A Holographic Model of the Quantum Hall Effect

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Holographic fermions

Many systems have strongly-coupled fermions

- QCD
- Condensed matter systems
  - High-T_c superconductors
  - Fractional quantum Hall effect (FQHE)
  - …

Interesting, but difficult

Holographic Approach:

Dp-Dq with #ND=6

- charged fermions in (p-1) dim
- Dq probe in Dp background
- ~~SUSY~~ → stability?

Example:

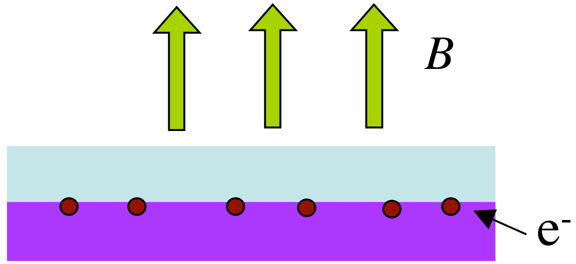
Sakai Sugimoto model: D4-D8-$\overline{\text{D8}}$

Rey
Kraus et al
Myers et al
Hong & Yee
…

Quantum Hall Effect (QHE)

Experimental Setup:

- e^- in 2+1 d
- high magnetic field B
- low temperature T

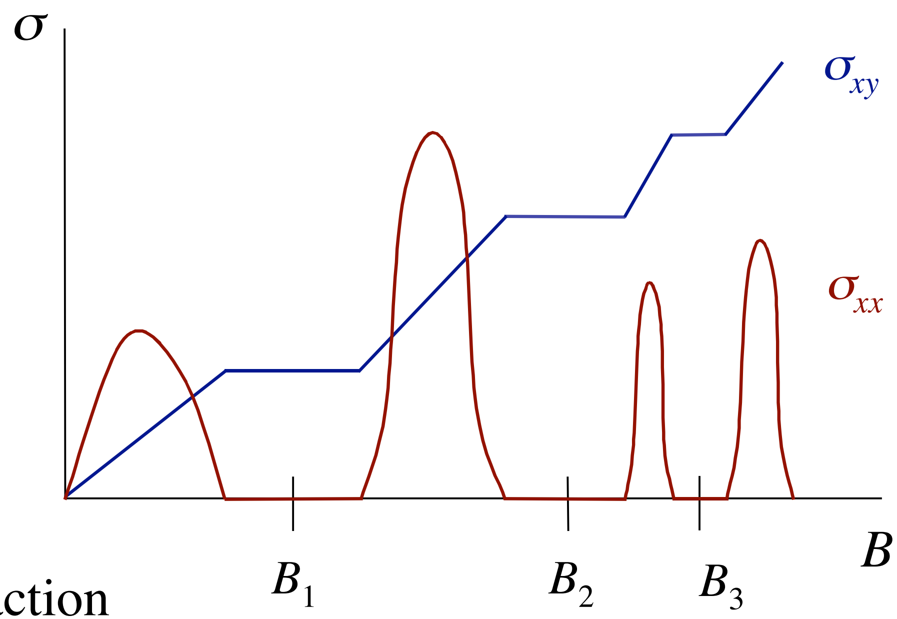


Conductivity

Longitudinal: $\sigma_{xx} = \frac{j_x}{E_x} = 0$

Hall: $\sigma_{xy} = \frac{j_y}{E_x} = \frac{e^2}{2\pi\hbar}\nu$

Filling fraction



Filling Fraction

$$\nu \equiv \frac{2\pi\hbar}{e}\frac{D}{B} \sim \frac{\#\text{ electrons}}{\#\text{ flux quanta}}$$

QHE states at $\nu = \nu_*$

$\nu_* \in \mathbf{Z}$ ⟹ Integer QHE

weakly-coupled electrons in filled Landau levels

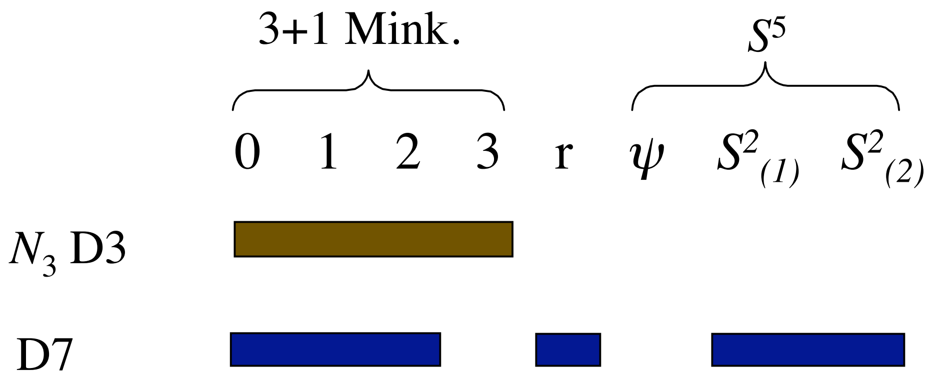
$\nu_* \notin \mathbf{Z}$ ⟹ Fractional QHE

strongly-coupled system, Laughlin wave function

Open questions:

- allowed ν_*'s
- transitions
- microscopic description

D3-D7′ system



$$d\Omega_5^2 = d\psi^2 + \cos^2\psi\, d\Omega_2^2 + \sin^2\psi\, d\Omega_2^2$$

D7 wraps $S^2 \times S^2 \subset S^5$

Embedding:

- x^3 constant
- solve for $\psi(r)$
- $\psi(r \to \infty) \equiv \psi_\infty = \pi/4$

Instability

- lowest mode for ψ: $m^2 = -8 < -9/4$ (BF bound in AdS_4)

Stabilization

Wrap n units of magnetic flux on $S^2_{(1)}$

$$F = \frac{L^2}{4\pi\alpha'} f \, d\Omega_2^{(1)}$$

where f is quantized: $f = \frac{2\pi\alpha'}{L^2} n$ with $n \in \mathbf{Z}$

Embedding: UV ansatz

$$\psi \to \psi_\infty + A r^{-\Delta}$$

Plug in to ψ eom

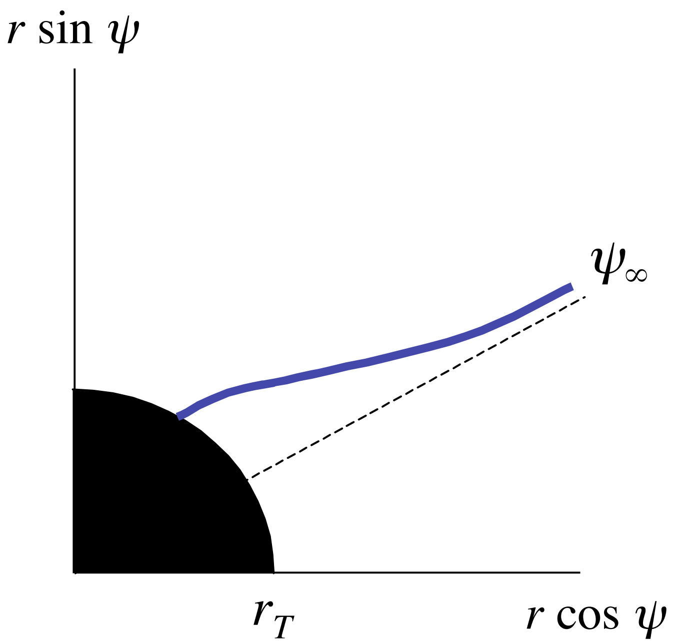
Stability (real Δ) requires:

$$f^2 = 4\sin^2\psi_\infty - 8\sin^4\psi_\infty$$

$$0 < \psi_\infty < \arcsin\left(\frac{5}{\sqrt{73}}\right) \approx \frac{\pi}{5} \qquad \text{and} \qquad 0 < f^2 < \frac{1}{2}$$

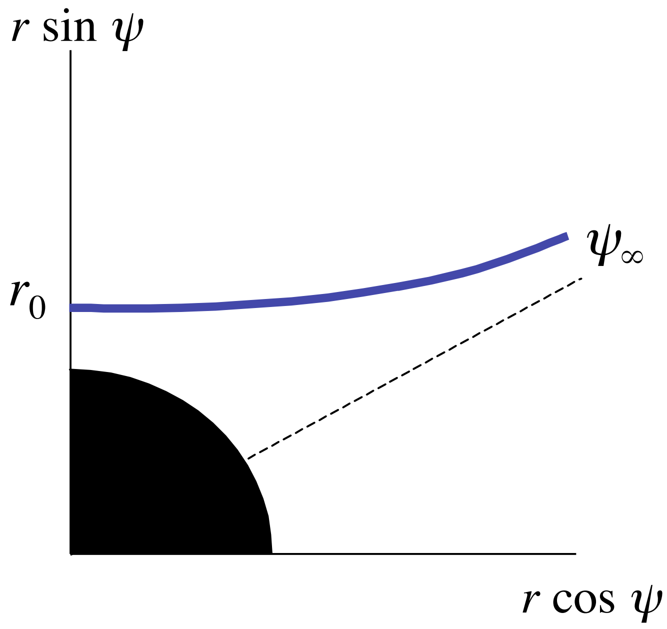
Embeddings

Black Hole (BH)



D7 enters horizon

Minkowski (MN)



D7 ends where S^2 shrinks

Add charges and magnetic field

Charge density

$$2\pi\alpha' F_{r0} = a_0'(r)$$

Magnetic field

$$2\pi\alpha' F_{xy} = B$$

Chern-Simons

$$S_{CS} \sim \int C_4 \wedge F \wedge F$$

$$\sim B \int dr\, c(r) a_0'(r)$$ where $$c(r) \sim \int_{S^2 \times S^2} C_4(r)$$

$$\approx \psi(r) - \psi_\infty$$

→ C_4 flux and B induce charge

Where's the charge?

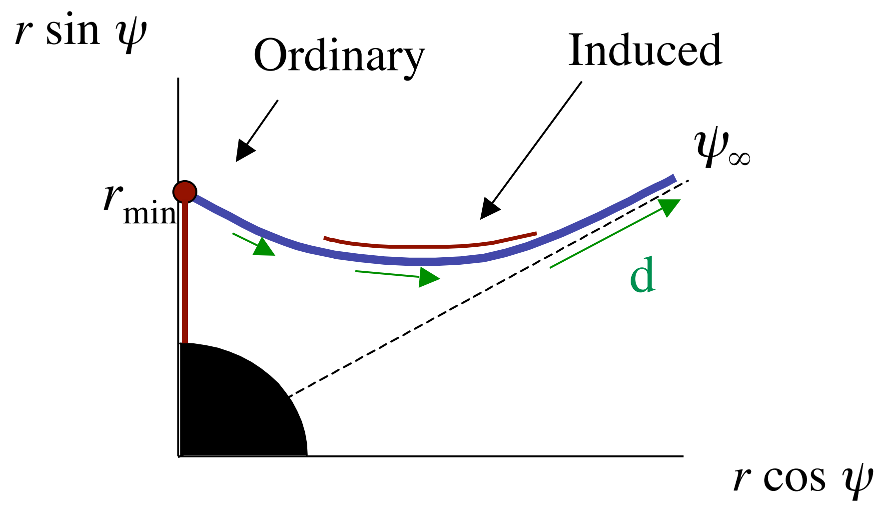
Electric displacement ⟺ radial charge distribution

$$d(r) \equiv \frac{\delta S_{D7}}{\delta a_0'} = d_\infty - 2Bc(r)$$

total charge: $D = d_\infty$

induced charge density: $2Bc' \approx 2B\psi'$

ordinary charge (F1 endpoints) : D - $2Bc(r_{\min})$

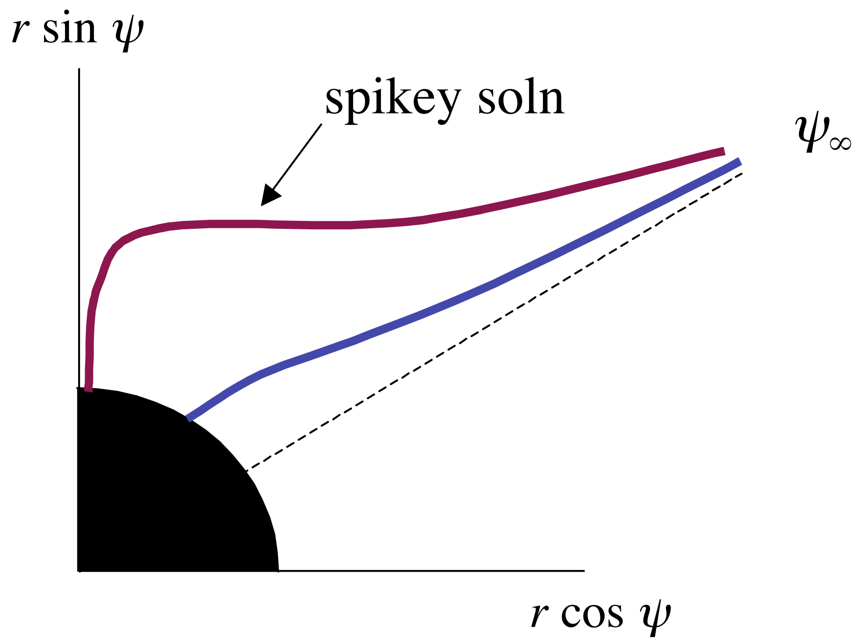


BH Embeddings

Metallic state

- gapless charged excitations
- longitudinal current (via Karch-O'Bannon)

Solutions become spikey as $\nu \equiv \frac{1}{\pi}\frac{D}{B} \longrightarrow 1 - \frac{2\psi_\infty}{\pi}$



MN Embeddings

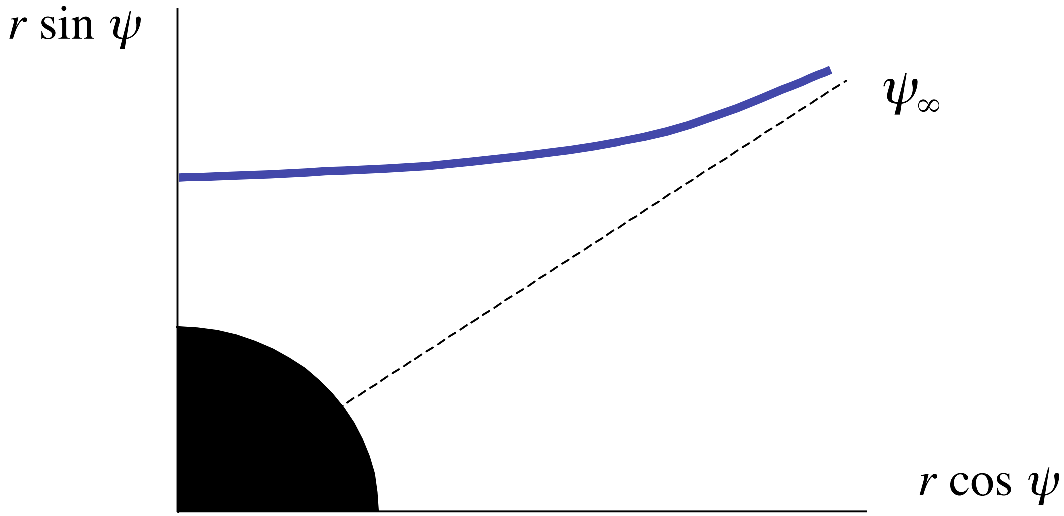
QHE state

- Filling fraction

$$\nu = \nu_* = 2c(r_0) \approx 1 - \frac{2\psi_\infty}{\pi}$$

ν_* fixed by f → quantized

- no sources at tip → D and B locked
- gap for charged excitations $m_g \sim r_0 - r_T$
- transition from BH - crossover



MN Conductivity

Modified Karch-O'Bannon

1. Turn on electric field and radial fields dual to j_x and j_y

2. But require regularity of gauge field at tip (no pseudohorizon)

3. Conductivity

$$\sigma_{xx} = 0$$
$$\sigma_{xy} = \nu_* / 2\pi$$

Summary

Holographic model:

D7 probe with flux in D3 background describes

- charged fermions in 2+1
- strongly-coupled gauge theory in 3+1
- nonzero T, D, B

MN embeddings $\Longleftrightarrow$ QHE state with quantized ν_*

To Do List

Transitions

ν_* fixed by f

scan in B with fixed D ⟹ only one QHE state

to vary ν_* ⟹ Legendre transform to magnetization dual to f

Fluctuations

Stability of QHE state

Spectrum

Metallic state

Thermodynamics

Phase structure

Other 2+1 dim, $\#ND = 6$ models

e.g. D2-D8