Effective Holographic Theories for Condensed Matter Systems

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ArXiv:1005.4690 (with C. Charmousis, B. Gouteraux, B.-S. Kim and E. Kiritsis)

Outline

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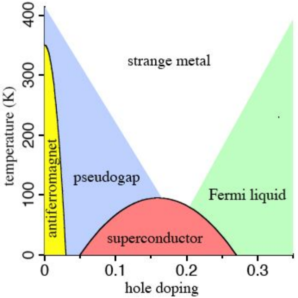
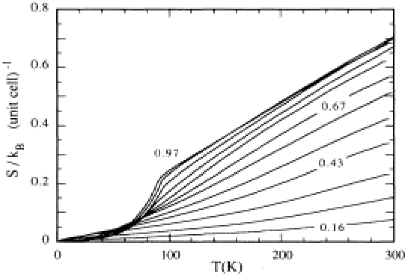
Motivation: Strange Metallic Behaviour

- Non-Fermi Liquid Behaviour: High Tc Cuprates (e.g. $La_{2-x}Sr_xCuO_4$), Heavy Fermion Systems (e.g. $CeCu_{5.9}Au_{0.1}$), Iron based High Tc compounds (e.g. $SmFeAsO_{1-x}F_x$)
- Properties of Strange Metal Region:

  $\rightarrow$ Linear DC Resistivity: $\rho = \rho_0 + AT$ (down to $T = 0$)
  $\rightarrow$ AC conductivity scaling: $\sigma(\omega) \sim \omega^{-2/3}$ for $T \ll \omega \ll \Lambda$
  $\rightarrow$ Electronic Specific Heat and Entropy: $C_v, S \sim T$





[Loram et. al. PRL 71, 11, 1993]

Motivation: Effective Holographic Theories

- EHT: p-dim. strongly coupled EFT $\leftrightarrow$ $p+1$-dim. weak gravity
- Minimal Ingredients for a High Tc EHT:
  - Finite Carrier Density: Global $U(1)$ current $J^\mu \leftrightarrow$ bulk gauge field A_μ
  - Leading Relevant Uncharged Operator: $\mathcal{O}_\Phi \leftrightarrow$ bulk scalar Φ
  - Energy-Momentum-Tensor: $T_{\mu\nu} \leftrightarrow$ bulk metric $g_{\mu\nu}$
- Two-Derivative Infrared Effective Holographic Action: (Sugra Intuition)

$$\begin{aligned} S &= M_{Pl}^{p-1} \int \mathrm{d}^{p+1}x \sqrt{-g} \left[R - \frac{1}{2}(\partial\Phi)^2 - V(\Phi) - \frac{Z(\Phi)}{4} F_{\mu\nu}^2 \right] \\ V(\Phi) &= 2\Lambda e^{-\delta\Phi}, \qquad Z(\Phi) = e^{\gamma\Phi} \end{aligned}$$

- (Un)Charged Black Hole Solutions: Important Observables
  - Thermodynamics (Entropy, Specific Heat, Phase Transitions)
  - DC/AC conductivities
  - Other transport coefficients, ...

On Naked Singularities

- In holography naked singularities may be acceptable, since they might not necessarily signal a breakdown of predictability of the theory, as long as they do not affect the field theory physics severely.
- Gubser gave the first criterion for good singularities : **They should be limits of solutions with a regular horizon.** [Gubser (2000)]
- The second criterion amounts to having a well-defined spectral problem for fluctuations around the solution: **The second order equations describing all fluctuations are Sturm-Liouville problems** (no extra boundary conditions needed at the singularity). [Gursoy+E.K.+Nitti (2008)]
- **The singularity is "repulsive" (like the Liouville wall).** It has an overlap with the previous criterion. It involves the calculation of ' 'Wilson loops" [Gursoy+E.K.+Nitti (2008)]
- It is not known whether the list is complete.

Conductivity & Charged Excitation Spectra I

- Conductivity is a main characteristic transport coefficient in a finite density system.

$$J^i(\omega,\vec{k}) = \sigma^{ij}(\omega,\vec{k})\; E_j(\omega,\vec{k})$$

- Can be calculated from a Kubo formula (in linear response)

$$\sigma^{ij}(\omega,\vec{k}) = \frac{G_R^{ij}(\omega,\vec{k})}{i\omega}$$

- Various limits are of experimental importance

$$\vec{k}\to 0 \quad \to \quad \sigma^{ij}(\omega,T) \quad \to \quad \text{AC conductivity}$$

$$\omega\to 0 \;\text{ and }\; \vec{k}\to 0 \quad \to \quad \sigma^{ij}(T) \quad \to \quad \text{DC conductivity}$$

Conductivity & Charged Excitation Spectra II

The linear response calculation at $\vec{k}=0$ can, in holography, be recast as a Schrödinger problem [Horowitz, Roberts 2009]. In the backreacted background

$$ds^2 = -D(r)dt^2 + B(r)dr^2 + C(r)(dx_i dx^i)\,, \quad A_t' = q\frac{\sqrt{D(r)B(r)}}{Z(\phi)C(r)^{\frac{p-1}{2}}}$$

we must turn on perturbations

$$A_i = a_i(r)e^{i(\omega t)}\,,\, g_{ti}(r,t) = z_i(r)e^{i\omega t}$$

From Einstein and Maxwell, we obtain a second order equation

$$\partial_r\left(ZC^{\frac{p-3}{2}}\sqrt{\frac{D}{B}}a_i'\right) + ZC^{\frac{p-3}{2}}\left(\sqrt{\frac{B}{D}}\omega^2 - \frac{q^2\sqrt{DB}}{ZC^{p-1}}\right)a_i = 0$$

It can be mapped to a Schrödinger equation

$$-\frac{d^2\Psi}{dz^2} + V_{eff}\Psi = \omega^2\Psi\,, \quad V_{eff} = \frac{q^2 D}{ZC^{p-1}} + \frac{1}{4}\left(\frac{\partial_z\bar{Z}}{\bar{Z}}\right)^2 + \frac{1}{2}\partial_z\frac{\partial_z\bar{Z}}{\bar{Z}}\,, \quad \bar{Z} = ZC^{\frac{p-3}{2}}$$

via a coordinate and wave function redefinition

$$\frac{dz}{dr} = \sqrt{\frac{B}{D}}\,, \quad a_i = \frac{\Psi}{\sqrt{\bar{Z}}}\,.$$

Conductivity & Charged Excitation Spectra III

The frequency dependent conductivity is given by [Roberts+Horowitz (2009), Kachru et.al. (2009)]

$$\sigma(\omega) = \frac{1-\mathcal{R}}{1+\mathcal{R}} - \frac{i}{2\omega}\frac{\dot{Z}}{Z}\Big|_{\text{boundary}}$$

The second term always vanishes in our cases. Several situations can now occur:

- $V \to 0$ in IR: Conductor, continuous spectrum
- $V \to V_0$ in IR: Conductor with continuous spectrum and mass gap
- $V \to \infty$ in IR, but not UV: Insulator with continuous spectrum (for $p = 3$ and $\Delta \geq 1$, with gap for $\Delta = 1$)
- $V \to \infty$ in both IR and UV: Insulator with discrete spectrum and mass gap (confined phase) (for $p > 3$ or $p = 3$ and $\Delta < 1$)

Outline

Uncharged Solutions

- Two Solutions with "good" IR singularities: [Gubser hep-th/0002160]
  - Small Planar Black Holes:

$$\mathrm{d}s^2 = r^{\frac{4}{(p-1)\delta^2}}\left(-f(r)\mathrm{d}t^2 + \mathrm{d}\vec{x}^2\right) + \frac{\mathrm{d}r^2}{f(r)}$$

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^{\frac{2p}{(p-1)\delta^2}-1}, \quad e^{\delta\Phi} = \frac{|\Lambda|\delta^4}{\frac{2p}{(p-1)} - \delta^2} r^2$$

  - Temperature: $\boxed{4\pi T = \left(\frac{2p}{(p-1)\delta^2} - 1\right) r_0^{\frac{2}{(p-1)\delta^2}-1}}$

$\to$ A) $\delta^2 < \frac{2}{p-1}$: $T \overset{r_0\to 0}{\to} 0$

$\to$ B) $\delta^2 = \frac{2}{p-1}$: $T \overset{r_0\to 0}{\to} \frac{p-1}{4\pi}$

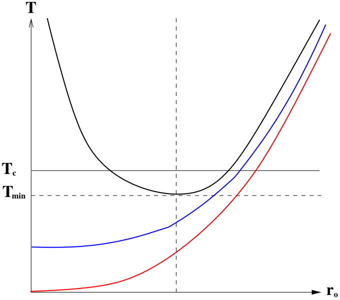
$\to$ C) $\delta^2 > \frac{2}{p-1}$: $T \overset{r_0\to 0}{\to} \infty$

  - Thermal Gas Solution: $f(r) \equiv 1$

- AdS UV completion: $V_{full}(\Phi) \overset{IR}{\sim} e^{-\delta\Phi}$, AdS Minimum in UV

$\to$ Large Planar Black Hole Branch for large r_0 with $T \sim r_0$

Uncharged Solutions



- A) $\delta^2 < \frac{2}{p-1}$: continuous PTs at $T = 0_+$, continuous & gapless spectrum
- B) $\delta^2 = \frac{2}{p-1}$: continuous PT at $T_{\text{min}} = \frac{p-1}{4\pi}$ depending on subleading terms in $V(\Phi)$ (n^{th} order or KT) [U. Gursoy's talk] , continuous & gapped
- C) $\delta^2 > \frac{2}{p-1}$: 1st order deconfinement transition, Insulator/Conductor
- D) $\frac{2p}{p-1} > \delta^2 > \frac{2(p+2)}{3(p-1)}$ Graviton fluctuation problem unacceptable
- E) $\delta^2 > \frac{2p}{p-1}$: Gubser's criterion violated

Uncharged Solutions: AC/DC Conductivity

- Probe DBI calculation yields ($g^S = e^{k\Phi} g^E$, $\kappa = \frac{4\pi}{\left(\frac{2p}{(p-1)\delta^2} - 1\right)}$)

$$\boxed{\sigma_{xx} = e^{-k\phi_0} (\kappa T)^{\frac{2k(p-1)\delta+4}{(p-1)\delta^2-2}} \sqrt{\langle J^t \rangle^2 + e^{2(\gamma+k)\phi_0} (\kappa T)^{\frac{4(p-1)[1+(\gamma+k)\delta]}{2-(p-1)\delta^2}}}}$$

♠ At low densities : $\rho \sim T^{\frac{2(p-1)\gamma\delta+2(p-3)}{(p-1)\delta^2-2}}$, $\gamma_{\text{linear}} \equiv \frac{\delta^2 - \frac{2(p-2)}{(p-1)}}{2\delta}$ (indep. of k)

♠ At high densities : $\rho \sim T^{\frac{2k(p-1)\delta+4}{2-(p-1)\delta^2}} \langle J^t \rangle$ (cannot be linear in the range $\delta^2 < 2(p+2)/(3(p-1))$)

♠ Interpolation between $\rho \sim T$ (NFL) and $\rho \sim T^2$ (FL) for $\delta = -k$

- Dragging string : $\rho \simeq \frac{T_f g^E_{xx}(r_h) e^{k\Phi(r_h)}}{J^t}$, $S \sim g^E_{xx}(r_h)^{\frac{p-1}{2}}$

$$\Rightarrow \boxed{\rho \sim S \sim C_V \quad \text{for} \quad p = 3\,, k = 0}$$

- AC conductivity at zero temperature ($\omega \gg T$, $p = 3$): [Kachru et. al. (2009)]

$$\boxed{\sigma \sim \omega^n\,, n = -\frac{2}{3} \text{ for } \gamma = \left\{ \frac{\delta^2 - 1}{3\delta}, \frac{2(\delta^2 - 1)}{3\delta} \right\}}$$

Outline

An exact charged solution for $\gamma\delta = 1$ [Charmousis et.al. 0905.3337]

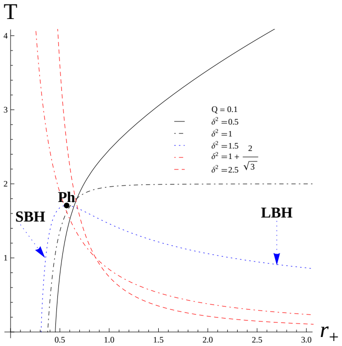
$$
\begin{aligned}
ds^2 &= -e^{\Phi/\delta} V(r) \frac{dt^2}{r^2} + e^{\delta\Phi} \frac{dr^2}{V(r)} + r^{3-\delta^2} e^{\frac{\delta^2-1}{2\delta}\Phi} (dx^2 + dy^2) \\
V(r) &= r^2 - 2mr^{\delta^2-1} + \frac{(1+\delta^2)q^2}{4\delta^2(3-\delta^2)^2} r^{2\delta^2-4}, \quad e^{\Phi} = r^{2\delta} \left[1 - \left(\frac{r_-}{r} \right)^{3-\delta^2} \right]^{\frac{4\delta(\delta^2-1)}{(3-\delta^2)(1+\delta^2)}} \\
(r_\pm)^{3-\delta^2} &= m \pm \sqrt{m^2 - \frac{(1+\delta^2)q^2}{4\delta^2(3-\delta^2)^2}}, \quad A_t = \mu - \frac{q}{(3-\delta^2)r^{3-\delta^2}}
\end{aligned}
$$

- For Black Hole Solutions: $\Lambda < 0$ and $\delta^2 < 3$ (cf. Gubser)
- AdS completable for all δ
- AdS-Schwarzschild Limit: $\delta \to 0$, $q \to 0$

$\gamma\delta = 1$: Solution Branches

- Temperature: $$T = (3-\delta^2) r_+^{1-\delta^2} \left[1 - \frac{4(1+\delta^2)Q^2}{\delta^2 (3-\delta^2)^2 r_+^{2(3-\delta^2)}} \right]^{1-\frac{2(\delta^2-1)^2}{(1+\delta^2)(3-\delta^2)}}$$

  - A) $\delta^2 \leq 1$: $T = 0$ at Extremality
  - B) $1 < \delta^2 < 1 + \frac{2}{\sqrt{3}}$: Two branches (SBH/LBH) + AdS-LBHs
  - C) $1 + \frac{2}{\sqrt{3}} \leq \delta^2 < 3$: T diverges at extremality, but AdS-LBHs take over

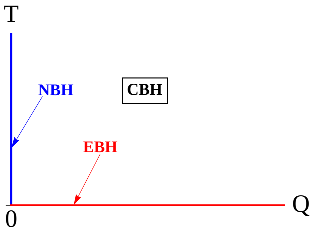


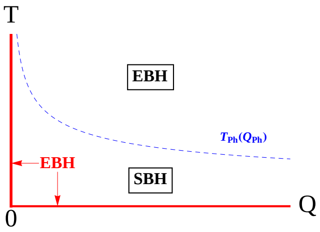
Spectrum

- $0 \leq \delta^2 < 1 + \frac{2}{\sqrt{3}}$: Conductor
- $1 + \frac{2}{\sqrt{3}} \leq \delta^2 < \frac{5+\sqrt{33}}{4}$: $V \sim |c|/z^2 \Rightarrow$ Mott-like insulator
- $\frac{5+\sqrt{33}}{4} < \delta^2 < 3$: $V \sim -|c|/z^2 \Rightarrow$ Conductor

$\gamma\delta = 1$: Phases and Low-Temperature Scaling

$\delta^2 \leq 1$ | $1 < \delta^2 < 1 + \frac{2}{\sqrt{3}}$



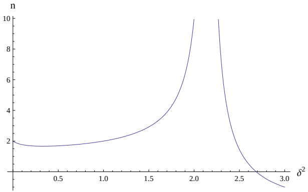


- Finite Q, $T \to 0_+$:
  - 2^{nd} order phase transition for $1 - \frac{2}{\sqrt{5}} < \delta^2 < 1 + \frac{2}{\sqrt{5}}$
  - 3^{rd} or higher order otherwise
- No residual entropy at zero temperature

$\gamma\delta = 1$: Transport Properties

- In the first two regimes $0 \leq \delta^2 \leq 1 + \frac{2}{\sqrt{3}}$, the AC conductivity is

$$\sigma(\omega) \sim \omega^n , \quad n = \frac{(3-\delta^2)(5\delta^2+1)}{|3\delta^4-6\delta^2-1|}$$



- DC resistivity from the dragging string calculation: At low temperatures

$$\rho_{\text{leading}} \sim \frac{T_f}{J^t} \left(\frac{q}{\ell}\right)^{\frac{2\delta(\delta(3-\delta^2)+(1+\delta^2)k)}{1+6\delta^2-3\delta^4}} (\ell T)^{\frac{2(\delta^2-1)(\delta^2-1+2k\delta)}{1+6\delta^2-3\delta^4}}$$

This is linear in temperature at $k = 0$ for $\delta^2 = 1 \pm \frac{2}{\sqrt{5}}$. We thus find

$$\boxed{S \sim C_Q \sim T \sim \rho_{DC}}$$

[Loram et. al. PRL 71 11 (1993)]

- The Karch/O'Bannon probe DBI calculation does not make sense in backreacted backgrounds.

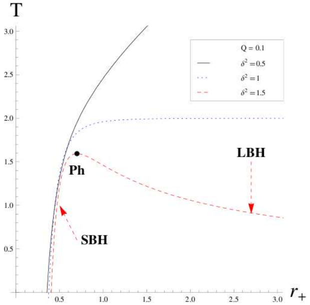
$\gamma = \delta$: RN-Like Charged Dilaton Black Holes

$$
\begin{aligned}
ds^2 &= -V(r)dt^2 + e^{\delta\Phi}\frac{dr^2}{V(r)} + r^2(dx^2 + dy^2) \\
V(r) &= r^2 - 2mr^{\delta^2-1} + \frac{q^2}{4(1+\delta^2)r^2}\,, \quad e^{\Phi} = r^{2\delta}\,, \quad r_e^4 = \frac{q^2}{4(3-\delta^2)} \\
A_t &= \mu - \frac{q}{(1+\delta^2)r^{1+\delta^2}}
\end{aligned}
$$

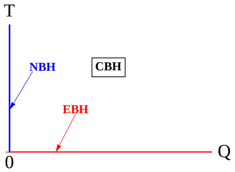
- For Black Hole Solutions: $\Lambda < 0$ and $\delta^2 < 3$ [Charmousis et.al. 0905.3337]
- AdS completable for all δ
- Coincides with $\gamma\delta = 1$ for $\gamma = \delta = 1$
- AdS-RN Limit: $\delta, \gamma \to 0$
- Residual Entropy: $\boxed{S = r_+^2 \simeq r_e^2 + \frac{T}{2}\frac{r_e^{1+\delta^2}}{3-\delta^2} + \dots}$

$$\boxed{C_Q \simeq \frac{T}{2}\frac{r_e^{1+\delta^2}}{3-\delta^2} + \dots\,, \qquad \rho_{DC} \simeq \rho_0 + AT + \dots}$$

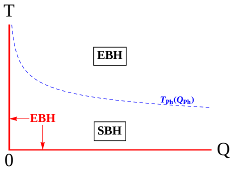
$\gamma = \delta$: Solution Branches and Phase Diagram



$\boxed{\delta^2 \leq 1}$



$\boxed{1 < \delta^2 < 3}$

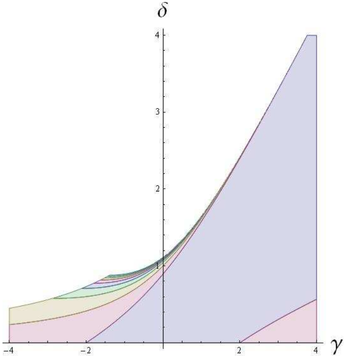


- $Q > 0, T \to 0_+$: EBHs dominate, no phase transition
- $T > 0, Q \to 0_+$:
  - $\delta^2 < 1$: 2^{nd} order phase transition to NBHs
  - $\delta^2 > 1$: 1^{st} order phase transition to EBHs
- AC Conductivity : $\sigma \sim \omega^2 \; \forall \delta$

Outline

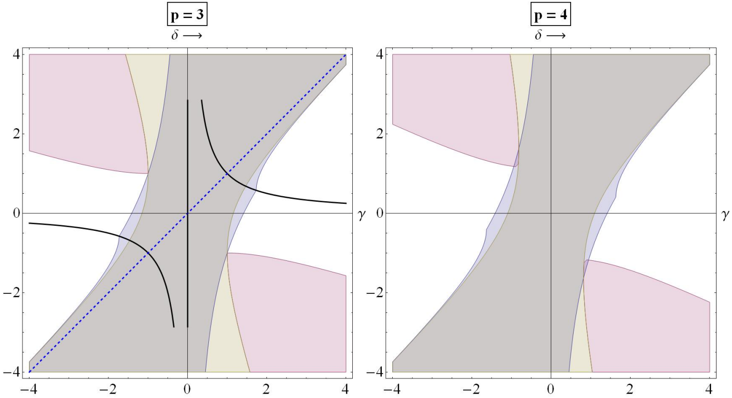
Near-Extremal Scaling Solutions

$$
\begin{aligned}
ds^2 &= r^{\frac{(\gamma-\delta)^2}{2}}\left[-f(r)dt^2 + d\vec{x}^2\right] + \frac{dr^2}{f(r)} \\
f(r) &= \frac{16(-\Lambda)}{wu^2} r^{1+\frac{3}{4}(\gamma-\delta)^2}\left(r^{\frac{wu}{4}} - 2m\right) \\
e^{\Phi} &= r^{\delta-\gamma}\,, \quad A_t = \mu + \frac{8}{wu}\sqrt{\frac{v\Lambda}{u}}\, r^{\frac{wu}{4}} \\
u &= \gamma^2 - \gamma\delta + 2\,, \quad v = \delta^2 - \gamma\delta - 2 \\
wu &= 3\gamma^2 - \delta^2 - 2\gamma\delta + 4 \overset{\delta^2<3}{>} 0
\end{aligned}
$$



- Near-Extremal Approximations to $\gamma\delta = 1$ and $\gamma = \delta$
- Lifshitz cases: $\delta = 0$, $z = 1 + \frac{4}{\gamma^2}$ [M. Taylor, 0812.0530]
- Entropy: $S \sim (2m)^{\frac{2(\gamma-\delta)^2}{wu}} \to 0$ except for $\gamma = \delta$
- Multitude of continuous $T \to 0_+$ Phase Transitions

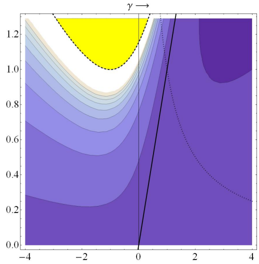
Spin 2 and Spin 1 constraints



Reliability constraints on the spin-2 fluctuations for $p = 3$ and $p = 4$. The blue region depicts the part of the (γ, δ) plane which satisfies Gubsers constraint. The yellow-brownish and purple regions are the allowed regions from spin 2 fluctuations. The purple region furthermore is thermodynamically unstable, and will be stabilised after AdS completion. This touches the Gubser allowed region for $p = 3$ in the point $(\pm 1, \mp 1)$, and has a small overlap for $p = 4$.

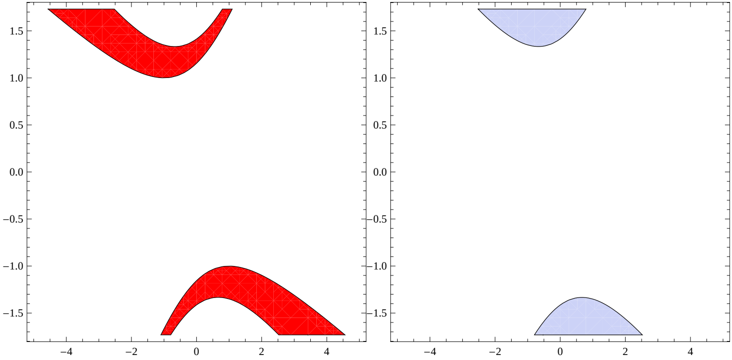
The extremal AC conductivity in $p = 3$

$$\sigma \sim \omega^n, \qquad n = \left| \frac{(\delta-\gamma)(3\gamma+5\delta)-12}{(\delta-\gamma)(\gamma+3\delta)-4} \right| - 1$$



Contour plot of the scaling exponent n in the (γ, δ) upper half plane for $p = 3$ Contours correspond to $n = 1.52, \ldots, 8.36$, starting with $n = 1.52$ in the upper right corner and increasing in steps of 0.76. The black solid line $\gamma = \delta$ is $n = 2$, and brighter colors correspond to larger n. The yellow region is thermodynamically unstable. The scaling exponent diverges to $+\infty$ along the dashed black line.

Mott-like States for $p = 3$



Left: The region on the $(\gamma\delta)$ plane where the IR black holes are unstable and the $c > 0$. Here the extremal finite density system has a mass gap and a discrete spectrum of charged excitations, when $\Delta < 1$. This resembles a Mott insulator and the figure provides the Mott insulator “islands" in the (γ, δ) plane. Right: The region where the IR black holes are unstable, and $c < 0$. In this region the extremal finite density system has a gapless continuous spectrum at zero temperature. In both figures the horizontal axis parametrizes γ, whereas the vertical axis δ.

Near-Extremal DC Conductivity

- For massive charge carriers the drag force calculation yields ($p = 3$)

$$\rho \sim T^m , \qquad m = \frac{4k(\delta - \gamma) + 2(\delta - \gamma)^2}{4(1 - \delta(\delta - \gamma)) + (\delta - \gamma)^2}$$

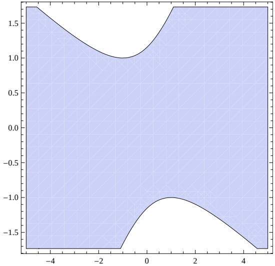
- The exponent becomes unity for two values of γ

$$\gamma_\pm = 3\delta + 2k \pm 2\sqrt{1 + (\delta + k)^2} .$$

- For a non-dilatonic scalar, $k = 0$ and the temperature dependence of the entropy and the resistivity are the same in $p = 3$. Therefore, the entropy also scales linearly with T.
- For the Lifshitz solutions, we must take $\delta = 0$ and $\gamma = -\sqrt{\frac{4}{(z-1)}}$. In this case we obtain that $m = \frac{2+k\sqrt{4(z-1)}}{z}$. When $k = 0$ this is in agreement with [Hartnoll+Polchinski+Silverstein+Tong]

Domination of Charged Entropy

$$S_q \sim T^{2\frac{(\gamma-\delta)^2}{wu-(\gamma-\delta)^2}}\,, \quad S_n \sim T^{\frac{2}{1-\delta^2}}$$



In the shaded region, the entropy at finite charge density dominates the one for zero density at very low temperatures. In the rest of the (γ, δ) diagram, the comparison cannot be made as both entropies are expected to be of $\mathcal{O}(1)$ instead of $\mathcal{O}(N^2)$. The vertical axis represents the value of δ, while the horizontal axis the value of γ.

Outline

Conclusions & Remarks

- Einstein-Maxwell-Neutral Scalar Theory is a good laboratory for strange metallic behaviour
- Describe: Interactions between a charged carrier sector and a leading relevant operator ("glue") $\rightarrow$ substrate in the High T_c normal state
- Interesting phase structures of uncharged and (fully backreacted) charged solutions of our system
- Uncharged solutions: 1^{st} order (De)confinement phase transitions appear with discrete and gapped spectra [Kiritsis, Gursoy, Nitti, Mazzanti on AdS/QCD]
- Charged solutions: Generically conductors, appearance of Mott-like insulating (confining) behaviour delayed to higher values of δ
- Well-definedness of the fluctuation problem poses interesting constraints on parameter space
- Continuous phase transitions at zero temperature (and charge)
- No residual entropy except for $\gamma = \delta$, charged entropy dominance
- Correlation between low-temperature scaling of entropy, (electronic) specific heat and DC resistivity observed in experiments