AdS in Light-Cone and Schrödinger space

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work in progress with E. Kiritsis, C. Panagopoulos, D. Yamada

Introduction & Motivations

• Schrödinger and Lifshitz spaces are interesting
  - generalization of the AdS/CFT for the non-relativistic setup,
  - potential applications to the strongly coupled CM systems with dynamical exponent z.

Son[2008], Balasubramanian McGreevy[2008]
Kachru Liu Mulligan[2008]

• Holographic renormalizations of those spaces are not well understood due to their difficulties such as degenerate boundary.

Hořava Melby-Thompson[2009]

Schrödinger space

Son[2008], Balasubramanian-McGreevy[2008]

$$ds^2 = r^2\left(d\vec{x}^2 - 2dx^+dx^- - r^2 dx^{+2}\right) + \frac{dr^2}{r^2}\ .$$

- Schrödinger isometry!

- Finite T generalizations with Null Melvin Twist (Alishahiha Ganor[2003])
  Herzog Rangamani Ross(HRR)[2008],
  Maldacena Martelli Tachikawa(MMT)[2008],
  Adams Balasubramanian McGreevy(ABM)[2008].

- Many efforts and several other developments $\cdots$, e.g.,
  Talk by M. Taylor in this conference,
  Rangamani Ross Son Thompson[2008],
  Ross Saremi[2009].

- Difficulties :
  - Degenerate : Holographic renormalization is not well understood.
  - Complicated metric : technical difficulties in computations.

Planar Schrödinger black hole

$$
\begin{aligned}
ds_E^2 \quad = \quad & r^2\, k(r)^{-\frac{2}{3}} \left(\left[\frac{1-f(r)}{4b^2} - r^2\, f(r) \right] dx^{+2} + \frac{b^2 r_+^4}{r^4}\, dx^{-2} - [1+f(r)]\; dx^+\, dx^- \right) \\
& + \; k(r)^{\frac{1}{3}} \left(r^2 d\mathbf{x}^2 + \frac{dr^2}{r^2\, f(r)} \right) ,
\end{aligned}
$$

where

$$
f(r) = 1 - \frac{r_+^4}{r^4} \,, \qquad k(r) = 1 + b^2\, r^2\, (1 - f(r)) = 1 + \frac{b^2\, r_+^4}{r^2} .
$$

with the massive vector and scalar

$$
A = \frac{r^2}{k(r)} \left(\frac{1+f(r)}{2}\, dx^+ - \frac{b^2 r_+^4}{r^4}\, dx^- \right), \quad e^{\phi} = \frac{1}{\sqrt{k(r)}} \,,
$$

Action which supports this metric is given by

$$
\mathcal{S} = \frac{1}{16\pi\, G_5} \int d^5x \sqrt{-g} \left(R - \frac{4}{3}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{4} e^{-8\phi/3} F_{\mu\nu} F^{\mu\nu} - 4\, A_\mu A^\mu - V(\phi) \right) ,
$$

where the scalar potential is

$$
V(\phi) = 4\, e^{2\phi/3} (e^{2\phi} - 4).
$$

- Light-Cone coordinate :

$$x^+ = t + x\,, \qquad x^- = \frac{1}{2}(t - x)\,.$$

- Two key procedures :

A. Identification of x^+, light-like coordinate, as time!

In the relativistic theory, light-cone direction can not be an appropriate time function. Thus assigning x^+ as time completely separate the system from the relativistic causal theory.

B. Projection onto a fixed momentum in the x^- direction.

Momentum projection in x^- provides a way to realize super-selection sector in Galilean invariant theories and also give "mass" of the non-relativistic particle.

$$ds^2 = r^2\left(d\vec{x}^2 - 2dx^+dx^-\right) + \frac{dr^2}{r^2}\ ,$$

- Realizing Schrödinger isometry without deformation $r^4 dx^{+2}$!
- Finite T generalization : Maldacena-Martelli-Tachikawa[2008]

$$x^+ = b(t+x)\ , \qquad x^- = \frac{1}{2b}(t-x)\ .$$

- But, no further developments along this line.
- Advantages :
  - Holographic renormalization is well understood as AdS case.
  - Calculations are as simple as AdS.
- Question : What's the precise connection between the Schrödinger and AdS in light-cone? Thermodynamics? Transport properties?

AdS planar black hole in light-cone

$$I = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(\mathcal{R} + \frac{12}{R^2}\right) - \frac{1}{8\pi G_5}\int d^4x\sqrt{-\gamma}\left(K + \frac{3}{R} + \frac{R}{4}\mathcal{R}_4\right),$$

$$ds^2 = \left(\frac{r}{R}\right)^2\left\{\frac{1-h}{4b^2}dx^{+2} - (1+h)dx^+dx^- + (1-h)b^2dx^{-2} + dy^2 + dz^2\right\} + \left(\frac{R}{r}\right)^2 h^{-1}dr^2\,, \qquad h = 1 - \frac{r_H^4}{r^4}$$

- b is boost parameter and directly related to the chemical potential of charge associated with the isometry along the direction x^-.

- Stress energy momentum tensor is finite with the counter terms used in AdS space. Balasubramanian Kraus[1999]

Brown-York : Two modifications due to dx^+dx^-

- ADM form :

$$ds^2 = \left(\frac{r}{R}\right)^2 \left\{ -\frac{h}{1-h} b^{-2} dx^{+2} + (1-h) b^2 \left(dx^- + V^-\right)^2 + dy^2 + dz^2 \right\}$$
$$+ \left(\frac{R}{r}\right)^2 h^{-1} dr^2 \,, \qquad V^- = -\frac{1}{2b^2}\frac{1+h}{1-h} \,.$$

- Mass and Charge along x^- : direct computations

$$M = \int d^3x \sqrt{\sigma} (N\epsilon - V^a j_a) \,, \qquad J = \int d^3x \sqrt{\sigma} j_- \,.$$
$$N = \left(\frac{r}{R}\right) \sqrt{\frac{h}{1-h}}\, b^{-1} \,, \quad u_\mu = -N\delta_\mu^+ \,, \quad \epsilon = u_i u_j T^{ij} \quad \text{and} \quad j_a = -\sigma_{ai} u_j T^{ij} \,,$$

where γ_{ij} with $i, j \in \{+, -, y, z\}$ and σ_{ab} with $a, b \in \{-, y, z\}$,

Thermodynamics : Planar black hole in light-cone

$$S = \frac{r_H^3 V_3}{4 G_5 R^3} b \,, \qquad \beta = \frac{1}{T} = \frac{\pi R^2}{r_H} b \,, \qquad \Omega_H = \frac{1}{2b^2} \,,$$

$$M = \frac{r_H^4 V_3}{16\pi G_5 R^5} \,, \qquad J = -\frac{r_H^4 V_3}{4\pi G_5 R^5} b^2 \,.$$

HRR[2008], MMT[2008], Yamada[2009]

- Checking the first law of thermodynamics by explicit calculations!!
- Identical to the previous results of Schrödinger black hole.
- AdS in light-cone might be an alternative candidate for investigating Schrödinger holography!

Thermodynamics : R-charged Balck holes in Light-Cone

$$\Omega_H = \frac{1}{2b^2}\ , \quad M = \frac{V_3}{16\pi G_5 R^3}\left(r_0^2 + \frac{2}{3}k\sum_i q_i + |k|\frac{R^2}{4}\right)\ , \quad N_i = \frac{g_i V_3}{8\pi G_5 R^3}b\ , \quad \mu_i = \frac{g_i}{r_H^2 + q_i}b^{-1}$$

$$J = -\frac{V_3}{4\pi G_5 R^3}\left(r_0^2 + \frac{2}{3}k\sum_i q_i + |k|\frac{R^2}{4}\right)b^2\ , \quad S = \frac{r_H^3 H_H^{1/2} V_3}{4 G_5 R^3}b\ , \quad \beta = \frac{H_H^{1/2}}{Q_k}\frac{\pi R^2}{r_H}b\ ,$$

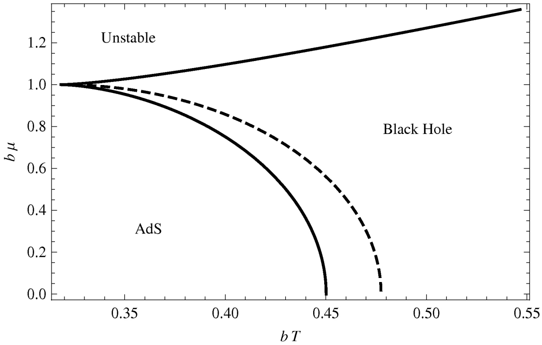
$$g_i = \sqrt{q_i(r_0^2 + kq_i)}\ , \quad Q_k := 1 + \frac{kR^2 + q_1 + q_2 + q_3}{2r_H^2} - \frac{q_1 q_2 q_3}{2r_H^6}\ .$$

• Thermodynamics and phase diagram of AdS R-charged black holes in light-cone with three independent charges are worked out for planar ($k = 0$), spherical ($k = 1$) and hyperbolic ($k = 0$) cases using the Brown-York.

• For $q_1 = q_2 = q_3 = q$, scalar fields decouple and the system becomes Einstein-Maxwell with cosmological constant. Thermodynamics of equal charge Scrödinger black hole was worked out in Imeroni Shina[2009], Adams Brown DeWolfe Rosen[2009].

• AdS in light-cone is as simple as the AdS case and the calculation is well defined and clear!

Phase Diagram : R-charged Black holes in Light-Cone



- Phase diagram of spherical BH ($k = +1$) in T-μ parameter space with charge $(q_1, q_2, q_3) = (q, 0, 0)$.
- The solid lines are the stability thresholds and the dashed curve is the Hawking-Page phase transition line.
- The curves merge and terminate at $(bT, b\mu) = (1/\pi, 1)$.

DC conductivity : Planar Schrödinger BH Ammon Hoyos O'Bannon Wu[2010]

- Probe $D7$ DBI Karch O'Bannon[2010] on $SchBH_5 \times S^5 \to$ embedding scalar $\theta(r)$.

- Gauge field :

$$A_+ = E_b\, y + h_+(r)\ , \quad A_- = 2\, b^2\, E_b\, y + h_-(r)\ , \quad A_y = A_y(r)\ ,$$

produces $F_{+y} = -E_b,\ F_{-y} = -2E_b b^2$, which is transformed to only electric field in the relativistic AdS case.

- Solve equation of motion in terms of three constants of motion $\langle J^+\rangle, \langle J^-\rangle, \langle J^y\rangle$ and plug them back into the DBI action. This onshell action is required to be real from the horizon to boundary. Karch O'Bannon[2010]

- $\langle J^-\rangle$ is proportional to $\langle J^+\rangle$ without dependence on E_b. There are two constraints that lead to a condition and conductivity through Ohm's law $\langle J^y\rangle = \sigma E_b$.

$$\begin{aligned} f(r_*) =& 4 r_*^2 b^2 E_b^2 [r_*^2 - b^2 f(r_*) \sin^2\theta(r_*)]\ , \\ \sigma_{sch} =& \sqrt{\frac{f(r_*)}{64 E_b^2 r_*^6} \cos^6\theta(r_*) + \frac{f(r_*)^2}{4 b^4 r_*^4 E_b^4} \langle J^+\rangle^2}\ . \end{aligned}$$

DC conductivity : Planar AdS BH in light-cone

• Aim : Want to compare the DC conductivity of planar AdS black hole in light-cone with that of planar Schrödinger BH.

$$\sigma_{lc} = 2\pi\alpha' \sqrt{\frac{\tilde{\mathcal{N}}^2 b^2 \cos^6\theta(r_*)}{16}\sqrt{4\tilde{E}_b^2 b^2 + R^4\pi^4 T^4 b^4} + \frac{4\langle J^+\rangle^2}{4\tilde{E}_b^2 b^2 + R^4\pi^4 T^4 b^4}}$$

- This result is different from Ammon Hoyos O'Bannon Wu(AHOW)[2010] .
- Yet, this result reproduces that of AHOW[2010] in the limiting cases, where the result is independent of $\theta(r)$. For example, $\tilde{E}_b \ll b(RT)^2$, $\theta(r_*) \approx \pi/2$,

$$\sigma_{lc} \approx \sigma_{sch} \approx 2\pi\alpha' \frac{2\langle J^+\rangle}{\pi^2 b^2 (RT)^2} \approx \frac{\langle J^+\rangle}{T}\frac{\Omega}{T} .$$

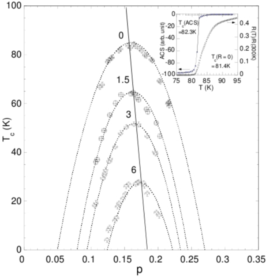
• Why different?
The relativistic background electric fields transform to our gauge field configuration with metric in light-cone, while NMT leads to a very different gravity solution than Schrödinger BH. Thus those two systems are, in fact, different!

• AdS in light-cone captures essential physics of transport properties independnet of embedding scalar.

Conclusions

- Thermodynamic and transport properties of AdS BH in light-cone is investigated and compared to those of Schrödinger BH.
- Thermodynamic quantities, calculated with Brown-York , are identical to available results of the Schrödinger BH. R-charged AdS BH in light-cone are also worked out.
- DC conductivity turns out to be identical to that of Schrödinger space when they are independent of embedding scalar.
- Thus, AdS in light-cone is an excellent candidate to investigate CM system with Schrödinger symmetry!

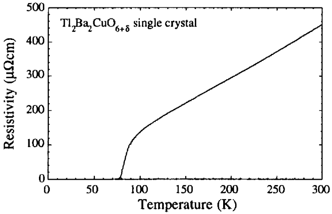
- Precise relation between AdS in light-cone and Schrödinger space is an important open problem, which might shed lights on holographic renormalization of Lifshitz as well as Schrödinger space.
- Holographic superconductor in the background of AdS in light-cone?
- Interesting results in the transport properties of AdS in light-cone! Next slide !

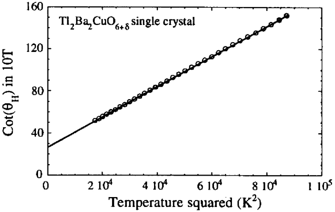


• Universal for several classes of unconventional superconductors, including YBCO, TBCO, LSCO, BSCCO, heavy fermion 115 family.

← S. H. Naqib et. al., Physica C 387, 365 (2003)

↓ A. W. Tyler and A. P. Mackenzie, Physica **C 282-287** (1997) 1185.





$\rho \sim T, \quad \cot\Theta_H \sim T^2$!

Work in progress ... [Kim, Kiritsis, Panagopoulos, Yamada]

- DC Resistivity, ρ, with:

$$A_+ = E_b y + h_+(r) \ , \quad A_- = 2b^2 E_b y + h_-(r) \ , \quad A_y = 2E_b b^2 x^- + h_y(r) \ ,$$

$$\rho = \frac{1}{\sigma} \approx \frac{\pi b \sqrt{\tilde{E}_b b}}{(2\pi\alpha')\langle J^+ \rangle} \ RT \ , \qquad \text{for} \qquad bR^2 T^2 \ll \tilde{E}_b = 2\pi\alpha' E_b \ .$$

Hartnoll Polchinski Silverstein Tong[2009], Ammon Hoyos O'Bannon Wu[2010], Charmousis Goutéraux Kim Kiritsis Meyer[2010], Lee Pang Park[2010]

- Inverse Hall Angle, $\cot\Theta_H$, with:

$$A_+ = E_b y + h_+(r), \quad A_- = 2b^2 E_b y + h_-(r), \quad A_y = 2b^2 E_b x^+ + h_y(r), \quad A_z = B_b y + h_z(r).$$

$$\cot\Theta_H = \frac{\sigma^{yy}}{\sigma^{yz}} \approx \frac{\pi^2 b^2 \sqrt{\left(\tilde{B}_b^2 + \tilde{E}_b^2 b^2\right)}}{\tilde{B}_b^2} \ R^2 T^2 \ ,$$

for $bR^2T^2 \ll \tilde{E}_b$ and $b^2R^2T^2 \ll \tilde{B}_b$.

- Comment : two limits $T \to 0$ and $B \to 0$ do not commute.