Fermions, Holography and Consistent Truncations

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Based on 1008.1423 and 1009.1615.
work with Jottar[2] and Pando Zayas[1],Bah[2],Faraggi[2].

AdS/CMT

- in the AdS/CFT correspondence, pure gravity corresponds to (neutral) hydrodynamics in the dual field theory
- there has been a great deal of work on “bottom-up" investigations, involving the addition of different types of bulk matter for different applications
  - $U(1)$ gauge fields give rise to chemical potential, charge density, conserved charge currents, magnetic fields, etc. in dual field theory
  - charged scalars model superfluids/superconductors
  - bulk fermions give rise to fermionic operators with interesting correlation functions
  - non-Abelian bulk gauge fields model more exotic superconducting order
  - all or some of the above
- these are ‘phenomenological’ approaches

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AdS/CMT

- there are several broad lessons to take from these investigations
- one is that continuous parameters (bulk masses, charges, etc.) give rise to continuous parameters in the dual theory (conformal dimensions, critical exponents, etc.)
- by and large, it is not clear to CM people what we are showing them
  - they are usually grounded in simple models that often have a weakly coupled limit
  - not understood how to 'engineer' real condensed matter systems in AdS/CFT
- there is a general feeling that 'top-down' models, that is models derived directly from string/M-theory constructions, may ameliorate some of these problems
- but even in bottom-up models, we can gain useful guidance from looking at top-down constructions

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- but even in bottom-up models, we can gain useful guidance from looking at top-down constructions

Top Down

- my focus will be on fermions, because of their interest in CM systems
  - e.g., strongly interacting electron systems – expect emergence of interesting IR physics
  - e.g., BCS superconductivity – fermions play a central role in symmetry breaking

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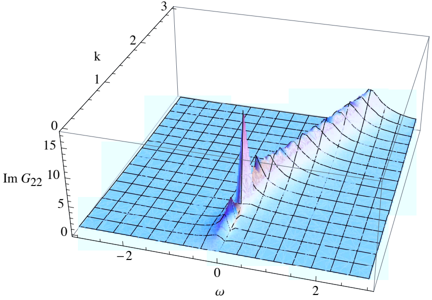

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  one can expect that in string/sugra models there will be generically gravitinoes, as well as (perhaps) 'spin-1/2' fermions (with or without supersymmetry) can these be decoupled in some sense? If not, is there something generic that we can say about them?

Consistent Truncations

- we wish to work just in the (super)gravity limit, and so we focus on *consistent truncations*
  - the fields that are kept cannot linearly source fields that are discarded
  - solutions lift up to higher dimensional solutions
- these are hard to find, and for a long time the known models involved projecting to pure gravity
- however, if one projects to the invariant sector of some symmetry, one can obtain interesting theories with charged matter, which do not correspond to just the lowest modes

Consistent Truncations

- in particular, we will consider two cases:

11d sugra compactified on squashed SE_7 to four dimensions
Type IIB sugra compactified on squashed SE_5 to five dimensions

- vacuum solutions related to supersymmetric and non-supersymmetric ('skew-whiffed') brane configs
- SE_{2n+1} can be thought of as a $U(1)$ bundle over a Kähler-Einstein space KE_n
  - e.g., S^7 is a $U(1)$ fibration over $\mathbb{CP}_3$ (susy enhanced though)
- the consistent truncation consists of projecting to the $SU(n)$-invariant sector
- the fields may be reduced making use of the Kähler form J and the holomorphic $(n, 0)$-form Ω

sSE_{2n+1} Spaces

- the compactification on a warped, squashed Sasaki-Einstein space has a metric of the form

$$e^{2W(x)}ds_E^2(M)_{(x)} + \left[e^{2U(x)}ds^2(KE)_{(y)} + e^{2V(x)}(\eta + A_1(x))^2\right]$$

  where $\eta = d\chi + \mathcal{A}(y)$, $d\mathcal{A} = 2J$ and $ds_E^2(M)_{(x)}$ is the Einstein-frame metric of the external spacetime (determines $W(x)$ in terms of $U(x)$ and $V(x)$)
- the bundle is non-trivial – the form Ω carries $U(1)$ charge, depending non-trivially on the fibre direction
- the same is true for spinors (we really have a $Spin_{\mathbb{C}}$ bundle (the base is not typically spin))
  - the $U(1)$ generator ∂_χ is given on spinors by $\sim -i\not{J}$
  - the gauge-covariantly constant $SU(n)$ singlet spinors $\varepsilon_\pm(y,\chi)$ have maximal eigenvalues $\pm\frac{n+1}{2}$, and are charge conjugates of each other

Consistent Truncations: 11d on sSE_7

- in 10+1, Clifford algebra can be written $\gamma^a \otimes 1_8$, etc.
- here we simply have the 11d gravitino Ψ_A

$$\begin{aligned}
\Psi_a(x,y,\chi) &= \psi_a(x) \otimes \varepsilon_+(y) e^{2i\chi} + \psi_a^{\mathbf{c}}(x) \otimes \varepsilon_-(y) e^{-2i\chi} \\
\Psi_\alpha(x,y,\chi) &= \lambda(x) \otimes \gamma_\alpha\, \varepsilon_+(y) e^{2i\chi} \\
\Psi_{\bar\alpha}(x,y,\chi) &= -\lambda^{\mathbf{c}}(x) \otimes \gamma_{\bar\alpha}\, \varepsilon_-(y) e^{-2i\chi} \\
\Psi_{\mathrm{f}}(x,y,\chi) &= \varphi(x) \otimes \varepsilon_+(y) e^{2i\chi} + \varphi^{\mathbf{c}}(x) \otimes \varepsilon_-(y) e^{-2i\chi}
\end{aligned}$$

  and a 4-form field strength F
- the equation of motion is

$$\Gamma^{ABC} \hat{D}_B \hat{\Psi}_C + \frac{1}{4}\frac{1}{4!}\left[\Gamma^{ADEFGC} F_{DEFG} + 12 \Gamma^{DE} F^{AC}{}_{DE}\right] \hat{\Psi}_C = 0$$

Consistent Truncations: 11d on sSE_7

- the most general $SU(3)$-invariant 4-form is [Gauntlett, Kim, Varela, Waldram]

$$F = f\,\mathrm{vol}_4 + H_3\wedge(\eta+A) + H_2\wedge J + dh\wedge J\wedge(\eta+A) + 2hJ^2 + \left[X(\eta+A_1)\wedge\Omega - \frac{i}{4}(dX - 4iA_1X)\wedge\Omega + \mathrm{c.c.}\right], \tag{1}$$

- so we have a real boson h, a charged boson X, the $U(1)$ gauge field A_1, a second $U(1)$ gauge field B_1, and an axion dual to H_3
- and as we have seen, there is a (Dirac) gravitino ψ_a and (Dirac) fermions λ, ϕ
- this $4d$ spectrum in fact is that of an $N=2$ gauged supergravity theory coupled to one vector and one hypermultiplet, and the evaluation of the full action confirms that (e.g. compare to general $N=2$ gauged sugra [Andrianopoli, et al]

Consistent Truncations: Type IIB on sSE_5

- in 9+1, Clifford algebra can be written as $\gamma^a \otimes 1_4 \otimes \sigma_1$, etc.
- here we have the 10d gravitino Ψ_A and the dilatino λ

$$
\begin{aligned}
\Psi_a(x,y,\chi) &= \psi_a^{(+)}(x) \otimes \varepsilon_+(y) e^{\frac{3}{2}i\chi} \otimes u_- + \psi_a^{(-)}(x) \otimes \varepsilon_-(y) e^{-\frac{3}{2}i\chi} \otimes u_- \\
\Psi_\alpha(x,y,\chi) &= \rho^{(+)}(x) \otimes \gamma_\alpha \varepsilon_+(y) e^{\frac{3}{2}i\chi} \otimes u_- \\
\Psi_{\bar\alpha}(x,y,\chi) &= \rho^{(-)}(x) \otimes \gamma_{\bar\alpha} \varepsilon_-(y) e^{-\frac{3}{2}i\chi} \otimes u_- \\
\Psi_{\mathrm{f}}(x,y,\chi) &= \varphi^{(+)}(x) \otimes \varepsilon_+(y) e^{\frac{3}{2}i\chi} \otimes u_- + \varphi^{(-)}(x) \otimes \varepsilon_-(y) e^{-\frac{3}{2}i\chi} \otimes u_- \\
\lambda(x,y,\chi) &= \lambda^{(+)}(x) \otimes \varepsilon_+(y) e^{\frac{3}{2}i\chi} \otimes u_+ + \lambda^{(-)}(x) \otimes \varepsilon_-(y) e^{-\frac{3}{2}i\chi} \otimes u_+
\end{aligned}
$$

and the usual array of RR and NSNS form fields, as well as the axion-dilaton

Consistent Truncations: Type IIB on sSE_5

- the most general $SU(2)$-invariant forms are [Gauntlett, Kim, Varela, Waldram]

$$
\begin{aligned}
F_{(5)} &= 4e^{8W+Z}\mathrm{vol}_5^E + e^{4(W+U)} * K_2 \wedge J + K_1 \wedge J \wedge J \\
&\quad + \left[2e^Z J \wedge J - 2e^{-8U} * K_1 + K_2 \wedge J\right] \wedge (\eta + A_1) \\
&\quad + \left[e^{4(W+U)} * L_2 \wedge \Omega + L_2 \wedge \Omega \wedge (\eta + A_1) + \text{c.c.}\right] \\
F_{(3)} &= G_3 + G_2 \wedge (\eta + A_1) + G_1 \wedge J + G_0 J \wedge (\eta + A_1) \\
&\quad + \left[N_1 \wedge \Omega + N_0 \Omega \wedge (\eta + A_1) + \text{c.c.}\right] \\
H_{(3)} &= H_3 + H_2 \wedge (\eta + A_1) + H_1 \wedge J + H_0 J \wedge (\eta + A_1) \\
&\quad + \left[M_1 \wedge \Omega + M_0 \Omega \wedge (\eta + A_1) + \text{c.c.}\right] \\
C_{(0)} &= a, \quad \Phi = \phi
\end{aligned}
$$

Consistent Truncations: Type IIB on sSE_5

- the 5d bosonic spectrum was explained in detail in papers by

  [Cassani, Dall'Agata, Faedo], [Gauntlett, Varela] (see also [Liu, Szepietowski, Zhao], [Skenderis, Taylor, Tsimpis])
- consistent with $N = 4$ 5d gauged supergravity coupled to two vector multiplets
- scalars parameterize $SO(1,1) \times (SO(5,2)/SO(5) \times SO(2))$, and a $Heis_3 \times U(1)$ subgroup is gauged
- in AdS_5 vacua, susy is broken to either $N = 2$ or $N = 0$, and the gauge group is Abelian

Fermion Actions

- it is hard to show the full fermionic actions here, but let me point to a few features that are likely important for holography
- first, one finds gravitino kinetic terms, and couplings between gravitinoes and other fermions
  - these must be present, as they are associated with the Stückelberg mechanism in any situation in which supersymmetries are broken
- fermions have couplings to scalars which give rise to masses, as well as various gauge couplings
- charged scalars are present, and there are interesting couplings between them and charged fermions
- there are Pauli (dipolar) couplings to gauge fields, e.g.

$$\ldots + \frac{3i}{4} e^{V-W} \bar{\eta} \left(\not{F}_2 - i\gamma_5 e^{-V-2U} \not{H}_2 \right) \eta + \ldots$$

$$S_F = K \int d^4x \sqrt{-g} \left[\bar{\zeta}_a \gamma^{abc} D_b \zeta_c + \frac{3}{2} \bar{\eta} \not{D} \eta + \frac{1}{2} \bar{\xi} \not{D} \xi + \mathcal{L}^{int}_{\bar{\psi}\psi} + \frac{1}{2} \left(\mathcal{L}^{int}_{\bar{\psi}\psi\mathbf{c}} + \text{c.c.} \right) \right],$$

$$\begin{aligned}
\mathcal{L}^{int}_{\bar{\psi}\psi} = & + \frac{3}{4} i (\partial_b h) e^{-2U-V} \bar{\zeta}_a \gamma_5 \gamma^{abc} \zeta_c + \frac{3}{8} i e^{-2U-V} \bar{\eta} \gamma_5 (\not{\partial} h) \eta - \frac{3}{8} i e^{-2U-V} \bar{\xi} \gamma_5 (\not{\partial} h) \xi \\
& + \frac{1}{4} e^{-2W-V} H_3{}^{abc} \bar{\zeta}_a \gamma_5 \gamma_b \zeta_c - \frac{3}{8} e^{-2W-V} \bar{\eta} \gamma_5 \not{H}_3 \eta + \frac{3}{8} e^{-2W-V} \bar{\xi} \gamma_5 \not{H}_3 \xi \\
& - \frac{i}{4} \bar{\zeta}_a \left[6 (\not{\partial} U) + e^{-2W-V} \gamma_5 \not{H}_3 \right] \gamma^a \xi + \frac{i}{4} \bar{\xi} \gamma^a \left[6 (\not{\partial} U) - e^{-2W-V} \gamma_5 \not{H}_3 \right] \zeta_a \\
& - \frac{3}{4} e^{-2U-V} \left[\bar{\zeta}_a \gamma_5 (\not{\partial} T) \gamma^a \eta - \bar{\eta} \gamma_5 \gamma^a (\not{\partial} T^\dagger) \zeta_a \right] \\
& + \frac{i}{4} \bar{\zeta}_a \left[-e^{V-W} (F + i\gamma_5 * F)^{ac} + 3 i e^{-W-2U} \gamma_5 (H_2 + i \gamma_5 * H_2)^{ac} \right] \zeta_c \\
& + \frac{3i}{4} e^{V-W} \bar{\eta} \left(\not{F} - i \gamma_5 e^{-V-2U} \not{H}_2 \right) \eta - \frac{i}{8} e^{V-W} \bar{\xi} \left(\not{F} + 3 i \gamma_5 e^{-V-2U} \not{H}_2 \right) \xi \\
& + \frac{3}{8} e^{V-W} \left[\bar{\zeta}_a \left(\not{F} - i \gamma_5 e^{-V-2U} \not{H}_2 \right) \gamma^a \eta + \bar{\eta} \gamma^a \left(\not{F} - i \gamma_5 e^{-V-2U} \not{H}_2 \right) \zeta_a \right] \\
& - 3 i e^{W-4U} \bar{\zeta}_a \gamma_5 T^\dagger \gamma^{ac} \zeta_c + 3 i e^{W-4U} \bar{\eta} \gamma_5 T^\dagger \eta + \frac{3}{2} e^{W-4U} \left(\bar{\zeta}_a \gamma^a \gamma_5 T \eta + \bar{\eta} T \gamma_5 \gamma^a \zeta_a \right) \\
& - \frac{9i}{2} e^{W-4U} \bar{\xi} \gamma_5 T \xi - 3 i e^{W-4U} (\bar{\eta} \gamma_5 T \xi + \bar{\xi} \gamma_5 T \eta) + 3 e^{W-4U} \left(\bar{\zeta}_a \gamma^a \gamma_5 T \xi + \bar{\xi} T \gamma_5 \gamma^a \zeta_a \right) \\
& + \frac{1}{4} i \left(\tilde{f} - 8 e^{W-V} \right) \left(i \bar{\zeta}_a \gamma^{ac} \zeta_c - 3 i \bar{\eta} \eta + \frac{3}{2} \bar{\zeta}_a \gamma^a \eta + \frac{3}{2} \bar{\eta} \gamma^a \zeta_a \right) \\
& + \frac{1}{8} \left(3 \tilde{f} + 8 e^{W-V} \right) \bar{\xi} \xi + \frac{3}{4} \tilde{f} \left(\bar{\eta} \xi + \bar{\xi} \eta \right) + \frac{1}{4} i \tilde{f} \left(\bar{\xi} \gamma^a \zeta_a + \bar{\zeta}_a \gamma^a \xi \right)
\end{aligned}$$

$$
\begin{aligned}
\mathcal{L}^{int}_{\bar{\psi}\psi^{\mathbf{c}}} = e^{-3U}\Big\{ &-\frac{i}{2}(D_b X)\bar{\zeta}_a\gamma_5\gamma^{abc}\zeta_c^{\mathbf{c}} - \frac{3i}{4}\bar{\eta}\gamma_5(\not{D}X)\eta^{\mathbf{c}} - \frac{1}{4}\bar{\zeta}_a\gamma_5(\not{D}X)\gamma^a\xi^{\mathbf{c}} + \frac{1}{4}\bar{\xi}\gamma^a\gamma_5(\not{D}X)\zeta_a^{\mathbf{c}}\Big\} \\
&+ Xe^{W-V-3U}\Big\{ 2i\bar{\zeta}_a\gamma_5\gamma^{ac}\zeta_c^{\mathbf{c}} - 6i\bar{\eta}\gamma_5\eta^{\mathbf{c}} - \bar{\zeta}_a\gamma_5\gamma^a\xi^{\mathbf{c}} + \bar{\xi}\gamma^a\gamma_5\zeta_a^{\mathbf{c}} \\
&- 3\Big[\bar{\zeta}_a\left(\gamma_5\gamma^a\right)\eta^{\mathbf{c}} + \bar{\eta}\left(\gamma_5\gamma^a\right)\zeta_a^{\mathbf{c}} + i\bar{\eta}\gamma_5\xi^{\mathbf{c}} + i\bar{\xi}\gamma_5\eta^{\mathbf{c}}\Big]\Big\},
\end{aligned}
$$

Further Truncations

- it is interesting to ask if there are further truncations that we can do to simplify the bulk theory
- in 4d, one such truncation is the (skew-whiffed) holographic superconductor [Gauntlett, Sonner, Wiseman]
- this retains a single $U(1)$ gauge field, a charged scalar and an additional neutral scalar – non-linear generalization of HHH
- we have not found a simple truncation of the fermionic spectrum to go along with this
- nevertheless, there are interesting couplings, such as

$$\ldots + -\frac{3i}{4}\bar{\eta}\gamma_5\left(\not{D}X + 8X\right)\eta^{\mathbf{c}} + \ldots$$

- these generalize terms that were found important in studies of fermion correlators in bottom-up superconductors

[Faulkner, Horowitz, McGreevy, Roberts, Vegh]

Further Truncations

- in 5d (IIB), truncations of the fermion sector seem more readily available. For example a truncation of the bosonic sector has been studied as a holographic superconductor. [Gubser, Herzog, Pufu, Tesileanu]
- in this case, there are several possible further consistent truncations of the fermionic sector. The simplest involves a single fermion $\lambda^{(+)}$
- in this case, the fermionic Lagrangian is simply

$$S_{4+1} = K_5 \int d^5x \sqrt{-g_5^E} \left[\frac{1}{2} \overline{\lambda^{(+)}} \not{D} \lambda^{(+)} + \mathcal{L}^{(+)}_{\bar{\psi}\psi} \right]$$

with

$$\mathcal{L}^{(+)}_{\bar{\psi}\psi} = -\frac{1}{2} \overline{\lambda^{(+)}} \left(\frac{3}{2} + \frac{1}{4} i \not{F}_2 + \frac{2 - 6|Y|^2 + Y^* \overleftrightarrow{\not{D}} Y}{1 - 4|Y|^2} \right) \lambda^{(+)}$$

Conclusions and Holography

- the study of fermion correlators in top-down approaches should include the full fermi sector
- we have computed this in some such consistent truncations of supergravity
- likely will have important implications for holographic correlators, but also can act as a guide in bottom-up model building
  - e.g., dilatonic black-holes
- generally, we expect rather complicated structure of correlators involving both spin-$3/2$ and spin-$1/2$ operators
  - analogous to the familiar mixed correlators of charge currents and stress-energy tensor