Recent Advances in 3d Holography

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Outline

- Review
- Regimes of large N gauge theories
- S^3 partition functions
- AdS_4 generalizations
- Cascades and s-rule

Motivations

- Understanding M-theory
- Physics of 3d quantum field theory
- CFT duals for the Landscape
- New exactly solvable theory
- Condensed matter applications

Last year's outlook

- Complete the analysis of $\mathcal{N}=2$ CSM duals to M2 branes on CY_4. Explore $\mathcal{N}=1$ theories, for example in the large N_f limit.
- Duals for landscape IIA vacua.
- Learn about the strong coupling limit of massive IIA.
- Explain $N^{3/2}$ - derive the $1/\sqrt{\lambda}$ suppression of dofs.
- More tests of integrability.
- Further explore the connection to 2+1 condensed matter systems.

Learn something new about M-theory?

M2 branes

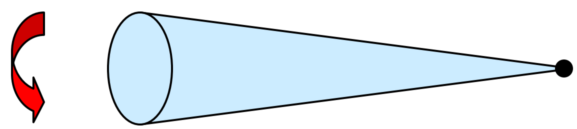
- The conformal field theory on M2 branes is the infrared limit of the $\mathcal{N} = 8$, 2+1 dimensional U(N) Yang-Mills theory of N D2-branes in flat space. The YM coupling in 3d is dimensionful, and the theory becomes strongly coupled in the IR. The IR R-symmetries that control the dimensions of operators are not manifest in the UV.

- The lack of adjustable coupling in M-theory suggested that the IR CFT had no Lagrangian description. Moreover, the black D2 brane has no smooth near horizon region in 10d SUGRA.

Why is there an M2 brane Lagrangian?

A background in which they become weakly coupled was found, due to the presence of a small circle.

$\theta = 2\pi/k$



Moreover, reducing to IIA string theory along the natural U(1) isometry results in a background in which the black D2 brane solution has a smooth AdS near horizon limit.

Black D2/M2 branes

- The black D2 gravity solution in asymptotically flat space does not have a smooth AdS near horizon limit.
- The string coupling blows up near the D2, so one lifts to M-theory. The black M2 solution in 11d has a smooth $AdS_4 \times S^7$ near horizon geometry with N units of flux in AdS.
- The effective worldvolume gauge theory on N D2 branes is the $N = 8$ super Yang-Mills with dimensionful coupling.

A different IIA reduction

- The $AdS_4 \times S^7/Z_k$ has a natural U(1) isometry, associated to the description of S^7 as a S^1 bundle over CP^3. In the 't Hooft limit one gets IIA on $AdS_4 \times CP^3$ with k units of F_2 flux.

[Nilsson Pope, Volkov Sorokin Tkach]

- This extends to the entire black M2 solution. This gives a background of IIA, with varying dilaton and F_2 flux, in which the black D2 **does** have a smooth near horizon.

- The string coupling is small if k is large.

$$(Nk)^{3/2}/k = \frac{N^2}{(N/k)^{1/2}}$$

Chern-Simons-matter theory

- We first consider the case with N=2 susy. It consists of a vector multiplet in the adjoint of the gauge group, and chiral multiplets in representations R_i

$$S_{CS}^{\mathcal{N}=2} = \frac{k}{4\pi}\int(A\wedge dA + \frac{2}{3}A^3 - \bar{\chi}\chi + 2D\sigma)$$

- The kinetic term for the chiral multiplets includes couplings $-\bar{\phi}_i\sigma^2\phi_i - \bar{\psi}_i\sigma\psi_i$

- There is the usual D term $\bar{\phi}_i D\phi_i$

We integrate out D, σ, and χ

$$S^{\mathcal{N}=2} = \int \frac{k}{4\pi}(A \wedge dA + \frac{2}{3}A^3) + D_\mu \bar{\phi}_i D^\mu \phi_i + i\bar{\psi}_i \gamma^\mu D_\mu \psi_i$$

$$-\frac{16\pi^2}{k^2}(\bar{\phi}_i T^a_{R_i}\phi_i)(\bar{\phi}_j T^b_{R_j}\phi_j)(\bar{\phi}_k T^a_{R_k} T^b_{R_k}\phi_k) - \frac{4\pi}{k}(\bar{\phi}_i T^a_{R_i}\phi_i)(\bar{\psi}_j T^a_{R_j}\psi_j)$$

$$-\frac{8\pi}{k}(\bar{\psi}_i T^a_{R_i}\phi_i)(\bar{\phi}_j T^a_{R_j}\psi_j).$$

Note that this action has classically marginal couplings. It is has been argued that it does not renormalize, up to shift of k, and so is a CFT.

N=3 CS-matter

- To obtain a more supersymmetric theory, begin with N=4 YM-matter. Then add the CS term, breaking to N=3.

- Thus we add an adjoint chiral multiplet, φ ,with no kinetic term in the CS limit, and the matter chiral multiplets, $\Phi_i, \tilde{\Phi}_i$, which must come in pairs.

- There is a superpotential, $W = -\frac{k}{8\pi} Tr(\varphi^2)$, needed to supersymmetrize the CS term.

- Integrating out φ one obtains the same action as before, but with a superpotential:

$$W = \frac{4\pi}{k}(\tilde{\Phi}_i T^a_{R_i} \Phi_i)(\tilde{\Phi}_j T^a_{R_j} \Phi_j)$$

- These N=3 theories are completely rigid, and hence superconformal. It is impossible to have more supersymmetry in a YM-CS-matter theory, but for particular choices of gauge groups and matter representations, the pure CSM can have enhanced supersymmetry.

[Zupnik, Khetselius, Kao, Lee, Lee, Schwarz, Gaiotto, Yin]

The N=6 CSM theory of N M2 branes in C^4/Z_k

- $U(N)_k$ x $U(N)_{-k}$ CSM with a pair of bifundamental hypermultiplets

[Aharony, Bergman, Maldacena, DLJ]

- Field content:

$$A_\mu, \tilde{A}_\mu \quad \text{gauge fields}$$
$$C_I, \psi^I \text{ in } (N, \bar{N}) \quad \text{matter fields}$$
$$(C_I)^*, (\psi^I)^* \text{ in } (\bar{N}, N) \quad \text{their conjugates}$$

$$C^I = (A_a, B^*_{\dot{a}}).$$

$$W = \frac{2\pi}{k}\epsilon_{ab}\epsilon_{\dot{a}\dot{b}}(A_a B_{\dot{a}} A_b B_{\dot{b}})$$

- SU(2) x SU(2) global symmetry, which does *not* commute with $SO(3)_R$, combining to form $SU(4)_R$

[ABJM, Benna, Klebanov, Klose, Smedback, Bandres, Lipstein, Schwarz, Schanbl, Tachikawa]

‘t Hooft Limit

- The gauge theory coupling is 1/k. Fix
  $\lambda = N/k,\ N \to \infty$
- Perhaps disappointingly, but unsurprisingly, the usual ‘t Hooft limit is a string theory.
- One obtains IIA on $AdS_4 \times CP^3$ with N units of F_4 and k units of F_2 in CP^3

$$R_{str}^2 = 2^{5/2}\pi\sqrt{\lambda + \left\{-\frac{1}{24}\left(1 - \frac{1}{k^2}\right) + \frac{\ell^2}{2k^2}\right\}}$$

Higher order curvature correction

[Bergman Hirano]

$$g_{IIA} \sim \frac{\lambda^{1/4}}{k}$$

Why is CSM a theory of M2 branes?

- There is an extra circle which emerges only at strong coupling, $N \gg k^5$, due to the monopole operators.

- If one gives an "eigenvalue" a VEV, $C_I = \begin{pmatrix} v & 0 \\ 0 & 0 \end{pmatrix}$, so one of the M2 branes is at distance $R = \ell_P^{3/2} v$, then the mass of the off-diagonal modes scales like
$$\frac{1}{k} v^2 = \frac{R}{k} \ell_P^{-3}$$
This is the area of a cone, rather than a length, as expected from a wrapped M2.

[Mukhi Papageorgakis, Lambert Tong, Distler Mukhi Papageorgakis van Raamsdonk, Berenstein Trancanelli]

Two natural generalizations

- Find other 7d conical backgrounds in IIA with vanishing dilaton at the origin, in which the black D2 brane will have a smooth near horizon.
- Most natural method for M2 branes on susy 8-manifolds.
- Marginally deform the CFT, or follow a relevant operator to a new fixed point, and identify the dual geometry.
- Typically gives vacua with fluxes on the internal manifold.

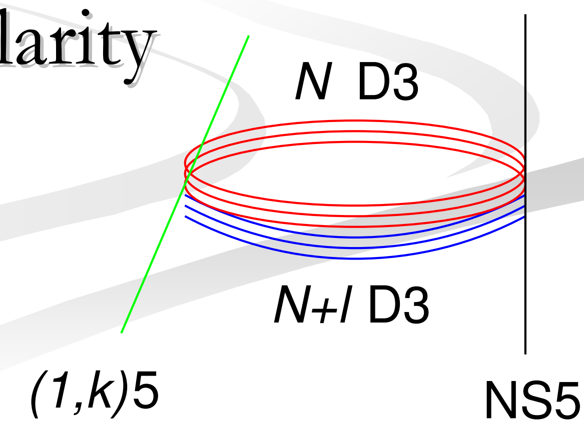
Fractional M2 branes

- One can also consider the $U(N+\ell)_k \times U(N)_{-k}$ CSM theory. This retains $\mathcal{N}=6$ supersymmetry.

  [Hosomichi Lee Lee Lee Park, Aharony Bergman DLJ]

- In the D-brane construction, it corresponds to unequal numbers of stretched D3 branes.
- In M-theory, one obtains N M2 branes, together with ℓ fractional M2 at the singularity

  [Aharony Bergman DLJ]



B-field in IIA

- In the IIA near horizon limit this corresponds to turning on the B field in the internal space.
- Roughly speaking, this is the reduction of the flat C-field. It doesn't affect the equations of motion, but does shift the quantization condition for F_4.
- Since supersymmetry requires that F_4 vanishes, B is dynamically quantized in units of 1/k J.
- Appears to be a mysterious shift by ½.

[Aharony Hashimoto Hirano Ouyang]

Massive IIA

- Consider deforming the N=6 CSM theory by the addition of a level a CS term for the second gauge group. $U(N)_k \times U(N)_{-k+a}$

- In this theory the monopole operators corresponding to D0 branes develop a tadpole, since the induced electric charge (k, a-k) cannot be cancelled with the matter fields.

- This motivates the idea that the total CS level should be related to the F_0 flux.

[Gaiotto Tomasiello, Fujita Li Ryu Takayanagi, Petrini Zaffaroni]

- The light U(1) on the moduli space has a level *a* Chern-Simons term, matching the coupling of the D2 worldvolume to the Romans mass.

$$kCS(A_1) + (a-k)CS(A_2) + |X|^2(A_1 - A_2)^2$$

- For such deformations of *N*=6 CSM, there are field theories with *N* = 3,2,1,0 differing by the breaking of the SU(4) into flavor and R-symmetry. Still have the topology $\mathrm{AdS}_4 \times \mathbb{CP}^3$

[Tomasiello; Gaiotto Tomasiello]

Monopoles and a new regime

- Three dimensional gauge theories have disorder operators that create magnetic vortices. Their dimensions can become small at strong coupling.
- Amazingly reflected in the lift of strongly coupled IIA to M-theory.

$1/k$ small N fixed	$\lambda = N/k$ small $N \to \infty$	λ large $N \to \infty$	k fixed $N \to \infty$
Perturbative CSM	Weakly coupled planar limit	Semi-classical IIA gravity	11d M-theory gravity limit

Regimes of general large N 3d U(N) gauge theories

$1 << N << k$	't Hooft limit N/k fixed	k fixed, $N \to \infty$
Deformations of Vasiliev higher spin gauge theory	Weakly coupled strings. Can be weakly or strongly curved.	Results in M-theory or again weakly coupled strings!

- Open question: how can one define large N intrinsically?

$$(M_{pl}/M_{AdS})^2 \sim \text{d.o.f.} \sim N^{3/2}k^{1/2}$$

‘t Hooft limit

- The string coupling always becomes weak.

$$g_s \sim \lambda/N \text{ in AdS}_5 \text{ and } g_s \sim \lambda^{5/4}/N \text{ in AdS}_4$$

- When λ is small, always have large curvature in string units. When λ is large, it depends – for example $\mathcal{N}$=2 $U(N)_k$ with g > 2 adjoints has a stringy dual.

- Open question: how can one tell given a CFT?

Weakly coupled planar limit

- Should be dual to the very weakly interacting limit of string theory in a highly curved background.

- Extreme example is the singlet sector of a free theory of N fields. This is dual to the Vasiliev's higher spin gauge theory. The massless higher spin fields correspond to the higher spin currents of the (trivially) integrable free theory.

[Klebanov Polyakov; Sezgin Sundell; Giombi Yin; …]

Vasiliev theory

- In the large N limit, it seems that the (M_{AdS}/M_{pl}) perturbative expansion of the nonlocal Vasiliev theory should be a change of variables from the 1/N expansion in the boundary theory, with a huge gauge redundancy. This is further suggested by the existence of a gauge in which the dependence on AdS drops out.

- On the other hand, can one engineer it in string theory?

Strict large N limit

- In the N=6 theory, this limit results in light monopole operators corresponding to the light D0 branes of IIA at strong coupling. There is an M-theory sugra description – which does not allow us to calculate much beyond supergravity.

- Not the only possible behavior: one can instead obtain weakly interacting strings again!

A new weakly coupled string regime

- Consider the massive IIA solution dual to $U(N)_k \times U(N)_{-k+a}$

$$R_{str} \sim \left(\frac{N}{a}\right)^{1/6}$$

$$g_s \sim \frac{1}{(Na^5)^{1/6}} \qquad \frac{1}{G_N} \sim N^{5/3} a^{1/3}$$

- This is in spite of the fact that the *N*=2 theory has light monopole operators.
- It would be interesting to understand the general behavior.

[Aharony DJ Tomasiello Zaffaroni]

Partition functions on S^3

- This partition function of the Euclidean theory is given in classical supergravity by minus the Euclidean Einstein action of the AdS.

$$S = -\frac{1}{16\pi G_N}\int d^4x\sqrt{g}(R-2\Lambda) + S_{surf} + S_{ct} = \frac{\pi}{2G_N}$$

[Balasubramanian, Emparan, Johnson, Kraus, Larsen, Myers, Siebelink,Taylor, …]

- The surface term is an integral of the extrinsic curvature of the boundary, the last term is a higher derivative boundary counterterm.
- Bulk CS terms would lead to an imaginary part.

Localization

[Kapustin Willett Yaakov]

- There is a holomorphic supersymmetry preserved on S^3, associated with a Killing spinor. $\nabla_\mu \epsilon = \frac{i}{2}\gamma_\mu \epsilon$

- Adding a Q-exact term to the action thus does not change the path integral, and, in the limit of a large coefficient, localizes the theory to configurations with vanishing fermion variations. The 1-loop saddle-point approximation becomes exact.

$$\delta A_\mu = -\frac{i}{2}\lambda^\dagger \gamma_\mu \epsilon, \quad \delta\sigma = -\frac{1}{2}\lambda^\dagger \epsilon, \quad \delta D = -\frac{i}{2}(D_\mu \lambda^\dagger)\gamma^\mu \epsilon + \frac{1}{4}\lambda^\dagger \epsilon + \frac{i}{2}[\lambda^\dagger, \sigma]\epsilon$$

$$\delta\lambda = \left(-\frac{1}{2}\gamma^{\mu\nu}F_{\mu\nu}D + i\gamma^\mu D_\mu \sigma - \sigma\right)\epsilon, \quad \delta\lambda^\dagger = 0$$

Matrix integral

- All fields are set to zero, except σ = - D, constant on the sphere. Results in a matrix integral after going to the eigenvalues.
- The gauge field 1-loop determinant together with the Vandermonde gives a measure factor of

$$\prod_{i<j} \sinh^2\left(\pi(\sigma_i - \sigma_j)\right)$$

- The matter sector localization requires that the fields have canonical dimensions (ie. $N = 3$).

1-loop determinant is $\det_R\left(\mathrm{sech}(\pi\sigma)\right)$ for a hyper in rep R.

$N^{3/2}$ from the field theory

$$\frac{1}{N!M!}\int\prod\frac{du_i}{2\pi}\frac{dv_j}{2\pi}\frac{\prod_{i<j}4\sinh^2(\frac{u_i-u_j}{2})4\sinh^2(\frac{v_i-v_j}{2})}{\prod_{i,j}4\cosh^2(\frac{u_i-v_j}{2})}e^{\frac{ik}{4\pi}(\sum u_i^2-\sum v_j^2)}$$

- Solved by Drukker Marino Putrov! The leading free energy when N = M is

$$-\frac{\pi\sqrt{2}}{3}k^2(\lambda-\frac{1}{24})^{3/2}$$

- This even captures the subleading corrections to the M2 charge. When B $\neq$0, there is a nontrivial phase, which remains to be understood in the gravity dual.
- Note that $N^{3/2}$ is special to this matrix model.

CSM from geometry and an isometry

- The theory of M2 branes probing a geometry with a U(1) isometry is part of a family made by taking Z_k quotients with weakly coupled gauge groups in the large k limit.

- Moreover, all of the theories now known to describe M2 branes have a baryonic U(1) associated to $\star \sum_i Tr(F_i)$. This includes those with pure Yang-Mills, where weak coupling is obtained in the large N_f limit, giving exactly a $\mathbb{Z}_{N_f} \subset U(1)_B$ quotient.

Varieties of $U(1)_B$ action

- 1) There are no fixed points away from the origin.
- 2) If the U(1) shrinks away from the origin, then there will be explicit D6 branes in the IIA reduction. Ex: Q^{111}, which is a circle bundle over S^2 x S^2 x S^2, with U(1) action generated by rotation of two of the spheres.
- 3) It is also possible that Y has non-isolated orbifold singularities. For example, C^4 with a $Z_q \subset U(1)$ acting with charges 1, -1, p, -p, which is related to NS5 – (p,q)5 configurations, has a nonisolated Z_p singularity in the IIA reduction.

The weakly coupled manifestly N=4 theory is mysterious in IIB. In principle determined by Gaiotto Witten

Corresponding field theories

- In the first case, the worldvolume theory will be just that of D2 branes in the 7d cone. With $N = 2$, they are D2 on CY 3-fold plus CS terms.
- The second case results in theories with flavors.
- The generic case 3 is probably described by strongly coupled CFT interacting via CS gauge theories.
- Changing the U(1) used gives a dual theory, generalizing 3d mirror symmetry.

Moduli space of N=2 CSM

- There are F-term equations of the usual type $\partial W = 0$. We have a D3 quiver on Y with n nodes, all fields are adjoints or bifundamentals.

- The D-term equations are replaced by cubic equations $\frac{1}{k_a}\mu_a T^i_b q_i = 0,$ where μ_a are the usual moment maps.

- The M-theory geometry is the solution $\mu_a = k_a r$, and the CS term implies that one only gauges the kernel of

$$\beta : (u_1, , u_n) \in U(1)^n \mapsto u_1^{k_1} ... u_n^{k_n}$$

- This is precisely the geometry X.

[Tomasiello DLJ, Martelli Sparks, Hanany Zaffaroni]

Monopoles in the chiral ring

- There are monopole operators in YM-CS-matter theories, which we follow to the IR CSM.

- In radial quantization, it is a classical background with magnetic flux $\int_{S^2} F_a = 2\pi n$, and constant scalar, $\sigma = n/2$. Of course, in the CSM limit, $\sigma_a = k^{-1}\mu$

- It is crucial that the fields in μ are not charged under a.

- This operator creates a vortex.

[Borokhov Kapustin Wu]

M2 branes on Calabi-Yau 4-folds

- Consider a conical Calabi-Yau 4-fold, X, with a U(1) isometry that leaves the holomorphic 4-form invariant. For example, a toric CY_4 will have 3 such U(1)'s.
- Then the Kähler quotient X//U(1) will be a CY_3, Y. Reducing M-theory on X to IIA on this U(1) gives a 7d cone which is Y fibered over a real line, with F_2 flux, varying dilaton, and $\theta_a^{FI} = k_a r$.
- The F_2 flux scales with k if one starts with X/Z_k.

[DLJ Tomasiello, Martelli Sparks, Hanany Zaffaroni, Aganagic]

N=2 CSM from D3 quivers

- We want to know the theory on N M2 branes on X. It is the IR limit of the theory on N D2 branes on the 7d cone X/U(1), with F_2 flux.

- Take the reduction to 2+1 of the quiver theory describing N D3 branes on Y. In the resolved geometry we can image this describes the fractional branes as D4 and D6 on holomorphic 2 and 4 cycles in Y. The CS terms arise from the D4 worldvolume coupling $\int F_2 \wedge S_{CS}(a)$

- The coupling $\frac{k}{2\pi} \int D\sigma$ must arise from the fibration of Y over R^1.

[Aganagic]

D6 branes in AdS_4

- Given this large class of $\mathcal{N} = 2, 3$ quiver CSM theories describing a stack of M2 branes at a (hyper) toric singularity. In the 't Hooft limit, the dual geometry is a warped product $\mathrm{AdS}_4 \times_w M_6$
- Introducing D6 branes wrapping an internal, homologically trivial, 3-cycle ($\mathbb{RP}^3$ in the $\mathbb{CP}^3$ case) adds fundamental hypermultiplets to the quiver. Choice of Z_2 Wilson line corresponds to which node the fundamental is attached.
- Interestingly, conformality is preserved.

[Hohenegger Kirsch, Gaiotto DLJ, Hikida Li Takayanagi, Fujita Tai]

Adding flavors

- The addition of fundamental flavors will alter the chiral ring, corresponding to the quantum correction of the moduli space.
- Mesonic operators are unaffected, but BPS monopole operators that appear in the chiral receive a quantum contribution to their dimension, and a corresponding change of the OPE.

Anomalous dimension
N=2 case

- We work in the UV to compute the 1-loop correction to the charge of a monopole operator under some flavor U(1). Following Borokhov-Kapustin-Wu, we find

$$-\frac{1}{2}\sum_{fermions}|q_e|Q_F$$

- This is an addition to the usual, mesonic charge of the operator.

Cancellation

- Doing the calculation in the UV YM-CSM theory, $\Delta \mathrm{dim} = \Delta R = -\frac{1}{2}\sum_{fermions} \text{R-charge}$, where R is the exact R-symmetry in the IR CSM. Note that in the toric case, each chiral field appears exactly twice in the superpotential. Therefore any flavor symmetry (which must leave W invariant) that might mix with the R-symmetry cancels in the monopole dimension.

- Therefore equivalent to anomaly cancellation in the 4d YM theory with the same quiver.

- More generally, anomalies of the 4d theory become quantum corrections to this monopole in the 3d theory.

[DLJ, Niarchos, Benna Klebanov Klose]

$\mathcal{N} = 2$ flavors

[Benini Closset Cremonesi, DLJ]

- Consider one of the quivers that descends from a toric CY 3-fold quiver. We add non-chiral flavors p and q to one node, with superpotential

$$W = qFp$$

- The fundamentals become massless when F = 0 and μ = 0. The location of the D6 brane is thus F = 0 in the 3-fold, Y.
- On the geometric branch, p = 0, q = 0.

Correction to the moduli space

- Suppose we have added N_f flavor pairs. The flavor symmetries of the original quiver act trivially on them, and their own flavor symmetries are always nonabelian groups, which cannot mix with the R-symmetry. Thus we are justified in computing the quantum correction to the naive UV R-charge of our monopole.

- This results in $-\frac{2N_f}{2}(d_{fund} - 1) = \frac{N_f}{2}\text{dimension}(F),$ since the total dimension of the superpotential q F p must be 2.
- The only consistent structure for the OPE is then

$$T\tilde{T} \sim F^{N_f}.$$

M-theory geometry

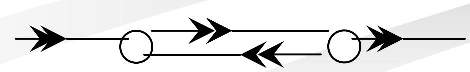
- The modified geometry is $X_f = (X \times \mathbb{C}^2)//U(1),\ t\tilde{t} = F^{N_f}$ where the U(1) acts as N_f times $U(1)_B$ on X, and with weights k, -k on C^2.

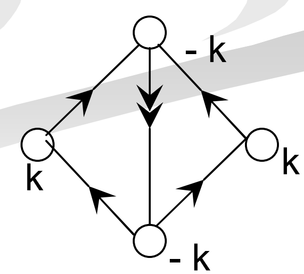
- Note that this U(1) has a fixed locus where t and $\tilde{t}$ are zero. This is exactly the location of the D6 branes, where the fundamentals are massless.

Chiral flavors

- F is now in the $(\bar{N}_i, N_j)$ rather than the adjoint.
- Using the formula for 1-loop corrections to the charges of monopoles, we now find that there is a quantum correction to the *gauge* charges, since the number of fields entered and leaving each node are not equal.
- Taking two such D6 branes, one can produce Q^{111}!

$$\mathcal{W}_{fl} = p_1 A_1 q_1 + p_2 A_2 q_2$$





Chiral example

- The cone over Q^{111} is the toric CY 4-fold, $C^6//U(1)^2$

	z_1	z_2	z_3	z_4	z_5	z_6
	1	1	-1	-1	0	0
	0	0	1	1	-1	-1
$U(1)_B$	1	0	-1	0	0	0

- The 3d Kahler quotient on which the quiver is based is then just the conifold. $a_1 = z_1 z_3,\ a_2 = z_2 z_4,\ b_1 = z_5,\ b_2 = z_6$
- There are two fixed loci defined by the equations $a_1 = 0, \quad a_2 = 0$
- Flavored CFT gives:

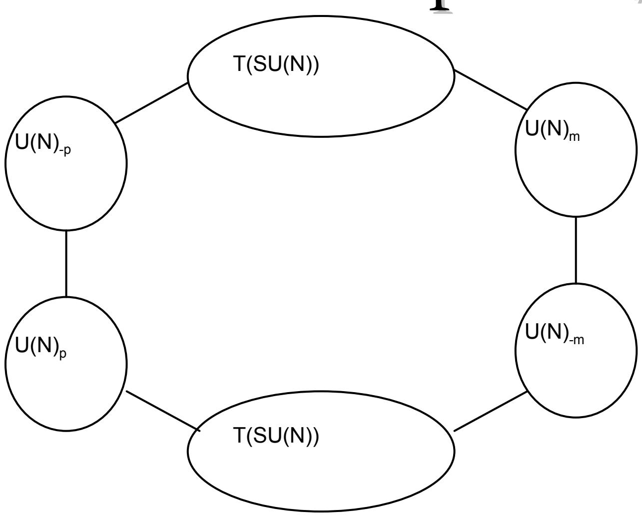
A_1	A_2	B_1	B_2	t	$\tilde{t}$
1	1	-1	-1	1	1

$$t\tilde{t} = A_1 A_2$$

- Assuming the 4 node quiver is quantum mechanically consistent, perhaps it describes the theory with a flat C-field turned on.

Generic orbi-bundle case

- The M-theory horizon manifold is smooth, but the reduction to IIA has singularities that are locally orbifolds. These can support fractional F_2 flux.
- For example C^4/Z_q acting by



$$(z_1, z_2, z_3, z_4) \mapsto (\zeta z_1, \zeta^{-1} z_2, \zeta^p z_3, \zeta^{-p} z_4)$$

$$T^p S T^{-m} \in SL(2, \mathbb{Z}) \quad q = mp + 1$$

maps NS5 to (p, mp+1)5

$$T\left(SU(N)\right) = U(1) \times U(2) \times ... \times U(N-1) \times U(N)_F$$

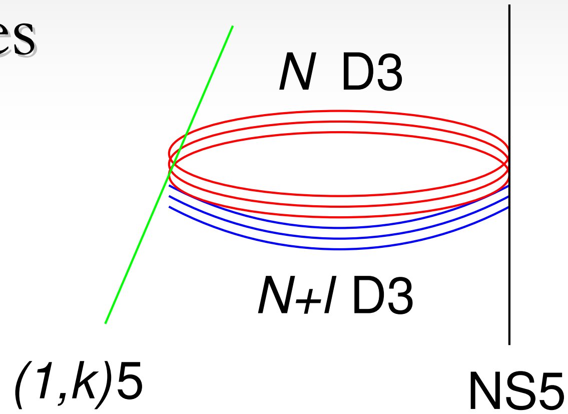
M5 branes on 3-manifolds

- Gives rise to interacting 3d SCFTs in the IR.
- On hyperbolic 3-manifolds, probably lack a Lagrangian description. Special case: mapping tori of large diffeomorphisms of Riemann surfaces. Very special case: circle bundles over Riemann surfaces.
- Can have large duals in M-theory with internal fluxes in the hyperbolic case.

[Gauntlett Conamhna Mateos Waldram]

$\mathcal{N}$ = 3 D-brane construction

IIB D3 intersecting fivebranes



UV field theory

N=3 YM-CS

M2 on $\mathbb{C}^4/\mathbb{Z}_k$

with torsion flux

IR field theory

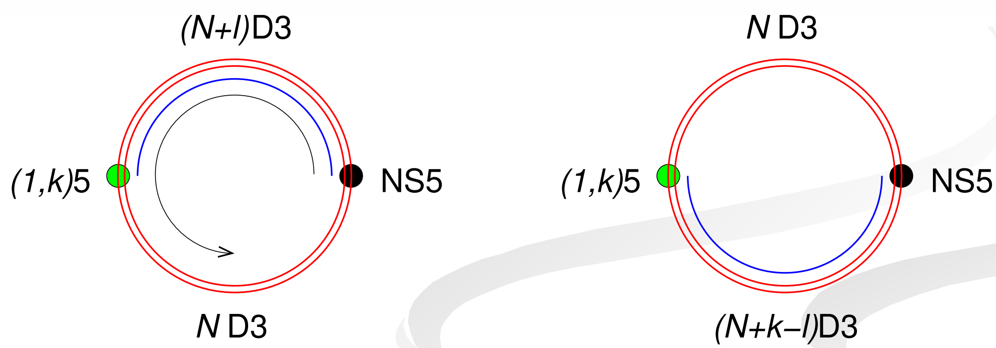
N=6 CSM

S-rule

- The discrete torsion fluxes live in $H^4(S^7/Z_k) = Z_k$
- Naively, ℓ can be any integer.
- But the Hanany-Witten s-rule implies that the moduli space is lifted if ℓ > k. This can be seen as the nonexistence of $SU(N)_k$ $N = 3$ CS theory for N > k.
- This is a strong coupling effect.

Parity

- The $U(N)_k \times U(N+\ell)_{-k}$ theory is equivalent to $U(N)_{-k} \times U(N+k-\ell)_k$ related by parity to $U(N)_k \times U(N+k-\ell)_{-k}$.



- Perhaps there is a cascade rather than supersymmetry breaking in certain configurations with $\ell > k$.

Cascades

- Evidence for a cascade when $2Nk \geq M(M-k)$ found by Aharony Hashimoto Hirano Ouyang; Evslin Kuperstein; Hashimoto Hirano Ouyang.
- Extra difference in the ranks is carried by D3 branes that wind several times.
- The supergravity dual has yet to be constructed for the flow from the YM-CS-matter through the cascade.

Cascades II

- A cascading solution was found for the *N*=4 configuration. This is a cascade starting with the 4d *N*=4 YM on a circle with domain walls.
- In the YM-CSM case, one needs to check when "flowing up" whether the cascade occurs before the field theory enters the weakly coupled regime.
- The s-rule remains to be fully understood. What is the potential generated on the moduli space?

For the future

- Embed Vasiliev theory into string theory.
- Understand when weakly coupled strings, small curvature, and/or weak gravity emerge.
- More applications of localization on S^3.
- A new window into the string landscape?
- Connect explicit duals with condensed matter?