Baryon with Massive Strangeness in Holographic QCD

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- arXiv:0910.1179 w/ K. Hashimoto, N. Iizuka and D. Kadoh
- arXiv:1009.0986

Realistic Holographic QCD ?

Real QCD: chiral symmetry + quark mass

Chiral Symmetry or/and Quark Mass

2 manners of doing holographic QCDs:

	Chiral sym.	Quark mass
D3-D7 D4-D6 etc.	unclear	easy
D4-D8/$\overline{\text{D8}}$ (Sakai-Sugimoto)	clear	technical

A route to realistic QCD: **SS model + mass**

Introducing quark masses by
- worldsheet instantons (D6 used) ← **I use this.**
- tachyon condensation

Flavor Symmetry: Good or Bad

In considering **three-flavor** (solitonic) **baryons**...

- If $SU(3)_f$ is **good**: $0 < m_{u,d} \simeq m_s$
  Baryons as the states of SU(3) flavor rotation

- If $SU(3)_f$ is **bad**: $0 < m_{u,d} \ll m_s$
  Hyperon = SU(2) baryon + Kaon: bound-state

	Skyrme	Sakai-Sugimoto
SU(3) rotation	80s	arXiv:0910.1179
Bound-state	Callan-Klebanov '85	arXiv:1009.0986

Plan of Talk

0. Summary of Sakai-Sugimoto model

1. SU(3) flavor rotation case
   mass splittings of baryons

2. Bound-state approach case
   a weakly bound baryon

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Sakai-Sugimoto Model

5 dim U(N_f) YM-CS theory [Sakai-Sugimoto]

$$S = -\kappa \int d^4x dz \, \mathrm{Tr} \left[\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right] + \frac{N_c}{24\pi^2} \int_{\mathrm{M}_5} \omega_5$$

[μ=0,1,2,3]

Confined phase of QCD in just 1 line

- Curved space: $h(z) = (1+z^2)^{-1/3}, \; k(z) = 1 + z^2$
- 2 parameters: $\kappa = 0.00746, \; M_{\mathrm{KK}} = 948 \, [\mathrm{MeV}] \to 1$
- Holographic dual of a large Nc **massless** QCD

Baryon is instanton-like soliton

$$S = -\kappa \int d^4x dz \, \mathrm{Tr}\left[\frac{1}{2}h(z)F_{\mu\nu}^2 + k(z)F_{\mu z}^2\right] + \frac{N_c}{24\pi^2}\int_{\mathrm{M}_5}\omega_5$$

Soliton solution (N_f=2 [Hata-Sakai-Sugimoto-Yamato])

$$A_\alpha^{\mathrm{inst}} = -if(\xi)g\partial_\alpha g^{-1}, \;\; A_0^{\mathrm{inst}} = \frac{1}{16\pi^2 a\lambda}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\rho^2+\xi^2)^2}\right]\mathbf{1}_2$$

SU(2) BPST [α=1,2,3,z]

U(1) Coulomb

- $D4_{\mathrm{baryon}}$ on $D8_{\mathrm{flavor}}$: instanton
- Instanton zeromode = collective coord. of baryon
- Instanton action ~ baryon mass: $S_{\mathrm{inst}} \sim \int dt \, M_{\mathrm{baryon}}$

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Baryon Mass Shift by Quark Mass

Mass term given by Wilson line

[Hashimoto-Hirayama-Lin-Yee, Aharony Kutasov]

$$S_{\text{mass}} = c \int d^4x \, \text{PTr} \left[M_q \left(e^{-i \int_{-\infty}^{\infty} A_z dz} - \mathbf{1}_3 \right) \right] + \text{c.c.}$$

Instanton action of mass term = baryon mass shift

$$S_{\text{mass}}^{(\text{inst.})} = \int dt \, \delta M_{\text{baryon}}$$

- Classical embedding to SU(3): $A_z = \begin{pmatrix} A_z^{\text{inst}} & 0 \\ 0 & 0 \end{pmatrix}$
- Instanton moduli gives baryon states

12 moduli = ~~**4** position~~ + **1** size + **7** SU(3)/U(1) rotation

$\langle \rho^n \rangle_{n_\rho, l}$ $\qquad U = G U_0 G^\dagger, \, G \in SU(3)$

Result

Leading order in meson-mass series

$$m_{\text{baryon}} = C_0 + \frac{C_1}{3}\left(a_0 m_{K^0}^2 + a_K m_{K^\pm}^2 + a_\pi m_{\pi^\pm}^2\right) + \cdots$$

The coefficient C_1 is obtained:

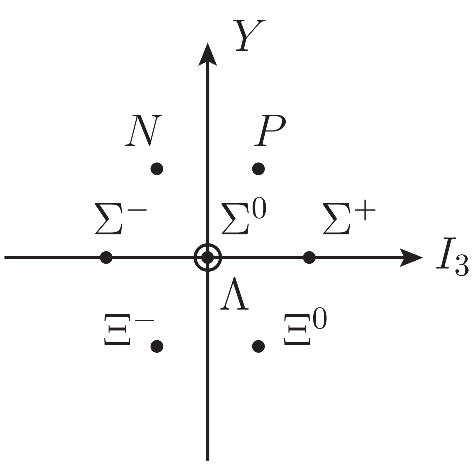
octet

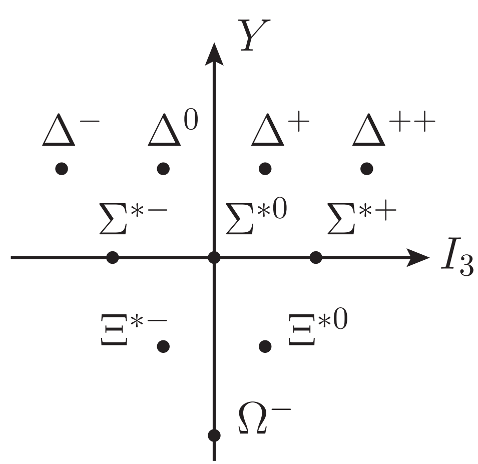
$$C_1^{\mathbf{8}} = 7.9\,[\text{GeV}^{-1}]$$

decuplet

$$C_1^{\mathbf{10}} = 9.5\,[\text{GeV}^{-1}]$$

input: f_π = 92.4 MeV





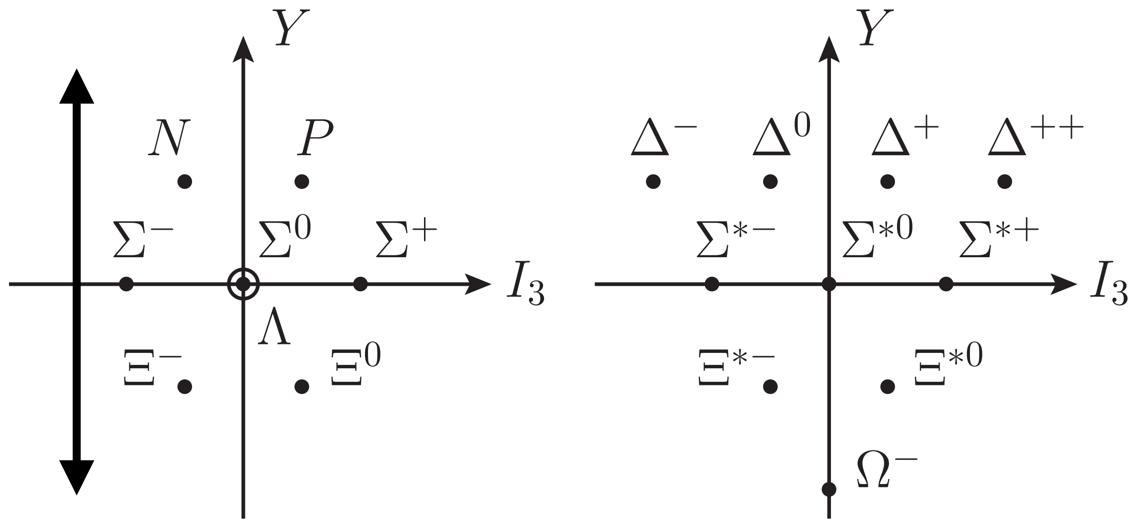
c.f.) 2-flavor case

[Hashimoto-Hirayama-Hong]

$$m_N = C_0 + C_1 m_\pi^2 + \cdots$$

$$C_1 = 4.1\,[\text{GeV}^{-1}]$$

Mass Splittings from Strangeness



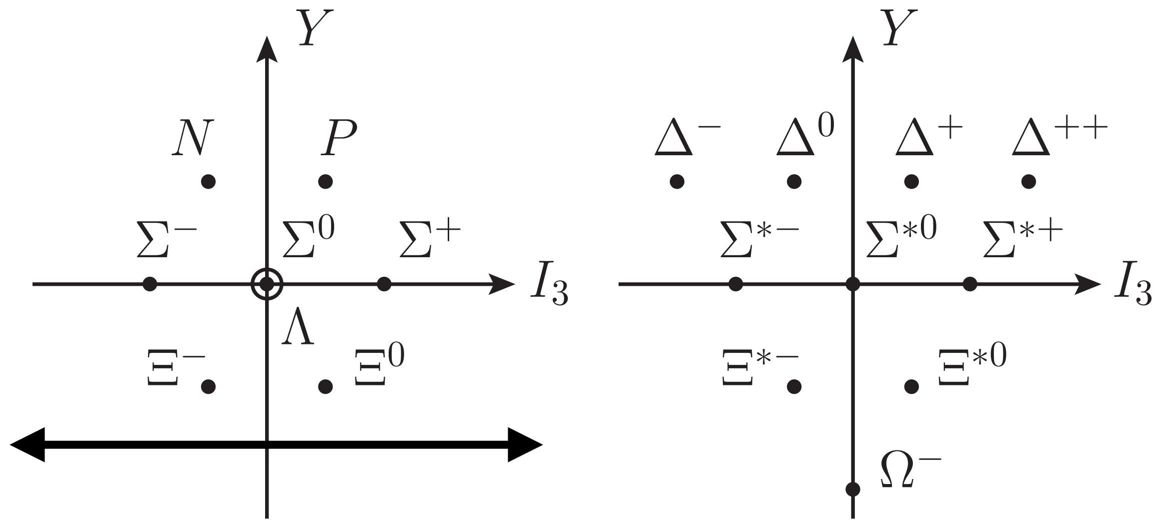
ΔY=1 or ΔI=1

8	$\Delta m_{\Lambda - N}$	$\Delta m_{\Sigma^0 - N}$	$\Delta m_{\Sigma^0 - \Lambda}$	$\Delta m_{\Xi^0 - \Lambda}$	$\Delta m_{\Xi^0 - \Sigma^0}$
theory [MeV]	2.4×10^2	4.8×10^2	2.4×10^2	3.5×10^2	1.2×10^2
experiment [MeV]	1.8×10^2	2.5×10^2	77	2.0×10^2	1.2×10^2

10	$\Delta m_{\Sigma^{*0} - \Delta^0}$	$\Delta m_{\Xi^{*0} - \Sigma^{*0}}$	$\Delta m_{\Omega^- - \Xi^{*0}}$
theory [MeV]	1.8×10^2	1.8×10^2	1.8×10^2
experiment [MeV]	1.5×10^2	1.5×10^2	1.4×10^2

inputs: $m_{\pi^\pm} = 140$ [MeV], $m_{K^\pm} = 494$ [MeV], $m_{K^0} = 498$ [MeV]

Mass Splittings from u, d Quarks



ΔI_3=1

8	Δm_{N-P}	$\Delta m_{\Sigma^- - \Sigma^0}$	$\Delta m_{\Sigma^0 - \Sigma^+}$	$\Delta m_{\Xi^- - \Xi^0}$
theory [MeV]	2.1	5.1	5.1	8.2
experiment [MeV]	1.3	4.8 ± 0.1	3.3 ± 0.1	6.9 ± 0.3

10	$\Delta m_{\Delta \text{baryons}}$	$\Delta m_{\Sigma^{*-} - \Sigma^{*0}}$	$\Delta m_{\Sigma^{*0} - \Sigma^{*+}}$	$\Delta m_{\Xi^{*-} - \Xi^{*0}}$
theory [MeV]	3.1	3.1	3.1	3.1
experiment [MeV]	$(\lesssim 2)$	3.5 ± 1.5	0.9 ± 1.4	3.2 ± 0.9

inputs: $m_{\pi^\pm} = 140$ [MeV], $m_{K^\pm} = 494$ [MeV], $m_{K^0} = 498$ [MeV]

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Review of Skyrme model case

Strategy: **Hyperon = SU(2) Skyrmion + kaon**

$$L \sim \mathrm{Tr}(U^{-1}\partial_\mu U)^2 + \mathrm{Tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2$$

ansatz: $U = \sqrt{U_\pi} U_K \sqrt{U_\pi}$

$$U_\pi = \begin{pmatrix} e^{iF(r)\hat{x}\cdot\tau} & 0 \\ 0 & 1 \end{pmatrix}, \quad U_K \sim \exp\left[\begin{pmatrix} 0 & K \\ K^\dagger & 0 \end{pmatrix}\right]$$

Quadratic in K is **a kaon in a potential** V(r):

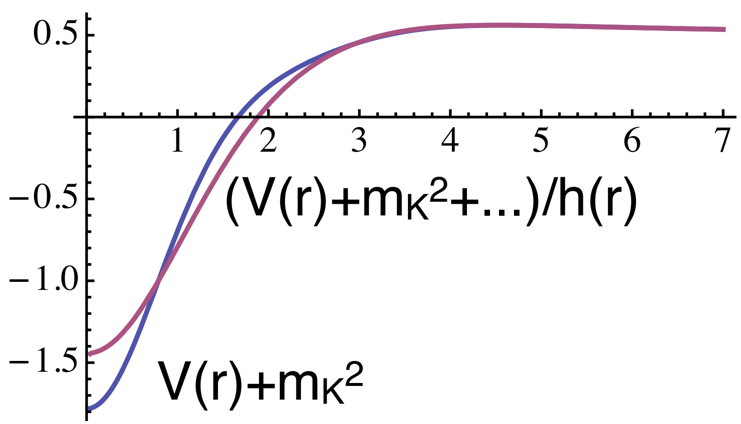
$$L \sim \int dr\, r^2 \left[f(r)\dot{k}^\dagger \dot{k} - h(r)\partial_r k^\dagger \partial_r k - (m_K^2 + V(r))k^\dagger k \right]$$

$$K(x^\mu) = k(r,t)Y(\Omega_2)$$

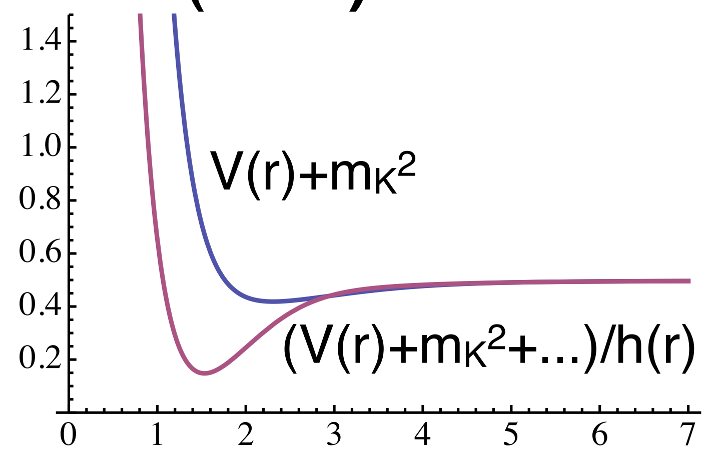
Eq. of motion ⇒ **bound-state potential problem**

$$-\left[\frac{1}{r^2}\partial_r(h(r)r^2\partial_r) - V(r) - m_K^2\right]k = \left[f(r)E_n^2 + 2\lambda(r)E_n\right]k$$

Λ, Σ, Σ* case



Λ(1405) case



	Λ	Σ	Σ*	Λ(1405)
mass (theory)	1048	1122	1303	1281
mass (exp)	1115	1190	1385	1405

The approach works well.

Holographic bound-state approach

Turn on kaon a_z in Sakai-Sugimoto model

2-flavor baryon: A^{inst}, Kaon: a_z $\quad a_z \sim K(x^\mu)\phi^{(0)}(z)$

$$A_\mu = \begin{pmatrix} A_\mu^{\text{inst}} & 0 \\ 0 & 0 \end{pmatrix}, \quad A_z = \begin{pmatrix} A_z^{\text{inst}} & a_z \\ a_z^\dagger & 0 \end{pmatrix}$$

Quadratic part in K: $S = S_{\text{inst}} + S_{\text{kaon}}$

$$S_{\text{kaon}} = -\int d^4x \left[\partial_\mu K^\dagger \partial^\mu K + K^\dagger \Upsilon K \right. \\ \left. + i\left(\partial^\mu K^\dagger\, \Psi_\mu K - K^\dagger\, \Psi_\mu \partial^\mu K\right)\right]$$

$$\Psi_\mu \equiv \frac{1}{\pi}\int dz \frac{1}{k(z)} A_\mu^{\text{inst}}, \quad \Upsilon \equiv \frac{1}{\pi}\int dz \frac{1}{k(z)} (A_\mu^{\text{inst}})^2$$

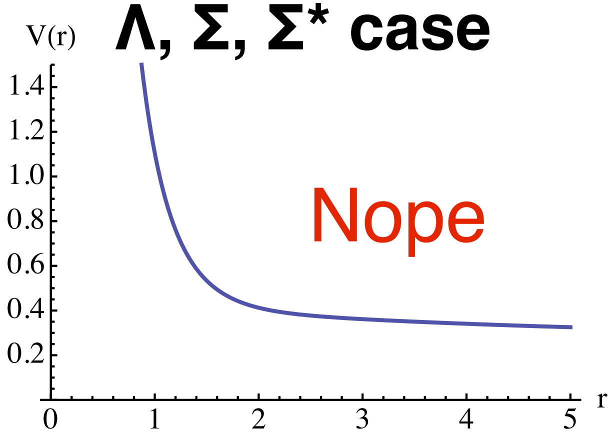
Is It Bound ?

Simply add canonical K mass: $S_{\text{kaon}} - \int d^4x\, m_K^2 K^\dagger K$

Similar to **but different** from the Skyrme model case

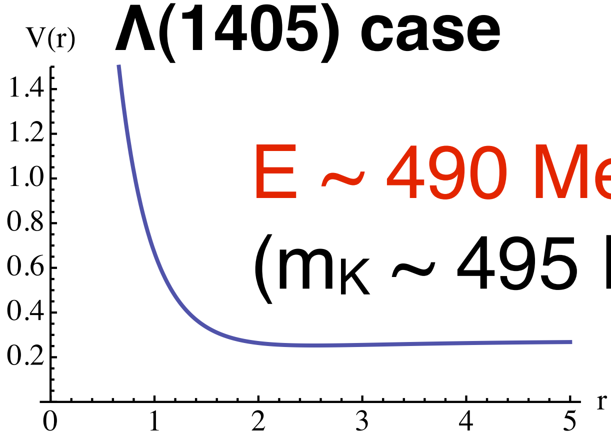
$$\left[-\frac{1}{r^2}\partial_r \left(r^2\partial_r\right) + V(r) + m_K^2 \right] k_n(r) = \left[E_n^2 + 2\Psi_0\, E_n \right] k_n(r)$$

Λ, Σ, Σ* case



Nope

Λ(1405) case



E ~ 490 MeV

(m_K ~ 495 MeV)

Thanks to practically large A_0^{inst}

Comment on Mass Term

Recall the mass term:

$$S_{\text{mass}} = c \int d^4x \, \mathrm{PTr}\left[M_q \left(e^{-i\int_{-\infty}^{\infty} A_z dz} - \mathbf{1}_3 \right) \right] + \text{c.c.}$$

Study kaon mass **in baryon background**

$$\mathrm{P}\exp\left[-i \int_{-\infty}^{\infty} A_z dz \right], \quad A_z = \begin{pmatrix} A_z^{\text{inst}} & a_z \\ a_z^\dagger & 0 \end{pmatrix}$$

Insert two a_z to the path-ordering

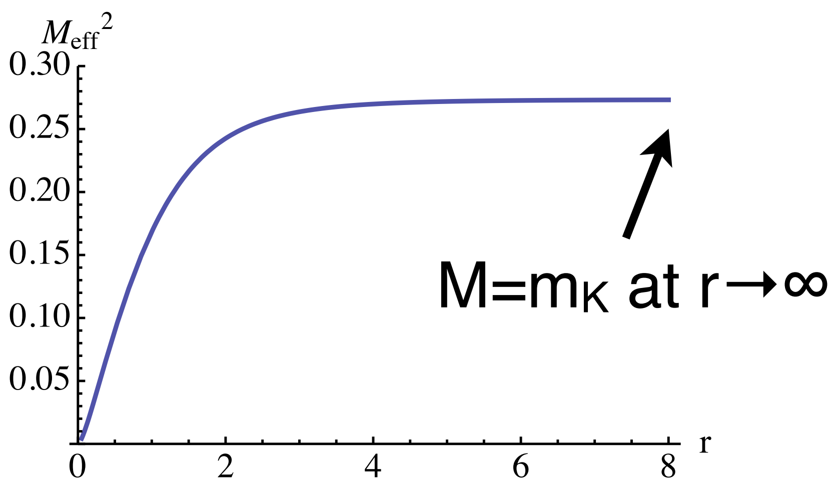
Distance-depend Kaon Mass

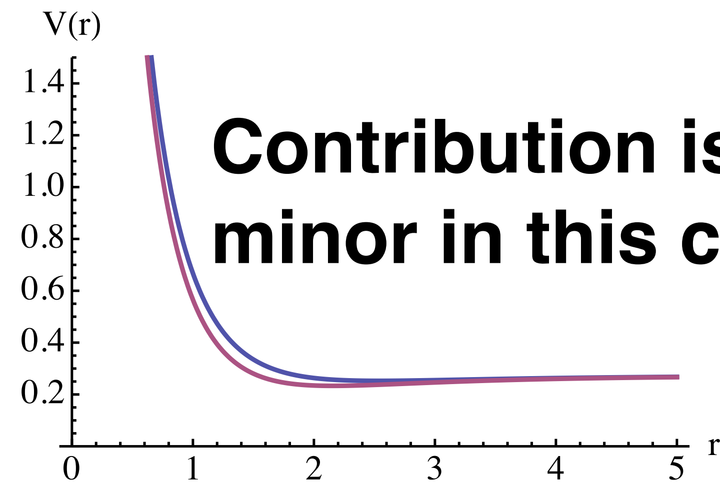
$$S_{\text{kaon mass}} = -\int d^4x\, M_{\text{eff}}^2(r) K^\dagger(x^\mu) K(x^\mu)$$

$$M_{\text{eff}}^2(r) = \frac{2}{\pi^2} \int_{-\infty}^{\infty} dz_2 \int_{-\infty}^{z_2} dz_1 \frac{1}{k(z_1)k(z_2)} \cos(I_{(z_1,z_2)})\, m_K^2$$

r = distance from the baryon

$$\int_a^b dz\, A_z^{\text{inst}} = I_{(a,b)} \hat{x}^i \tau_i$$





Contribution is minor in this case.

Summary

Baryons from SU(3) collective coordinates

- Mass splittings of baryons calculated
- Qualitatively good

Bound-state approach to strangeness

- Weakly Bound Λ(1405)

  (Is Λ(1405) a N-$\bar{K}$ weak bound-state?)

- Can these two methods be interpolated?
- Three-flavors in other holographic QCD models?
- Meson-baryon system is challenging.