d^* = -\frac{\pi^{d/2} \, \tilde{L}^{d+1}}{2\Gamma\left(\frac{d}{2}+1\right)} \, \mathcal{L}|_{AdS}$$

eq. of motion: $$\frac{\delta \mathcal{L}}{\delta R^{ab}{}_{cd}}\Big|_{AdS} \delta^b{}_d = -\frac{\tilde{L}^2}{4} \, \mathcal{L}|_{AdS} \, \delta^a{}_b$$

$$\boxed{S = \frac{2\pi}{\pi^{d/2}} \Gamma(d/2) \, \frac{a_d^*}{\tilde{L}^{d-1}} \times \int_{defect} d^{d-1}x \, \sqrt{h}}$$

entanglement entropy: $S = \frac{2\pi}{\pi^{d/2}} \Gamma\left(d/2\right) \; \frac{a_d^*}{\tilde{L}^{d-1}} \, V\left(H^{d-1}\right)$

$$ds^2 = \tilde{L}^2 \left[\frac{du^2}{1+u^2} + u^2 \, d\Omega_2^{d-2} \right]$$

$$S \; = \; a_d^* \, \frac{4\pi^{\frac{d-3}{2}}}{(d-2)\Gamma\left(\frac{d-1}{2}\right)} \; \underbrace{\left(\frac{\tilde{L}}{\delta}\right)^{d-2}}_{} \; + \; \cdots$$

“area law” for d-dimensional CFT

entanglement entropy: $S = \frac{2\pi}{\pi^{d/2}}\Gamma\left(d/2\right)\ \frac{a_d^*}{\tilde{L}^{d-1}}\,V\left(H^{d-1}\right)$

$$ds^2 = \tilde{L}^2\left[\frac{du^2}{1+u^2} + u^2\,d\Omega_2^{d-2}\right]$$

$$S = \cdots + (-)^{\frac{d}{2}-1}4\,a_d^*\ \log\left(\tilde{L}/\delta\right) + \cdots \quad \text{for even d}$$

$$\cdots + \underbrace{\quad (-)^{\frac{d-1}{2}}2\pi\,a_d^* \quad}_{\text{universal contribution}} + \cdots \quad \text{for odd d}$$

Conjecture:

- place CFT on S^{d-1} X R and divide sphere in half along equator
- entanglement entropy of ground state has universal contribution

$$S_{univ} = \begin{cases} (-)^{\frac{d}{2}-1}\,4\,a_d^*\,\log(L/\delta) & \text{for even } d \\ (-)^{\frac{d-1}{2}}\;2\pi\,a_d^* & \text{for odd } d \end{cases}$$

(any gravitational action)

- in RG flows between fixed points

$$(a_d^*)_{UV} \;\geq\; (a_d^*)_{IR}$$

(constrained models)

⟶ gives framework to consider c-theorem for **odd** or even *d*

⟶ behaviour discovered for holographic model but conjecture that result applies generally (outside of holography)

and Beyond:

- Susskind & Witten: density of degrees of freedom in N=4 SYM connected to area of holographic screen at large R in AdS_5

$$\frac{N_c^2}{\delta^3} \times V_3 \;\sim\; \frac{A(R)}{\ell_P^3}$$

cut-off scale defined by regulator radius: $\frac{1}{\delta} = \frac{R}{L^2}$

- given higher curvature bulk action, natural extension is to evaluate Wald entropy on holographic screen at large R

$$S = -2\pi \oint d^{d-1}x\sqrt{h}\;\hat{\varepsilon}^{ab}\,\hat{\varepsilon}_{cd}\;\frac{\partial \mathcal{L}_{bulk}}{\partial R^{ab}{}_{cd}}$$

a_d^* and Entanglement Entropy

- puzzle pieces:

entanglement entropy: $S = -2\pi \, \frac{\partial \mathcal{L}}{\partial R^{ab}{}_{cd}} \varepsilon^{ab} \, \varepsilon_{cd} \Big|_{AdS} \times \int_{defect} d^{d-1}x \, \sqrt{h}$

trace anomaly: $$a_d^* = -\frac{\pi^{d/2} \, \tilde{L}^{d+1}}{2\Gamma\left(\frac{d}{2}+1\right)} \, \mathcal{L}|_{AdS}$$

eq. of motion: $$\frac{\delta \mathcal{L}}{\delta R^{ab}{}_{cd}} \Big|_{AdS} \delta^b{}_d = -\frac{\tilde{L}^2}{4} \, \mathcal{L}|_{AdS} \; \delta^a{}_b$$

and Beyond:

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- given higher curvature bulk action, natural extension is to evaluate Wald entropy on holographic screen at large R

$$S = -2\pi \oint d^{d-1}x\sqrt{h}\ \hat{\varepsilon}^{ab}\ \hat{\varepsilon}_{cd}\ \frac{\partial \mathcal{L}_{bulk}}{\partial R^{ab}{}_{cd}}$$

- with previous results, straightforwardly evaluate "entropy" density

$$\boxed{s \ = \ \frac{2}{\pi}\, a_d^*\, \frac{1}{\delta^{d-1}}}$$

for any covariant action: $\mathcal{L}_{bulk} = \mathcal{L}_{bulk}\left(g^{ab}, R^{ab}{}_{cd}, \nabla_e R^{ab}{}_{cd}, \cdots\right)$

Conclusions:

- AdS/CFT correspondence (gauge/gravity duality) has proven an excellent tool to study strongly coupled gauge theories
- toy theories with higher-R interactions extend class of CFT's

$\longrightarrow$ maintain calculational control with GB or quasi-top. gravity

- consistency (causality & positive fluxes) constrains couplings
- provide interesting insights into RG flows
- naturally support Cardy's version of a-theorem with d even
- suggests extension of a-theorem to d odd
- what are details in the "fine print" of a-theorem??
- a_d^* seems to play a privileged role in holography
- further implications for holographic dualities??

Lots to explore!