Stringy effects in black hole scattering

$\Sigma\alpha\mu\acute{\eta}\rho$ Μ$o\acute{\upsilon}\rho\theta\eta$

LPTHE, Paris

Ο$\rho\theta\acute{o}\delta o\xi o\varsigma$ Α$\kappa\alpha\delta\eta\mu\acute{\iota}\alpha$, Κ$\rho\acute{\eta}\tau\eta\varsigma$
September 12, 2010.

Based on `S. Kuperstein, Σ.M., 1008.0813.`

References (among participants)

Banerjee, Dutta

Kulaxizi, Parnachev

Myers

Moura

Policastro

...

Two pictures of black holes in string theory

Macroscopic

String theory (being a theory of quantum gravity) reduces to general relativity coupled to other fields at low energies. This low energy theory admits a black hole solution carrying charges $\{q_i\}$.

Microscopic

A microscopic description of a collection of states all carrying the same charges $\{q_i\}$, perhaps in a different regime of parameter space (weak coupling).

Compare various physical quantities computed in the two pictures.

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Comparison of macroscopic and microscopic pictures

Early success (1996)

Strominger-Vafa, three charge BPS black hole (Q1,Q5,n) in type II string theory:

- On the macroscopic side, find the black hole solution carrying three charges, and measure the Bekenstein-Hawking entropy

$$S_{BH} = \frac{A}{4G} = 2\pi\sqrt{Q_1 Q_5 n} \,, \tag{1}$$

- On the microscopic side, the generic state carrying the same three charges is a chiral excitation of a two dimensional superconformal field theory. Estimate density of states.

$$\Omega \approx \exp(2\pi\sqrt{Q_1 Q_5 n}) \,. \tag{2}$$

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Non-trivial finite size effects

- Now, corrections to formula in inverse powers of charges.

$$\log \Omega = 2\pi \sqrt{Q_1 Q_5 n} \left(1 + \frac{3}{2n} + \cdots \right) \,, \tag{3}$$

in the *dilute gas regime* $Q_1 Q_5 \gg n \gg 1$, ... ; Castro, Σ.M. 2008; Banerjee 2008.

- Detailed understanding on macroscopic as well as microscopic sides.

- Macro: Higher derivative corrections to supergravity action coming from string theory + Wald formula,

  Castro, Davis, Kraus, Larsen 2007 based on Cardoso, de Wit, Mohaupt 1999.

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Scattering cross sections and decay rates

- Black hole decays into a (scalar) field mode of frequency ω.
- The decay rate and the absorption cross section are related as:

$$\Gamma(\omega) = \sigma_{abs}(\omega) \frac{1}{e^{\omega/T} - 1} \ . \tag{4}$$

- For the D1-D5-P system, the cross sections were computed in the infinite charge limit for $\omega/T \to 0$, $T \to 0$, Das, Mathur 1996

$$\sigma_{micro} = \sigma_{macro} = 4G_N \, 2\pi \sqrt{Q_1 Q_5 n} = A_H \ . \tag{5}$$

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Question

Do the macroscopic and microscopic cross sections agree beyond the leading order?
(Note: inherently non-BPS quantity).

Aim of the talk

In this talk, I will answer this question in the affirmative for the scattering of a scalar field off a $T \to 0$ black hole in $\mathcal{N} = 4$ string theory

$$\sigma_{macro}(\omega \to 0) = \sigma_{micro}(\omega \to 0) \ , \tag{6}$$

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Technique of computation

Microscopic

Absorption by the effective D1-D5 string. Honest scattering amplitude in asymptotically Minkowski space.

Macroscopic

- Radial slicing + Green functions defined on cutoff at boundary r_0.
- So far this has been done most rigorously in asymptotically AdS space, (but see recent work Bredberg, Keeler, Lysov, Strominger 2010).
- Idea is to take $\Lambda \to 0$ limit (or take away the regulator $r_0 \to \infty$).
- Define the limit such that (see Polchinski 1999, Minwalla,Seiberg 1999)

$$\lim_{r_0 \to \infty} G_R(k_\mu; r = r_0) \to \mathcal{A}(k_\mu; k_r = \sqrt{k_\mu k^\mu}) \,. \tag{8}$$

- Then use

$$\sigma_{abs}(r_0) := \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im}\, G_R(k_\mu = 0, \omega, r_0) \,. \tag{9}$$

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Radial non-evolution

Macroscopic (contd)

- Equations for the "radial flow" in the two derivative theory have been written down for such quantities Iqbal, Liu 2008.
- In the limit $\omega \to 0$ (less than all other scales in the theory),

$$\frac{d}{dr_0}\sigma_{abs}(r_0) = 0 \ . \tag{10}$$

- This result remains true even after including higher derivative corrections Paolos 2009.
- We can therefore focus on the near-horizon region.
  This is captured by the nice formula Paolos 2009:

$$\sigma_{abs} = -8\pi T \lim_{\omega\to 0} \frac{\mathrm{Res}_{r_0=r_H}\,\mathcal{L}(\delta\phi = e^{-i\omega t})}{\omega^2} \ . \tag{11}$$

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A purely flat space derivation

Unruh Method

- Divide black hole spacetime into three regions – near-horizon, asymptotic and an intermediate region.
- Wave equation solvable in the asymptotic region – same solution as the two derivative theory:

$$\varphi = e^{-i\omega t}(A + B/r^{d-3}) \, . \qquad (12)$$

- In the near horizon region, *regularity* of the black hole horizon

$$\Rightarrow S_{\rm eff} = \frac{1}{2}\int d^d x \, \frac{\sqrt{g}}{\lambda(r)} \, g^{\mu\nu}(r) \, (\partial_\mu \varphi \, \partial_\nu \varphi) \, , \qquad (13)$$

Purely ingoing boundary conditions:

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where x is the tortoise coordinate $dx = \lambda(r) dr/f(r)$.

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Matching the solutions

- Assume slow variation of the effective coupling $\lambda(r)$.
- At very low energies $\omega \ll 1/R_H, T_H$, one can match the solutions across the three regions.
- The ratio B/A (and therefore the scattering amplitude) can be computed from the pure near horizon data.

$$\sigma_{abs} = \frac{A_H}{\lambda(r_H)} \,. \tag{15}$$

- This can be rewritten as

$$\sigma_{abs} = -8\pi T \lim_{\omega \to 0} \frac{\mathrm{Res}_{r_0 = r_H} \mathcal{L}(\delta\phi = e^{-i\omega t})}{\omega^2} \,. \tag{16}$$

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The string theory setup

$\mathcal{N} = 4$ string theory in five dimensions

- Type IIB string theory on $K3 \times S^1$. Equivalently, heterotic string theory on $T^4 \times S^1$.

- In the type II frame, the black hole carries
  Q_1 units of D1-brane charge wrapped on S^1,
  Q_5 units of D5-brane charge wrapped on $K3 \times S^1$, and
  n units of momentum on S^1.

- In the heterotic frame, the black hole carries
  Q_1 units of momentum along S^1,
  Q_5 units of F-string charge wrapped on S^1, and
  n units of NS5-brane charge wrapped on $T^4 \times S^1$.

Low energy effective action and solution

- The ten-dimensional action for the metric, dilaton and the two form potential is

$$S_{\rm het} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g}\, e^{-2\Phi} \left(R + 4\,(\partial\Phi)^2 - \frac{1}{2}\,|H_3|^2 \right) . \qquad (17)$$

- The solution for the three-charge black hole is Horowitz, Maldacena, Strominger 1996:

$$\begin{aligned} \ell_s^{-2} ds_{10}^2 \;&=\; \frac{1}{\sqrt{h_1(r)h_5(r)}} \left(-f(r)dt'^2 + dx_5'^2 \right) + \sqrt{\frac{h_1(r)}{h_5(r)}} dx_i dx^i \\ &\quad + \sqrt{h_1(r)h_5(r)} \left(\frac{dr^2}{f(r)} + r^2 d\Omega_{S^3}^2 \right) . \qquad (18) \end{aligned}$$

where $f(r)$ and $h_{1,5}(r)$ are the harmonic functions encoding the temperature and the charges Q_1, Q_5.

- The solution asymptotes to ten dimensional flat space.

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Near-horizon limit

- For the very-near horizon region the solution in the $T \to 0$ limit is $AdS_2 \times S^1 \times S^3 \times T^4$.

$$\begin{aligned} ds^2 &= \frac{v_1}{16}\left(-(z^2-z_0^2)\,dt^2+\frac{dz^2}{z^2-z_0^2}\right)+\frac{v_2}{16}\left(d\theta^2+\sin^2\theta\,d\phi^2\right) \\ &\quad + u_1^2\left(d\chi_1+\frac{1}{2}e_1(z-z_0)dt\right)^2+u_2^2\left(d\chi_2-\frac{1}{2}\cos\theta\,d\phi\right)^2 \\ &\quad + dx_i dx_i \;. \end{aligned} \tag{19}$$

where the parameters are fixed by the charges (attractor mechanism).

$$v_{1,2}^{(0)} = 4n\,, \quad u_1^{(0)} = \sqrt{\frac{Q_1}{Q_5}}\,, \quad u_2^{(0)} = \sqrt{n}\,,$$
$$u_S^{(0)} = \sqrt{\frac{Q_1\,Q_5}{n}} \quad e_1^{(0)} = \sqrt{\frac{nQ_5}{Q_1}}\,. \tag{20}$$

Stringy corrections

Four derivative action

The ten-dimensional heterotic string action to four derivative order is Metsaev, Tseytlin, 1987:

$$S = S_0 + S_1 \tag{21}$$

where S_0 is the two derivative action (17) with $H = dB + \frac{\alpha'}{4}\omega_{LCS}$ and

$$\begin{aligned} S_1 \;=\; & \frac{1}{2\kappa^2}\frac{\alpha'}{8}\int d^{10}x\sqrt{-\det g}\; e^{-2\phi}\,\Big(R_{klmn}R^{klmn} \\ & -\frac{1}{2}R_{klmn}H_p{}^{kl}H^{pmn} - \frac{1}{8}H_k{}^{mn}H_{lmn}H^{kpq}H^l{}_{pq} \qquad (22) \\ & +\frac{1}{24}H_{klm}H^k{}_{pq}H_r{}^{lp}H^{rmq}\Big)\,. \qquad (23) \end{aligned}$$

In the extremal limit, the near-horizon solution retains the same form, but the parameters receive corrections.

Absorption cross section

Calculation of σ

- Consider the 5d scalar perturbation h_{67} and compute the perturbed action $\delta\mathcal{L}(h_{67})$.
- The absorption cross section for this scalar can be computed as discussed:

$$
\begin{aligned}
\sigma_{abs}/G_N &= 4z_0 \lim_{\omega\to 0} \frac{1}{\omega^2} \mathrm{Res}_{z\to z_0}\, \delta\mathcal{L}(h_{67}) \ , && (24)\\
&= 2\pi u_S\, v_2 \left(1 + \frac{\alpha'}{2v_1}\left(1 - \frac{2u_1^2 e_1^2}{v_1}\right)\right) \ , && (25)\\
&= 8\pi\sqrt{Q_1 Q_5 n}\left(1 + \frac{3}{2n}\right) \ , && (26)\\
&= 4\, S_{Wald} \ . && (27)
\end{aligned}
$$

The microscopic model

- D1-D5-P system as a $1+1$-dimensional gas of open strings living on the branes wrapping a circle of radius R. The decay process is that of open strings annihilating to form a closed string (graviton) which runs off to infinity.
- At low energies, this is dominated by massless open strings annihilating to form a graviton.
- One then has to average over initial states and sum over final states.
- In the limit $R/\ell_s \gg 1$, the spectrum is well approximated by a continuum, and there is a non-zero coupling to a graviton of arbitrarily low energies via the Born-Infeld action.
- In the dilute gas approximation $n \ll c = 6Q_1 Q_5$, one can treat the left and right movers as independent.

Choice of ensemble and Entropy-energy relation

- The $\omega \to 0$ limit naturally leads to the *canonical ensemble* with fixed temperature

$$\frac{1}{T} \equiv \left.\frac{\partial S(E+\omega)}{\partial \omega}\right|_{\omega=0} , \tag{28}$$

and the canonical distribution function

$$\rho(E) = \frac{1}{e^{E/T} \mp 1} \tag{29}$$

- In a theory with an entropy-energy relation $S = 2\pi\sqrt{\frac{c}{6}ER + b}$ the temperature is

$$\frac{1}{T} = \frac{\partial S}{\partial E} = \frac{2\pi R\, c/6}{2\sqrt{\frac{c}{6}ER + b}} \equiv \frac{L\pi}{S} ,$$

$$\Rightarrow S = \pi L T \ . \tag{30}$$

The decay rate

- Decay rate for a pair of open strings of momenta (p_0, p_1), (q_0, q_1) to produce a closed string with momentum $(k_0, k_1 = 0, \vec{k})$ is

$$\Gamma(p,q,k) = \frac{8\pi^3 G_N}{L}\delta(p_0+q_0-k_0)\delta(p_1+q_1-k_1)\frac{|\mathcal{A}|^2}{p_0 q_0 k_0}\frac{d^d k}{(2\pi)^4}\ , \quad (31)$$

where the amplitude $\mathcal{A} = \sqrt{2}\, p \cdot q$, and $L = 2\pi Q_1 Q_5 R$.

- To compute the total decay rate, one has to average over initial configurations:

$$\begin{aligned}\Gamma(k) &= \int_{-\infty}^{\infty} \frac{L dp_1}{2\pi} \int_{-\infty}^{\infty} \frac{L dq_1}{2\pi}\, \Gamma(p,q,k)\, \rho(q_0,q_1)\, \rho(p_0,p_1)\ , \\ &= 8\pi G_N L\, \frac{\omega}{4}\, \rho_L(\omega/2)\, \rho_R(\omega/2)\, \frac{d^d k}{(2\pi)^d}\ . \end{aligned} \quad (32)$$

The absorption cross section

- The effective string is at temperature T_H with $2T_H^{-1} = (T_L^{-1} + T_R^{-1})$.

- Using the detailed balance condition, one can read off the absorption cross section of a graviton with four momentum $k = (\omega, k_i = 0)$:

$$\sigma_{abs}(\omega) = 8\pi G_N L \, \frac{\omega}{4} \, \frac{\rho_L(\omega/2)\, \rho_R(\omega/2)}{\rho(\omega)} \ . \tag{33}$$

- In the limit $T_H \to 0$, $T_H \approx 2T_R \ll T_L \Rightarrow \rho_R(\omega/2) = \rho(\omega)$.

- As $\omega \to 0$, $\rho_L(\omega/2) = 2T_L/\omega$, and

$$\begin{aligned} \sigma_{abs} &= 4\pi G_N L \, T_L \\ &= 4 G_N S_L \ . \end{aligned} \tag{34}$$

using the the entropy-temperature relation (30) for the left movers.

Discussion

1. Can one take into account higher α' effects in σ_{abs}?
   For entropy, possible to sum up *all* α' corrections both in macro and micro pictures $S = 2\pi\sqrt{Q_1 Q_5 (n+3)}$ Dabholkar, Gomes, Σ.M., Sen, to appear.

2. Non-renormalization theorem of moduli space metric Maldacena 1996.
   4d black holes $\Rightarrow$ Extension to $(0,4)$ theories?

3. Better understanding of small (2-charge) black hole solution for which cross section to leading order vanishes?

4. Understand the equation $\sigma_{abs} = 4 G_N S_{Wald}$ better.
   What is the general $\mathcal{N} = 2$ black hole story?
   Compare with mechanism in AdS space.

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ΩΡΑ ΦΑΓΗΤΟΥ !

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