Correlation functions of operators dual to classical strings

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$\longrightarrow$ RJ, P. Surówka, A. Wereszczyński: 1002.4613
Related talks by A. Tseytlin, M. Costa...

Main questions for a CFT

- Find the spectrum of conformal weights
  $\equiv$ eigenvalues of the dilatation operator
  $\equiv$ (anomalous) dimensions of operators

$$\langle O(0)O(x)\rangle = \frac{1}{|x|^{2\Delta}}$$

- Find the OPE coefficients C_{ijk} defined through

$$\langle O_i(x_1)O_j(x_2)O_k(x_3)\rangle = \frac{C_{ijk}}{|x_1-x_2|^{\Delta_i+\Delta_j-\Delta_k}|x_1-x_3|^{\Delta_i+\Delta_k-\Delta_j}|x_2-x_3|^{\Delta_j+\Delta_k-\Delta_i}}$$

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Anomalous dimensions of operators	$\equiv$	Energies of the corresponding string states in $AdS_5 \times S^5$

- One computes directly energies of string states
- Use integrability... $\longrightarrow$ lots of information... see the lecture of V. Kazakov
- But on the gauge theory side there is also an alternative (and equivalent) way using 2-point correlation functions

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- This is well understood *only* for operators dual to supergravity fields ($\equiv$ massless string states) – there one uses Green's functions of the corresponding fields
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Ultimate goal: Develop methods for computing correlation functions of operators corresponding to *massive* string states

- This is certainly very difficult for generic string states
- We will consider classical string states — spinning strings in $AdS_5 \times S^5$
- For these states, correlation functions should be accessible by a classical computation
- In this work we concentrated mainly on 2-point functions, for which we know the result
$$\langle O(0) O(x) \rangle = \frac{1}{|x|^{2\Delta}}$$
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Classical spinning strings

- The string action in $AdS_5 \times S^5$ has the form

$$S_{string} = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \, (\text{Polyakov action})$$

- Spinning strings are classical solutions at strong coupling with nonzero angular momenta (of order $\sqrt{\lambda}$).
- These solutions looks generically like a rotating string in the center of $AdS_5 \times S^5$ — very far from the boundary
- On the gauge theory side this string configuration corresponds to a 'long' operator composed of very many fields
- The energy is a function of the angular momentae

$$E = \Delta = \sqrt{\lambda}\, F(J_i, \ldots)$$

- The corresponding two point correlation function should be equal to

$$\langle O(0)O(x)\rangle = \frac{const.}{|x|^{2\Delta}} \sim e^{-2\sqrt{\lambda}\cdot F(J_i,\ldots)\cdot \log|x|}$$

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Two point correlation functions

- In the AdS/CFT correspondence the prescription for two point functions involves essentially the Green's function of the corresponding field

- We expect to have, for each spinning string solution and any two arbitrary points on the boundary, a **new** classical string solution which should determine the two point correlation function giving the same anomalous dimension
- Proceed first to the analog of an ordinary point particle exchange...

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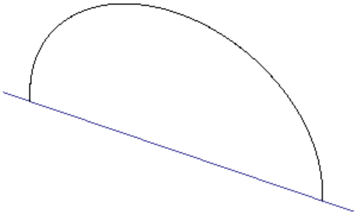

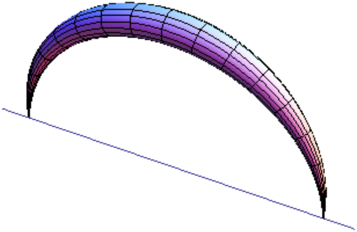

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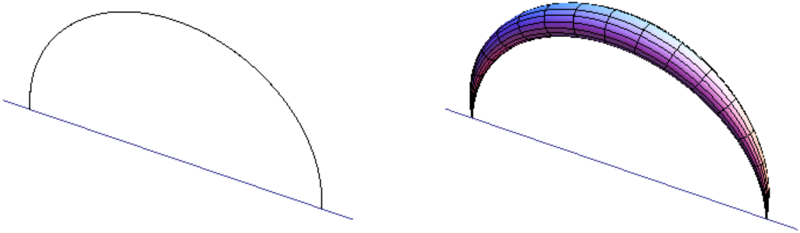

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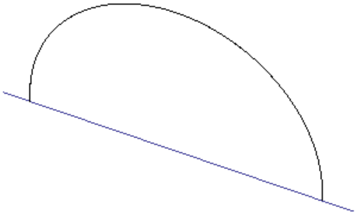

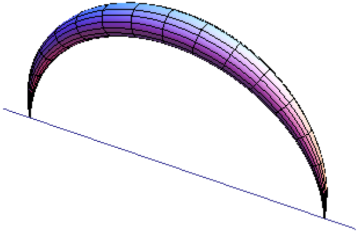

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A point particle example — flat space

- The Green's function $G(x,y)$ for a particle of mass m can be found by evaluating the (Polyakov) worldline action leading to

$$\int_0^\infty ds \int [dx^\mu](measure) \exp\left(-\frac{1}{2}\int_0^s \left(\dot{x}^2 + m^2\right) dt\right)$$

- Evaluate by saddle point:
  i) use $x^\mu(t) = (y^\mu - x^\mu)t/s + x^\mu$ giving

$$S_P = \frac{1}{2}\left(\frac{|x-y|^2}{s} + m^2 s\right)$$

  ii) perform the saddle point w.r.t. the modular parameter s

$$G(x,y) \sim e^{-m|x-y|}$$

- The standard scalar $G(x,y)$ can be obtained by evaluating the path integral exactly [Cohen, Moore, Nelson, Polchinski]

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String cylinder amplitude — flat space

- The string analog of the preceeding setup is a *cylinder amplitude*
- A calculation in flat space in Euclidean signature (with pointlike boundary conditions) was performed by Cohen, Moore, Nelson, Polchinski

$$\int_0^\infty \frac{ds}{s^{13}} \underbrace{e^{4\pi s} \prod (1-e^{-4\pi n s})^{-24}}_{\textit{fluctuation determinant}} \cdot \underbrace{e^{-\frac{(\Delta x)^2}{4\pi s}}}_{e^{-S_P(\Delta x)}}$$

- This expression can be directly rewritten in terms of Green's functions of the intermediate string states

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$$x^1 + ix^2 = a_1 \sin n_1 \sigma e^{in_1 \tau} \qquad x^3 + ix^4 = a_2 \sin n_2(\sigma + \sigma_0) e^{in_2 \tau}$$

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- Problem: It is not a solution of Euclidean equations of motion!
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- How to reconcile this with the standard path integral treatment???

Essentially the same apparent problem appears in ordinary quantum mechanics..

- A state with definite energy evolves in time with the phase factor e^{-iET}. For a classical state $E \sim E_{class}$ with E_{class} being the energy of the classical trajectory
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Semiclassical propagator revisited

- We have to implement convolution with an initial semi-classical (WKB) wavefunction

$$\int dx_i\, \Psi(x_i) \cdot e^{iS_{class}[x_i,x_f,T]} = \int dx_i\, e^{i\int^{x_i} p(x)dx} \cdot e^{iS_{class}[x_i,x_f,T]}$$

- Evaluate the x_i integral by saddle point

$$p(x_i) + \frac{\partial S_{class}[x_i,x_f,T]}{\partial x_i} = p(x_i) - p = 0$$

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Prescription for the Green's function of a classical string state

- Compute the cylinder amplitude with modular parameter s by finding a suitable solution of the classical equations of motion
- Implement projection on the wavefunction by the additional factor

$$\exp(iS_{class}[in, out, s]) \cdot \exp\left(-i \int d\sigma d\tau\, (\pi - \pi_0) \cdot (\dot{x} - \dot{x}_0)\right)$$

where π_0 and $\dot{x}_0$ are the zero mode parts of the canonical momentum and velocity i.e.

$$\pi_0(\tau) \equiv \frac{1}{2\pi} \int_0^{2\pi} d\sigma\, \pi(\tau, \sigma) \qquad \dot{x}_0(\tau) \equiv \frac{1}{2\pi} \int_0^{2\pi} d\sigma\, \dot{x}(\tau, \sigma)$$

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Rotating string in S^5

- The Polyakov action for a cylinder takes the form

$$S_P = -\frac{\sqrt{\lambda}}{4\pi} \int_{-\frac{s}{2}}^{\frac{s}{2}} d\tau \int_0^{2\pi} d\sigma \left\{ -\frac{\dot{x}^2 + \dot{z}^2}{z^2} + S^5 \text{ part} \right\}$$

- The classical solution for $x(\tau), z(\tau)$ will be as for the point particle described earlier with the S^5 part identical to a conventional spinning string
- Evaluating the action for the simplest circular string in S^5 and implementing the wavefunction projectors yields

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Rotating strings in AdS_5

- Here we face a couple of difficulties...
- The rotation of the string interferes with the bending of the string necessary for the classical solution to approach two given points on the boundary
- *A-priori* it seems to be difficult to find such solutions...
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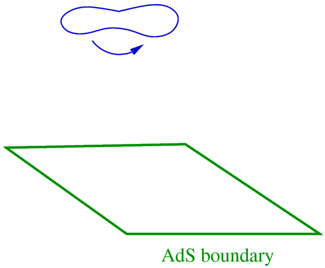
Original spinning string in the center of AdS_5

In global ('embedding') coordinates:

$$
\begin{aligned}
Y_0 &\propto \sin \kappa t \\
Y_i &= \ldots \\
Y_5 &\propto \cos \kappa t
\end{aligned}
$$



A spinning string emanating from the boundary and propagating into the bulk

Substitute:

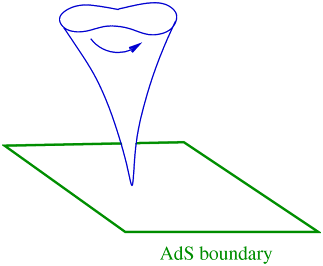
$$Y_0 \leftrightarrow iY_4 \qquad \kappa \to i\kappa$$

this exchanges

$$iD \leftrightarrow \frac{1}{2}(P_0 + K_0)$$

As a result

$$z = e^{\kappa t}$$



A spinning string approaching two given points on the boundary

Perform a special conformal transformation

$$x^\mu \to \frac{x^\mu + b^\mu(x^2+z^2)}{1+2xb+b^2(x^2+z^2)}$$
$$z \to \frac{z}{1+2xb+b^2(x^2+z^2)}$$

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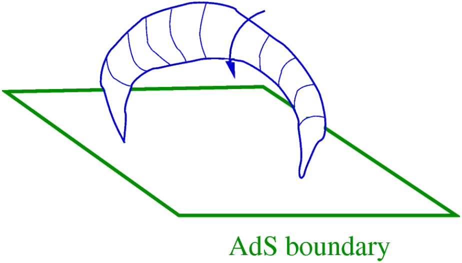

Rotating strings in AdS_5

- Since the string motion is in the same space as the arguments of the correlation function, one has to decouple the zero mode

  recall

$$\exp(iS_{class}[in, out, s]) \cdot \exp\left(-i \int d\sigma d\tau \, (\pi - \pi_0) \cdot (\dot{x} - \dot{x}_0)\right)$$

- However in curved spacetime this notion is ambiguous and depends on the coordinate system!
- We found that *one* natural choice exists which is compatible with the $so(2,4)$ symmetry of AdS_5:

$$-Y_0^2 + Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 - Y_5^2 = -1$$

- We use the Y_A coordinates to define the zero modes
- With these choices, we found for a couple of examples that one recovers the correct two point correlation functions

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Three point correlation functions

- The main point of the two point correlation function computation was to define how does the string solution look like close to the operator insertion point
- We obtain a formulation of a three point correlation function:
  - a classical solution with the topology of a punctured sphere
  - the asymptotics close to each puncture can be read off from the two point correlation function computation
  - Moreover one has to deal with the wavefunction projectors
- Still a very difficult (but interesting) problem...

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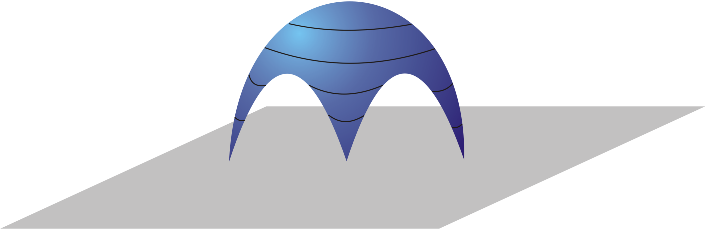

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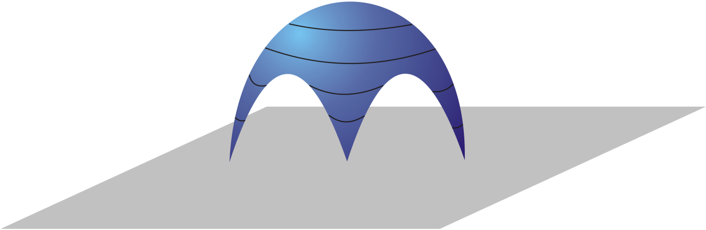

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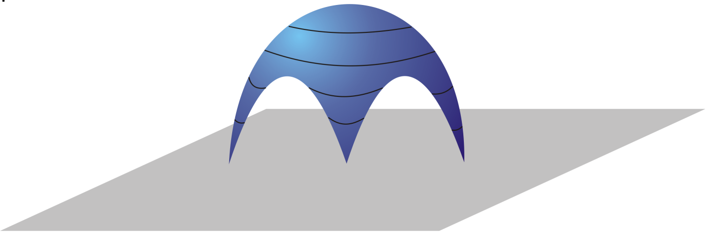

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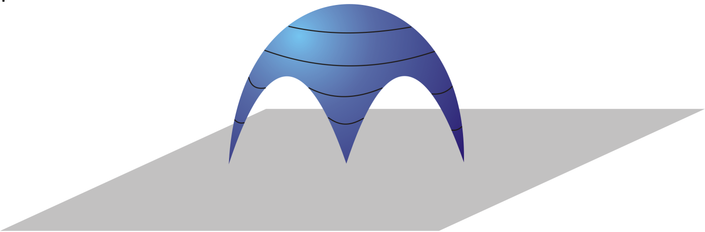

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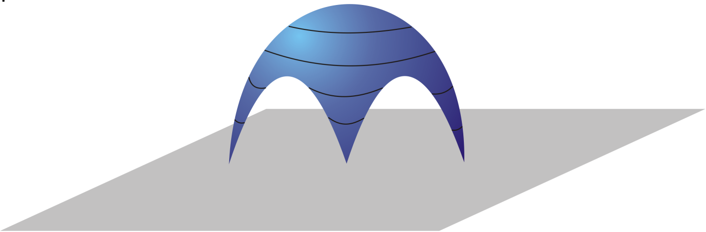

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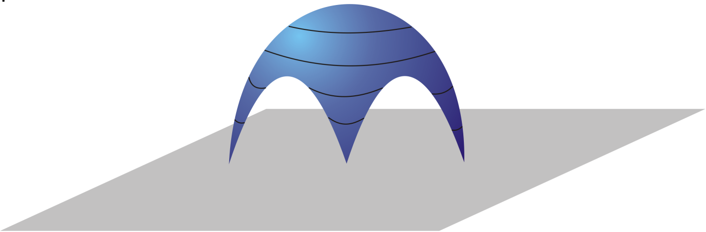

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- Numerous open problems
- Issues of the existence of a solution relevant for the three point function with given external operators
- Interrelations with the integrability of the classical string in $AdS_5 \times S^5$
- Analog of the algebraic curve construction of spinning strings??
- More generally — investigate integrable quantum field theories on various Riemann surfaces:
  - Cylinder — the spectrum
  - Disc (or plane) — Wilson loops/scattering amplitudes
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  - Higher genus — $1/N$ corrections (?)

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