D-Brane Decay
and
Coulomb Gas Electrostatics

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arXiv:0911.0339, arXiv:1003.3663, arXiv:1008.4743

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* Thanks for template slides an figs

Outline

- Mapping D-brane decay to electrostatics
  - D-brane decay: open string worldsheet theory approach
  - Connection to Coulomb gas thermodynamics
  - Electrostatics and high energy string production
  - The potential problem
- Results

String emission by a decaying brane

- Basic time-dependent process in string theory
- This work: calculation of two and higher point (tree-level bosonic) string emission amplitudes from a decaying Dp-brane
- Open strings may be emitted parallel to the decaying brane
- Basic question: which states dominate?

Introduction to open string worldsheet approach

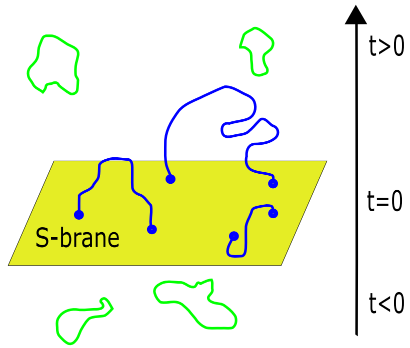
Two prototypes for the exactly marginal tachyonic deformation δS_{bdry}:

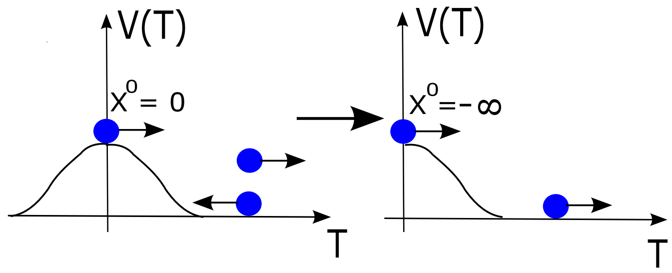
- $\lambda \oint_{\partial\Sigma} dt \cosh X^0(t)$ full S-brane

[Sen]

- $\lambda \oint_{\partial\Sigma} dt e^{X^0(t)}$ half S-brane

[Larsen-Naqvi-Terashima]





Worldsheet action for (bosonic) open strings

$$Z_{\mathrm{open}} = \int \mathcal{D}X e^{-S_P - \delta S_{\mathrm{bdry}}} = \int \mathcal{D}X e^{-\frac{1}{2\pi}\int_\Sigma \partial X \cdot \bar{\partial} X - \lambda \oint_{\partial\Sigma} dt e^{X^0(t)}}$$

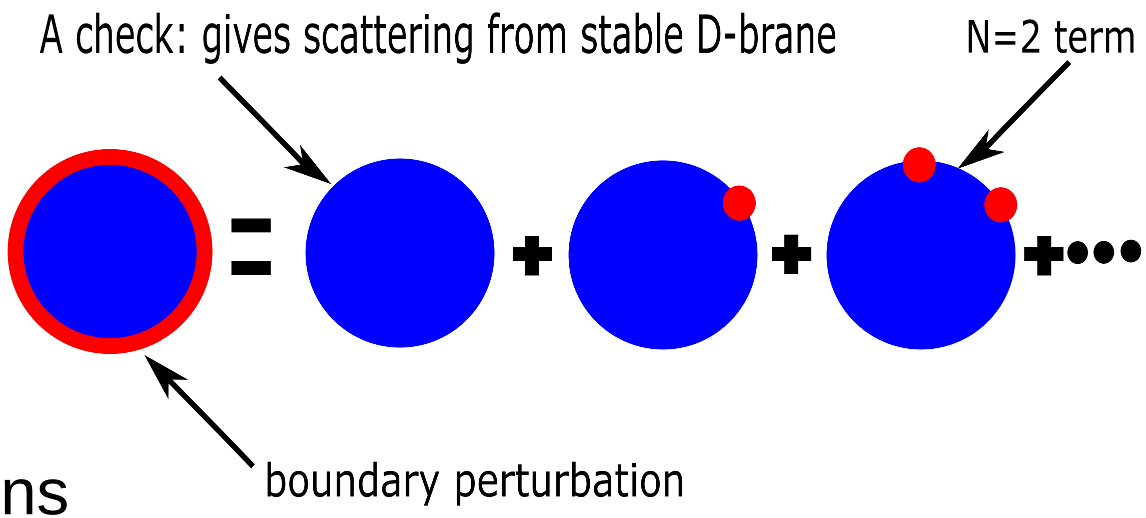
Direct calculation in the timelike theory

Straightforward calculational strategy (for half S-brane):

[Larsen-Naqvi-Terashima, Balasubramanian-KeskiVakkuri-Kraus-Naqvi]

① Isolate the zero mode: $X^0 = x^0 + X'^0(t)$, where x^0 is the target space time.

② Taylor expand in λ



③ Work out the contractions

④ Do the remaining integrals $\leftrightarrow$ relation to random matrix theory

⑤ Sum back the series

Direct calculation in the timelike theory

Emission amplitude for n_c closed, n_o open strings after ① ② ③:

$$A_{n_c+n_o} = \int dx^0 d^p \vec{x}^{\parallel} e^{i \sum_a k_a^\mu x_\mu} \bar{A}_{n_c+n_o}(x^0)$$

$$\bar{A}_{n_c+n_o}(x^0) = \sum_{N=0}^{\infty} (-z)^N \int \prod_{a=1}^{n_c} \frac{d^2 w_a}{2\pi} \prod_{a=n_c+1}^{n} \frac{d\tau_a}{2\pi} \prod_{1\le a<b\le n} |w_a - w_b|^{2\vec{k}_a^{\parallel}\cdot\vec{k}_b^{\parallel}}$$
$$\times \prod_{1\le a<b\le n_c} |w_a - w_b|^{-k_a^{\parallel}\cdot k_b^{\parallel} + \vec{k}_a^{\perp}\cdot\vec{k}_b^{\perp}} \prod_{a,b=1}^{n_c} |1 - w_a \bar{w}_b|^{\frac{1}{2}(k_a^{\parallel}\cdot k_b^{\parallel} - \vec{k}_a^{\perp}\cdot\vec{k}_b^{\perp})}$$
$$\times Z_{n_c+n_o}(\{w_a, k_a\}; N)$$

$$Z_{n_c+n_o}(\{w_a, k_a\}; N) = \frac{1}{N!} \prod_{1\le a<b\le n} |w_a - w_b|^{2\xi_a \xi_b}$$
$$\times \int \prod_{i=1}^{N} \frac{dt_i}{2\pi} \prod_{1\le i<j\le N} |e^{it_i} - e^{it_j}|^2 \prod_{a=1}^{n} |1 - w_a e^{-it_i}|^{2\xi_a}$$

- Infinite series of complicated coupled integrals. Difficult!

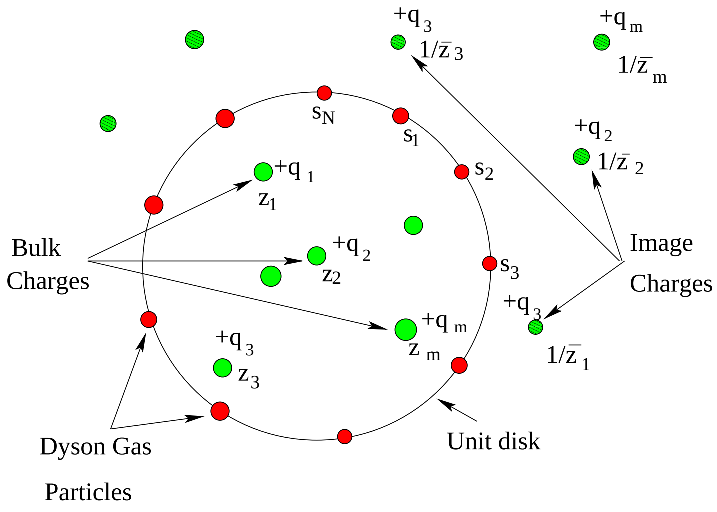
Direct calculation in the timelike theory

- Try to develop insight by studying the problem from various directions:
  - Expectation values in random matrix ensemble
  - Selberg's integral formulae
  - Toeplitz determinants
  - Thermodynamics of a classical Coulomb gas

Coulomb gas approach

Classical gas of N unit charges on unit circle

$$E = E_{\text{kin}} + \sum_{i<j} \underbrace{(-\log|e^{it_i} - e^{it_j}|)}_{=V}$$



Add external charges (ξ_a) at w_a

$$\begin{aligned} H(\{\xi_a\};N) &= -\sum_{1\le i<j\le N} \log|e^{it_i} - e^{it_j}| - \sum_{i=1}^{N}\sum_{a=1}^{n} \xi_a \log|e^{it_i} - w_a| \\ &\quad - \sum_{1\le a<b\le n} \xi_a\xi_b \log|w_a - w_b| \end{aligned}$$

Coulomb gas approach

Coulomb gas at $\beta = 2$ is the same as decaying brane

[Balasubramanian-KeskiVakkuri-Kraus-Naqvi]

$$\begin{aligned} A_{n_c+n_o} &= \int dx^0 \prod_a d^2 w_a d\tau_a \bar{\gamma}(x_o; \{w_a\}) \sum_{N=0}^{\infty} (-z)^N Z_{n_c+n_o}(\{\xi_a\}; N) \\ Z_{n_c+n_o}(\{\xi_a\}; N) &= \int \prod_i \frac{dt_i}{2\pi} \exp\left[-2H(\{\xi_a\}; N)\right] \end{aligned}$$

- Closed strings: external particles in the bulk with imaginary charge $\xi_a \leftrightarrow$ string energy ω_a $\;(\xi_a = -i\omega_a)$
- Open strings: external charges on the circle $w_a = e^{i\tau_a}$, $\xi_a = -i\omega_a$
- Also linked to the ensemble of random unitary matrices

Coulomb gas approach

Important observation: x_0 integral can be done first

[Zamolodchikov-Zamolodchikov, Balasubramanian-KeskiVakkuri-Kraus-Naqvi]

$$\int dx^0 e^{\sum_a \xi_a x^0} \sum_{N=0}^{\infty} (-z)^N Z_{n_c+n_o}(\{\xi_a\}; N)$$

$$= \quad \frac{\pi (2\pi\lambda)^{-\sum_a \xi_a}}{\sin\left(\pi \sum_a \xi_a\right)} Z_{n_c+n_o}\left(\{\xi_a\}; N = -\sum_a \xi_a\right)$$

- ❏ $\Rightarrow A_{n_c+n_o}$ is “proportional” to the canonical Coulomb gas partition function, continued analytically to complex N
- ❏ Proof by contour integration

[Jokela-MJ-KeskiVakkuri arXiv:0806.1491]

- ❏ Large $N \leftrightarrow$ large string energies ($\xi_a = -i\omega_a$)

Coulomb gas electrostatics

Large N expansion of Coulomb gas partition function

$$\log Z_{n_c+n_o}(N) = C_0 N^2 + C_1 N^1 + C_2 N^0 + \cdots$$

- $N = -\sum_a \xi_a = i\sum_a \omega_a \longrightarrow$ large ω expansion of (the logarithm of) the string emission amplitude
- Leading term is $C_0 N^2 = -2\mathcal{E}$, given by the (total) energy of the continuum limit of the gas at electrostatic equilibrium

$$\begin{aligned}\mathcal{E} &= -\frac{1}{2}\int dt_1 dt_2 \rho_0(t_1)\rho_0(t_2)\log\left|e^{it_1} - e^{it_2}\right| \\ &\quad -\sum_{a=1}^{n}\xi_a \int dt \rho_0(t)\log\left|e^{it} - w_a\right| - \sum_{1\le a<b\le n}\xi_a\xi_b \log|w_a - w_b|\end{aligned}$$

$\rho_0 =$ continuum charge density at equilibrium

Coulomb gas electrostatics

Conclusion: the leading (saddle-point) term of the emission amplitude at high energy reads

$$A_{n_c+n_o} = \frac{\pi(2\pi\lambda)^{-\sum_a \xi_a}}{\sin\left(\pi \sum_a \xi_a\right)} \int (dw d\bar{w} d\tau)\gamma(w,\bar{w},\tau) \times \exp\left[-2\,\mathcal{E}\big|_{N=-\sum_a \xi_a}\right]$$

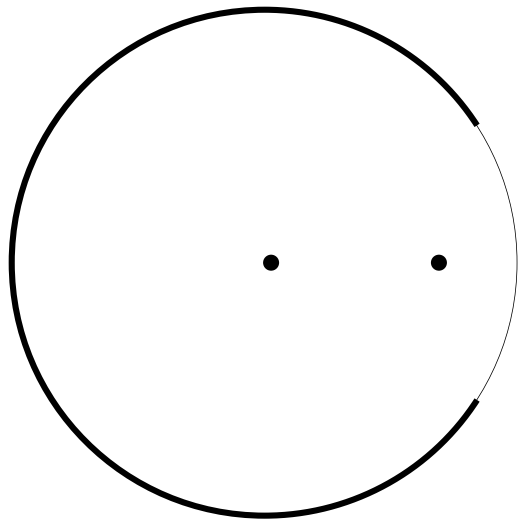
where γ is a known elementary function

Remaining task: solve the potential problem, necessary to evaluate $\mathcal{E}$

Potential problem: gaps

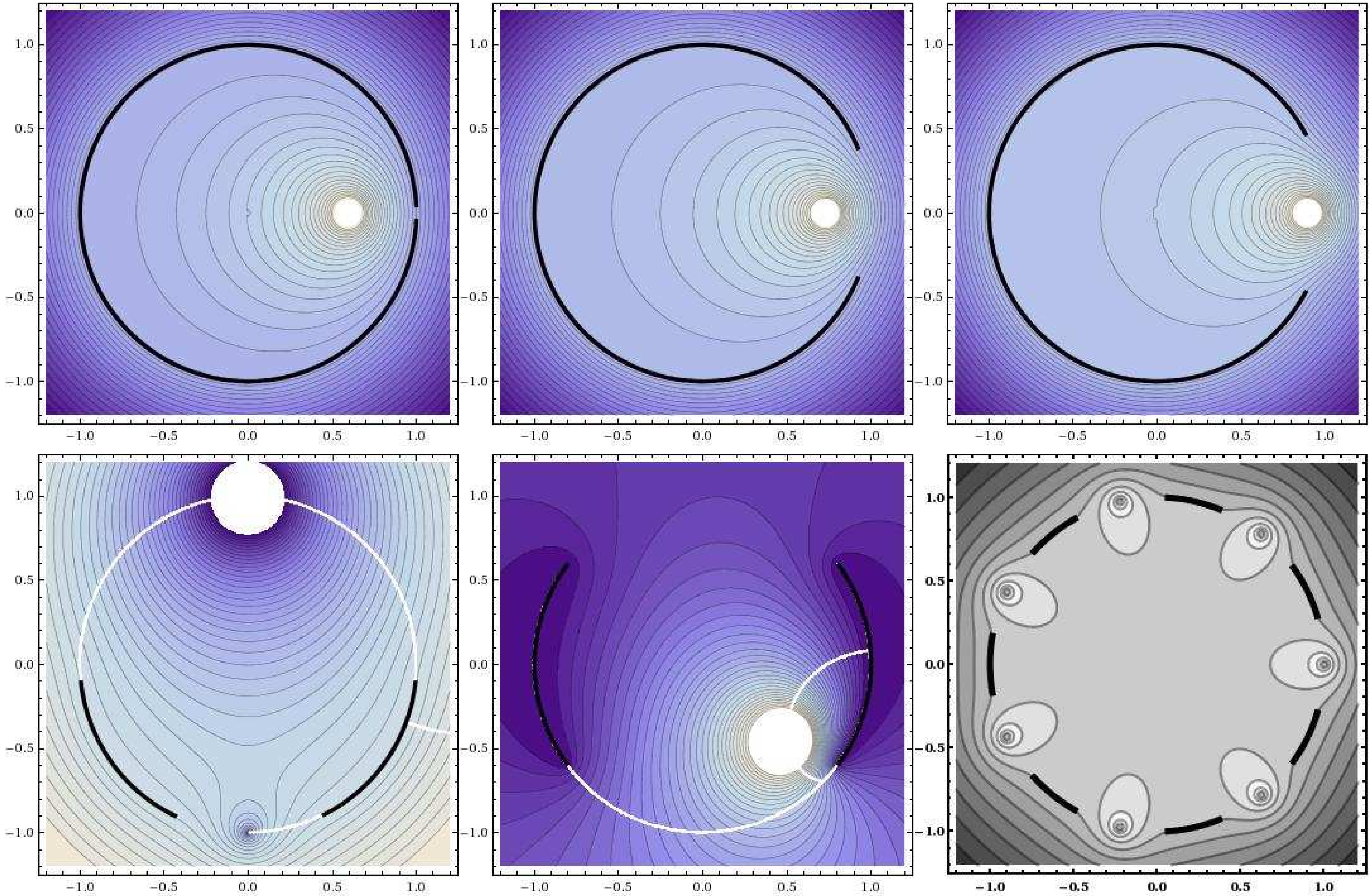
Start with large positive external charges ξ_a

Coulomb gas includes only positive (unit) charge carries $\Rightarrow$ gaps in the vicinity of external charges



$\Rightarrow$ solutions of potential problem with conducting arcs and gaps on the unit circle required

Potential problem: solutions



Application to string emission

Calculate the total energy $\mathcal{E}$, continue analytically to imaginary $\xi_a = i\omega_a \Rightarrow$ results for high energy string emission

Miraculously, amplitudes stay bounded at high energies

- We checked the results we were able to derive case by case: amplitudes vanish for high energies in the kinematically allowed region (and blow up basically everywhere else)
- Not obvious from any of the intermediate steps

Results

① Emission of two closed strings, having equal energies

$$A_2 \simeq -\frac{i\pi e^{i\frac{\pi s}{2}}(2\pi\lambda)^{2i\omega}}{2\sinh 2\pi\omega}\left[e^{i\frac{\pi u}{2}}\frac{\Gamma\left(\frac{t}{2}-1\right)}{\Gamma\left(\frac{t+u}{2}-2\right)}+\frac{\Gamma\left(\frac{s}{2}+1\right)}{\Gamma\left(\frac{s+u}{2}\right)}\right]\Gamma\left(\frac{u}{2}-1\right)$$

- ❍ Pole structure ok
- ❍ At nonequal energies total mess, not exchange symmetric, conformal invariance broken??

② Emission of a closed and an open string

$$A_{1+1}(\omega_o,\omega_c) \simeq \frac{i\pi\left(2\pi\lambda\right)^{i(\omega_c+\omega_o)}}{\sinh\pi\left(\omega_c+\omega_o\right)}\exp\left[\omega_o^2\log\frac{\omega_o}{\omega_c+\omega_o}+\omega_c^2\log\frac{\omega_c}{\omega_c+\omega_o}\right.$$
$$\left.-\frac{1}{2}\left(\omega_c-\omega_o\right)^2\log\frac{\omega_c-\omega_o}{\omega_c+\omega_o}+\omega_o^2\left(2\log 2\pm i\pi\right)\right]$$

- ❍ Agrees with the known exact result [Balasubramanian-KeskiVakkuri-Kraus-Naqvi]

Results

③ Emission of two open strings (saddle point)

$$A_{0+2}(\omega) \simeq \frac{i\pi\,(2\pi\lambda)^{2i\omega}}{\sinh 2\pi\omega}\, e^{\pm i\pi\omega^2}$$

- Agrees with previously suggested asymptotics [Gutperle-Strominger]

④ Emission of one closed and two open strings

$$\begin{aligned}
A_{1+2}(\omega_c,\omega_1,\omega_2) &= \frac{i\pi\,(2\pi\lambda)^{i\sum_a \omega_a}}{\sinh\left(\pi\sum_a \omega_a\right)} \\
&\quad \times \exp\left\{(\omega_1+\omega_2+\omega_c)^2\left[\Xi(\alpha_1,\alpha_2,\phi)\right]\right\} \times \text{phase} \\
\Xi(\alpha_1,\alpha_2,\phi) &= -\frac{1}{2}\left[F(|1-2\alpha_1-2\alpha_2|)+F(1-2\alpha_1)+F(1-2\alpha_2)\right] \\
&\quad +\frac{1}{2}\left[F(2\alpha_1+2\alpha_2)+F(2\alpha_1)+F(2\alpha_2)\right] \\
&\quad -2F(\alpha_1+\alpha_2)+2F(1-\alpha_1-\alpha_2)-2\,(1-\cos\phi)\,\alpha_1\alpha_2\log 2
\end{aligned}$$

$$\alpha_a = \omega_a/\textstyle\sum_b \omega_b; \quad F(x) = (x^2\log x)/2$$

- Locations of vertex operators fixed in a particular way

Conclusions

- We developed a relation between (tree level) brane decay and electrostatics of Coulomb gas
- We solved the high energy amplitudes for emission of a few strings in the half S-brane background
  - Amplitudes vanish at high energies as $\sim \exp\left[-\pi \sum_a \omega_a\right]$ or faster, as expected
- Generalizations to full S-brane and superstrings, as well as appications to other systems possible