Crete Conference
On Gauge Theories And The Structure Of Spacetime

The Orthodox Academy of Crete, Kolymvari, Crete
11-18 September , 2010

Holographic QCD
-gluon condensation & dense media effects-

Bum-Hoon Lee

Center for Quantum Spacetime
Sogang University
Seoul, Korea

서강대학교
SOGANG UNIVERSITY

CQUeST
Center for Quantum SpaceTime

Contents

I. Introduction
- Motivation
- AdS/CFT Holography

II. AdS/QCD : Brief Review
- Top-down & Bottom up approaches
- AdS/QCD at finite temperature and dual geometries

III. Holographic QCD for
- gluon condensation effects
- finite (arbitrary) density

(towards the dual geometries of AdS/QCD)

IV. Summary

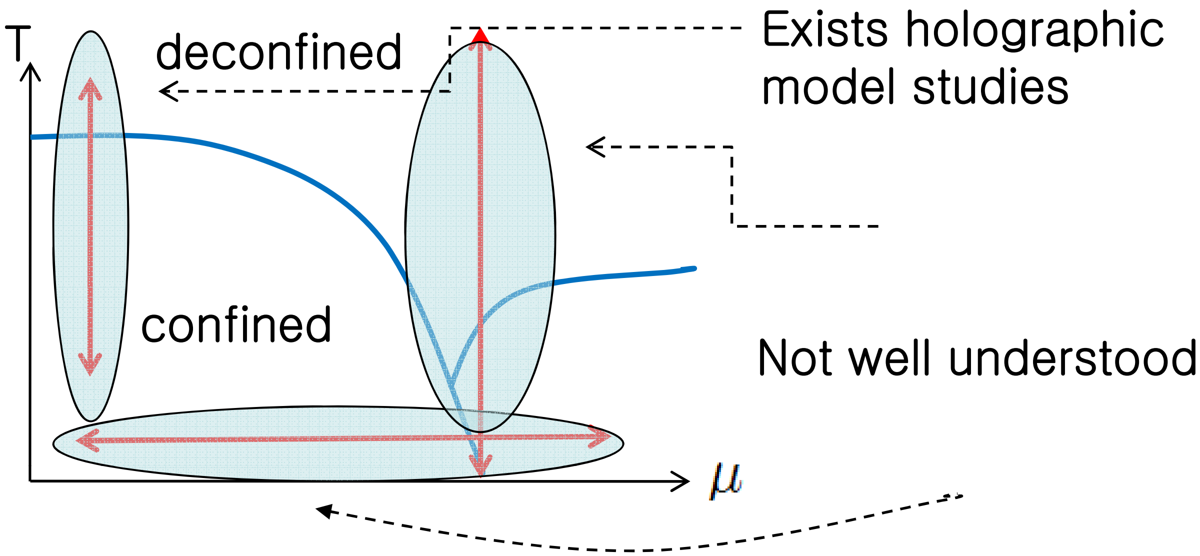
I. Introduction

Motivation

- AdS-CFT Holography : Useful tool for strongly interacting system such as QCD, Condensed Matter, etc.

- How to explain properties such as phases, confinement, chiral symmetry breaking, gluon condensates, chemical potentials, etc. ?

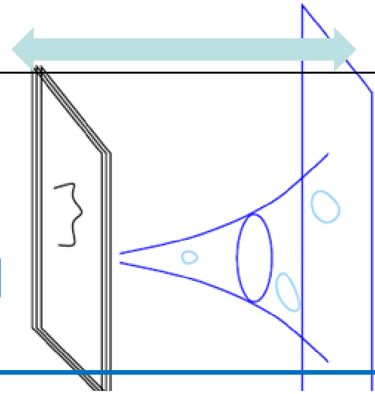
  Ex) meson mass dependence on condensates, chem. Potential, etc.



AdS/CFT Holography

Ex) D3 branes (# of Nc)

N=4 SU(Nc) SYM ⟷ SUGRA on AdS5 x S5



4D QFT (CFT) | Classical Gravity

Conformal x R-symmetry I | Isometries of AdS x S5

SO(4,2) x SO(6) | SO(4,2) x SO(6)

Parameters (g_{YM}^2 , N) $$4\pi g_s N = \frac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N$$ (g_s , R)

Extension of the AdS/CFT

- Finite T QFT — Black Hole back ground
- with $\beta(g^2) \to 0$ (asym. freedom) — in asymptotic AdS
- less or no SUSY — D-branes, b.g. fluxes, etc.
- Chemical potential — bulk gauge field
- Fundamental matters — flavor brane
  - quenched approx. — w/o back reaction
- QCD in dense media, . with gluon condensates, etc. — in ??

AdS/CFT Dictionary

Witten 98;
Gubser,Klebanov,Polyakov 98

Partition function of bulk gravity theory (semi-classial)

$$Z = \int_{\phi_0} \mathcal{D}\phi \exp\left(-S[\phi, g_{\mu\nu}]\right)$$

$$\phi(t, \mathbf{x};\ u = \infty) = u^{\Delta-4}\phi_0(t, \mathbf{x})$$

ϕ_0 bdry value of the bulk field ϕ

Generating functional of bdry QFT for operator $\mathcal{O}$

$$Z = \left\langle \exp \int_{boundary} d^d x \phi_0 \mathcal{O} \right\rangle$$

$$= \int \mathcal{D}[\Phi] \exp\{iS_4 + i\int \phi_0(x)\mathcal{O}\}$$

ϕ_0: source of the bdry op. $\mathcal{O}$

- ϕ: scalar → $S = \int d^4x du \sqrt{-g}\left(g^{ab}\partial_a\phi\partial_b\phi - m^2\phi^2\right)$ $\phi(u) \sim u^{4-\Delta}\phi_0 + u^{\Delta}\langle\mathcal{O}\rangle$
- Correlation functions by $\dfrac{\delta^n Z_{\text{string}}}{\delta\phi_0(t_1, \mathbf{x}_1)\cdots\delta\phi_0(t_n, \mathbf{x}_n)} = \left\langle T\,\mathcal{O}(t_1, \mathbf{x}_1)\cdots\mathcal{O}(t_n, \mathbf{x}_n)\right\rangle_{\text{field theory}}$
- Radial coord. r in the bulk is proportional to the energy scale E of QFT
- 5D bulk field ϕ ←→ Operator $\mathcal{O}$
  w/ 5D mass m_5 ←→ w/ Operator dimesion Δ
- 5D gauge symmetry ←→ Current (global symmetry)
- Large r (small z) ←→ Large Q

$\mathcal{O}$ (Operator in QFT) <-> ϕ (p-form Field in 5D)

$$(\Delta - p)(\Delta + p - 4) = m_5^2$$

Δ : Conformal dimension

m_5^2 : mass (squared)

Note : the fluctuation field ϕ on the bulk space corresponds to a source for the QCD Operator $\mathcal{O}$.

Ex)

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3
$\langle \mathrm{Tr} G^2 \rangle$ Gluon cond .	dilaton	0	4	0
$\bar{q}_L \gamma^\mu q_L$, $\bar{q}_R \gamma^\mu q_R$ baryon density	vector w/ U(1)	1	3	0

fields in gravity

- massless dilaton
- scalar field with $m^2 = -\frac{3}{R^2}$
- m=0 vector field A_μ in the $SU(N_f)$ gauge group

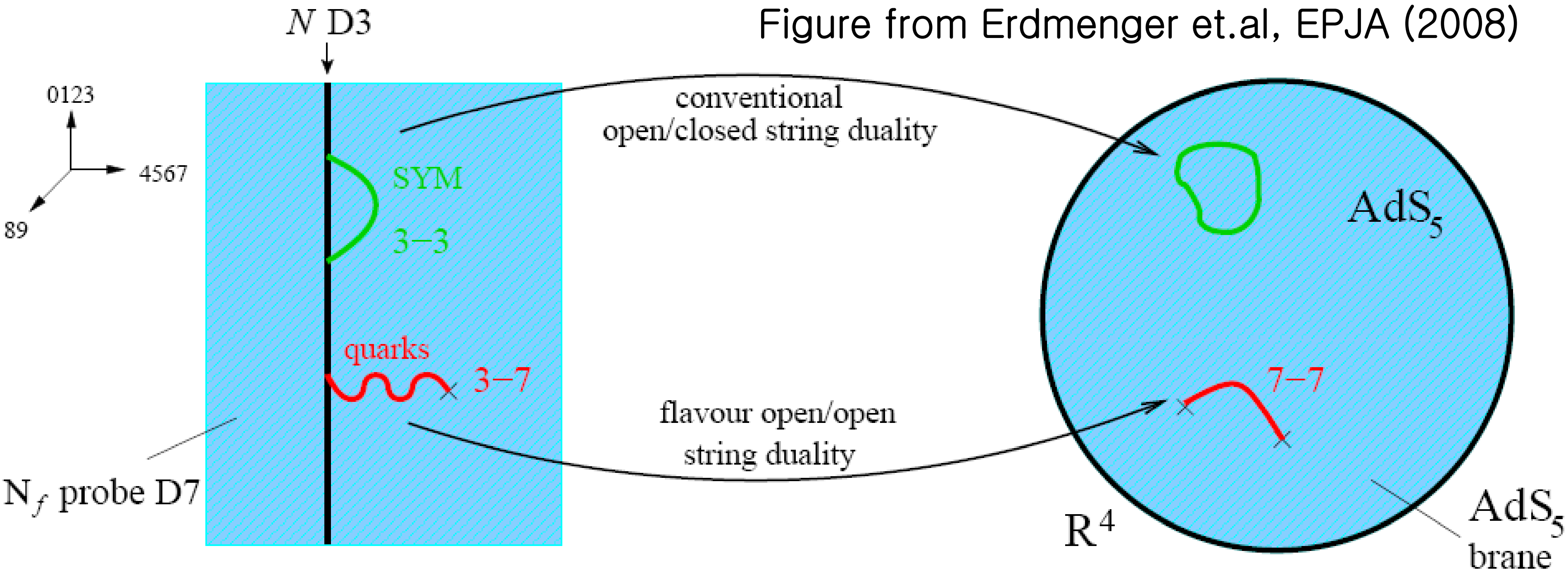
<-> dual

operators of QCD

- gluon condensation $\langle \mathrm{Tr} G^2 \rangle$
- chiral condensation $\bar{q}_R q_L$
- mesons in the $SU(N_f)$ flavor group

Toward the more realistic models
: AdS/CFT with flavors – Intersecting D-Branes

Figure from Erdmenger et.al, EPJA (2008)



"Boundary" QFT ⟺ "Bulk" Gravity

3–3 open strings ("gluons")
3–7 open strings ("quarks")

closed strings (AdS5 geo.)
7–7 open strings ("mesons")

- If Nf/Nc → 0 then probe approximation (back reaction neglected) (quenched approx. ("quark" loops neglected)
- If Nf/Nc finite then beyond probe (quenched) approx.

Probe brane：Neglect back reaction to the metric

7−7 open strings：Low energy dynamics for D7 branes (DBI action)

$$S_{D7} = -\mu_7 \int d^8\xi \sqrt{-\det\left(P[G]_{ab} + 2\pi\alpha' F_{ab}\right)} + \frac{(2\pi\alpha')^2}{2}\mu_7 \int P[C^{(4)}] \wedge F \wedge F$$

$$\mu_7 = [(2\pi)^7 g_s \alpha'^4]^{-1}$$

Metric of AdS5 x S5

$$ds^2 = \frac{r^2}{R^2}\eta_{ij}dx^i dx^j + \frac{R^2}{r^2}(d\rho^2 + \rho^2 d\Omega_3^2 + dw_5^2 + dw_6^2)$$

$$\rho^2 = w_1^2 + \ldots + w_4^2$$

$$r^2 = \rho^2 + w_5^2 + w_6^2$$

action of D7 (for static embedding)

$$S_{D7} = -\mu_7 \int d^8\xi \, \rho^3 \sqrt{1 + \dot{w}_5^2 + \dot{w}_6^2}\,,$$

$\rho \to \infty$, AdS5xS3
$\rho \to 0$, rad (S3)−>0

D7 embedding in 10 dimension ： wraps S3 of S5 and lies flat

w_5=0, w_6 = L = constant (→ non renormalization of the mass)

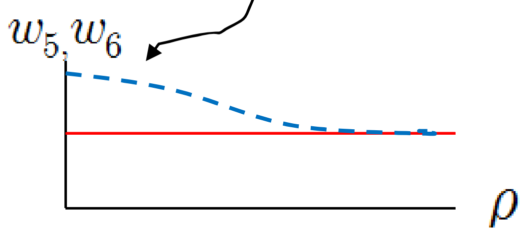
Mesons：Fluctuation of the D7 branes

(In general, $w = L + \frac{c}{\rho^2} + \ldots$)

$$w_5 = 0 + \delta w_5\,, \qquad w_6 = L + \delta w_6$$ etc.

$$\Phi = \phi(\rho)e^{ik\cdot x}\mathcal{Y}^\ell(S^3) \qquad \nabla^i\nabla_i\mathcal{Y}^\ell = -\ell(\ell+2)\mathcal{Y}^\ell$$

$$= \frac{2L}{R^2}\sqrt{(n+\ell+1)(n+\ell+2)}$$



$$-k^2 = M_s(n,\ell) = \frac{2L}{R^2}\sqrt{(n+\ell+1)(n+\ell+2)}$$

Beyond Probe approximation ：back reaction to the metric

D3/D7 SUGRA Solution

$$ds_{10}^2 = h^{-1/2}(r,\rho)\, dx_\mu^2 + h^{1/2}(r,\rho)\left(dr^2 + r^2 d\Omega_3^2 + e^{\Psi(\rho)}(d\rho^2 + \rho^2 d\varphi)\right)$$

$$\chi(\varphi) = \frac{N_f}{2\pi}\varphi \qquad \rho^2 = w_1^2 + \ldots + w_4^2$$

$$r^2 = \rho^2 + w_5^2 + w_6^2$$

$$e^{-\phi(\rho)} = e^{\Psi(\rho)} = -\frac{N_f}{2\pi}\log\frac{\rho}{\rho_L}$$

with

$$h(r,\rho) = 1 + Q_{D3}\int_0^\infty dq \frac{(qr)^2 J_1(qr)}{2r^3}\sum_{n=0}^\infty \lambda^n e^{2nx} p_n(x) \qquad x = \log(\rho/\rho_L) \qquad \rho_L = e^{\frac{2\pi}{g_s N_f}}$$

$$\left(4n^2 + 4n\frac{d}{dx} + \frac{d^2}{dx^2}\right) p_n = x p_{n-1} \qquad p_0(x) = -x - \log(\rho_L q/2) - \gamma$$

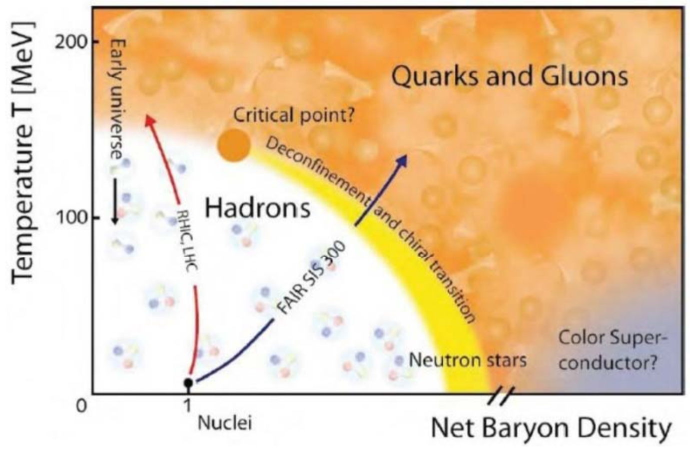
Mesons ：fluctuation of D7

Complicated !

II. AdS/QCD : Brief Review

Holography idea of AdS/CFT applied to QCD is called AdS/QCD

Goal : Using the 5 dim. dual gravity theory (AdS/CFT correspondence) try to understand 4dimensional QCD properties such as spectra & Phases (confining, deconfining, etc.), etc., in terms of parameters (Nc, Nf, m_q, T and μ, χ−symm., gluon condensation, etc.)



Needs the dual geometry of QCD.

Approaches :

- Top−down Approach : rooted in string theory
  Find brane config. or SUGRA solution giving the gravity dual

- Bottom−up Approach : phenomenological
  Introduce fields, etc. as needed based on the AdS/CFT
  - * Hard Wall Model − Introduce IR brane for confinement
  - * Soft Wall Model − dilaton running

cf. Light−Front : radial direction of AdS <−> Parton momenta (Brodsky, de Teramond, 2006)

Top-Down Approach

Start with D-brane configuration (or 10dim. Supergravity).
Get the corresponding geometries.
May put the probe brane in the above geometry.

Observation：

- Nc of D3 branes : AdS5 x S5 <-> N=4 SUSY YM
- Nc of D3 / orbifold, etc. : AdS5 x X <-> N=2, 1 YM
- Flavors (fundamental rep.) : flavor branes etc.

QFT with Asymptotic AdS SUGRA Duals

- N=1* Polchinski & Strassler,hep-th/0003136
- Cascading Gauge Theory Klebanov & Strassler, JHEP 2000
- Nc of D3 + M of D7 system Kruczenski ,Mateos, Myers, Winters 2004
- Nc of D4 + M of D8 system Sakai & Sugimoto 2005

  -> closely related to QCD
- 10Dim. SUGRA solution etc.

Ghoroku,Sakaguchi,Uekusa,Yahiro, hep-th/0502088

Far from the real QCD yet inspite of much success.

Bottom-Up Approach – Phenomenological

Introduce the contents (fields, etc.) as needed based on the AdS/CFT

Karch,Katz,Son,Stephanov, PRD(2006)

- Kaluza-Klein modes

  radial excitations of hadrons, identified by the symmetries

- Confinement realized by "IR cut-off"
  - * Hard Wall Model – by introducing IR brane for confinement
  - * Soft Wall Model – by dilaton running

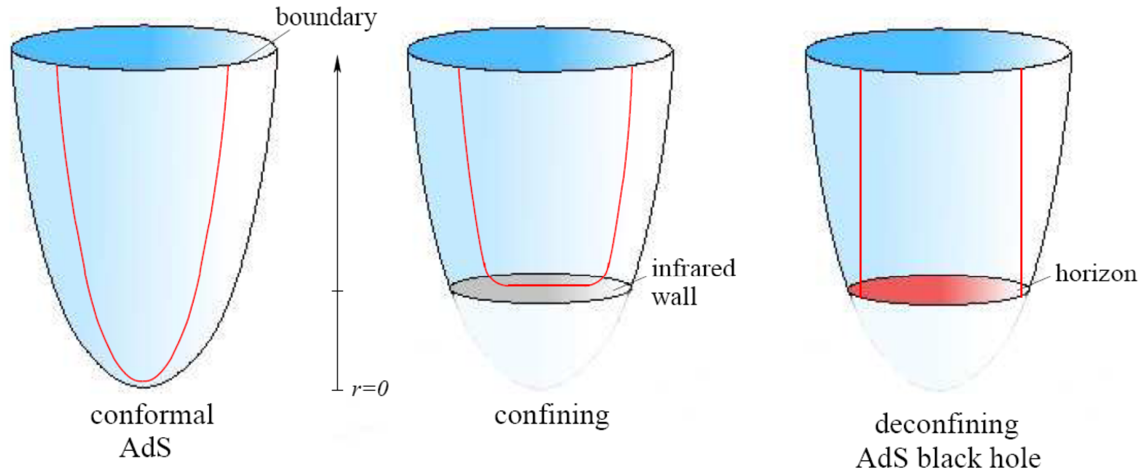
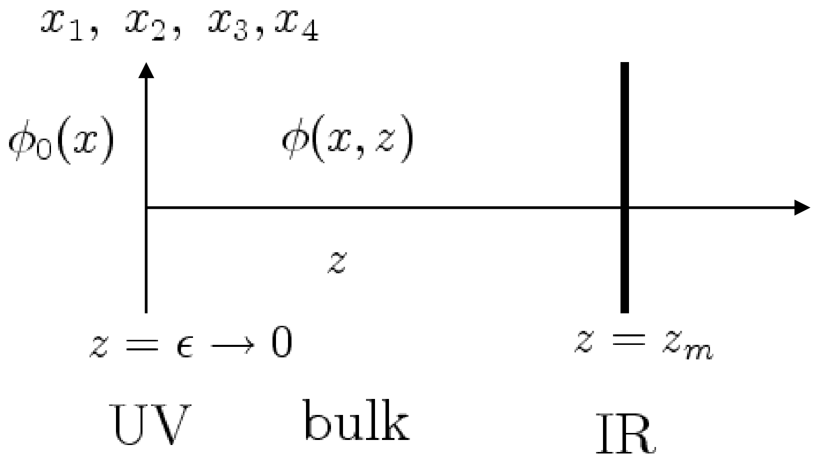


Figure from Erdmenger et.al, EPJA (2008)

Ex) Hard wall Model   Erlich,Katz,Son,Stephanov PRL(2005), Da Rold, Pomarol NPB(2005)

Infrared Brane at $z = z_m$

⟹ Confinement



Metric – Slice of AdS metric

$$ds^2 = \frac{1}{z^2}(-dz^2 + dx^\mu dx_\mu), \qquad 0 < z \le z_m$$

5D action (Nf=2)

$$S = \int d^5x\sqrt{-g}\left(-\frac{1}{2g_5^2}\mathrm{Tr}\left(L_{MN}L^{MN} + R_{MN}R^{MN}\right) + \mathrm{Tr}\left(|D_M X|^2 + m_X^2|X|^2\right)\right)$$

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3$$

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

Observable	Measured (MeV)	M (MeV)	(MeV)
m_π	139.6±0.0004 [8]	139.6*	141
m_ρ	775.8±0.5 [8]	775.8*	832
m_{a_1}	1230±40 [8]	1363	1220
f_π	92.4±0.35 [8]	92.4*	84.0
$F_\rho^{1/2}$	345±8 [15]	329	353
$F_{a_1}^{1/2}$	433±13 [6, 16]	486	440
$g_{\rho\pi\pi}$	6.03±0.07 [8]	4.48	5.29

Parameters

$m_q \quad \sigma \quad z_m$

$$g_5^2 = \frac{12\pi^2}{N_c}$$

Witten '98

2. AdS/CFT at finite temperature

(for the pure Yang-Mills theory without quark matters)

1) Low T (confining phase) : tAdS — (thermal) AdS space, no stable AdS black hole

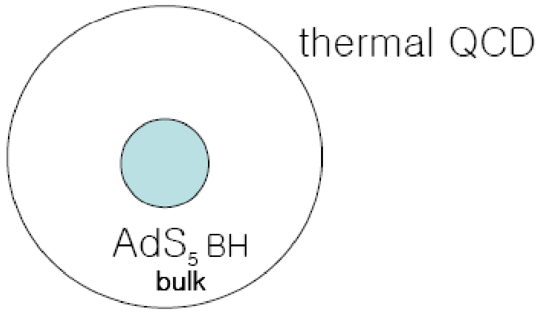
at β(=1/Tc) ↕ QCD Phase transition ⟺ dual ⟺ ↕ Hawking-Page transition =Transition of bulk geometry

2) High T (deconfining phase) : AdS BH — Schwarzschild AdS black hole is stable

E. Witten (1998)

$$ds_5^2 = \frac{1}{z^2}\left(f^2(z)dt^2 - (dx^i)^2 - \frac{1}{f^2(z)}dz^2\right),$$

$$f^2(z) = 1 - \left(\frac{z}{z_T}\right)^4 \qquad T = \frac{1}{\pi z_T}$$



3) Hawking-Page phase transition [Herzog , Phys.Rev.Lett.98:091601,2007]

The geometry is described by the following action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{g}\,(-\mathcal{R}+2\Lambda)$$

$\Lambda = -\frac{6}{R^2}$: cosmological constant

: AdS radius

- Calculate the free energy (= the gravity on-shell action) for tAdS &AdSBH.
- The geometry with smaller action is the stable one for given T.

The regularized on-shell action

1) for the tAdS, $S_{tAdS} = \frac{4R^3}{\kappa^2}\int_0^{\beta'} dt \int_\epsilon^{z_{IR}} dz \frac{1}{z^5}$

β' : arbitrary

2) for the AdS BH $S_{AdSBH} = \frac{4R^3}{\kappa^2}\int_0^{\pi z_h} dt \int_\epsilon^{z_h} dz \frac{1}{z^5}$

To remove the divergence at $z = 0$ introduce a UV cut-off ϵ

the period in the t-direction of tAdS β'
= the period in t-direction of AdS BH at z ϵ

$\Longrightarrow \beta' = \pi z_h \sqrt{f(\epsilon)}$

Using this, the difference of two actions :

$$\Delta S = \lim_{\epsilon\to 0}(S_{AdSBH} - S_{tAdS}) = \frac{\pi z_h R^3}{\kappa^2}\left(\frac{1}{z_{IR}^4} - \frac{1}{2z_h^4}\right)$$

> 0 for T < Tc
< 0 for T>Tc

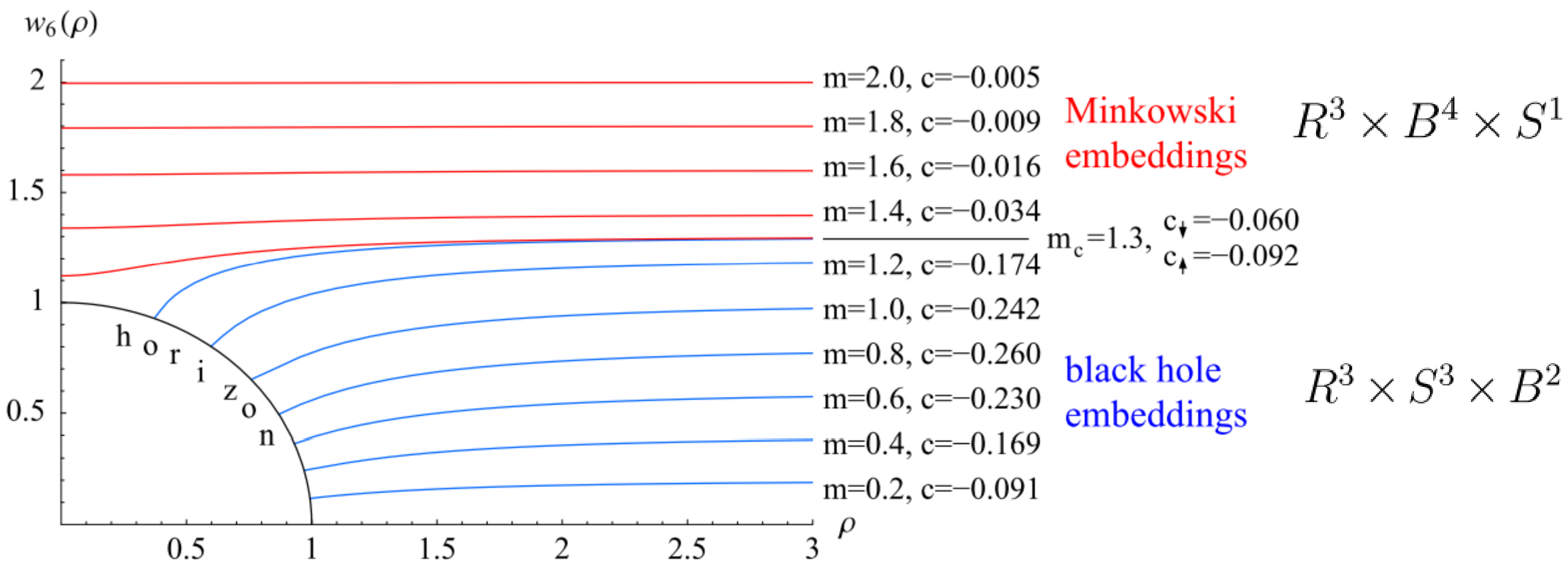
• The Hawking-Page (or deconfinement) transition occurs at

$$z_h = \frac{z_{IR}}{2^{1/4}} \quad \text{or} \quad T_c = \frac{2^{1/4}}{\pi z_{IR}}$$

• At the low temperature $T < T_c$ the tAdS space is stable (confining phase) .

• At the high temperature $T > T_c$ the AdS BH is more stable (deconfining phase)

Note : Phenomenology such as meson spectrum, etc. can be studied by embedding D7, etc.



III. Holographic QCD for
– gluon condensation & finite (arbitrary) density effects –

towards the dual geometry of AdS/QCD

1. Gluon Condensate Background

4dim gluon condensate $\leftrightarrow$ the dilaton in 5 dim.

Action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{G}\left(-\mathcal{R}+2\Lambda+\frac{1}{2}\partial_M\phi\partial^M\phi\right) \qquad \Lambda = -\frac{6}{R^2} \qquad \frac{1}{\kappa^2} = \frac{4(N_c^2-1)}{\pi^2 R^3}$$

Dilaton wall solution (cf. dilaton black hole solution)

Csaki & Reece, hep-th/0608266,

$$ds^2 = \frac{R^2}{z^2}\left(\sqrt{1-c^2z^8}\,\delta_{\mu\nu}dx^\mu dx^\nu + dz^2\right),$$

$$\phi(z) = \phi_0 + \sqrt{\frac{3}{2}}\log\left(\frac{1+cz^4}{1-cz^4}\right),$$

singular at $z_c \equiv \frac{1}{c^{1/4}}$

Perturbative expansion near the boundary z → 0

$$\phi = \phi_0 + \sqrt{6}\,\frac{z^4}{z_c^4} + \mathcal{O}(z^8)\,.$$

Gluon condensate

$$\langle \mathrm{Tr}\, G^2\rangle = \frac{8\sqrt{3(N_c^2-1)}}{\pi}\frac{1}{z_c^4},$$

T-independent

General solution with metric back reaction

$$ds^2 = \frac{R^2}{z^2}\left[dz^2 + (1-f^2z^8)^{1/2}\left(\frac{1+fz^4}{1-fz^4}\right)^{a/2f}\left(d\vec{x}^2 - \left(\frac{1-fz^4}{1+fz^4}\right)^{2a/f} dt^2\right)\right]$$

$$\phi(z) = \phi_0 + \frac{c}{f}\sqrt{\frac{3}{2}}\log\left(\frac{1+fz^4}{1-fz^4}\right) \qquad f^2 = a^2 + c^2 \qquad 0 < z < f^{-1/4} := z_f$$

Kim,BHL, Park, Sin, hep-th/0702131 (JHEP 09(2007))

Note :

- For a=0, the solution reduces to the dilaton-wall solution.
- For c=0, becomes the AdS Schwarzschild black hole solution.

  with T by $a = \frac{1}{4}(\pi T)^4$

- Hence, describes the finite temperature with the gluon condensation with the metric having an essential singularity at $z = f^{-1/4}$
- Thermodynamics with gluon condensation

- Gluon condensate is sensitive to the QCD deconfinement transition.
- The heavy quark potential becomes deeper as the gluon condensate value decreases.

Kim, BHL, Park, Sin, arXiv:0808.1143 (PRD80,2009).

Meson spectra in the gluon condensate background

Ko, BHL, Park, JHEP 1004, (2010) (arXiv:0912.5274)

Action

$$\Delta S = \int d^5x \sqrt{G}\ \mathrm{Tr}\left[|DX|^2 - \frac{3}{R^2}|X|^2 + \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right], \qquad \frac{1}{g_5^2} = \frac{N_c}{12\pi^2 R},$$

With axial gauge

$$V_z = 0,$$

Eq. of motion

$$0 = \frac{1}{\sqrt{G}}\partial_M \sqrt{G} G^{MP} G^{ij} \partial_P V_i, \qquad V_i = \int \frac{d^4k}{(2\pi)^4}\ e^{-i\omega_n t + i\vec{p}_n \vec{x}}\ V_i^{(n)}(z),$$

becomes

$$0 = \partial_z^2 V_i^{(n)} - \frac{1+3c^2z^8}{z(1-c^2z^8)}\partial_z V_i^{(n)} + \frac{m_n^2}{\sqrt{1-c^2z^8}} V_i^{(n)}$$

IR cutoff for Confined phase by

1) Hard wall at z_c or 2) braneless approach

hard wall approach

Various meson masses depending on the gluon condensation

z_c (1/GeV)	$\langle \mathrm{Tr} G^2 \rangle$ (GeV^4)	m_ρ (GeV)	m_A (GeV)	m_π (GeV)
∞	0	0.7767	1.3582	0.13961
1/0.176	0.012	0.7767	1.3583	0.13961
1/0.200	0.020	0.7767	1.3584	0.13961
1/0.250	0.049	0.7762	1.3589	0.13964
1/0.280	0.077	0.7755	1.3599	0.13970
1/0.320	0.131	0.7724	1.3612	0.13999

- As the gluon condensation increases
  mass of the vector meson decreases slightly while
  masses of the axial vector meson and pion increase very slowly

Braneless approach – singlularity identified with the IR cutoff

Boundary condition $V_i^{(1)} = 0$ at z=0, $\partial_z V_i^{(1)} = 0$ at z=zc

Fixing zc by m ρ = 776 MeV gives zc = 1/325 MeV

Gluon condensation (for Nc = 3)

$$\langle \mathrm{Tr}\, G^2 \rangle \approx 0.139[\mathrm{GeV}^4]$$

decreases as T increases

Cf. Lattice calculation

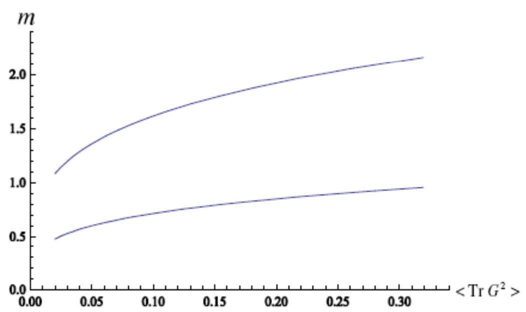
$$\langle \mathrm{Tr}\, G^2 \rangle \approx 0.012[\mathrm{GeV}^4]$$

Miller, hep-ph/0608234 (Phys. Rept, 2007)

Meson masses

z_c (1/GeV)	$\langle \mathrm{Tr} G^2 \rangle$ (GeV^4)	m_ρ (GeV)	m_A (GeV)	m_π (GeV)
1/0.200	0.020	0.4780	1.4081	0.13796
1/0.250	0.049	0.5975	1.4057	0.13808
1/0.325	0.139	0.7768	1.3574	0.14020
1/0.378	0.253	0.9035	1.2880	0.14743
1/0.400	0.319	0.9561	1.2715	0.15302

Rho-meson masses



- meson spectra well defined in spite of the singularity
- meson masses similar to those in EKSS model
  for gluon condensation larger than that of lattice calculation.
- Meson spectra significantly depend on the gluon condensate
- As the gluon condensation becomes large,
  masses of the vector meson and pion increase while
  masses of the axial vector mesons decrease

2. QCD in dense media

- Upto the normal nuclear density, most of the properties of hadronic properties are relatively well understood

  thanks to the rich experimental data & many body techniques.

  Ex) (unusually long) life time (5700 years) of carbon 14 may be explained by the 15% hadron mass reduction inside the nucleus.

  Holt, Brown, Kuo,Holt,Machleidt, arXiv :0710.0310 (PRL 100 (2008)

- Little understanding beyond the normal nuclear density due to
  - no reliable theoretical input
  - limited experimental data
  - difficulties in lattice calculation, etc.

- Holographic description may shed some lights

  Karch & Katz, JHEP 06 (2002) hep-th/0205236
  Kim, Sin, & Zahed, hep-th/0608046
  Horigome & Tanii, JHEP 01(2007) hep-th/0608198
  etc.

Dual geometry for finite chemical potential

bulk		boundary
field A_0	⟷	dual operator $\bar{\psi}\gamma_0\psi = \psi^\dagger\psi$
		(quark number density)

Chamblin−Emparan−Johnson−Myers,1999
Cvetic−Gubser,1999

5−dimensional action dual to the gauge theory with quark matters

$$S = \int d^5x\sqrt{G}\left[\frac{1}{2\kappa^2}(-\mathcal{R}+2\Lambda)+\frac{1}{4g^2}F_{MN}F^{MN}\right]$$

Euclidean

Wick rotation $t \to -i\tau$

Equations of motion

1) Einstein equation $\quad \mathcal{R}_{MN}-\frac{1}{2}G_{MN}\mathcal{R}+G_{MN}\Lambda=\frac{\kappa^2}{g^2}\left(F_{MP}F_N^P-\frac{1}{4}G_{MN}F_{PQ}F^{PQ}\right)$

2) Maxwell equation $\quad 0=\partial_M\sqrt{-G}G^{MP}G^{NQ}F_{PQ}$

Ansatz :

$$\begin{cases} ds^2 = \dfrac{R^2}{z^2}\left(f(z)dt^2+d\vec{x}^2+\dfrac{1}{f(z)}dz^2\right) \\ A_0 = A(z) \quad \text{and other are zero.} \end{cases}$$

Solutions

S.-J. Sin, 2007

- most general solution, which is RNAdS BH (RN AdS black hole)

$$f(z) = 1 - mz^4 + q^2 z^6$$
$$A(z) = i\left(\mu - Qz^2\right)$$

m	black hole mass
q	black charge
μ	quark chemical potential
$Q = \sqrt{\frac{3g^2R^2}{2\kappa^2}}\, q$	quark number density

corresponds to the deconfining phase (quark-gluon plasma)

Note

1) The value of A_0 at the boundary ($z = 0$) corresponds to the quark chemical potential μ of QCD.

2) The dual operator of A_0 is denoted by Q, which is the quark (or baryon) number density operator.

3) We use $\frac{1}{2\kappa^2} = \frac{N_c^2}{8\pi^2 R^3}$ and $\frac{1}{g^2} = \frac{N_c N_f}{4\pi^2 R}$

- What is the dual geometry of the confining (or hadronic) phase ?

find non-black hole solution (BHL, Park ,Sin JHEP 0907,(2009))

$$f(z) = 1 + q^2 z^6$$
$$A(z) = i\left(\mu - Qz^2\right)$$

We call it tcAdS (thermal charged AdS space)

- baryonic chemical potential

$$\mu_B = 3\mu$$

- baryon number density

$$Q_B = Q/3$$

Note : Solutions in both phases are valid for arbitrary densities

Hawking-Page transition

- The difference of the on-shell actions for RN AdS BH and tcAdS

$$\Delta S = S^D_{RN} - S^D_{tc}$$
$$= \frac{V_3 R^3}{\kappa^2} \frac{1}{T_{RN}} \left(\frac{1}{z_{IR}^4} - \frac{1}{2z_+^4} + \frac{3\kappa^2}{2g^2R^2} \frac{\mu^2}{z_{IR}^2} - \frac{\kappa^2}{3g^2R^2} \frac{\mu^2}{z_+^2} \right)$$

- When $\Delta S = 0$, Hawking-Page transition occurs

- Suppose that $\Delta S = 0$ at a critical point $z_+ = z_c$

 1) For $z_+ < z_c$, ΔS becomes negative. $\Longrightarrow$ deconfining phase

 2) For $z_c < z_+ \leq z_{IR}$, tcAdS is stable. $\Longrightarrow$ confining phase

For the fixed chemical potential

- dimensionless variables

$$\tilde{z}_c \equiv \frac{z_c}{z_{IR}}, \qquad \tilde{\mu}_c \equiv \mu_c z_{IR}, \qquad \tilde{T}_c \equiv T_c z_{IR},$$

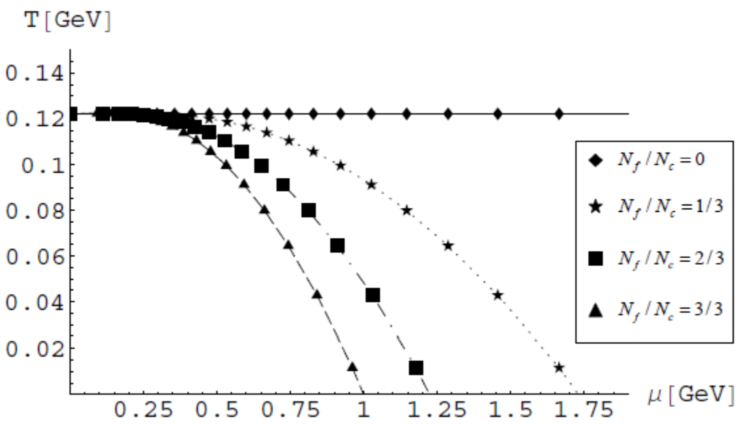
For the fixed number density

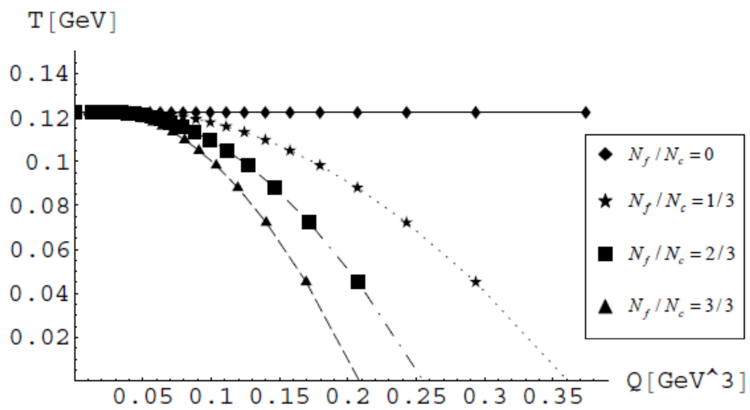
- Legendre transformation,

the Hawking–Page transition occurs at

$$\begin{cases} \tilde{\mu}_c = \sqrt{\dfrac{3N_c}{N_f}\,\dfrac{(1-2\tilde{z}_c^4)}{\tilde{z}_c^2(9\tilde{z}_c^2-2)}}\,, \\ \tilde{T}_c = \dfrac{1}{\pi\tilde{z}_c}\left(1-\dfrac{1-2\tilde{z}_c^4}{9\tilde{z}_c^2-2}\right). \end{cases}$$

$$\begin{cases} \tilde{Q}_c = \sqrt{\dfrac{3N_c}{2N_f}\,\dfrac{(1-2\tilde{z}_c^4)}{\tilde{z}_c^4(5\tilde{z}_c^2-2)}}\,, \\ \tilde{T}_c = \dfrac{1}{\pi\tilde{z}_c}\left[1-\dfrac{\tilde{z}_c^2}{2}\dfrac{(1-2\tilde{z}_c^4)}{(5\tilde{z}_c^2-2)}\right] \end{cases}$$



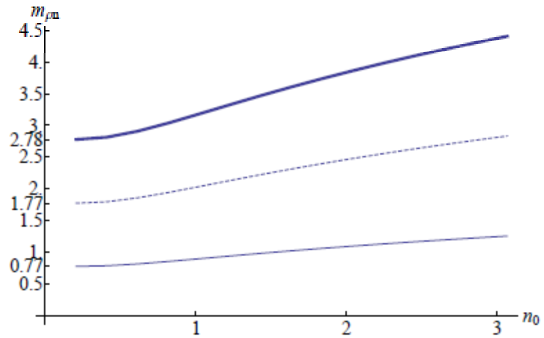


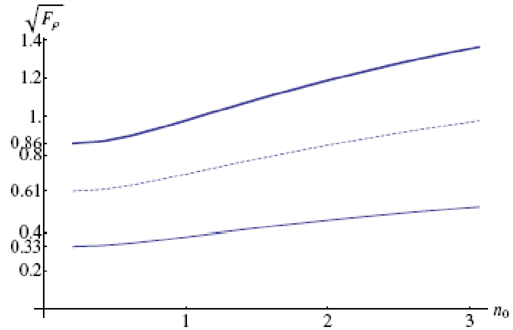
Light meson spectra in the hadronic phase

Turn on the fluctuation in bulk corresponding the meson spectra in QCD

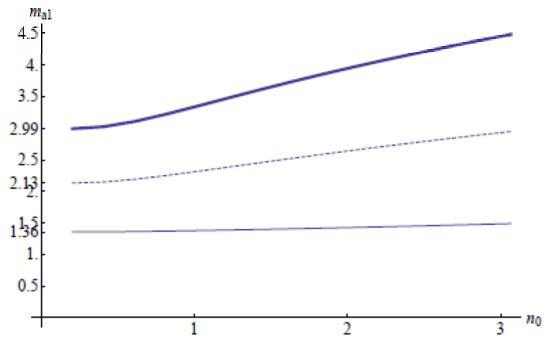
$$\Delta S = \int d^5x \sqrt{G}\, \mathrm{Tr}\left[|DX|^2 - \frac{3}{R^2}|X|^2 + \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right) \right]$$

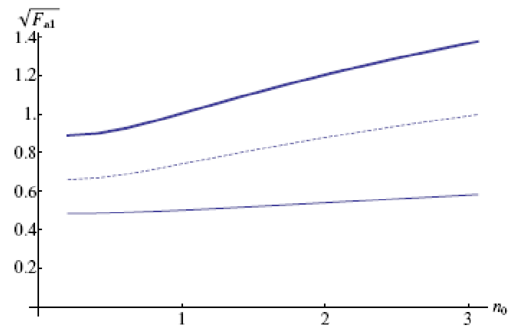
1. Vector meson



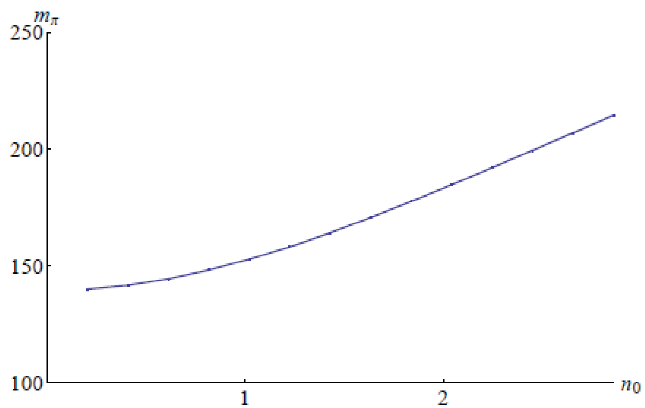


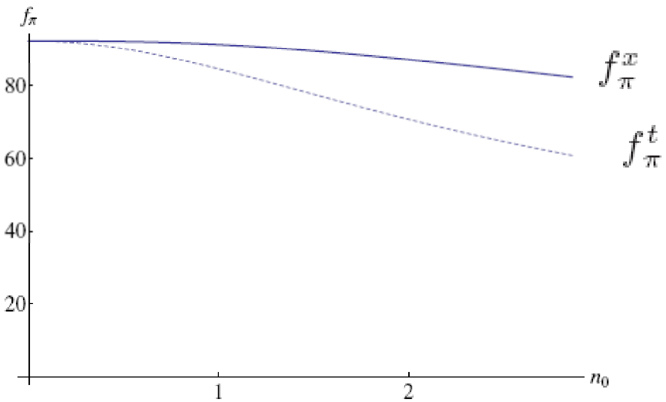
2. Axial vector meson





3. pion





IV. Summary

- Holographic Principles :

(d+1 dim.) (classical) Sugra ↔ (d dim.) (quantum) YM theories

- AdS/QCD : can be a powerful tool for QCD

– Top-down Approach & Bottom-up Approach

- Holographic QCD using dual Geometry (w/o chemical potential)

QCD phase: confined phase ↔ deconfined phase transition

Geometry : thermal AdS ↔ AdS BH

Hawking-Page transition

IV. Summary – continued

- the gluon condensate background and the meson spectra
  - backreaction of the dilaton causes naked singularity
- Hard wall – introduce the IR cutoff screening the singularity
  - As the gluon condensation increases
    mass of the vector meson decreases slightly while
    masses of the axial vector meson and pion increase very slowly
- Braneless approach – singlularity identified with the IR cutoff
  - meson spectra well defined in spite of the singularity
  - meson masses similar to those in EKSS model
    for gluon condensation larger than that of lattice calculation.
  - Meson spectra significantly depend on the gluon condensate
  - As the gluon condensation becomes large,
    masses of the vector meson and pion increase while
    masses of the axial vector mesons decrease

IV. Summary – continued

- Dual Geometry and phase transition in Dense matter (μ : arbitrary)

  – U(1) chemical potential → baryon density

  – deconfined phase by RNAdS BH

  hadronic phase by tcAdS (Zero black-hole mass limit of RN AdS)

  – there exists Hawking-Page phase transition

  – As the density is increasing in the hadronic phase,

  all the meson mass go up

  pion decay constants decreases