Holographic Hydrodynamics from 5d Dilaton–Gravity

Liuba Mazzanti
(University of Santiago de Compostela)

based on:
U. Gursoy, E. Kiritsis, L. M., F. Nitti, [arXiv:1006.3261]
U. Gursoy, E. Kiritsis, L. M., F. Nitti, [arXiv:0903.2859]
U. Gursoy, E. Kiritsis, L. M., F. Nitti, [arXiv:0812.0792]
U. Gursoy, E. Kiritsis, L. M., F. Nitti, [arXiv:0804.0899]
see also:
U. Gursoy, E. Kiritsis, G. Michalogiorgakis, F. Nitti, [arXiv:0906.1890]
U. Gursoy, E. Kiritsis, F. Nitti, [arXiv:0707.1349]
U. Gursoy, E. Kiritsis, [arXiv:0707.1324]

Kolymbari, September 12, 2010

Motivations and Goals: Gluon Plasma and Holography

1. **Motivations**: phase diagram & hydrodynamics of the plasma
   - 4D gauge theory: deconfinement phase transition
   - lattice results for thermodynamic functions (equation of state, . . .)
   - heavy ion collision experiments for hydrodynamic and transport coefficients

2. **Goals**: 5d holography for the plasma
   - setup: confinement & running coupling for 4D YM
   - can it match the thermodynamics from lattice?
   - if yes, can it give the hydrodynamic and transport coefficients ?

Outline

5d dilaton–gravity setup: zero T and finite T (TG & BH)

- **Phase transition and thermodynamics**
- **Drag force from worldsheet background**
- **Diffusion constants and "jet quenching parameters"**

Numerics

Thermodynamics and Hydrodynamics

- Finite temperature: gauge/gravity correspondence Hawking-Page'83,Witten'98

Gravity solutions with periodic time: thermal gas and black hole

$\Updownarrow$ gauge/gravity

Gauge theory at finite temperature

- Langevin diffusion: momentum broadening

Casalderrey-Teany'06, Gubser'06,deBoer-Hubeny-Rangamany-Shigemori'08, Son-Teaney,Giecold-Iancu-Müller'09

Thermodynamics and Hydrodynamics

- Finite temperature: gauge/gravity correspondence Hawking-Page'83,Witten'98

Gravity solutions with periodic time: thermal gas and black hole

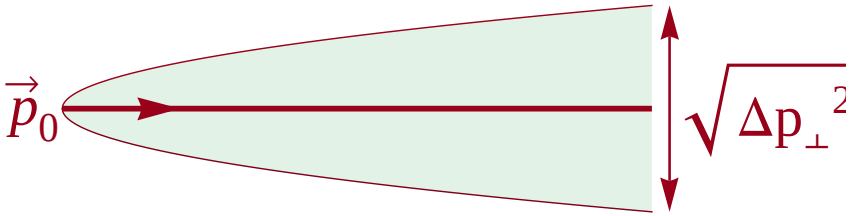
$\Updownarrow$ gauge/gravity

Gauge theory at finite temperature

- Langevin diffusion: momentum broadening

Casalderrey-Teany'06, Gubser'06,deBoer-Hubeny-Rangamany-Shigemori'08, Son-Teaney,Giecold-Iancu-Müller'09

quark moving		worldsheet
drag force	$\Leftrightarrow$	background
diffusion		fluctuations



5d Dilaton–Gravity Holography: Action Gursoy-Kiritsis'07

Field/operator correspondence

$$g^{\mu\nu} \Leftrightarrow T^{\mu\nu}$$
$$\phi \Leftrightarrow \mathrm{tr}F^2$$

- Action in the Einstein frame

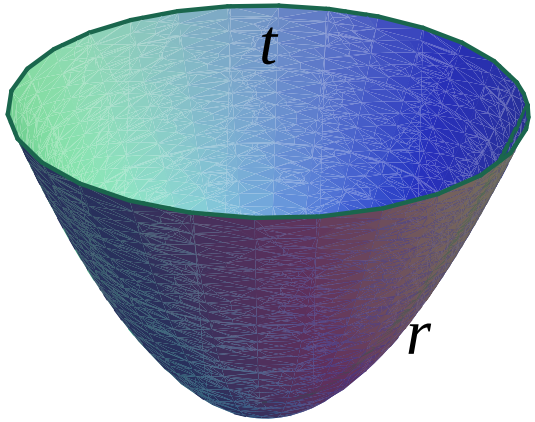
$$S_E = -\frac{M^3 N_c^2}{2} \int \mathrm{d}^5x \sqrt{-g} \left[R - \frac{4}{3}(\partial\phi)^2 + V(\phi) \right]$$

- Dilaton potential determined phenomenologically: **bottom-up**

Black Hole Solution for 5d Dilaton–Gravity Gursoy-Kiritsis-Mazzanti-Nitti'08

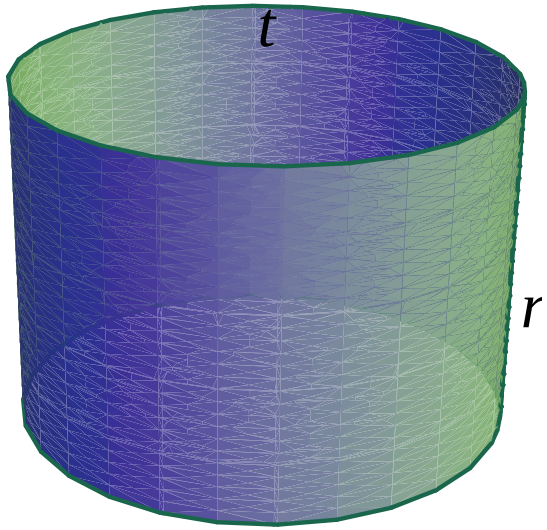
Finite T: asymptotic AdS BH + dilaton

$$\mathrm{d}s^2 = b(r)^2 \left[\frac{\mathrm{d}r^2}{f(r)} - f(r)\mathrm{d}t^2 + \mathrm{d}x^2 \right]$$
$$\lambda = \lambda(r), \quad V = V(\lambda)$$



Uniqueness: integration constants

1. $\Lambda \Leftrightarrow$ strong coupling scale
2. $\lambda_h \Leftrightarrow T$
3. "good singularity"
4. UV normalization $\Rightarrow f(0) = 1$
5. unphysical



Zero T: Asymptotic Freedom and Confinement

UV geometry $\Leftrightarrow$ YM β–function $\quad (V = \frac{4}{3}W_{o,\phi}^2 - \frac{64}{27}W_o^2) \quad (r \to 0)$

$$\beta(\lambda) = -b_0\lambda^2 - b_1\lambda^3 + \ldots = -\frac{9}{4}\lambda^2\,\partial_\lambda \log W_o \quad \text{as } \lambda \to 0$$

- b_0 and b_1 fixed from YM
- $\lambda = N_c e^{\phi} \simeq -\frac{1}{b_0 \log r\Lambda}$, $b \simeq \frac{\ell}{r}\left(1 + \frac{4}{9}\frac{1}{\log r\Lambda}\right)$ and $V \simeq V_0 + V_1\lambda$

IR geometry $\Leftrightarrow$ confinement via Wilson loop $\quad (r \to \infty)$

$$W_o \sim \lambda^{\frac{2}{3}}(\log\lambda)^{P/2}$$

- singularity at $\infty \Rightarrow P \geq 0$ ($P = 1/2$ for linear confinement)
- magnetic screening and discrete gapped glueballs for all confining backgrounds

Zero T: Asymptotic Freedom and Confinement

UV geometry $\Leftrightarrow$ YM β–function $\quad (V = \frac{4}{3}W_{o,\phi}^2 - \frac{64}{27}W_o^2)$ $\quad (r \to 0)$

$$\beta(\lambda) = -b_0\lambda^2 - b_1\lambda^3 + \ldots = -\frac{9}{4}\lambda^2\,\partial_\lambda \log W_o \quad \text{as } \lambda \to 0$$

- b_0 and b_1 fixed from YM
- $\lambda = N_c e^{\phi} \simeq -\frac{1}{b_0 \log r\Lambda}$, $b \simeq \frac{\ell}{r}\left(1 + \frac{4}{9}\frac{1}{\log r\Lambda}\right)$ and $V \simeq V_0 + V_1\lambda$

IR geometry $\Leftrightarrow$ confinement via Wilson loop $\quad (r \to \infty)$

$$W_o \sim \lambda^{\frac{2}{3}}(\log\lambda)^{P/2}$$

- singularity at $\infty \Rightarrow P \geq 0$ ($P = 1/2$ for linear confinement)
- magnetic screening and discrete gapped glueballs for all confining backgrounds

Black Hole: UV and IR Horizon

UV: same log expansion as the thermal–gas $(r_h \to 0)$

$$\lambda(r) = \lambda_o(r)\left[1 + \tfrac{45}{8}\mathcal{G}(T) r^4 \log \Lambda r + \dots\right]$$
$$f(r) = 1 - \tfrac{1}{4}\mathcal{B}(T) r^4 + \dots$$

Holography

- $\mathcal{G}(T)$: vev of the $\Delta = 4$ operator $\Rightarrow$ gluon condensate $\mathcal{G}(T) \sim \frac{T^4}{\log^2 T}$
- $\mathcal{B}(T)$: thermodynamic quantity $\mathcal{B}(T) = T\mathcal{S}$

IR: **good singularity** singled out $(r_h \to \infty)$

$$b(r) \sim b_o(r), \quad \lambda(r) \sim \lambda_o(r)$$

Black Hole: UV and IR Horizon

UV: same log expansion as the thermal–gas $(r_h \to 0)$

$$\lambda(r) = \lambda_o(r)\left[1 + \tfrac{45}{8}\mathcal{G}(T) r^4 \log \Lambda r + \dots\right]$$
$$f(r) = 1 - \tfrac{1}{4}\mathcal{B}(T) r^4 + \dots$$

Holography

- $\mathcal{G}(T)$: vev of the $\Delta = 4$ operator $\Rightarrow$ gluon condensate $\mathcal{G}(T) \sim \frac{T^4}{\log^2 T}$
- $\mathcal{B}(T)$: thermodynamic quantity $\mathcal{B}(T) = T\mathcal{S}$

IR: **good singularity** singled out $(r_h \to \infty)$

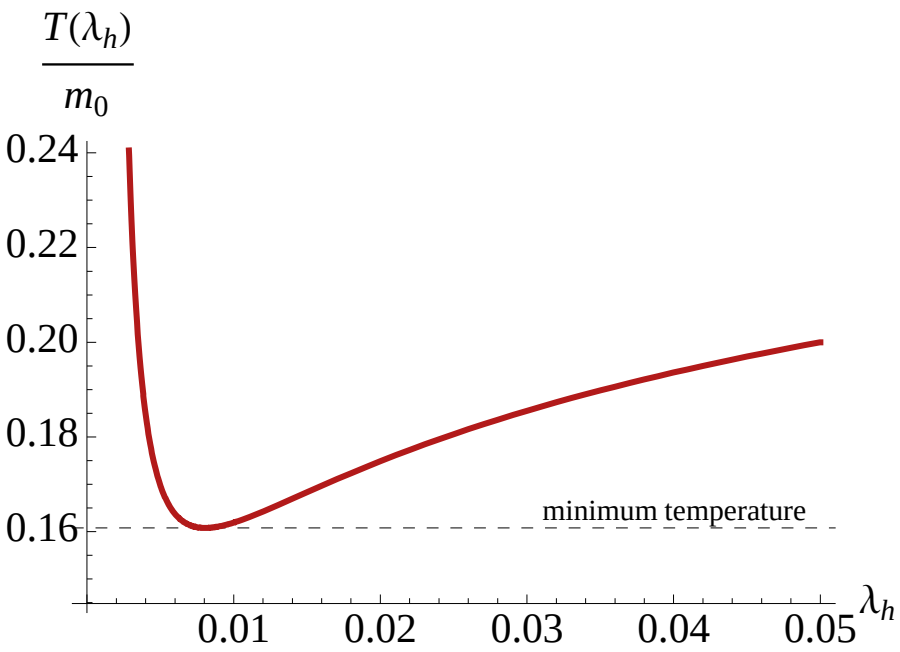
$$b(r) \sim b_o(r), \quad \lambda(r) \sim \lambda_o(r)$$

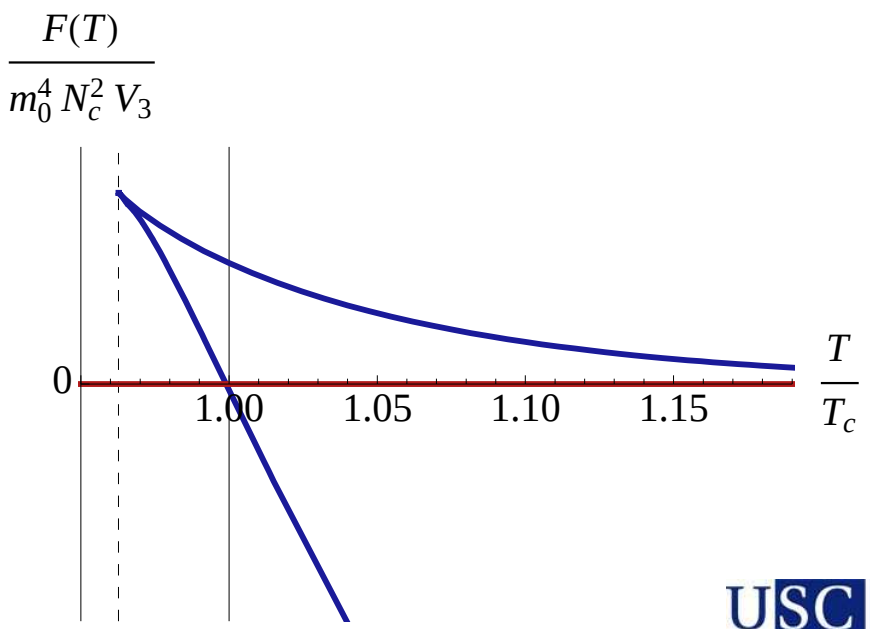
Phase Transition

$$\frac{\mathcal{F}}{M_P^3 V_3} = \frac{S - S_o}{\beta M_P^3 V_3} = 15\mathcal{G}(T) - \frac{\mathcal{B}(T)}{4}$$

$$\Downarrow$$

First order phase transition $\Leftrightarrow$ confining backgrounds





Holographic Thermodynamics

Trace Anomaly vs. Gluon Condensate: the Trace Anomaly Equation

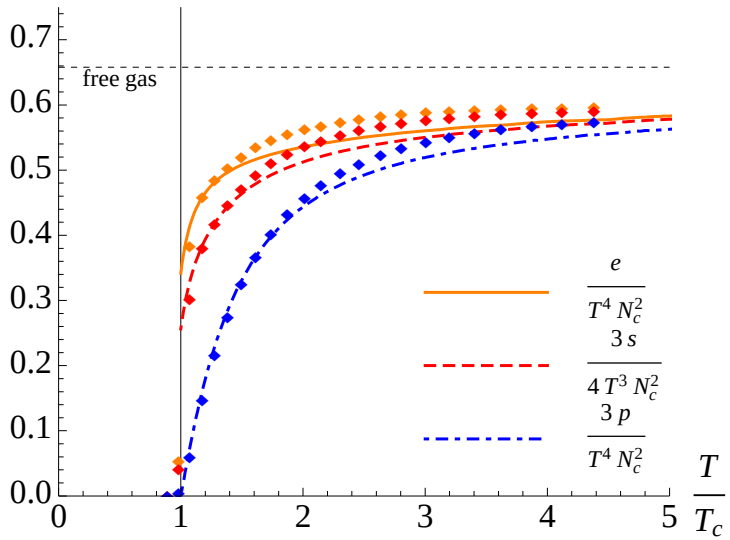
From dilaton fluctuation and field/operator correspondence

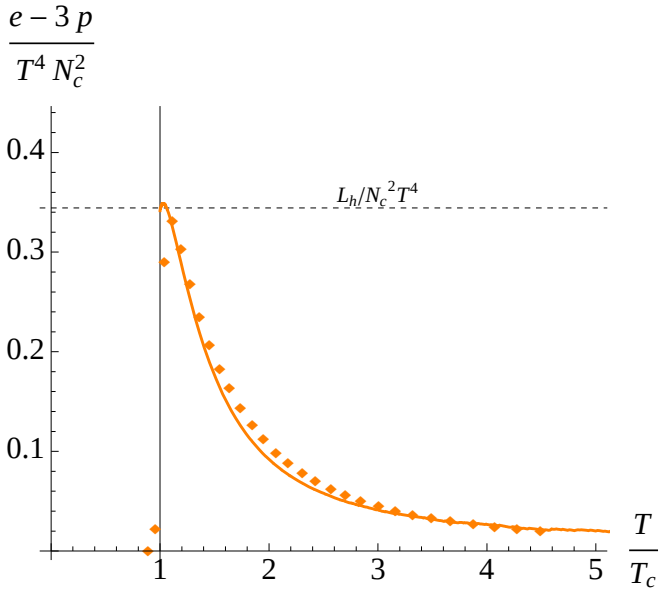
$$\frac{1}{4}\frac{\beta(\lambda)}{\lambda^2}\langle \mathrm{tr} F^2\rangle = e - 3p = 60\mathcal{G}(T)M^3N_c^2$$

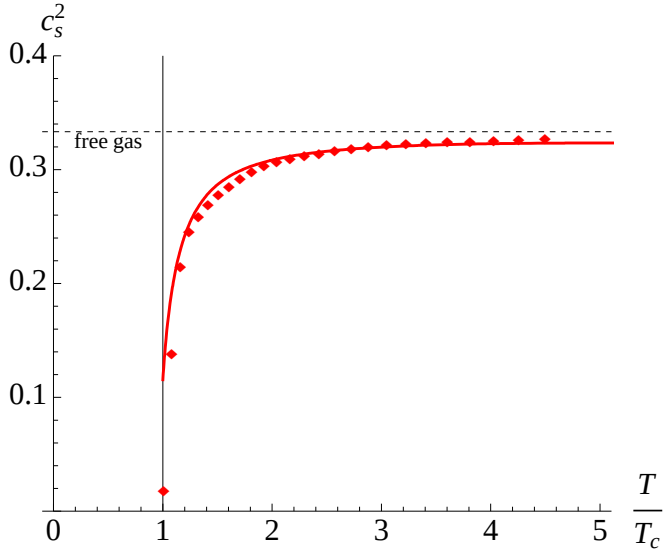
- **Trace**: $e - 3p \propto$ condensate $\Rightarrow$ latent heat $\sim N_c^2$
- **Pressure, Energy, Entropy**: $p, e, s \sim N_c^2$ for $T > T_c \Rightarrow$ deconfinement
- **Sound speed**: $c_s^2 \to 1/3$ at high–T, small at T_c

Thermodynamic Result

Boyd-et al.'05 ($SU(3)$), Lucini-Teper-Wenger'05 (large–N_c),







Holographic Langevin Diffusion

Heavy quark dynamics:

$$\frac{d\vec{p}}{dt} + \eta_D \vec{p} = \vec{\xi}, \quad \langle \xi^i \xi^j \rangle = \kappa^{ij} \delta(t - t')$$

$\uparrow$ viscous force

$\uparrow$ diffusion constants

$\Leftrightarrow$ momentum broadening

$$\eta_D = -\frac{1}{\gamma M \omega} \mathrm{Im} G_R(\omega)|_{\omega=0}, \quad \kappa = G_{sym}(\omega)|_{\omega=0}$$

- G_{sym} and G_R are correlators of $\mathcal{F}(t)$, the instantaneous force on the quark

Field/operator correspondence

$$\mathcal{F} \Leftrightarrow X^M$$

- correlators are computed holographically from the string fluctuations δX^M

Holographic Langevin Diffusion

Heavy quark dynamics:

$$\frac{d\vec{p}}{dt} + \eta_D \vec{p} = \vec{\xi}, \quad \langle \xi^i \xi^j \rangle = \kappa^{ij} \delta(t - t')$$

$\uparrow$ viscous force

$\uparrow$ diffusion constants

$\Leftrightarrow$ momentum broadening

$$\eta_D = -\frac{1}{\gamma M \omega} \mathrm{Im} G_R(\omega)|_{\omega=0}, \quad \kappa = G_{sym}(\omega)|_{\omega=0}$$

- G_{sym} and G_R are correlators of $\mathcal{F}(t)$, the instantaneous force on the quark

Field/operator correspondence

$$\mathcal{F} \Leftrightarrow X^M$$

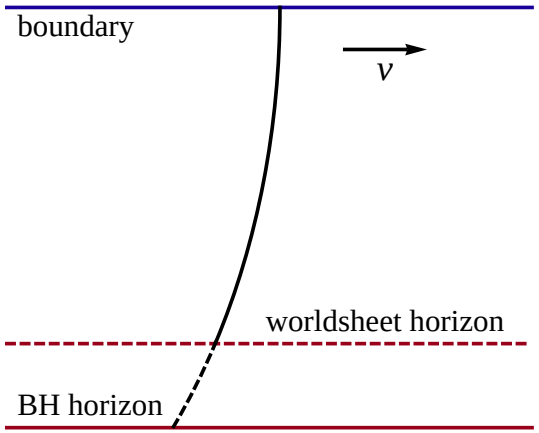
- correlators are computed holographically from the string fluctuations δX^M

Trailing String Gursoy-Kiritsis-Mazzanti-Nitti'10

- Worldsheet background:

$$X^1 = vt + x(r), \quad X^2 = X^3 = 0$$

$$S_{NG} = -\frac{1}{2\pi\ell_s^2}\int \mathrm{d}r\mathrm{d}t\; b^2\sqrt{1-\frac{v^2}{f}+f\dot{x}^2}$$



Worldsheet Horizon

The induced metric has a horizon at r_s with temperature: $f(r_s) = v^2$

$$4\pi T_s = \sqrt{f\dot{f}}\sqrt{4\frac{\dot{b}}{b}+\frac{\dot{f}}{f}}\Bigg|_{r_s}$$

Worldsheet Background: the Drag Force

- Drag force ⇒ classical momentum lost by the moving string to the horizon

$$\eta_D = -\frac{\pi_x}{\gamma v M} = \frac{1}{\gamma M}\frac{b^2(r_s)}{2\pi\ell_s^2}$$

- Diffusion time ⇒ attenuation time for the momentum

$$\tau_D \equiv \frac{1}{\eta_D} = \gamma M \frac{2\pi\ell_s^2}{b^2(r_s)}$$

Comparison to AdS_5 **(λ_{AdS} fixed)**

- $T_{s,AdS} = T/\sqrt{\gamma}$
- $\tau_{D,AdS} = \frac{2M}{\pi\sqrt{\lambda_{AdS}}T^2}$ momentum–independent

Worldsheet Fluctuations

- Worldsheet onshell action to second order

$$X^1 = vt + x(r) + \delta X^1, \quad X^2 = \delta X^2, \quad X^3 = \delta X^3$$

$$S^{(2)}_{NG} = -\frac{1}{2\pi\ell_s^2}\int \mathrm{d}r\mathrm{d}t\ \frac{G^{\alpha\beta}}{2}\partial_\alpha\delta X\,\partial_\beta\delta X$$

$$G^{\alpha\beta}_{\perp} = Z^2 G^{\alpha\beta}_{\parallel} = \frac{b^2}{Z^3}\begin{pmatrix} -\frac{Z^2 f + v^2}{f^2} & v\dot{x} \\ v\dot{x} & f - v^2 \end{pmatrix}, \quad Z \equiv b^2\frac{\sqrt{f-v^2}}{\sqrt{b^4 f - b_s^4 v^2}}$$

Leading Asymptotics from *eom* $I = \perp, \parallel$

- **boundary**: $\delta X^I \sim c^I_{sour} + c^I_{vev} r^3$
- **horizon**: $\delta X^I \sim c^I_{in}\,(r_s - r)^{-\frac{\mathrm{i}\omega}{4\pi T_s}} + c^I_{out}\,(r_s - r)^{\frac{\mathrm{i}\omega}{4\pi T_s}}$

- Retarded wave functions Ψ_R: $c_{out} = 0$ and $c_{sour} = 1$

Diffusion Constants and Jet Quenching Gursoy-Kiritsis-Mazzanti-Nitti'10

$$G_R = -\frac{1}{2\pi\ell_s} G^{r\alpha}\Psi_R^* \partial_\alpha \Psi_R\Big|_{\text{bound.}}, \quad G_{sym} = -\coth\left(\frac{\omega}{2T_s}\right)\mathrm{Im}G_R$$

$$\Downarrow$$

$$\kappa = \lim_{\omega\to 0} G_{sym} = \lim_{\omega\to 0}\coth\left(\frac{\omega}{2T_s}\right) J^r$$

- J^r is a conserved current (number current)
- $\hat{q} \equiv \langle \Delta p^2\rangle / L$ at strong coupling

$$\Downarrow$$

$$\kappa_\perp = \frac{1}{2} v\hat{q}_\perp = \frac{1}{\pi\ell_s^2} b_s^2 T_s, \qquad \kappa_\parallel = v\hat{q}_\parallel = \frac{16\pi}{\ell_s^2}\frac{b_s^2}{\dot{f}_s^2} T_s^3$$

Generalized Einstein relation to diffusion time:

$$\tau_D \kappa_\perp = 2\gamma M T_s$$

Diffusion Constants and Jet Quenching Gursoy-Kiritsis-Mazzanti-Nitti'10

$$G_R = -\frac{1}{2\pi\ell_s} G^{r\alpha}\Psi_R^* \partial_\alpha \Psi_R\Big|_{\text{bound.}}, \quad G_{sym} = -\coth\left(\frac{\omega}{2T_s}\right)\mathrm{Im}G_R$$

$$\Downarrow$$

$$\kappa = \lim_{\omega\to 0} G_{sym} = \lim_{\omega\to 0}\coth\left(\frac{\omega}{2T_s}\right) J^r$$

- J^r is a conserved current (number current)
- $\hat{q} \equiv \langle \Delta p^2\rangle / L$ at strong coupling

$$\Downarrow$$

$$\kappa_\perp = \frac{1}{2} v\hat{q}_\perp = \frac{1}{\pi\ell_s^2} b_s^2 T_s, \qquad \kappa_\parallel = v\hat{q}_\parallel = \frac{16\pi}{\ell_s^2}\frac{b_s^2}{\dot{f}_s^2} T_s^3$$

Generalized Einstein relation to diffusion time:

$$\tau_D \kappa_\perp = 2\gamma M T_s$$

WKB Approximation for Large Frequencies

1. WKB for $\omega r_s \gg 1$: $\Psi_R \sim C_1 \cos\left[\int \frac{\omega Z}{f-v^2} + \theta_1\right] + C_2 \sin\left[\int \frac{\omega Z}{f-v^2} + \theta_2\right]$
2. Coefficients determined by boundary and horizon behavior

Spectral Densities

$\rho \equiv -\frac{1}{\pi}\mathrm{Im}G_R$

- **Infinite mass**: cubic in ω

$$\rho_\perp \simeq \gamma^{-2}\rho_\parallel \simeq \frac{\ell^2\gamma^2}{\pi^2\ell_s^2}\omega^3\lambda_{tp}^{\frac{4}{3}}$$

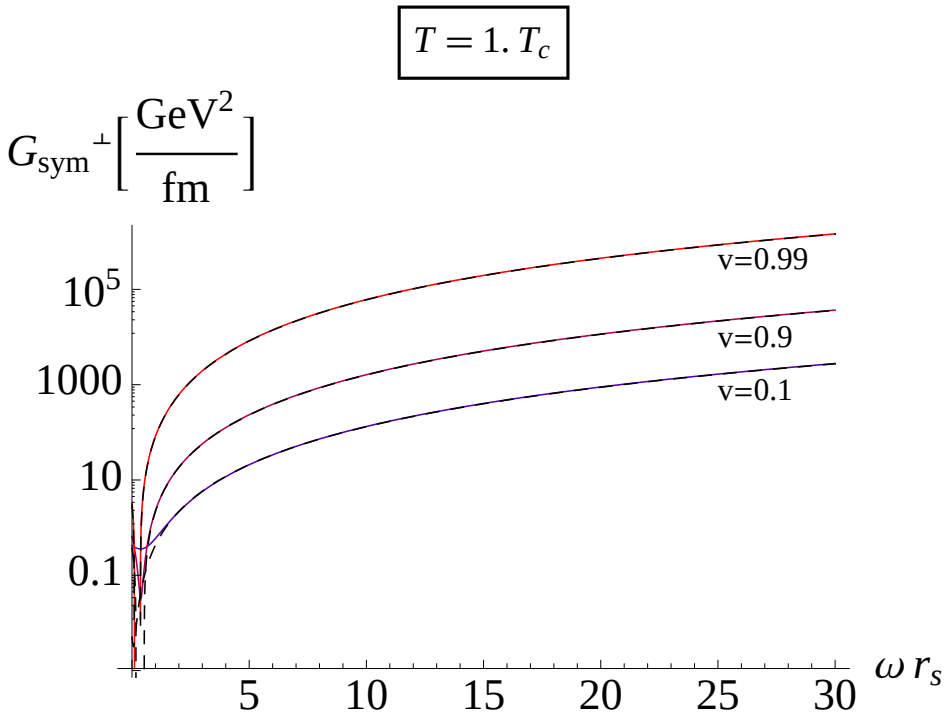
- **Finite mass**, high velocities ($\gamma\omega r_Q \gg 1$): linear in ω

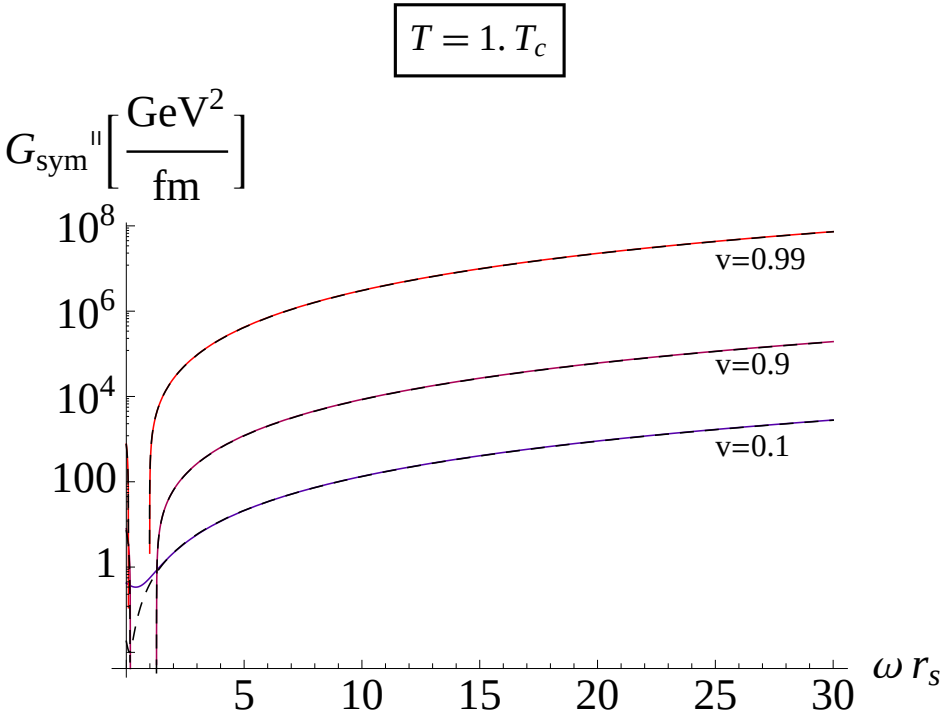
$$\rho_\perp = \gamma^{-2}\rho_\parallel \simeq \frac{\ell^2\gamma^2}{\pi^2\ell_s^2}\omega^3 r_Q^2 b_Q^2 \lambda_Q^{\frac{4}{3}}\left[1+(\gamma\omega r_Q)^2\right]^{-1}$$

$M \sim 1/r_Q$,$\lambda_{tp} = \lambda$ at turning point

Spectral Densities — Infinitely Massive Quarks

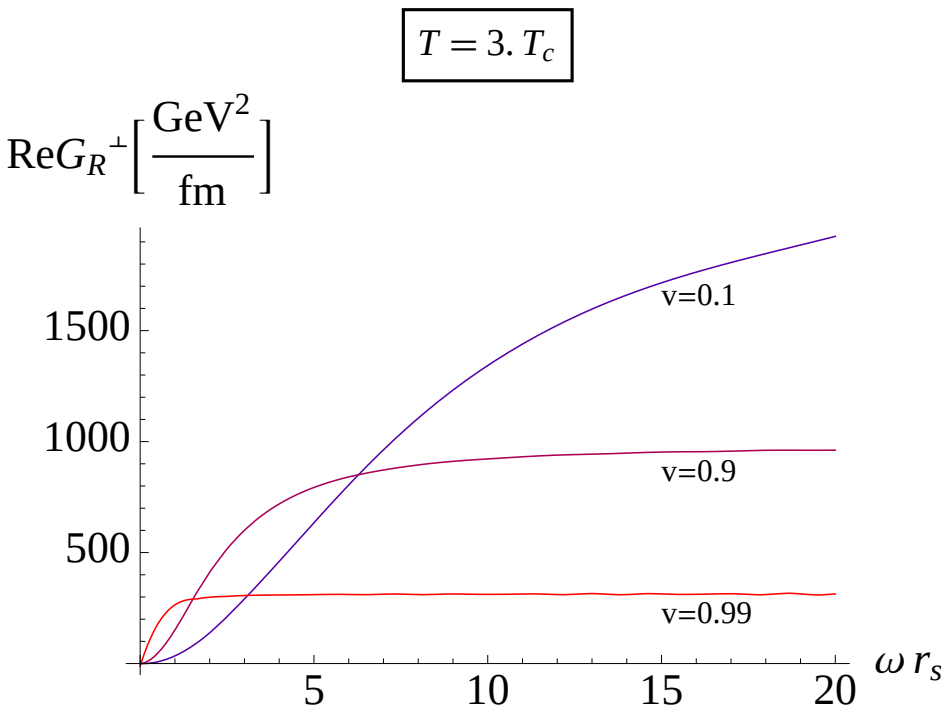
Symmetric Correlator

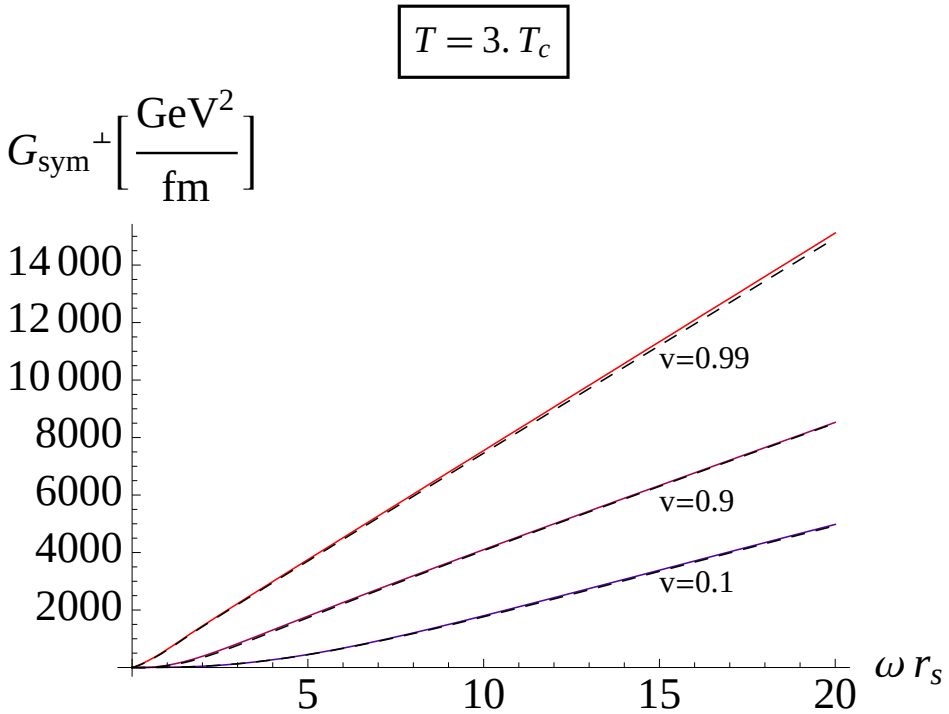




Spectral Densities — Finite Massive Quarks

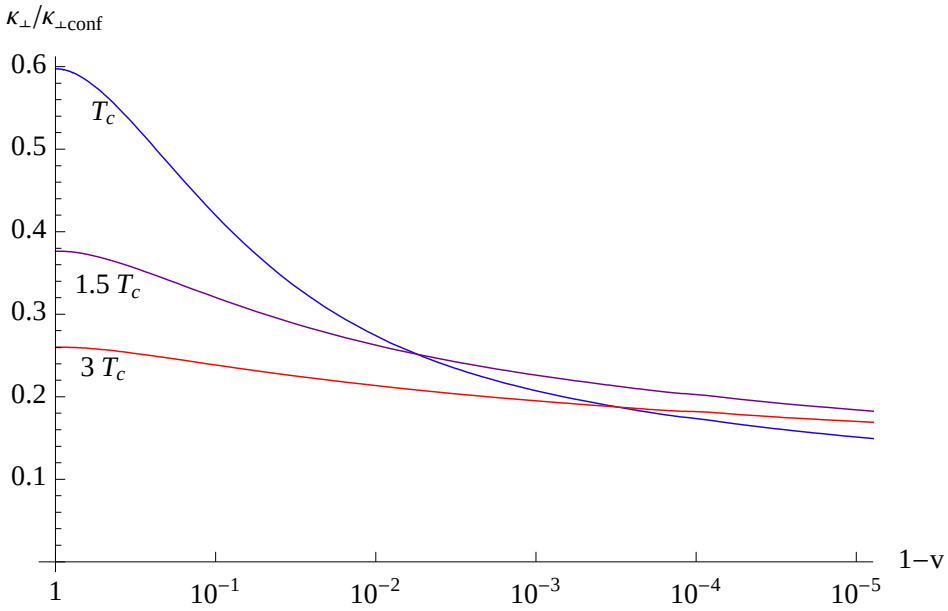
Retarded and Symmetric Correlator — Charm

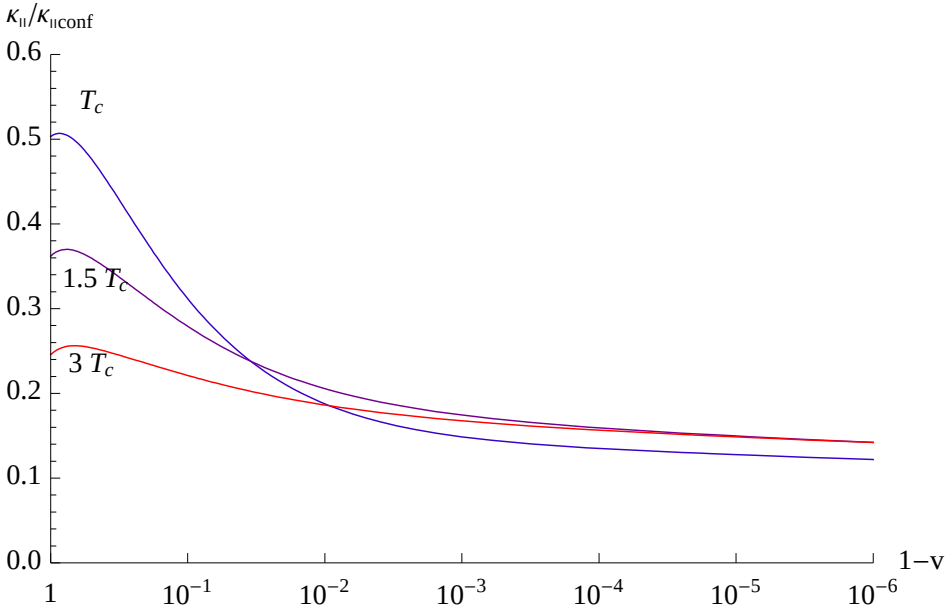




Diffusion Constants

Ratio to AdS

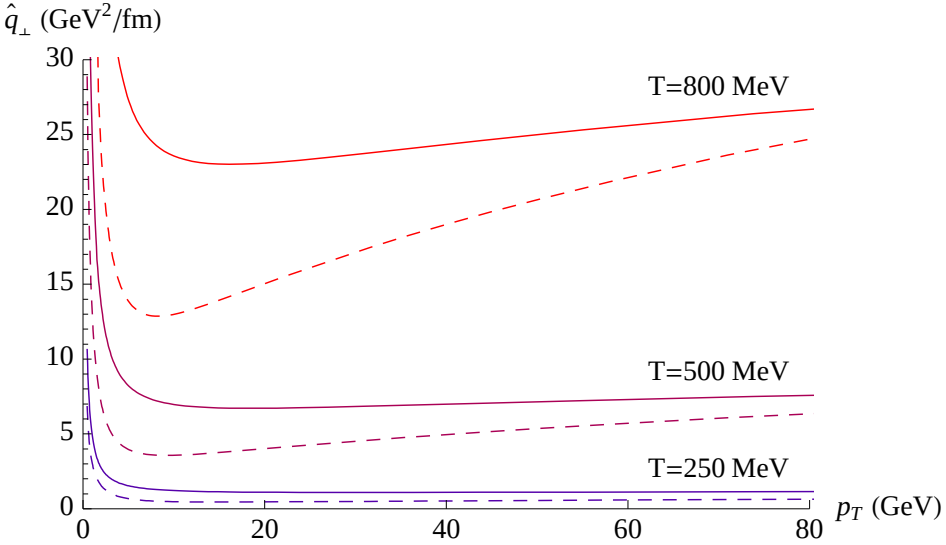




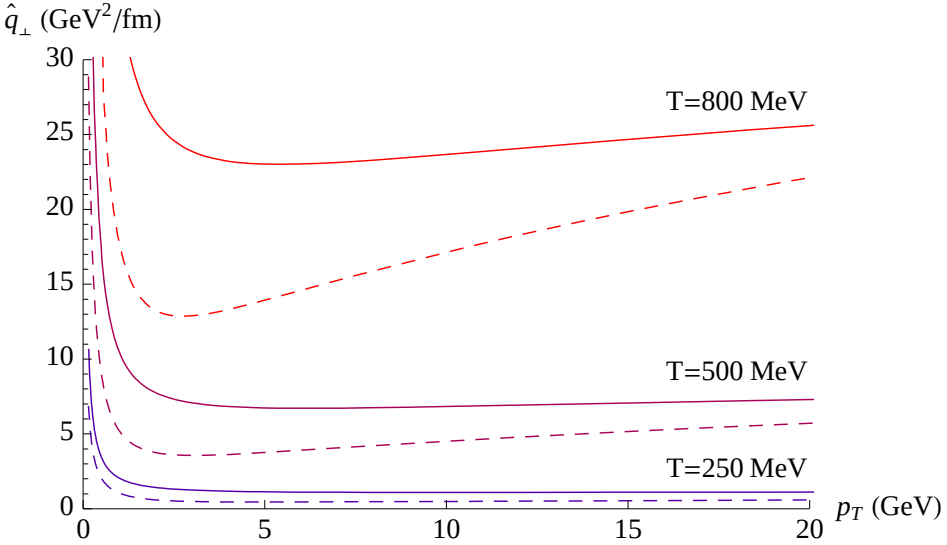
Jet Quenching Parameter

Bottom and Charm Quarks: $\hat{q}$ vs. momentum

bottom



charm



Summary

5D dilaton–gravity $\Leftrightarrow$ 4D holographic large-N_c gauge theory

$\Downarrow$

Confinement with discrete spectrum in agreement with lattice at low–T

1. **Phase transition** first order Hawking–Page confinement/deconfinement
   - only for confining backgrounds
2. **Thermodynamics** in good agreement with lattice
3. **Hydrodynamics** compatible with phenomenological models and experiments

Further issues

1. Flavors
2. Chemical potentials

Ansatz for the Potential Gursoy-Kiritsis-Mazzanti-Nitti'09

$$V(\lambda) = \frac{12}{\ell^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}\left[\log\left(1 + V_2\lambda^{4/3} + V_3\lambda^2\right)\right]^{1/2}\right\}$$

1. Monotonic
2. Asymptotic freedom and confinement
3. Linear Regge trajectories
4. YM for V_0, V_2

$$+$$

- $V_1 = 14$ $\quad p/T^4$ at $T = 2T_c$
- $V_3 = 170$ $\quad e/T^4$ at $T = T_c$ (latent heat)

Thermodynamic Results

Summary of Parameters

	HQCD	$N_c = 3$	$N_c \to \infty$	Parameter
$m_{0^{++}}/\sqrt{\sigma}$	**3.37**	3.56 *	**3.37** **	$\ell_s/\ell = 0.15$
$\left[\frac{p}{(N_c^2 T^4)}\right]_{T\to\infty}$	$\pi^2/45$	$\pi^2/45$	$\pi^2/45$	$M\ell = [45(2\pi)^2]^{-1/3}$
$\left[\frac{p}{(N_c^2 T^4)}\right]_{T=2T_c}$	**1.2**	**1.2** ••	-	$V1 = 14$
$\frac{L_h}{(N_c^2 T_c^4)}$	**0.31**	0.28 •	**0.31** ••	$V3 = 170$

Table: *=Chen-et al.'05, **=Lucini-Teper'01, •=Boyd-et al.'96, ••=Lucini-Teper-Wenger'05

Thermodynamic Results

Summary of Results

	HQCD	$N_c = 3$	$N_c \to \infty$
$m_{0^{*++}}/m_{0^{++}}$	1.61	1.56(11) *	1.90(17) **
$m_{2^{*++}}/m_{2^{++}}$	1.36	1.40(4) *	1.46(11) **
$T_c/m_{0^{++}}$	0.167	-	0.177(7) ••

Table: *=Chen-et al.'05, **=Lucini-Teper'01, •=Boyd-et al.'96, ••=Lucini-Teper-Wenger'05