Universal type IIA de Sitter solutions at tree-level

Based on 1003.3590 with Ulf Danielsson and Thomas Van Riet,
and older work: 0812.3551 with Caviezel, Körs, Lüst, Wrase and Zagermann
http://itf.fys.kuleuven.be/~koerber/talks.html

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Goal: construct a completely explicit and simple dS solution

- Classical in $\alpha', g_S \to$ solution of supergravity eoms
- Clear 10d interpretation: compact internal manifold
- Limit ingredients to what is well-understood: avoid e.g. non-geometric fluxes

Motivation II

In this talk:

- Focus on type IIA supergravity
- Derive simple set of conditions on the geometry for obtaining dS solutions
- Present a one-dimensional family of examples on SU(2)×SU(2)

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Do not expect too much!

- Still far from realistic
- In particular: tachyonic modes
- Smeared orientifolds

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But there is more. . .

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  Impose duality condition
- Two opposite chirality gravitino fields $\Psi_M^{1,2}$ and dilatino fields $\lambda^{1,2}$

Compactification ansatz

- Metric:

$$\mathrm{d}s^2 = e^{2A(y)} g_{(4)\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}(y)\mathrm{d}y^m \mathrm{d}y^n \ ,$$

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  with $g_{(4)}$ flat Minkowski, AdS$_4$ metric or dS$_4$ metric, A warp factor

- NSNS-flux H: purely internal
- RR-fluxes:

$$F = \sum_l F_l = \hat{F} + e^{4A}\mathrm{vol}_4 \wedge F_{\mathrm{el}} \ , \qquad (F_{\mathrm{el}} = \star_6 \sigma(\hat{F}))$$

  where σ reverses the indices of a form

Learn from $N = 1$ susy AdS solutions with SU(3)-structure

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$$\Omega = \frac{1}{3!||\eta||^2}\eta_-^\dagger \gamma_{i_1 i_2 i_3}\eta_+ \mathrm{d}x^{i_1} \wedge \mathrm{d}x^{i_2} \wedge \mathrm{d}x^{i_3}$$

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- Susy conditions translate into condition $\mathrm{d}J$ and $\mathrm{d}\Omega$
- More general: η^1, η^2: SU(3)×SU(3)-structure

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Lüst, Tsimpis

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$$\mathrm{d}J = \frac{3}{2}\mathrm{Im}\left(\mathcal{W}_1\Omega^*\right) + \mathcal{W}_4 \wedge J + \mathcal{W}_3$$
$$\mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5^* \wedge \Omega$$

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with $\mathcal{W}_1, \mathcal{W}_2$ purely imaginary
$\Rightarrow$ put $\mathcal{W}_1 = iW_1$, $\mathcal{W}_2 = iW_2$,

- Form-fluxes:

$$H = \frac{2m}{5}e^{\Phi}\mathrm{Re}\Omega$$
$$e^{\Phi}\hat{F}_2 = -\frac{W_1}{4}J - W_2$$
$$e^{\Phi}\hat{F}_4 = \frac{3m}{10}J\wedge J$$
$$e^{\Phi}\hat{F}_6 = \frac{9W_1}{4}\mathrm{vol}_6$$

AdS$_4$ superpotential W:

$$\nabla_\mu\zeta_- = \frac{1}{2}W\gamma_\mu\zeta_+ \quad \text{definition}$$
$$We^{i\theta} = -\frac{1}{5}e^{\Phi}m - \frac{3i}{4}W_1$$
$$R_4 = 4\Lambda = -12|W|^2$$

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- Leads to

$$e^{2\Phi} m^2 = \mu + \frac{5}{16}\left(3(W_1)^2 - 2(W_2)^2\right) \geq 0$$

with $\mu > 0$: net orientifold charge, $\mu < 0$: net D-brane charge

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Geometries:

- Nearly-Kähler solutions *Behrndt, Cvetič* $\mathcal{W}_2 = 0$
  The only examples known are the homogeneous manifolds:
  SU(2)×SU(2), $\frac{\mathrm{G}_2}{\mathrm{SU}(3)} = S^6$, $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))} = \mathbb{CP}^3$, $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$
- Families of solutions on the above manifolds with $\mathcal{W}_2 \neq 0$
  Tomasiello;*PK,Lüst,Tsimpis*
- $\mu \neq 0$: e.g. Iwasawa manifold

How to construct dS solutions?

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  Approximation of constant warp factor & smeared orientifolds

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- Let's try adding $W_3 \to$ half-flat manifold
- The simplest dS solution we can find: put $W_2 = 0$
- Orientifold sources: like in susy case $\to$ calibrated

The ansatz II

Geometry: SU(3)-structure with

$$\begin{aligned} \mathrm{d}J &= \frac{3}{2} W_1 \mathrm{Re}\Omega + W_3 \,, \\ \mathrm{dRe}\Omega &= 0 \,, \\ \mathrm{dIm}\Omega &= W_1 J \wedge J. \end{aligned}$$

Put $A = 0$, Φ constant

The ansatz II

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Flux & sources:

$$e^{\Phi}\hat{F}_0 = f_1 \qquad H = f_5 \mathrm{Re}\Omega + f_6 W_3$$

$$e^{\Phi}\hat{F}_2 = f_2 J \qquad e^{\Phi} j = j_1 \mathrm{Re}\Omega + j_2 W_3$$

$$e^{\Phi}\hat{F}_4 = f_3 J \wedge J$$

$$e^{\Phi}\hat{F}_6 = f_4 \mathrm{vol}_6$$

Supergravity equations of motion

Sugra eom with calibrated sources (taken from *PK, Tsimpis*)
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$$\mathrm{d}\hat{F}_2 + H\hat{F}_0 = -j, \qquad \text{(Bianchi } \hat{F}_2\text{)}$$

$$\mathrm{d} \star_6 \hat{F}_4 - H \wedge \star_6 \hat{F}_6 = 0, \qquad \text{(eom } \hat{F}_4\text{)}$$

$$\mathrm{d}(e^{-2\Phi} \star_6 H) - (\star_6 \hat{F}_2)\hat{F}_0 - (\star_6 \hat{F}_4) \wedge \hat{F}_2 - (\star_6 \hat{F}_6) \wedge \hat{F}_4 = 0, \qquad \text{(eom } H\text{)}$$

$$2(R_4 + R_6) - H^2 - e^{\Phi} \star_6 (\mathrm{Im}\Omega \wedge j) = 0, \qquad \text{(eom } \Phi\text{)}$$

$$R_4 + e^{2\Phi} \sum_n (\tilde{F}^2_{(n)}) + e^{\Phi} \star_6 (\mathrm{Im}\Omega \wedge j) = 0, \qquad \text{(external Einstein)}$$

$$-\tfrac{1}{2}H^2 + \tfrac{1}{4}e^{2\Phi} \sum_n (5-n)\hat{F}^2_{(n)} + \tfrac{3}{4}e^{\Phi} \star_6 (\mathrm{Im}\Omega \wedge j) = 0, \qquad \text{(trace Einstein/eom } \Phi\text{)}$$

$$R_{ij} - \tfrac{1}{2}H_i \cdot H_j - \tfrac{1}{4}e^{2\Phi} \sum_n \left(\hat{F}_{(n)i} \cdot \hat{F}_{(n)j} - \tilde{F}_{(n)i} \cdot \tilde{F}_{(n)j}\right)$$

$$+ \tfrac{1}{4}e^{\Phi} \left\{ -g_{ij} \star_6 (\mathrm{Im}\Omega \wedge j) + 2 \star_6 \left[(g_{k(i}\mathrm{d}x^k \wedge \iota_{j)}\mathrm{Im}\Omega) \wedge j \right] \right\} = 0. \qquad \text{(Einstein/eom } \Phi\text{)}$$

Ricci tensor in terms of SU(3)-structures I

Bedulli, Vezzoni

Decompose R_{ij} SU(3)-reps

$$R_{ij} = \frac{s(R_{ij})}{6} g_{ij} + R^{+}_{ij} + R^{-}_{ij}$$

$s(R_{ij})$ trace

R^{+}_{ij} (1,1)-part: $\mathbf{8}$ of SU(3)

R^{-}_{ij} (2,0)+(0,2)-part: $\mathbf{6} + \bar{\mathbf{6}}$ of SU(3)

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R^-_{ij} (2,0)+(0,2)-part: $\mathbf{6} + \bar{\mathbf{6}}$ of SU(3)

Associated primitive two- and three-form to R^+ and R^-

$$P_2(R_{ij}) = \frac{1}{2} J^k{}_i R^+_{kj} \mathrm{d}x^i \wedge \mathrm{d}x^j, \quad P_3(R_{ij}) = \frac{1}{2} R^-_{il} \Omega^l{}_{jk} \mathrm{d}x^i \wedge \mathrm{d}x^j \wedge \mathrm{d}x^k$$

Ricci tensor in terms of SU(3)-structures II

$$s(R_{ij}) = \frac{15}{2}(W_1)^2 - \frac{1}{2}(W_2)^2 - \frac{1}{2}(W_3)^2 \,,$$

$$P_2(R_{ij}) = -\frac{1}{4} \star_6 (W_2 \wedge W_2) - \frac{1}{2} \star_6 d \star_6 \left(W_3 - \frac{1}{2} W_1 \mathrm{Re}\Omega \right) \,,$$

$$P_3(R_{ij}) = 2W_1 W_3|_{(2,1)} + 2\mathrm{d}W_2|_{(2,1)} - \frac{1}{4} Q(W_3, W_3)$$

with

$$Q(W_3, W_3) = \left(\Omega^{ijk} \iota_j \iota_i W_3 \wedge \iota_k W_3 \right)_{(2,1)} \,,$$

Conditions

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Conditions on torsion classes:

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And equations for the constants f_i, j_i in the ansatz

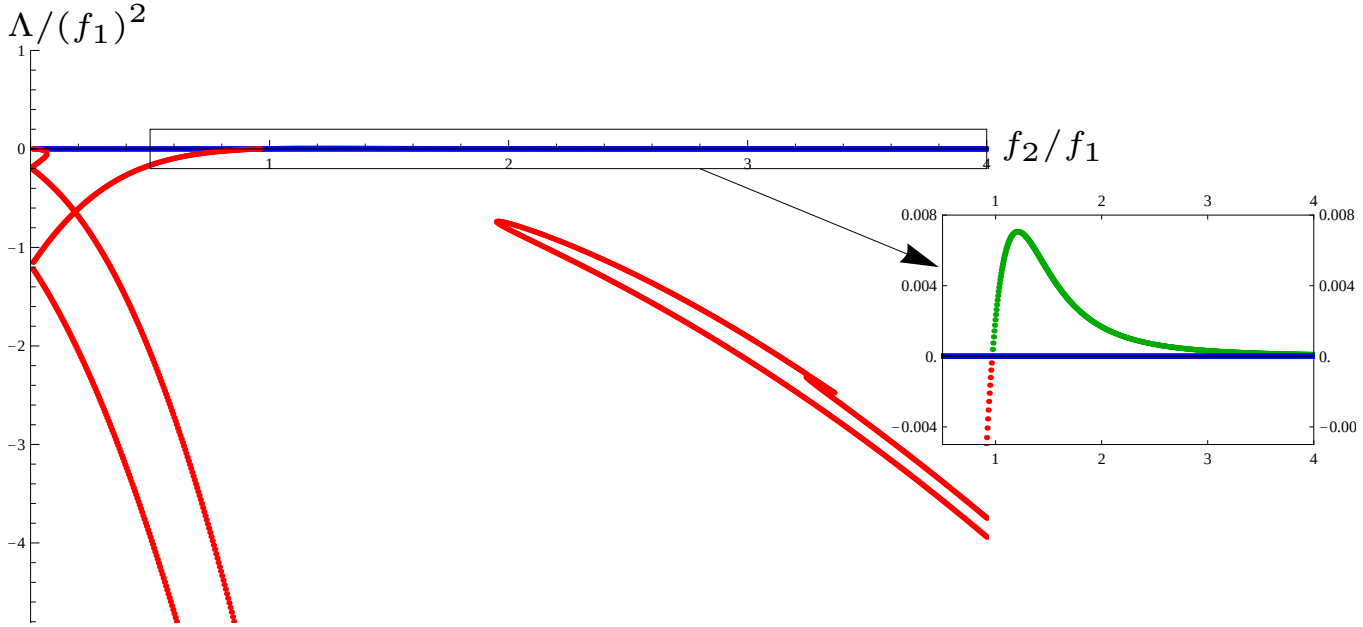
Solutions I

Standard orientifold ansatz sets $q/w_3 = 8/\sqrt{3}$

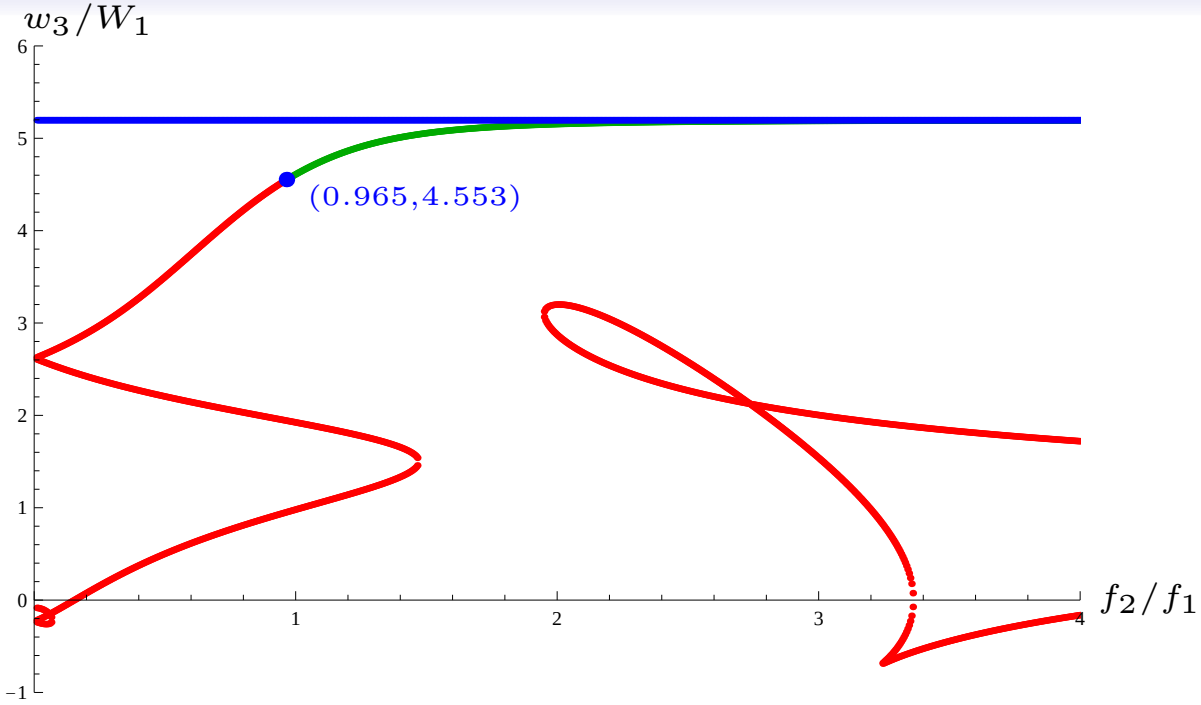
We then only find dS solutions with $f_3 = f_4 = 0$

Parameters: overall scale, dilaton and one extra parameter

Take e.g. f_2/f_1



Solutions II



dS solutions with $4.553 < w_3/W_1 < 3\sqrt{3}$

Example: SU(2)×SU(2)

Example on group manifold SU(2)×SU(2)
On group manifold: left-invariant forms e^i:

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Four orientifolds (compatible SU(3)-structure)

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Solution on SU(2)×SU(2) reads

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J &= a(e^{16} - e^{24} + e^{35}), \\
\mathrm{Re}\,\Omega &= v_1(e^{456} + e^{236} + e^{125}) + \left(a^6/(v_1)^3\right) e^{134} \\
\mathrm{Im}\,\Omega &= \left(a^3/v_1\right)\left(e^{123} + e^{145} - e^{346}\right) - \left((v_1)^3/a^3\right) e^{256}
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Parameters: a (scale), v_1 shape

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It follows:

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g &= \mathrm{diag}\left(a^4/(v_1)^2, (v_1)^2/a^2, a^4/(v_1)^2, a^4/(v_1)^2, (v_1)^2/a^2, (v_1)^2/a^2\right),\\
W_1 &= \frac{a^6 + v_1^4}{4\,a^5 v_1}, \qquad W_2 = 0,\\
W_3 &= \frac{a^6 - 3(v_1)^4}{8\,a^5(v_1)^4}\left[(v_1)^4(e^{456} + e^{236} + e^{125}) - 3\,a^6 e^{134}\right]\\
&\Rightarrow w_3 = \pm|W_3| = -\frac{\sqrt{3}(a^6 - 3(v_1)^4)}{4a^5 v_1}, \quad q/w_3 = 8/\sqrt{3}
\end{aligned}
$$

Solution II

One easily checks conditions torsion classes:

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w_3/W_1 = \frac{\sqrt{3}(3(v_1)^4 - a^6)}{a^6 + (v_1)^4}\,,
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takes values between $-\sqrt{3}$ and $3\sqrt{3}$ (boundaries not included)

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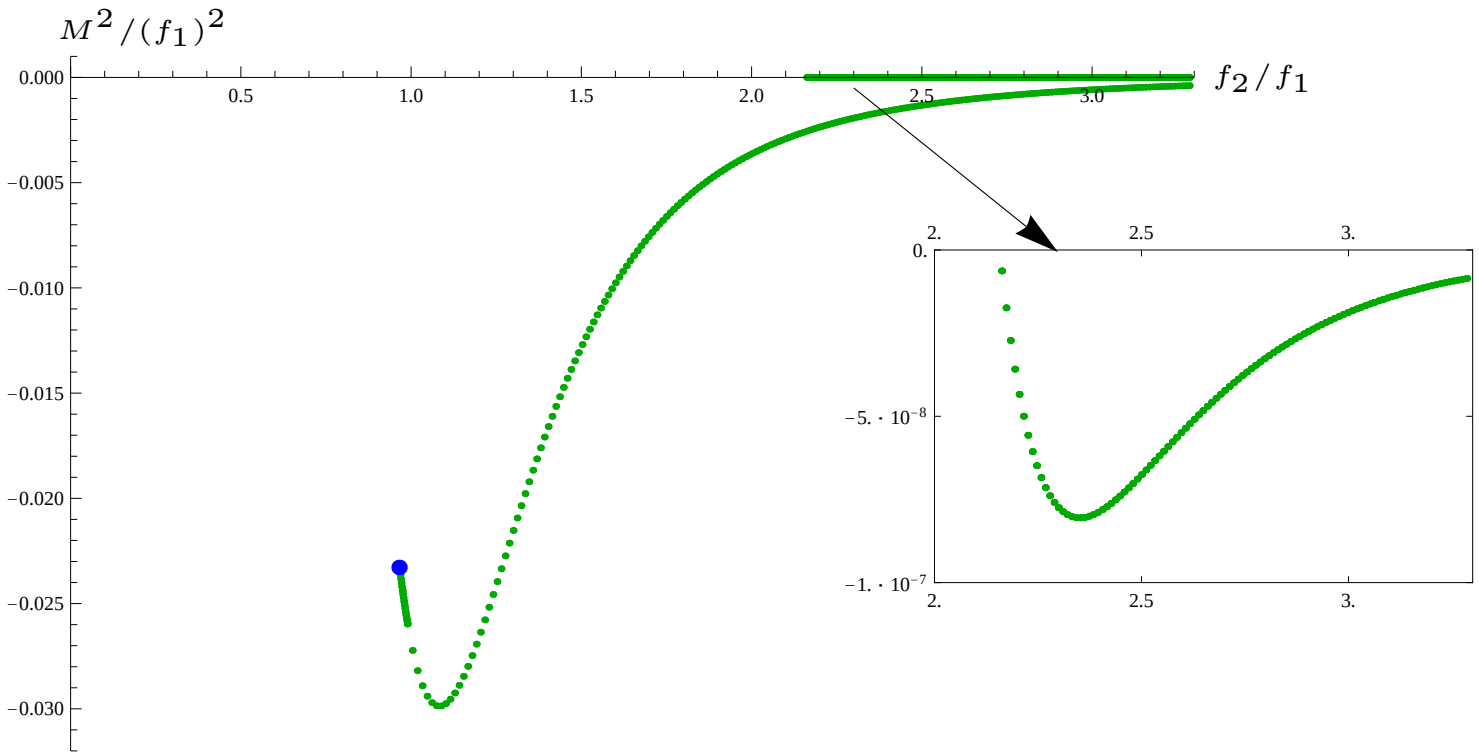
- SU(2)×SU(2) seems to know about upper limit $3\sqrt{3}$
- Minkowski solution not present (we would need: $v_1/a^{3/2} \to \infty$)

Spectrum

Using 4D $N = 1$ supergravity technology we can calculate spectrum of left-invariant fluctuations. Contains tachyonic modes:



Conclusions

- We found simple conditions on geometry for having classical dS solutions
- We found geometry on SU(2)×SU(2) satisfying these conditions
- Problems: tachyonic modes & smeared orientifolds
- Are tachyons generic? (e.g. *Gómez-Reino, Louis, Scrucca*)
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- Further work: systematic scan manifold & include $W_2 \neq 0$
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The end... The end... The end...