An AdS/QCD model from Sen's tachyon action

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September 13, 2010

Based on hep-ph/1003.2377 and work on progress.

Outline

General

- AdS/CFT states that 4 dimensional $\mathcal{N} = 4$ SYM with $SU(N_c)$ gauge symmetry at large N_c and strong coupling is dual to weakly coupled type IIB supergravity in $AdS_5 \times S^5$.
- Are other strongly coupled gauge theories possible to be described holographically?
- QCD is strongly coupled in the IR. At zero temperature it is also a confining gauge theory.
- Until now, the strongly coupled regime of QCD has been studied using lattice methods and chiral perturbation theory.
- We are interested in phenomena related to the fundamental degrees of freedom of QCD.

D_3/D_7 model

In D_3/D_7 model D_7 flavor branes are embedded in $AdS_5 \times S^5$ spacetime. $U(1)_A$ symmetry is realized as an isometry of the internal space. It breaks due to a quark mass and a quark condensate. It is not generalized to $U(N_f)_L \times U(N_f)_R$ symmetry. (Badington Erdmenger Evans Guralnik Kirsch, Kruczenski Mateos Myers Winters)

Sakai-Sugimoto model

In Sakai-Sugimoto model pairs of D_8 branes and $\bar{D}_8$ antibranes are put in the spacetime which is generated by D_4 branes. The full chiral symmetry breaking is described, i.e. $U(N_f)_L \times U(N_f)_R \to U(N_f)_V$. No quark mass and quark condensate.

AdS/QCD

Phenomenological Lagrangians in AdS_5 backround are proposed. They are not derived from any D brane construction. The spectrum of low energy mesons is in agreement with experiment within 10%, Gell Mann-Oakes-Renner relation is satisfied and linear Regge trajectories are found. (Erlich Katz Son Stephanov, Da Rold Pomarol)

Improvements compared to the soft wall model

This model belongs to the general class of AdS/QCD models but with some inspiration from string theory and particularly Sen's tachyon-DBI action.

Compared to the soft wall model of AdS/QCD, the present model has some advantages

- Chiral symmetry breaking is realized dynamically.
- The vector meson masses grow linearly with quark mass.
- The model is formulated in a confing background in the usual holographic sence.
- Adding the appropriate WZ term in the action provides with a correct descriprion of the chiral anomaly.

The action

$$S = \int d^6x \sqrt{g_{(6)}} \left[e^{-2\phi} \left(R + 4(\partial\phi)^2 + \frac{c}{\alpha'} \right) - \frac{1}{2}\frac{1}{6!} F_{(6)}^2 \right]$$

The solution

- The metric reads $ds_6^2 = \frac{R^2}{z^2} \left[dx_{1,3}^2 + f_\Lambda^{-1} dz^2 + f_\Lambda \, d\eta^2 \right]$
- The RR field strength $F_{(6)} = \frac{Q_c}{\sqrt{\alpha'}} \sqrt{g_{(6)}} \, d^6x$
- And the dilaton $e^\phi = \frac{1}{Q_c} \sqrt{\frac{2c}{3}}$
- where $f_\Lambda = 1 - \frac{z^5}{z_\Lambda^5}$, $\eta \sim \eta + \frac{4\pi}{5} z_\Lambda$ and $R^2 = \frac{30}{c} \alpha'$

(Kuperstein Sonnenschein)

The worldvolume action

We consider N_f ($N_f << N_c$) coincident pairs of D_4 brane and $\bar{D}_4$ antibrane at fixed η which are described by Sen's tachyon action

$$S = -\int d^4x dz V(|T|)\left(\sqrt{-\det \mathbf{A}_L} + \sqrt{-\det \mathbf{A}_R}\right)$$

where

- $\mathbf{A}_{(L/R)MN} = g_{MN} + \frac{2\pi\alpha'}{g_V^2} F_{MN}^{(L/R)} + \pi\alpha'\lambda\left((D_M T)^*(D_N T) + (D_N T)^*(D_M T)\right)$
- $D_M T = (\partial_M + iA_M^L - iA_M^R)T$
- $V = \mathcal{K} e^{-\frac{1}{2}\mu^2\tau^2}$

The bulk fields and their duals.

Bulk Field	$U(N_f)_L$	$U(N_f)_R$	Dual Operator
T	$\bar{N}_f$	N_f	$\bar{q}_L q_R$
A_L	Adjoint	1	$\bar{q}_L \gamma_\mu q_L$
A_R	1	Adjoint	$\bar{q}_R \gamma_\mu q_R$

- The complex tachyon will be denoted by $T = \tau e^{i\theta}$.
- τ is dual to $\bar{q}q$ and θ to $\bar{q}\gamma_5 q$.
- We will study the simpler abelian case.

The parameters of the model

- The parameters g_V and λ determine the normalization of the bulk fields.
- $\mathcal{K}$ is related to the tension of the $D4$ branes.
- μ^2 can be absorbed in τ.
- There are also two parameters from the background metric, z_Λ and R.

The Vacuum

The action

The fields θ, A_L and A_R are set to zero in the vacuum. The only non trivial field in the vacuum is $\tau(z)$, which causes the chiral symmetry breaking. The action then reads

$$S = -2\mathcal{K} \int d^4x dz e^{-\frac{1}{2}\mu^2\tau^2} g_{tt}^{\frac{1}{2}} g_{xx}^{\frac{3}{2}} \sqrt{g_{zz} + 2\pi\alpha'\lambda(\partial_z\tau)^2}$$

Equation of motion in the confining background

$$\tau'' - \frac{4\mu^2 z f_\Lambda}{3}\tau'^3 + (-\frac{3}{z} + \frac{f'_\Lambda}{2f_\Lambda})\tau' + \left(\frac{3}{z^2 f_\Lambda} + \mu^2\tau'^2\right)\tau = 0 \qquad (1)$$

The Vacuum

The vev of the tachyon in the UV

- Near the boundary ($z \to 0$)

$$\tau = c_1 z + \frac{\mu^2}{6} c_1^3 z^3 \log z + c_3 z^3 + \mathcal{O}(z^5) \tag{2}$$

- c_1 corresponds to the source of the $\bar{q}q$ operator (i.e. the quark mass) and c_3 is related to its vev.
- A non trivial vev of $\bar{q}q$ leads to the chiral symmetry breaking of the dual quantum field theory.
- Since τ is dual to a dimension 3 operator, hence $\frac{\mu^2 R^2}{2\pi\alpha'\lambda} = 3$.

The vev of the tachyon in the IR

- Near the IR ($z \to z_\Lambda$)

$$\tau = \frac{c_\Lambda}{(z_\Lambda - z)^{\frac{3}{20}}} - \frac{13}{6\, c_\Lambda\, \mu^2}(z_\Lambda - z)^{\frac{3}{20}} + \dots \tag{3}$$

- We solve the eom [1] numerically requiring that the solution asymptotes to Eqs.[2] and [3] in the UV and IR, respectively. This fixes c_3 in terms of c_1

Vev versus mass

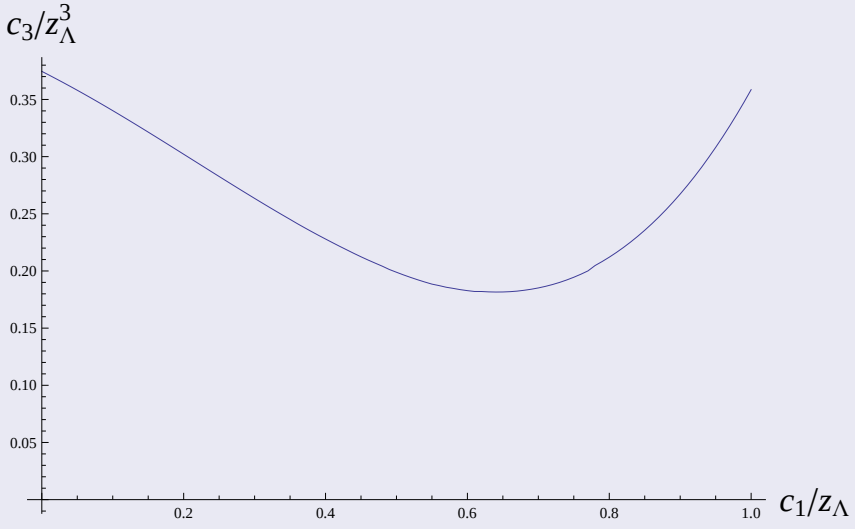

Figure: *c_3 versus c_1 for the confining background.*

Spectrum

Methodology

- We consider excitations of the bulk fields around the vacuum.
- To find the mass spectrum we need to find the quadratic action and the linear equation of motion for each excitation field.
- We define the vector and the axial vector fields

$$V_M = \frac{A_M^L + A_M^R}{2} \,, \qquad A_M = \frac{A_M^L - A_M^R}{2}$$

- We use the gauge $V_z = A_z = 0$

The fields fluctuations

$$\begin{aligned} V_\mu(x,z) &= \psi^V(z)\,\mathcal{V}_\mu(x) \\ A_\mu(x,z) &= \psi^A(z)\,\mathcal{A}_\mu(x) - \varphi(z)\,\partial_\mu(\mathcal{P}(x^\mu)) \\ \theta(x,z) &= 2\,\vartheta(z)\,\mathcal{P}(x^\mu) \\ \tau(x,z) &= \tau_{vac}(z) + \psi^S(z)\,\mathcal{S}(x) \end{aligned}$$

Vector Meson Masses

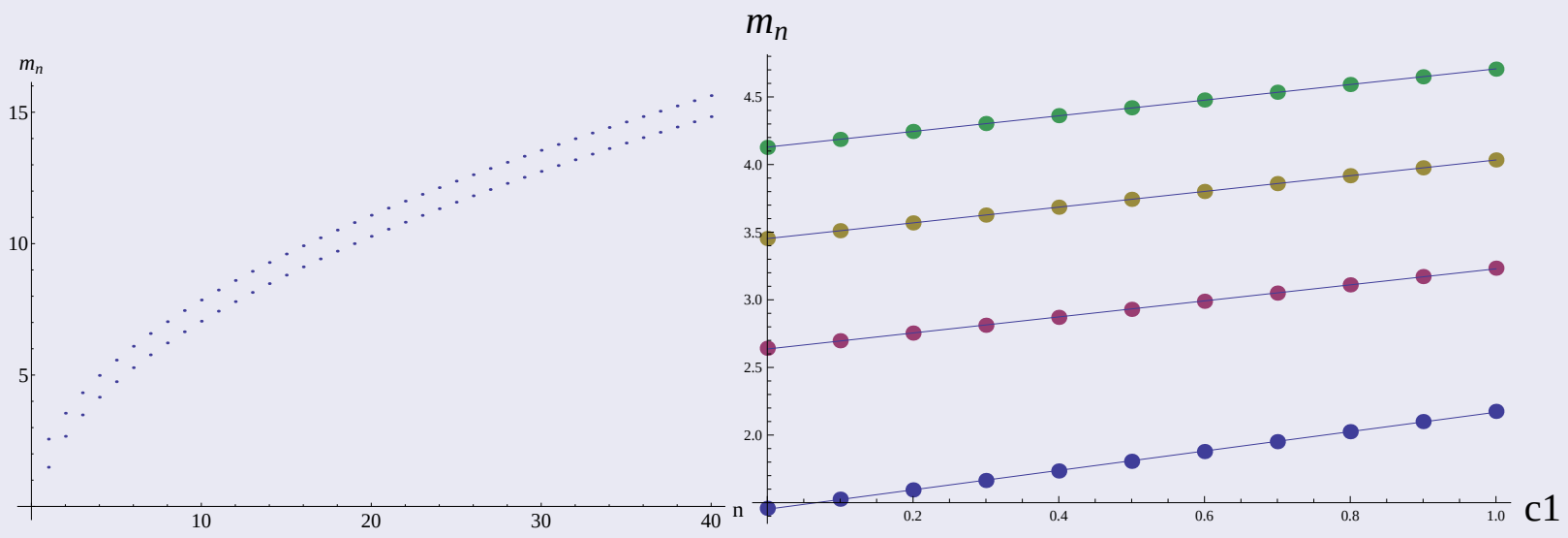

Figure: The plot on the left depicts the masses of the first fourty vector states with $c_1 = 0.05$ and $c_1 = 1.5$. The masses of the four first excited modes as a function of the bare quark mass (namely of c_1). The masses are plotted in units of z_Λ^{-1}.

Vector Meson Decay Constants

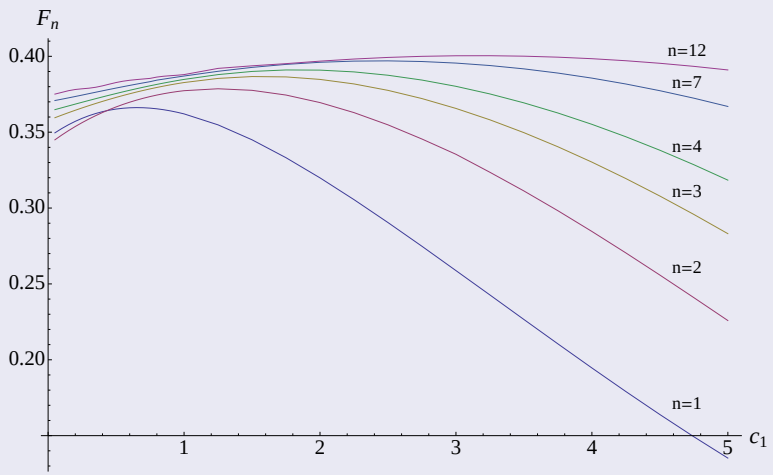

Figure: The decay constant of various vector meson modes in terms of the quark mass in units of z_Λ^{-1}. We notice that the dependence of the decay conatant on n is mild for small quark mass and for large number of modes.

Meson masses in terms of the quark mass

- The masses of all the fluctuations depend linearly on the quark mass (c_1).
- Except for the lowest pseudoscalar mode which has the following mass

$$z_\Lambda m_P^{(1)} = \sqrt{3.53c_1^2 + 6.33c_1}$$

which is in accordance with the Gell-Mann-Oakes-Renner relation

$$m_\pi^2 f_\pi^2 = -2m_q \langle \bar{q}q \rangle \tag{4}$$

- When $c_1 = 0$ the spectrum contains N_f^2 massless Goldstone bosons

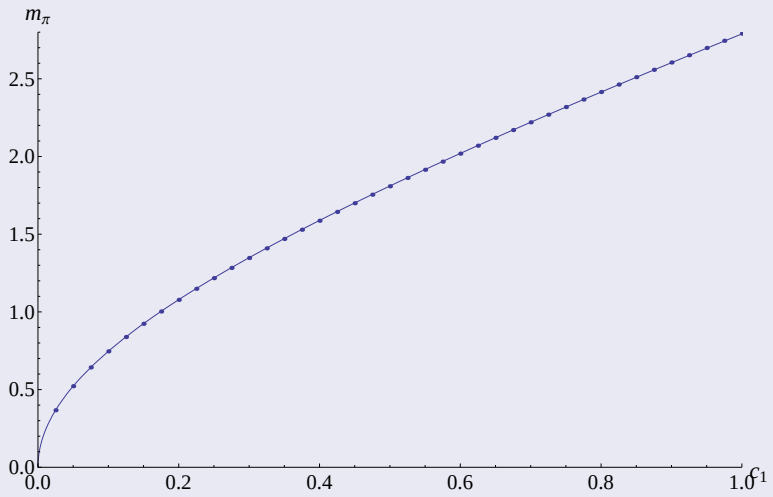

Figure: The lightest pseuposcalar mass as a function of the quark mass, c_1.

Comparison with Experiment

- We fit the three parameters which affect the spectra $z_\Lambda^{-1} = 549 MeV$ and $c_1 = 0.0094$, $k = \frac{18}{\pi^2}$, where $k = \frac{4R^4 g_V^4}{3(2\pi\alpha')^2}$.
- We consider mesons which consist of the lightest quarks (u,d) with isospin 1.

J^{CP}	Meson	Measured (MeV)	Model (MeV)	$100\|\delta O\|/O$
1^{--}	$\rho(770)$	775	800	3.2%
	$\rho(1450)$	1465	1449	1.1%
1^{++}	$a_1(1260)$	1230	1135	7.8%
0^{-+}	π_0	135.0	134.2	0.5%
	$\pi(1300)$	1300	1603	23.2%
0^{++}	$a_0(1450)$	1474	1360	7.7%

Table: The results of the model and the experimental values for light unflavored meson masses.

J^{CP}	Meson	Measured (MeV)	Model (MeV)	$100\|\delta O\|/O$
1^{--}	$\rho(770)$	216	190	12%
1^{++}	$a_1(1260)$	216	228.5	5.8%
0^{-+}	π_0	127	101.3	20.2%

Table: A comparison of the results to the experimental values for the decay constants of light unflavored mesons.

Results

Summary of results

- We constructed a concrete model realizing the general analysis of $hep-th/0702155$.
- A bifuntamental scalar field is contained in our model. It's condensation causes the chiral symmetry breaking $U(N_f)_L \times U(N_f)_R \to U(N_f)_V$.
- When the quark mass is zero there are N_f^2 massless Goldstone bosons in the spectrum of the model.
- The model is confining.
- The mass of the mesons depends linearly on the quark mass.
- The mass of vector mesons depends linearly on the number of mode, $m_n^2 \sim n$.
- The Gell-Mann-Oakes-Renner relation is satisfied when the quark mass is small.
- We find good agreement with the experiment when we compute the masses of light mesons and some decay constants.

Open Problems

Future directions

- Generalize Sen's action to consider the full non abelian case.
- Study the model with finite baryon density.
- Find a top down approach incorporating a bifundamental scalar which controls chiral symmetry breaking (Niarchos talk).
- Add a large number of flavors, $N_f \sim N_c$, to investigate how the backreaction of the flavor branes affects the background geometry.

Thank you