Gravitational Bremsstrahlung in Transplanckian Scattering

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Our aim : Gravitational bremsstrahlung during collisions of ultra-relativistic massive particles $\begin{cases} D-dim \;\; flat \;\; space \\ ADD \;=\; 4-dim \;+\; compact\;(large) \end{cases}$

Our result : Explicit expressions for the radiated energy in terms of the parameters (masses, initial velocities, impact parameter, couplings, scales)

<u>IMPORTANT</u> : $\gamma-$ factor dependence $\qquad \Delta E \sim \frac{G_D^3 m^2 m'^2}{b^{3D-9}} \gamma^{D-1}$

$\Rightarrow$ enhancement of radiated energy

- Standard 4-dim gravity not significant for elementary particles , $E_{P\ell} = c^2 M_{P\ell} = \sqrt{\frac{c^5\hbar}{G_4}} = 10^{19} GeV$
(LHC $E \sim 10^4 GeV$, cosmic rays $E \sim 10^{11} GeV$)

- Extra dimensions , M_* , $L \sim 1TeV$, $1mm$ (ADD)

$\Rightarrow$ gravity significant for elementary particles

$\Rightarrow$ microscopic test of gravity

Multidimensional gravitational radiation from scattering processes interesting for BH at LHC, supernovae, cosmic rays, etc.

Scattering at $E \sim M_*$ $\rightsquigarrow$ $full\ quantum\ gravity$ (?)
Scattering at $E \gg M_*$ $(transplanckian)$ $\rightsquigarrow$ $classical\ general\ relativity$

▶ Black holes in LHC are believed to be copiously produced with $M_{BH} \sim E \gg M_*$, $\sigma(2 \to BH) \sim \pi R_S^2$

Thorne's hoop conjecture : BH is formed if $R < R_S = \frac{2GE}{c^4}$

D'Eath-Payne : shock waves

Standard 4 stages of BH : formation of a closed trapped surface
balding phase
Hawking evaporation
quantum gravity

We focus on the <u>first</u> stage

Gravitational bremsstrahlung calculation misses from other approaches :

— second phase of gravitational emission during collapse

— based on the assumption that a BH already exists

— results of linearized theory

— weakly relativistic numerical simulations

— collisions of waves in 4-dim

— Veneziano-Amati-Ciafaloni-et all

SCALES OF THE PROBLEM

General Relativity $D = 4 + \delta$ dimensions

$\underline{G_D = V_\delta G_4}$, $V_\delta = L^\delta$

$r \gg L \rightsquigarrow F_4 \gg F_D$ 4-dimensional gravity

$r \ll L \rightsquigarrow F_D \gg F_4$ D-dimensional gravity

$\underline{M_* = (\frac{\hbar^{\delta+1}}{c^{\delta-1} G_D})^{\frac{1}{\delta+2}}}$, $M_{P\ell} = \sqrt{\frac{c\hbar}{G_4}} = \frac{\hbar}{c\,\ell_{P\ell}}$ $(M_{P\ell} \sim 10^{19} GeV, \ell_{P\ell} \sim 10^{-33} cm)$

$\frac{M_{P\ell}}{M_*} = (\frac{L}{\ell_{P\ell}})^{\frac{\delta}{\delta+2}}$, $L \gg \ell_{P\ell} \rightsquigarrow M_* \ll M_{P\ell}$

(e.g. $L \sim 1mm$, $\delta = 2 \rightsquigarrow M_* \sim TeV$)

$\underline{\ell_* = (\frac{G_D \hbar}{c^3})^{\frac{1}{\delta+2}} = \frac{\hbar}{cM_*}}$ (e.g. $\ell_* \sim 10^{-17} cm$)

- $2E = \sqrt{s}$

$\lambda_B = \frac{\hbar c}{\sqrt{s}}$, $\frac{\ell_*}{\lambda_B} = \frac{\sqrt{s}}{c^2 M_*}$ (De Broglie)

$R_S \sim (\frac{G_D \sqrt{s}}{c^4})^{\frac{1}{\delta+1}}$, $\frac{R_S}{\ell_*} \sim (\frac{\sqrt{s}}{c^2 M_*})^{\frac{1}{\delta+1}}$ (Schwarzschild)

▶ Classical physics $\hbar \to 0$ ($G_D, \sqrt{s}$ fixed) $\rightsquigarrow$ $M_* \to 0$ $\rightsquigarrow$ $E \gg M_*$ (transplanckian)

$\hbar \to 0$: $\ell_*, \lambda_B \to 0$, R_S finite

$E \gg M_*$: $\lambda_B \ll \ell_* \ll R_S$

Scattering angle $\delta\alpha \sim \frac{G_D \sqrt{s}}{b^{\delta+1}} \sim (\frac{R_S}{b})^{\delta+1}$ $\rightsquigarrow$ $b \gg R_S$

- One more scale for $D > 4$: $b_c = (\frac{G_D\, s}{c^5 \hbar})^{\frac{1}{\delta}} = R_S (\frac{\sqrt{s}}{c^2 M_*})^{\frac{\delta+2}{\delta(\delta+1)}}$

$\hbar \to 0$: $b_c \to \infty$, $b \lesssim b_c$

classicality of orbit : $\Delta\theta \lesssim \theta$, $\Delta b \lesssim b$ $\rightsquigarrow$ $b \lesssim b_c$

Hierarchy : $\underline{\ell_{P\ell} \ll \lambda_B \ll \ell_* \ll R_S \ll b \lesssim b_c}$

- One more scale : $\lambda_C = \frac{\hbar}{cm}$ (Compton)

classicality of radiation : $\hbar\omega_{cr} \ll c^2 m\gamma$, $\omega_{cr} \sim \frac{c\gamma}{b}$ $\rightsquigarrow$ $\underline{b \gg \lambda_C}$

[consistent if $\lambda_C < b_c$]

$$\hookrightarrow \quad Spin-1 \quad : \quad M_* \ , \ \ell_* \quad \rightsquigarrow \quad M_e = [\frac{(\hbar c)^{\delta+1}}{e^2}]^{\frac{1}{\delta}} \ , \ \lambda_e = (\frac{e^2}{\hbar c})^{\frac{1}{\delta}}$$

$$R_S \quad \rightsquigarrow \quad R_e = (\frac{e^2}{\sqrt{s}})^{\frac{1}{\delta+1}}$$

$\hbar \to 0$ ($e^2, \sqrt{s}$ fixed) : $\lambda_e \to \infty$ (quantum fluctuations not suppressed)

$$\delta\alpha \sim \frac{e^2}{\sqrt{s}\, b^{\delta+1}} \quad \rightsquigarrow \quad b \to 0$$

our action

$$S = \frac{1}{2\kappa_D^2}\int_M R\sqrt{|g|}d^Dx + \sum_{(n)}\int dt_{(n)}\frac{1}{2}\left(e_{(n)}g_{\mu\nu}\frac{dz^\mu_{(n)}}{dt_{(n)}}\frac{dz^\nu_{(n)}}{dt_{(n)}} - \frac{m^2_{(n)}}{e_{(n)}}\right)$$

$$\kappa_D^2 = 8\pi G_D$$

full equations of motion

$$G^{\mu\nu} = \kappa_D^2 T^{\mu\nu} = \frac{\kappa_D^2}{\sqrt{|g|}}\sum_{(n)}\int dt_{(n)}e_{(n)}\dot{z}^\mu_{(n)}\dot{z}^\nu_{(n)}\delta_D(x;z_{(n)}) \quad , \quad \dot{z}^\mu_{(n)} = \frac{dz^\mu_{(n)}}{dt_{(n)}}$$

$$\frac{d}{dt_{(n)}}(e_{(n)}\dot{z}^\mu_{(n)}) + e_{(n)}\Gamma^\mu{}_{\nu\lambda}\dot{z}^\nu_{(n)}\dot{z}^\lambda_{(n)} = 0$$

$$m^2_{(n)}e^{-2}_{(n)} = -g_{\mu\nu}\dot{z}^\mu_{(n)}\dot{z}^\nu_{(n)}$$

our method

SOLVE PERTURBATIVELY IN κ_D^2 STARTING WITH MINKOWSKI BACKGROUND WHEN THE PARTICLES ARE VERY FAR

ALL CALCULATIONS IN FOURIER SPACE (COMMON IDEA IN PHYSICS BUT NOT IN GRAVITY). Contact for $D = 4$ with gravitational wave production in astrophysics, various methods there

VARIOUS ENCOUNTERED INTEGRALS (similar to QFT) SUCCESSFULLY DONE

- No connection with radiation-reaction problem

Our particles follow geodesics in the final unknown metric

In self-force considerations, particles follow other equations (e.g. Japanese)

Methods in astrophysical gravitational wave generation

— Quadrupole-moment formalism $\rightsquigarrow$ low-velocity $(v < 0.3)$, also large deflections , simple

— Post-Newtonian wave-generation formalism $\rightsquigarrow$ $v < 0.5$, also large deflections , relatively easy

— Linear perturbations of Schwarzschild (Peters) $\rightsquigarrow$ small deflections , large b , $m \ll m'$, not easy , $\Delta E = 20\frac{(mm')^2}{b^3}\gamma^3$
[$\Delta E_{our}(D=4) = (10\pi + \frac{35}{2}\tilde{G} - \frac{211}{12})\frac{(mm')^2}{b^3}\gamma^3$]

— Postlinear formalism (Thorne-Kovacs) $\rightsquigarrow$ $\frac{m_1+m_2}{bv^2} \ll 1$, small deflection angles , large b , arbitrary velocities , very complicated , we agree

— Colliding plane waves (D'Eath) $\rightsquigarrow$ $\gamma \gg 1$, also small b , very complicated , spectrum ?

— Virtual quanta (Matzner) $\rightsquigarrow$ $\gamma \gg 1$, $m \ll m'$, easy , high-frequency almost correct , low-frequency wrong , total energy ?

— Zero-frequency limit (Smarr) $\rightsquigarrow$ arbitrary velocities , easy , low-frequency spectrum , total energy ?

— quantum (Feynman-diagram) approach $\rightsquigarrow$ $\frac{G_4 m m'}{c\hbar} \simeq \frac{m m'}{(10^{-5} gr)^2}$, elementary particles

Expansion of fields

$$g_{\mu\nu} = \eta_{\mu\nu} + \underset{(1)}{h}_{\mu\nu} + \underset{(2)}{h}_{\mu\nu} + \ldots \quad , \quad z^{\mu}_{(n)} = \underset{(0)}{z}^{\mu}{}_{(n)} + \underset{(1)}{z}^{\mu}{}_{(n)} + \ldots \quad , \quad e = \underset{(0)}{e} + \underset{(1)}{e} + \ldots \quad ,$$

$$\underset{(p)}{\psi}_{\mu\nu} = \underset{(p)}{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\underset{(p)}{h}^{\kappa}{}_{\kappa}$$

Perturbed Einstein equations for De Donder gauge

$$G_{\mu\nu} = \kappa_D^2 T_{\mu\nu} \Leftrightarrow \underset{(p)}{G}_{\mu\nu} = \kappa_D^2 \underset{(p-1)}{T}_{\mu\nu}$$

$$\underset{(1)}{G}_{\mu\nu} = \kappa_D^2 \underset{(0)}{T}_{\mu\nu} \Leftrightarrow -\frac{1}{2}\Box_\eta \underset{(1)}{\psi}_{\mu\nu} = \kappa_D^2 \underset{(0)}{T}_{\mu\nu} \qquad \text{(no radiation)}$$

$$\underset{(2)}{G}_{\mu\nu} = \kappa_D^2 \underset{(1)}{T}_{\mu\nu} \Leftrightarrow -\frac{1}{2}\Box_\eta \underset{(2)}{\psi}_{\mu\nu} = \kappa_D^2 (\underset{(1)}{T}_{\mu\nu} + \underset{(1)}{S}_{\mu\nu}) \qquad \text{(radiation)}$$

$$\underset{(1)}{S}_{\mu\nu} = \frac{1}{2\kappa_D^2}[\underset{(1)}{h}^{\kappa\lambda}(\underset{(1)}{h}_{\kappa\mu,\nu\lambda} + \underset{(1)}{h}_{\kappa\nu,\mu\lambda} - \underset{(1)}{h}_{\kappa\lambda,\mu\nu} - \underset{(1)}{h}_{\mu\nu,\kappa\lambda}) - \frac{1}{2}\underset{(1)}{h}_{\mu\nu}\underset{(1)}{h}^{\lambda}{}_{\lambda,\rho}{}^{,\rho} - \frac{1}{2}\underset{(1)}{h}^{\kappa\lambda}{}_{,\mu}\underset{(1)}{h}_{\kappa\lambda,\nu} -$$
$$\underset{(1)}{h}^{\kappa}{}_{\mu,\lambda}\underset{(1)}{h}_{\nu\kappa}{}^{,\lambda} + \underset{(1)}{h}^{\kappa}{}_{\mu,\lambda}\underset{(1)}{h}^{\lambda}{}_{\nu,\kappa} + \eta_{\mu\nu}(\underset{(1)}{h}^{\kappa\lambda}\underset{(1)}{h}_{\kappa\lambda,\rho}{}^{,\rho} + \frac{3}{4}\underset{(1)}{h}^{\kappa\lambda}{}_{,\rho}\underset{(1)}{h}_{\kappa\lambda}{}^{,\rho} - \frac{1}{2}\underset{(1)}{h}^{\kappa\lambda}{}_{,\rho}\underset{(1)}{h}^{\rho}{}_{,\kappa,\lambda})]$$

$\underset{(1)}{S}{}^{\mu\nu}$ *energy-momentum "tensor" of the gravitational field :*

- $\underset{(1)}{S}{}^{\mu\nu}$ not tensor , $\langle \underset{(1)}{S}{}^{\mu\nu} + \underset{(1)}{T}{}^{\mu\nu} \rangle$ invariant

- $T^{\mu}_{\nu;\mu} = 0 \quad \rightsquigarrow \quad \underset{(0)}{T}{}^{\mu}_{\nu,\mu} = 0$, $2\underset{(1)}{T}{}^{\mu}_{\nu,\mu} = \underset{(1)}{h}{}^{\kappa}_{\mu,\nu}\underset{(0)}{T}{}^{\mu}_{\kappa} - \underset{(1)}{h}{}^{\kappa}_{\kappa,\mu}\underset{(0)}{T}{}^{\mu}_{\nu}$ energy-exchange between gravity and matter

- De Donder : $(\underset{(1)}{T}{}^{\mu}_{\nu} + \underset{(1)}{S}{}^{\mu}_{\nu})_{,\mu} = 0 \quad \rightsquigarrow \quad \underset{(1)}{\tau}{}^{\mu\nu} = \underset{(1)}{T}{}^{\mu\nu} + \underset{(1)}{S}{}^{\mu\nu}$ total energy-momentum "tensor" of matter-gravitation

- $\underset{(1)}{h}{}^{\mu\nu} \sim \frac{1}{r^{D-3}} \quad (r \to \infty) \quad \rightsquigarrow \quad \int d^{D-1}x \, \underset{(1)}{S}{}^{\mu\nu}$ finite

- $\int d^{D-1}x \, \underset{(1)}{S}{}^{00}$ positive (ADM mass)

- $\int d^{D-1}x \, \underset{(1)}{S}{}^{0\mu}$ invariant under diffeomorphisms + additive

- Global Lorentz transformations $\rightsquigarrow$ $\underset{(1)}{S}{}^{\mu\nu}$ properly

0th ORDER

$$\underset{(o)}{z}^{\mu}_{(n)}(\tau_{(n)}) = z^{\mu}_{(n)}(0) + u^{\mu}_{(n)}\tau_{(n)}$$

$$\underset{(o)}{e}_{(n)} = m_{(n)}$$

$$\underset{(o)}{T}^{\mu\nu}(x) = \sum_{(n)} \frac{p^{\mu}_{(n)}p^{\nu}_{(n)}}{m_{(n)}} \int_{-\infty}^{+\infty} d\tau_{(n)}\delta_D(x - \underset{(o)}{z}_{(n)})$$

$$\underset{(o)}{\tilde{T}}^{\mu\nu}(k) = \frac{1}{(2\pi)^{\frac{D}{2}-1}} \sum_{(n)} p^{\mu}_{(n)}p^{\nu}_{(n)} e^{-ik\cdot z_{(n)}(0)}\delta(k \cdot p_{(n)})$$

1st ORDER

gravity :

$$\underset{(1)}{\tilde{\psi}}{}_{\mu\nu}(k) = \frac{2\kappa_D^2}{k^2}\underset{(0)}{\tilde{T}}{}_{\mu\nu}(k)$$

$$\begin{aligned}\underset{(1)}{h}{}_{\mu\nu}(x) &= \frac{2\kappa_D^2}{(2\pi)^{D-1}}\sum_{(n)}\left(p_{(n)\mu}p_{(n)\mu} + \frac{m_{(n)}^2}{D-2}\eta_{\mu\nu}\right)\int_{-\infty}^{+\infty}\frac{d^Dk}{k^2}e^{ik\cdot(x-z_{(n)}(0))}\delta(k\cdot p_{(n)}) \\ &= \underset{(1)}{h}{}^{(m)}_{\mu\nu}(x) + \underset{(1)}{h}{}^{(m')}_{\mu\nu}(x)\end{aligned}$$

vierbein :

$$\underset{(1)}{e}(n) = \frac{m_{(n)}}{2}\left[\underset{(1)}{h}{}_{\mu\nu}u^{\mu}_{(n)}u^{\nu}_{(n)} + 2u_{(n)\mu}\underset{(1)}{\dot{z}}{}^{\mu}_{(n)}\right]$$

orbit :

$$\ddot{z}^{\mu}_{(1)(n)} = \beta \mathcal{P}^{\mu\nu}_{(n)} \Lambda_{(n)\nu} + (1-\beta)\Lambda^{\mu}_{(n)}$$

$$\mathcal{P}^{\mu\nu}_{(n)} = \eta^{\mu\nu} + u^{\mu}_{(n)} u^{\nu}_{(n)} \quad , \quad \Lambda_{(n)\mu} = (\tfrac{1}{2} h_{(1)\nu\lambda,\mu} - h_{(1)\mu\nu,\lambda}) u^{\nu}_{(n)} u^{\lambda}_{(n)}$$

$$[\mathcal{P}^{\mu\nu}_{(n)} \ddot{z}_{(1)(n)\nu} = \mathcal{P}^{\mu\nu}_{(n)} \Lambda_{(n)\nu} \;\Rightarrow\; \ddot{z}^{\mu}_{(1)(n)} - \Lambda^{\mu}_{(n)} \propto u^{\mu}_{(n)}]$$

$\beta = 0 \quad , \quad t_{(n)}$ proper time

$$z^{\mu}_{(1)(n)}(\tau_{(n)}) = \frac{2i\kappa_D^2}{(2\pi)^{D-1}} \sum_{n \neq n'} \int_{-\infty}^{+\infty} d^D k \frac{e^{ik\cdot u_{(n)}\tau_{(n)}}}{k^2 (k\cdot p_{(n)})} e^{ik\cdot\Delta_{n,n'}} \delta(k\cdot p_{(n')})(p_{(n)}\cdot p_{(n')})$$

$$\left[p^{\mu}_{(n')} + \frac{m^2_{(n')}}{D-2} \frac{p^{\mu}_{(n)}}{p_{(n)}\cdot p_{(n')}} - \frac{D-3}{2(D-2)} (1 + \frac{v^2_{n,n'}}{D-3}) \frac{p_{(n)}\cdot p_{(n')}}{k\cdot p_{(n)}} k^{\mu} \right] + c^{\mu}_{(n)} \tau_{(n)} + \tilde{c}^{\mu}_{(n)}$$

$$\Delta^{\mu}_{n,n'} = z^{\mu}_{(n)}(0) - z^{\mu}_{(n')}(0)$$

Deflection angle :

$\beta = 1$, $t_{(n)}$ not proper time

$x^M = (x^\mu, y^i)$, $\mu = 0, \ldots, 3$, $i = 1, \ldots \delta$ (torus T^δ)

$y_j \to y_j + 2\pi L$: $\underset{(1)}{h}_{MN}(x^P) = \sum_{n_1=-\infty}^{+\infty} \cdots \sum_{n_\delta=-\infty}^{+\infty} \frac{\overset{h^n}{\underset{(1)}{}}_{MN}(x)}{\sqrt{V_\delta}} \exp\left(i \frac{n_i y^i}{L}\right)$

$(mass)^2 = p_T^i p_{Ti} = \frac{n^i}{L}\frac{n_i}{L} = \frac{\vec{n}^2}{L^2}$

$T_{MN}(x^P) = \delta_M^\mu \delta_N^\nu T_{\mu\nu}(x) \delta^\delta(\mathbf{y})$

frame of m' : $p'^\mu = m'(1, 0, 0, 0)$, $p^\mu = m\gamma(1, 0, 0, v)$, $\gamma = \frac{1}{\sqrt{1-v^2}}$

$$\underset{(1)}{\dot{z}}^{\mu}(t) = \frac{2}{\pi^2 m' m M_*^{2+\delta} V_\delta} \int \sum_n \frac{e^{ik\cdot\Delta - ik\cdot pt/m}}{\mathbf{k}^2 + p_T^2} Q^\mu \Big|_{k^0 = |\mathbf{k}|} d^3k \quad , \quad Q^\mu = Ak^\mu + Bp^\mu + Cp'^\mu$$

$$\sum_n \to \frac{V_\delta S_{\delta-1}}{(2\pi)^\delta} \int_0^\infty p_T^{\delta-1} dp_T \quad , \quad S_{\delta-1} = \frac{2\pi^{\delta/2}}{\Gamma(\delta/2)}$$

Born amplitude $\mathcal{M}_{\text{Born}}(s,t) = \frac{s^2 \kappa_D^2}{2(2\pi)^\delta} \int \frac{d^\delta p_T}{-t + p_T^2}$ diverges

Natural classical cut-off

$$s = (P + P')^2 \simeq (p + p')^2 \qquad \text{(c.m. energy)}$$

$$t = q^2 = (\bar{P} - P)^2 = m^2 \left(\lim_{t\to\infty} \underset{(1)}{\dot{z}}^{\mu}(t) - \lim_{t\to-\infty} \underset{(1)}{\dot{z}}^{\mu}(t) \right)^2 \qquad \text{(momentum transfer)}$$

$$-t = 2^4\Gamma^2(\tfrac{\delta}{2}+1)\frac{m^2m'^2}{\pi^\delta\gamma^2v^2M_*^2}\frac{1}{(M_*b)^{2(\delta+1)}}\left(\frac{(p\cdot p')^2}{m^2m'^2}-\frac{1}{\delta+2}\right)^2$$

$$\gamma \gg 1 \quad : \quad -t = \frac{2^2\Gamma^2(\delta/2+1)}{\pi^\delta(M_*b)^{2(\delta+1)}}\frac{s^2}{M_*^2}$$

differential cross-section $\underline{\underline{\frac{d\sigma}{dt} = \frac{1}{(\delta+1)(-M_*^2t)}\left(-4\pi\Gamma^2(1+\delta/2)\frac{s^2}{M_*^2t}\right)^{1/(\delta+1)}}}$

$\delta = 0 \quad \rightsquigarrow \quad$ GR $\quad \frac{d\sigma}{dt} = \frac{4\pi G_4^2 s^2}{t^2} \quad \rightsquigarrow \quad$ Deibel, Schucker 1991

small scattering angles : $\underline{\underline{b \gg b_{cr}}}$, $b_{cr} = \frac{1}{M_*}(\frac{m'}{M_*})^{\frac{1}{\delta+1}}$

If $M_* \sim TeV$, $m' = m_p \sim 1GeV$, then $b \gg 10^{-19}cm$.

Eikonal : $\mathcal{M}_{\text{eik}}(s, t \simeq -\mathbf{q}^2) = 2is \int e^{i\mathbf{q}\cdot\mathbf{b}} \left(1 - e^{i\chi(s,b)}\right) d^2b$

$$\chi(s,b) = \frac{1}{2s} \int e^{-i\mathbf{q}\cdot\mathbf{b}} \mathcal{M}_{\text{Born}}(s,t) \, \frac{d^2q}{(2\pi)^2} = (\frac{b_c}{b})^\delta$$

$$\mathcal{M}_{\text{eik}}(s,t) = \frac{4\sqrt{\pi} s e^{i(qb_s - \pi/2)}}{q\sqrt{\delta+1}} \left(\frac{2\sqrt{\pi} s \Gamma(\delta/2+1)}{M_*^{\delta+2} q}\right)^{\frac{1}{\delta+1}} \quad , \quad b_s = (\frac{\delta \, b_c^\delta}{q})^{\frac{1}{\delta+1}}$$

$$\underline{\underline{\frac{d\sigma_{\text{eik}}}{dt}}} = \frac{1}{(\delta+1) M_*^2 |t|} \left(4\pi \Gamma^2 (1 + \delta/2) \frac{s^2}{M_*^2 |t|}\right)^{1/(\delta+1)} = \underline{\underline{\frac{d\sigma}{dt}}}$$

Up to now :

— at transplanckian energies, classical scattering trustable

— classically no divergencies , quantum tree-level divergencies

— classical elastic cross-section $\equiv$ quantum eikonal cross-section

— e.g. QED, classical relativistic bremsstrahlung $\leftrightsquigarrow$ non-perturbative quantum

$\Rightarrow$ second order ultra-relativistic gravitational bremsstrahlung

2nd ORDER

Start with Φ (easier , other scales , powers of γ , $\Delta E_\Phi \sim \gamma^{D-2}$)

Gravity $\frac{E}{M_*} \sim 10$ $\Rightarrow$ Extreme bremsstrahlung ($\Delta E_{gr} \sim \gamma^{D-1}$)

$$\underset{(2)}{\tilde{\psi}}{}_{\mu\nu}(k) = \frac{2\kappa_D^2}{k^2}[\underset{(1)}{\tilde{T}}{}_{\mu\nu}(k) + \underset{(1)}{\tilde{S}}{}_{\mu\nu}(k)] = \frac{2\kappa_D^2}{k^2}\tilde{\tau}_{\mu\nu}(k)$$

- $P^\mu = \frac{\kappa_D^2}{2(2\pi)^{D-1}} \int_0^\infty d\omega\, \omega^{D-3} \int_{S^{D-2}} d\Omega\ k^\mu\ (\tilde{\tau}^*_{\mu\nu}\tilde{\tau}^{\mu\nu} - \frac{|\tilde{\tau}^\mu_\mu|^2}{D-2})\Big|_{k^0=|\vec{k}|}$ total emission

$$\frac{d^2E}{d\omega d\Omega} = \frac{\kappa_D^2}{2(2\pi)^{D-1}}\omega^{D-2}(\tilde{\tau}^*_{\mu\nu}\tilde{\tau}^{\mu\nu} - \frac{|\tilde{\tau}^\mu_\mu|^2}{D-2})\Big|_{k^0=|\vec{k}|}$$ energy spectrum

$$\tilde{\tau}^*_{\mu\nu}\tilde{\tau}^{\mu\nu} - \frac{|\tilde{\tau}^\mu_\mu|^2}{D-2} = \Lambda^{\mu\nu\kappa\lambda}\tau_{\mu\nu}\tau^*_{\kappa\lambda} = \sum_{A=1}^{\frac{D(D-3)}{2}} |\tilde{\tau}_{\mu\nu}(k)\varepsilon^{\mu\nu}_{(A)}|^2$$

- $P^\mu = \frac{\kappa_D^2}{2(2\pi)^{D-1}} \sum_{A=1}^{\frac{D(D-3)}{2}} \int_0^\infty d\omega\, \omega^{D-3} \int_{S^{D-2}} d\Omega\ k^\mu\ |\tilde{\tau}_{\mu\nu}(k)\varepsilon^{\mu\nu}_{(A)}|^2\Big|_{k^0=|\vec{k}|}$

Construction of polarizations :

$\Lambda_{\mu\nu\kappa\lambda}$ projector in the space of symmetric traceless matrices ("polarizations") constructed by products of vectors orthonormal to u'^{μ}, k^{μ}

$$\exists\ e^{\mu}_{(\alpha)} \quad , \quad \alpha = 3, 4, ...D-2 \quad : \quad e_{(\alpha)} \perp k, p', p, \Delta \quad , \quad e_{(\alpha)} \cdot e_{(\beta)} = \delta_{\alpha\beta}$$

$$e^{\mu}_{(1)} = \frac{1}{v\gamma \sin_{\theta}}[\frac{(k\cdot p)p'^{\mu} - (k\cdot p')p^{\mu}}{m(k\cdot p')} - \frac{m' v\gamma \cos\theta}{k\cdot p'} k^{\mu}] \quad , \quad \theta = (\widehat{\vec{v}_{1,2}, \vec{k}})$$

$$e^{\mu}_{(2)} = -\frac{1}{mv\gamma(k\cdot p')\sin\theta} \epsilon^{\mu\nu\kappa\lambda\sigma_5 ... \sigma_{D-2}} p_{\nu} p'_{\kappa} q_{\lambda} e_{(3)\sigma_3} ... e_{(D-2)\sigma_{D-2}}$$

$$\Longrightarrow \quad e_{(i)} = \{e_{(1)}, e_{(2)}, e_{(\alpha)}\} \ , \ i = 1, 2, 3, ..., D-2$$

$$e_{(i)} \cdot k = e_{(i)} \cdot u' = 0 \ , \ e_{(i)} \cdot e_{(j)} = \delta_{ij}$$

$$\varepsilon^{\mu\nu}_{(a')} \ (\alpha' = 3, 4, ..., D-3) \ : \quad \varepsilon_{(\alpha')} \in \langle \tilde{\varepsilon}_{(\alpha')} \rangle \ , \ \varepsilon^{\mu\nu}_{(\alpha')} \varepsilon_{(\beta')\mu\nu} = \delta_{\alpha'\beta'}$$

$$\tilde{\varepsilon}^{\mu\nu}_{(\alpha)} = (D-5) e^{\mu}_{(\alpha)} e^{\nu}_{(\alpha)} - \sum_{\beta \neq \alpha} e^{\mu}_{(\beta)} e^{\nu}_{(\beta)}$$

$$\varepsilon^{\mu\nu}_{(I)} = \frac{1}{\sqrt{2}}(e^{\mu}_{(1)}e^{\nu}_{(1)} - e^{\mu}_{(2)}e^{\nu}_{(2)})$$

$$\varepsilon^{\mu\nu}_{(III)} = \sqrt{\frac{2(D-4)}{D-2}}\left[\frac{1}{D-4}\sum_{\alpha} e^{\mu}_{(\alpha)}e^{\nu}_{(\alpha)} - \frac{1}{\sqrt{2}}(e^{\mu}_{(1)}e^{\nu}_{(1)} + e^{\mu}_{(2)}e^{\nu}_{(2)})\right] \quad (\sharp \text{ for } D=4)$$

$$\rightarrow \quad \varepsilon^{\mu\nu}_{(i')} = \{\varepsilon^{\mu\nu}_{(I)}\ ,\ \varepsilon^{\mu\nu}_{(III)}\ ,\ \varepsilon^{\mu\nu}_{(\alpha')}\} \quad , \quad i' = 1,2,3,...,D-3$$

$$\rightarrow \quad \varepsilon^{\mu\nu}_{(ij)} = \frac{1}{\sqrt{2}}(e^{\mu}_{(i)}e^{\nu}_{(j)} + e^{\mu}_{(j)}e^{\nu}_{(i)}) = \{\varepsilon_{(12)} = \varepsilon_{(II)}\ ,\ \varepsilon_{(1\alpha)}\ ,\ \varepsilon_{(2\alpha)}\ ,\ \varepsilon^{\alpha<\beta}_{(\alpha\beta)}\}$$

$$\frac{(D-2)(D-3)}{2}$$

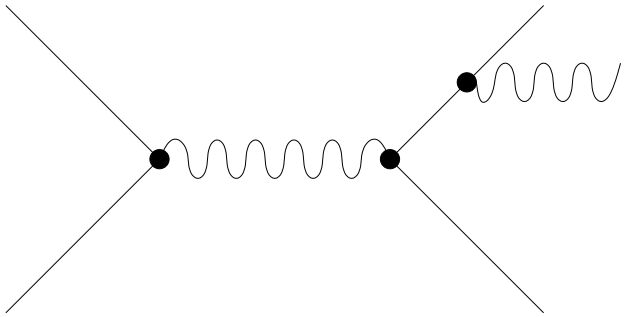
$$\varepsilon_{(\mathcal{A})} = \{\varepsilon_{(I)}\ ,\ \varepsilon_{(III)}\ ,\ \varepsilon_{(\alpha')}\ ,\ \varepsilon_{(II)}\ ,\ \varepsilon_{(1\alpha)}\ ,\ \varepsilon_{(2\alpha)}\ ,\ \varepsilon^{\alpha<\beta}_{(\alpha\beta)}\} \qquad \frac{D(D-3)}{2}$$

$$\Longrightarrow \quad \varepsilon_{(A)} = \{\varepsilon_{(+)}\ ,\ \varepsilon_{(-)}\ ,\ \varepsilon_{(III)}\ ,\ \varepsilon_{(\alpha')}\ ,\ \varepsilon_{(1\alpha)}\ ,\ \varepsilon_{(2\alpha)}\ ,\ \varepsilon^{\alpha<\beta}_{(\alpha\beta)}\}$$

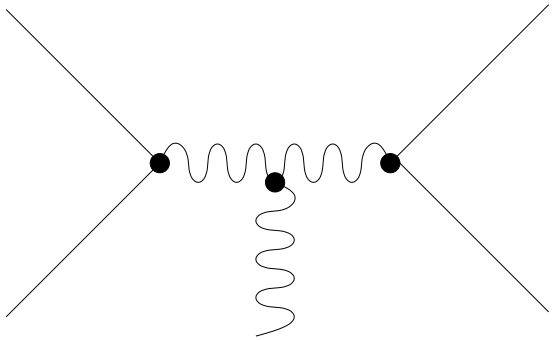
$$\varepsilon^{\mu\nu}_{(\pm)} = \frac{1}{\sqrt{2}}(\varepsilon^{\mu\nu}_{(I)} \pm i\varepsilon^{\mu\nu}_{(II)}) = e^{\mu}_{(\pm)}e^{\nu}_{(\pm)}$$

- $P^{\mu} = \frac{\kappa_D^2}{2(2\pi)^{D-1}}\int_0^{\infty} d\omega\, \omega^{D-3}\int_{S^{D-2}} d\Omega\ k^{\mu}\ (|\tilde{\tau}_{\mu\nu}\varepsilon^{\mu\nu}_{(+)}|^2 + |\tilde{\tau}_{\mu\nu}\varepsilon^{\mu\nu}_{(-)}|^2 + |\tilde{\tau}_{\mu\nu}\varepsilon^{\mu\nu}_{(III)}|^2)$

$$\underset{(1)}{T}{}^{\mu\nu}$$



$$\underset{(1)}{S}{}^{\mu\nu}$$



energy :

- MacDonald functions in $\underset{(1)}{\tilde{T}}{}^{\mu\nu}$, $\underset{(1)}{\tilde{S}}{}^{\mu\nu}$ $\rightsquigarrow$ cutoff $\omega_{cr} \sim \frac{\gamma}{b}$

- Destructive interference between dominant powers of $\underset{(1)}{T}{}^{\mu\nu}$, $\underset{(1)}{S}{}^{\mu\nu}$

$$\underline{\underline{\Delta E = C_D \frac{\kappa_D^6 m^2 m'^2}{b^{3D-9}} \gamma^{D-1}}} \quad \text{(D-flat)}$$

$$C_D = \frac{2^{2D-17} \pi^{\frac{D-1}{2}}}{(2\pi)^{D-2}(D-2)^2} \frac{\Gamma(\frac{D-4}{2})\Gamma(\frac{3D-7}{2})\Gamma^2(\frac{2D-5}{2})\Gamma(\frac{D-3}{2})}{\Gamma(\frac{D+4}{2})\Gamma(\frac{D+3}{2})\Gamma(2D-3)} H$$

$$H = 108D^{11} - 2124D^{10} + 19995D^9 - 114628D^8 + 412742D^7 - 858540D^6 + 659362D^5 + 1098506D^4 - 2995767D^3 + 2278902D^2 - 170404D - 308280$$

Extra powers from the prefactor H !!

$$\Delta E_{(D=4)} = (10\pi + \frac{35}{2}\tilde{G} - \frac{211}{12}) \frac{G_4^3 m^2 m'^2}{b^3} \gamma^3 \quad \text{(Thorne-Kovacs 1977)}$$

$$\underline{\underline{\Delta E = C'_D \, \frac{\kappa_D^6 m^2 m'^2}{b^{3\delta+3}} \, \gamma^{\delta+3}}} \qquad \text{(Compact } D = 4+\delta\text{)}$$

LAB to CM : $\underline{\epsilon \equiv \frac{\Delta E}{E} = C_\delta (\frac{R_S}{b})^{3(\delta+1)} \gamma_{\text{cm}}^{2\delta+1}}$ radiation efficiency

$R_S \ll \lambda_C \ll b_c \quad : \quad b = \lambda_C \quad \rightsquigarrow \quad \epsilon = B_\delta (\frac{m\,s}{M_*^3})^{\delta+2}$

$m\,s \gtrsim M_*^3 \rightsquigarrow$ extreme bremsstrahlung

e.g. m = 200GeV, $\sqrt{s} = 10$TeV, $\delta = 2$

$\epsilon > 1$ $(unphysical)$ $\rightsquigarrow$ improve the approximation by including for instance radiation back reaction

Other characteristic features :

— $\frac{dE}{d\omega}(\omega)$ starts from 0 at $\omega = 0$, has peak and then exponential fall-off

— cutoff of $\frac{dE}{d\omega}(\omega)$ at $\omega_{cr} \sim \frac{\gamma}{b}$ due to destructive interference for $\frac{\gamma}{b} < \omega < \frac{\gamma^2}{b}$ (electromagnetism : $\omega_{cr} \sim \frac{\gamma^2}{b}$)

— almost all energy in the cone of half-angle $\theta \sim \frac{1}{\gamma}$ in forward direction (electromagnetism similar)

— Gravity has less forward peaking than electromagnetism

Summary

- We have calculated the gravitational bremsstrahlung in transplanckian collisions in the presence of extra dimensions. We found an extreme radiation loss

- Maybe this has negative implications for the black hole production (e.g. at LHC)

- The results could be used elsewhere, e.g. to constraint parameters of the theory (number of extra dimensions, fundamental mass scale, cosmological constant, e.t.c.)

- Future work : next-order corrections to confirm the validity of the calculations, consideration of radiation reaction, massless particles, consideration of quantum corrections (comparison), for phenomenology take into account the structure of particles, e.t.c.