Holographic QCD in three dimensions

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Pusan National University

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Based on arXiv:1003.1306

Why 3D QCD?

- High temperature limit of 4D QCD.
- Toy model for QCD
- Universality class with strongly coupled planar systems (e.g. Hubbard model at half filling or High T_c cuprates)

$$U(2n) \mapsto U(n) \times U(n)$$

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Planar fermions:

- In 3D spin-1/2 fermions are described by two-component (Weyl) spinors:

$$\mathcal{L}_{\text{free}} = \bar{\psi}\left(i\gamma^{\mu}_{2\times 2}\partial_{\mu} - m\right)\psi\,. \tag{1}$$

- The 2×2 Dirac matrices are given as:

$$\gamma^{0}_{2\times 2} = \sigma^{3}, \quad \gamma^{1}_{2\times 2} = i\sigma^{1}, \quad \gamma^{2}_{2\times 2} = i\sigma^{2}$$

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Planar fermions:

- Under the parity, P_2, $x = (t, x_1, x_2) \mapsto x' = (t, -x_1, x_2)$, the fermions transform to

$$\psi'(x') = e^{i\delta}\gamma^1_{2\times 2}\psi(x)\,.$$

- The mass term changes its sign:

$$P_2^{-1}\mathcal{L}_{\text{free}}P_2 = \bar{\psi}'(x')\left(i\gamma^\mu_{2\times 2}\partial'_\mu + m\right)\psi'(x')\,.$$

- Lagrangian is P_2-invariant, if $m = 0$.

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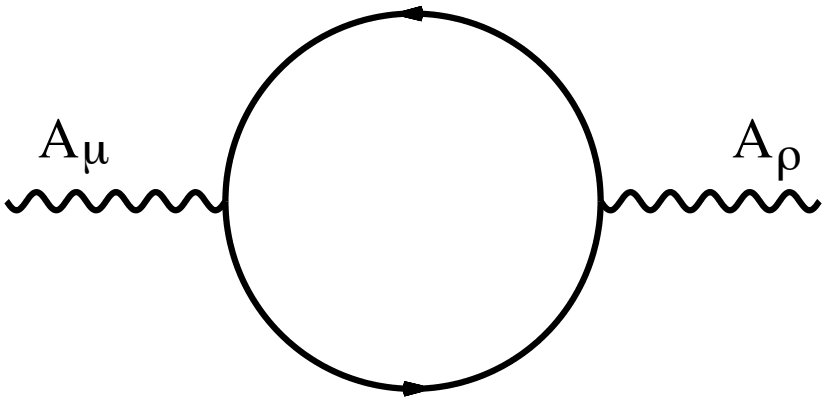
Parity Anomaly:

- However the parity is broken at the quantum level,



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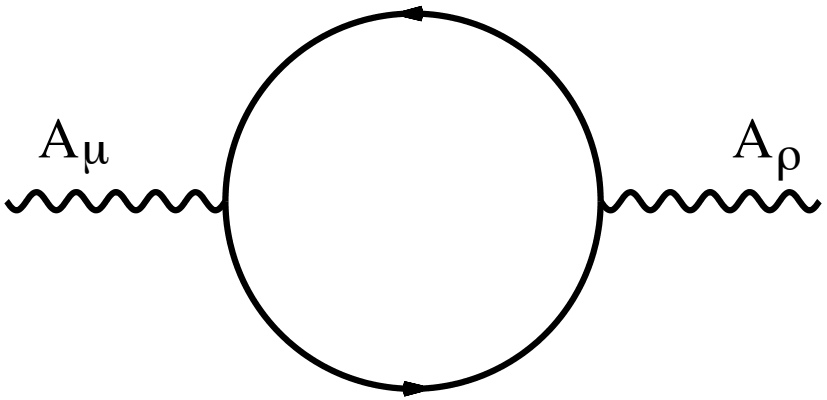

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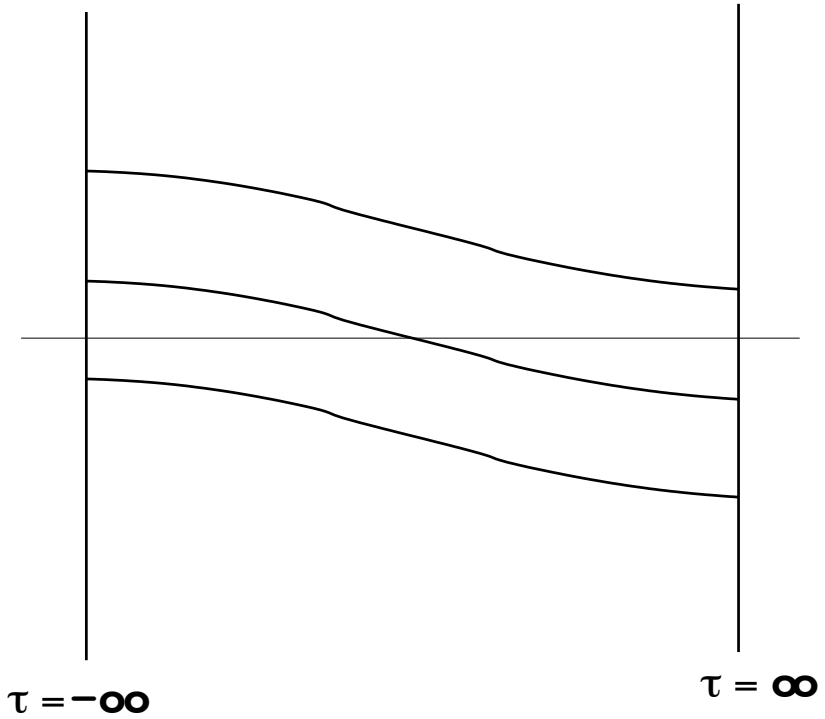
Parity Anomaly in QCD_3:

- Effective action is not gauge-invariant:

$$\det\left(\not{\partial}+\not{A}\right) \longrightarrow (-1)^n \det\left(\not{\partial}+\not{A}\right)$$



Parity Anomaly in QCD_3:

- To restore the gauge invariance one should add a term which cancels $(-1)^n$,

$$S_{\rm ct} = \pi \int \omega_3(A) \longrightarrow S_{\rm ct}(A) + n\,\pi\,.$$

- For even number of flavors, one can define a Dirac spinor, $\Psi = (\psi_L, \psi_R)^T$, which has a parity-invariant Dirac mass. The effective action is hence parity-invariant.

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Dynamical mass generation

- For an even number $(2n)$ of flavors

$$\Psi_i = \begin{pmatrix} \psi_i \\ \psi_{i+n} \end{pmatrix}, \quad \gamma^\mu = \gamma^\mu_{2\times 2} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

- We have $\mathrm{U}(2)$ 'chiral symmetry', generated by $\mathbf{1}_{4\times 4}$, γ^3, γ^5, $[\gamma^3, \gamma^5]$.

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Dynamical mass generation

- For $2n$ massless flavors we have $\mathrm{U}(2n)$ 'chiral' symmetry and non-anomalous P_4 parity:

$$\Psi_i(x) \underset{P_4}{\longmapsto} \Psi'_i(x') = e^{i\delta} \begin{pmatrix} 0 & \gamma^1_{2\times 2} \\ \gamma^1_{2\times 2} & 0 \end{pmatrix} \Psi_i(x)\,.$$

Symmetry breaking pattern

- QCD_3 is strongly coupled at low energy and confining. (KKN)
- Schwinger-Dyson analysis shows quarks get dynamical mass (Appelquist and Nash '90).
- The chiral symmetry is spontaneously broken and quarks get dynamical mass: $U(2n) \mapsto U(n) \times U(n)$.
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Vafa-Witten:

- The gauge-invariant regularization gives non-positive measure:

$$\det\left(i\not{D}\right) = \pm\sqrt{\det\left(i\not{D}_4\right)}\,.$$

- But, for $N_f = 2n$ flavors the quark determinant is positive ($m \to 0$):

$$\det\left[(i\not{D}+im)(i\not{D}-im)\right]^{\frac{N_f}{2}} = \det\left[-(\not{D})^2+m^2\right]^{\frac{N_f}{2}} \geq 0$$

- The vector symmetry $U(n)\times U(n)$ can not be spontaneously broken in P_4 invariant 3D theory. (Vafa-Witten '84)

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Coleman-Witten:

- Suppose the order parameter is a quark bilinear, $M_i^j = \langle \bar{\psi}_i \psi^j \rangle$, $g \in \mathrm{U}(2n)$:

$$M \longmapsto g^\dagger M g\,;\; M \underset{P_4}{\longmapsto} P_4^{-1} M P_4 = -I_1 M I_1\,.$$

- The vacuum energy in the large N_c limit

$$V = N_c \,\mathrm{Tr}\, F(M^2) = N_c \sum_i F(\lambda_i)\,,$$

- The minimum occurs at $\lambda_i = \kappa^2$ and P_4 invariance requires $\mathrm{Tr} M = 0$. The unbroken symmetry is $U(n) \times U(n)$, if $\kappa \neq 0$.

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Low Energy Effective Lagrangian of QCD_3

- Consider composite fields for $g \in U(2n)$

$$\phi(x) = \lim_{y \to x} \frac{|x-y|^\gamma}{\kappa} \psi(y)\bar{\psi}(x) \longmapsto g\phi g^\dagger \,.$$

- Ground state: $\langle \phi \rangle = I_3 = \mathrm{diag}\left(\mathbf{1}_{n\times n}, -\mathbf{1}_{n\times n}\right)$
- Nambu-Goldstone bosons are described by

$$\mathcal{L}_{\mathrm{B}} = \frac{f_\pi^2}{2} \mathrm{Tr}(\partial_\mu \phi)^2 = \mathrm{Tr}\left[\left(\partial_\mu - i\bar{A}_\mu\right) g^\dagger \left(\partial_\mu + i\bar{A}_\mu\right) g\right] \,.$$

- Redundancy of $g(x)$ is removed by gauge sym.:

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Effective Largangian

- The effective Lagrangian should match P_2-anomaly. Consider two-point functions of $j_i^\mu = \bar{\psi}_i \gamma^\mu_{2\times 2} \psi_i$ $(i = 1, \cdots, 2n)$:

$$\left\langle j_i^\mu(k)\, j_j^\nu(-k) \right\rangle = \lim_{m\to 0} \frac{m_i}{|m_i|} \delta_{ij} \frac{N_c}{4\pi} \epsilon^{\mu\lambda\nu} k_\lambda \,,$$

- To match the parity anomaly we need to include CS terms such a way that preserves P_4 parity (Rajeev et al '92),

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_B + \frac{N_c}{4\pi} \mathcal{L}_{\text{CS}}(\bar{A}_1) - \frac{N_c}{4\pi} \mathcal{L}_{\text{CS}}(\bar{A}_2) + \cdots \,,$$

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Baryon as a vortex

- The manifold of Nambu-Goldstone fields of 3d QCD has a nontrivial topology

$$\Pi_2\left(\frac{\mathrm{SU}(2n)}{\mathrm{SU}(n)\times SU(n)\times \mathrm{U}(1)_3}\right) = \Pi_1\left(\mathrm{U}(1)_3\right) = Z\,.$$

- It should allow a vortex (baby Skyrmion),

$$Q = \int \mathrm{d}^2x\, J_0 = \frac{1}{2\pi}\int \mathrm{d}^2x\, \epsilon_{0ij}\partial_i \bar{A}_{3j}\,.$$

- $\mathrm{U}(1)_3$ vorticity is the baryon number:

$$\left\langle J^\mu(k)\, J^\nu_{35}(-k)\right\rangle = \frac{N_c}{2\pi}\epsilon^{\mu\lambda\nu}\, k_\lambda + \mathcal{O}(k^2)\,.$$

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Brane setup and geometry

- N_c D3 branes wrapping S^1 and N_f probe D7:

	0	1	2	3	4	5	6	7	8	9
D3	○	○	○	○	×	×	×	×	×	×
D7	○	○	○	×	○	○	○	○	○	×

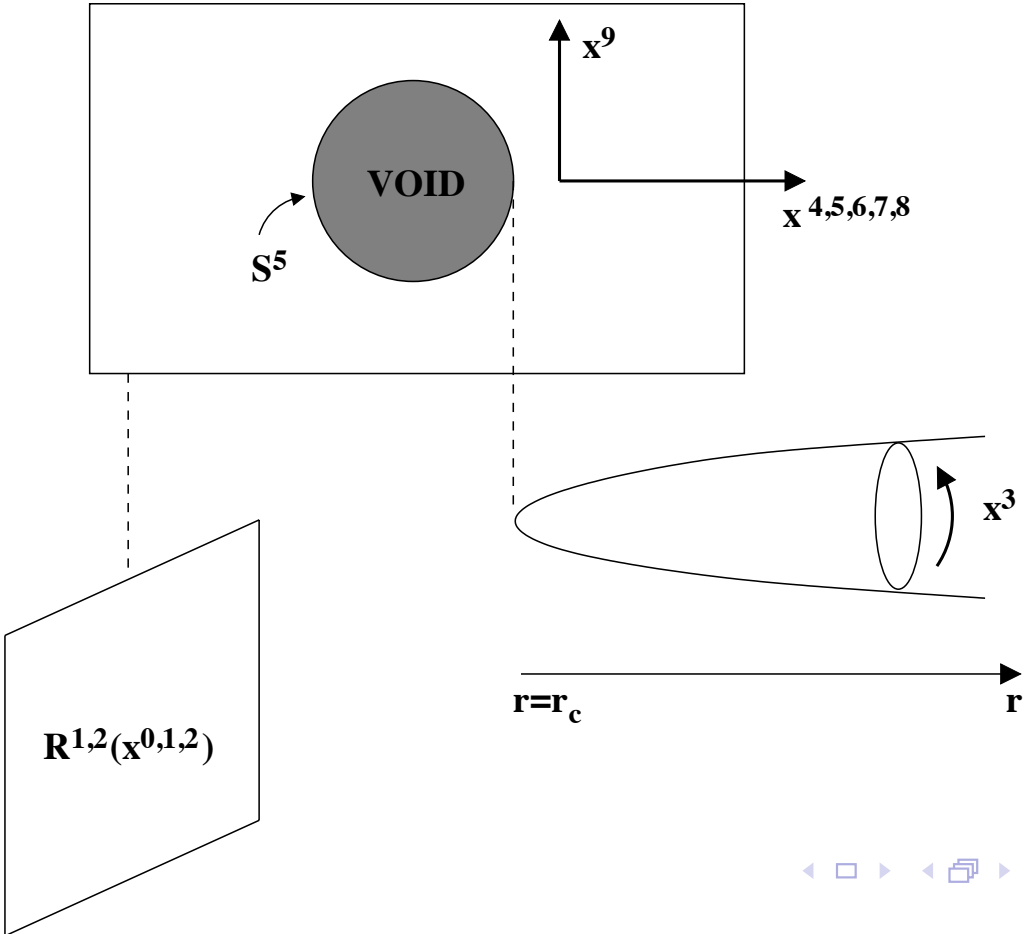
$$ds^2 = \frac{r^2}{L^2}\left(f(r)\left(dx^3\right)^2 + \left(dx^\mu\right)^2\right) + \frac{L^2}{r^2}\frac{dr^2}{f(r)} + L^2 d\Omega_5^2$$

$$F_5^{RR} = \frac{\left(2\pi l_s\right)^4 N_c}{\mathrm{Vol}(S^5)}\epsilon_5\,, \qquad e^\phi = g_s\,.$$

$$\left(\epsilon_5 = \sin^4\theta\, d\theta \wedge \epsilon_4\right)$$

Brane setup and geometry

$$L^4 = 4\pi g_s N_c l_s^4 \ , f(r) = 1 - \frac{M_{KK}^4 L^8}{16 r^4} \ , x^3 \sim x^3 + \frac{2\pi}{M_{KK}} \ .$$



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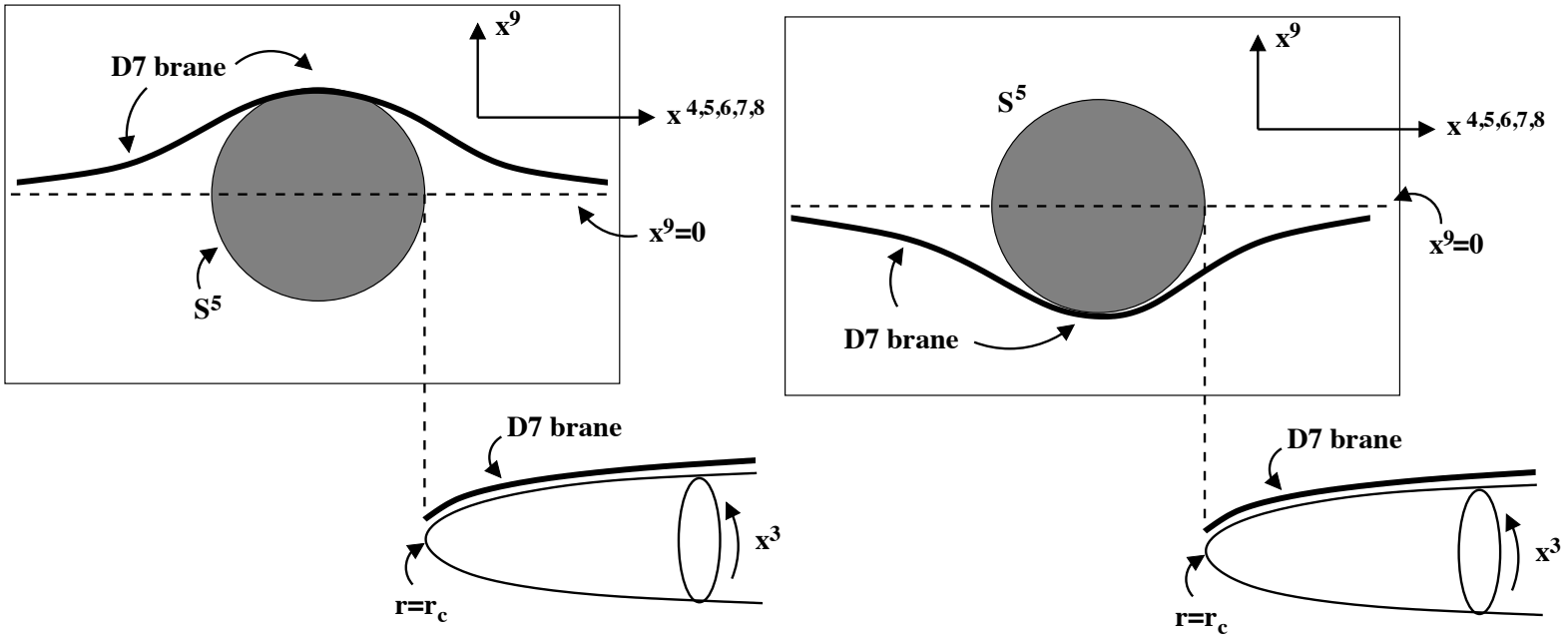

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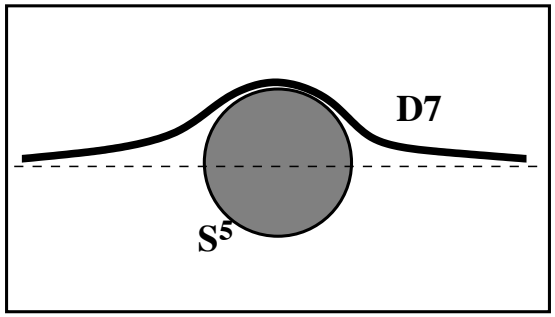
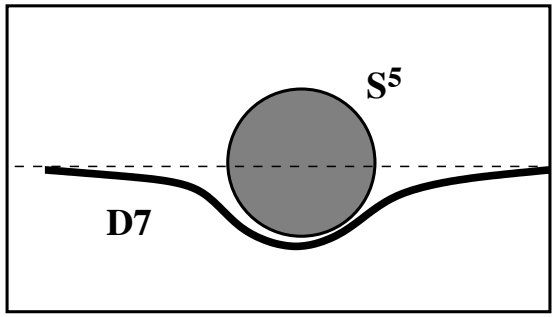
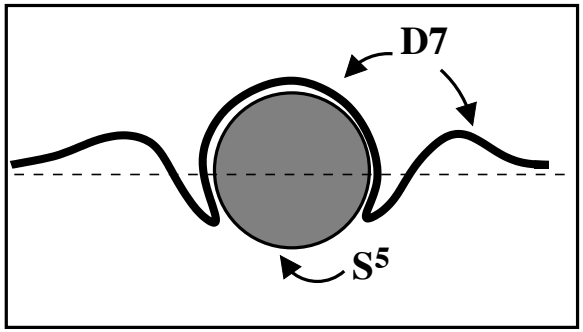
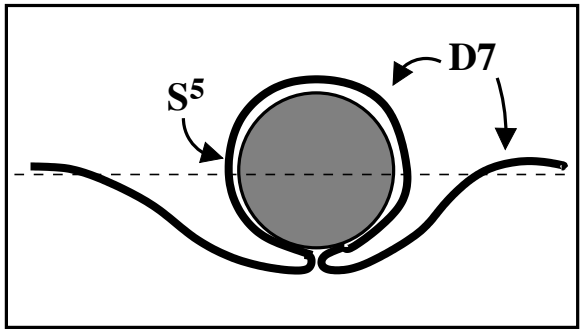


(a)



(b)





(c)

Holographic parity anomaly:

- A D7 wrapping S^5 and sitting at $r = r_c$ introduces one unit of C_0^{RR} monodromy along x_3: D3 worldvolume action contains for k units (uv data)

$$\mu_3 \frac{(2\pi l_s^2)^2}{2!} \int_{D3} C_0^{RR} \wedge F \wedge F = \frac{k}{4\pi} \int_{\mathbb{R}^{1,2}} A \wedge F \quad .$$

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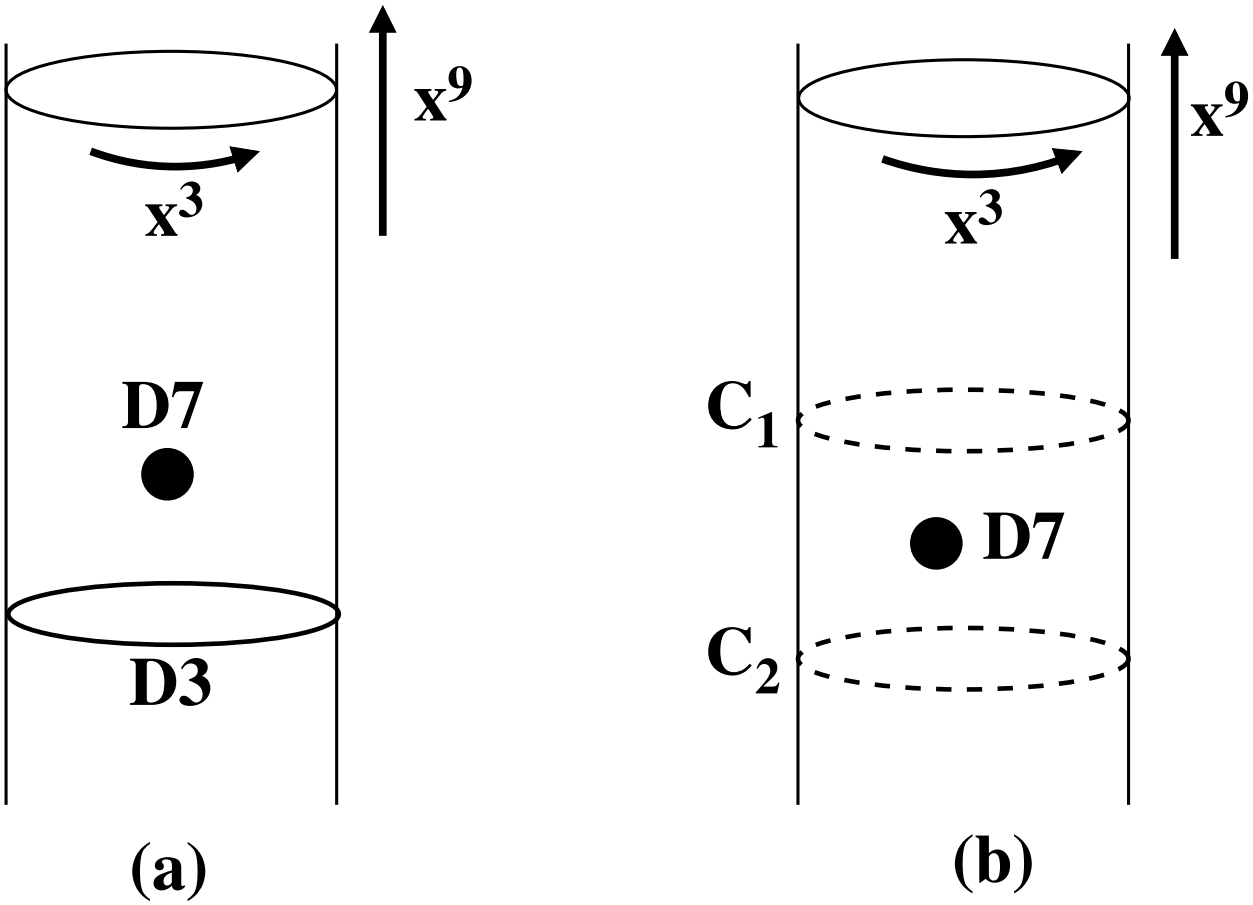
Weak-coupling D-brane picture

- Another view of our D-brane setup:



Weak-coupling D-brane picture of parity anomaly:

- $C_1 - C_2$ is homological to a circle on (x_3, x_9) around D7:

$$\int_{C_1} F_1^{RR} - \int_{C_2} F_1^{RR} = 1$$

- The D3 world-volume gauge theory contains a piece induced by background C_0^{RR}

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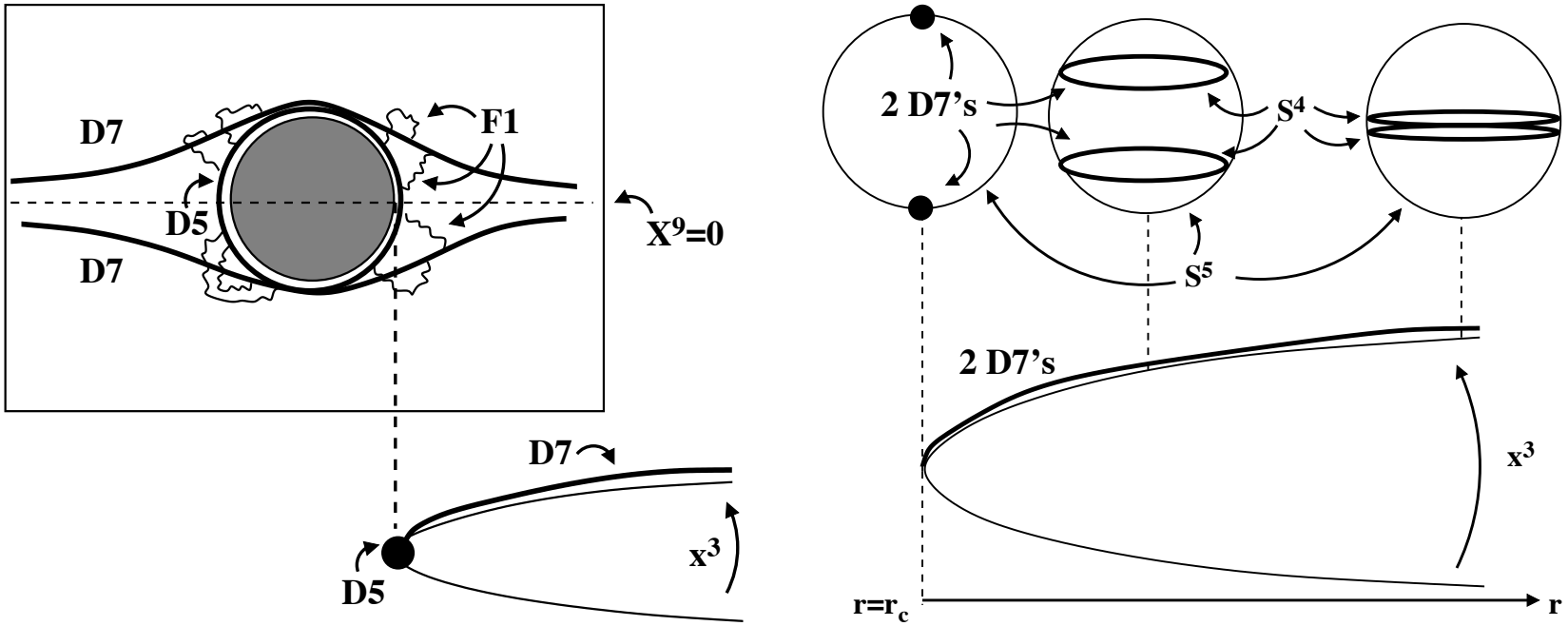
Baryons as D5 wrapping S^5



- S^5 wrapped by D5 carries $\int_{S^5} F_5^{RR} = N_c$ fundamental string charges. D7 wraps S^4 inside S^5. ($N_f = 2n$ from now on.)

Baryons as D5 wrapping S^5

- D5 wrapping S^5 at r_c ends on D7 at two intersecting points, as monopoles in D3/D1 (Callan+Maldacena, '97)
- D5, suspended between two sets of D7 branes at $r > r_c$, can be identified as carrying a monopole charge $(+1, -1)$ with respect to $U(n) \times U(n)$ gauge symmetry on the D7 branes world-volume, where the charges sit in the trace part of $U(n)$.
- Since S^4 is common, D5 is a monopole in $(r, x^{0,1,2})$ on D7 after integrating over $S^4 \subset S^5$.

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- $\theta(r)$ describe D7 emb. $(d\Omega_5^2 = d\theta^2 + \sin^2\theta d\Omega_4^2)$
- Induced metric on D7 is

$$g^* = \frac{r^2}{L^2}(dx^\mu)^2 + \left(\frac{L^2}{r^2 f} + L^2\left(\frac{d\theta}{dr}\right)^2\right) dr^2 + L^2 \sin^2\theta d\Omega_4^2$$

- Worldvolume action on D7 is

$$S = S_{\mathrm{DBI}} + \mu_7 \frac{(2\pi l_s^2)^2}{2!} \int C_4^{RR} \wedge F \wedge F$$

$$C_4^{RR} \sim N_c \left[\left(\theta - \frac{\pi}{2} \pm \frac{\pi}{2}\right) - \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta\right] \epsilon_4$$

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Electric charge of Monopole

- Consider a D7, embedded in the upper hemisphere of S^5 ($x^9 > 0$). ($0 \leq \theta \leq \frac{\pi}{2}$) with

$$\tilde{\mu}_7 \int_{S^4} C_4^{RR} \wedge F \wedge F = \frac{\Theta(r)}{8\pi^2} F \wedge F$$

$$\Theta(r) = \frac{16}{3} N_c \left(\frac{3}{8}\theta - \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta \right)$$

- Integrating Θ over r, it leads to (flavor) CS term of boundary theory:

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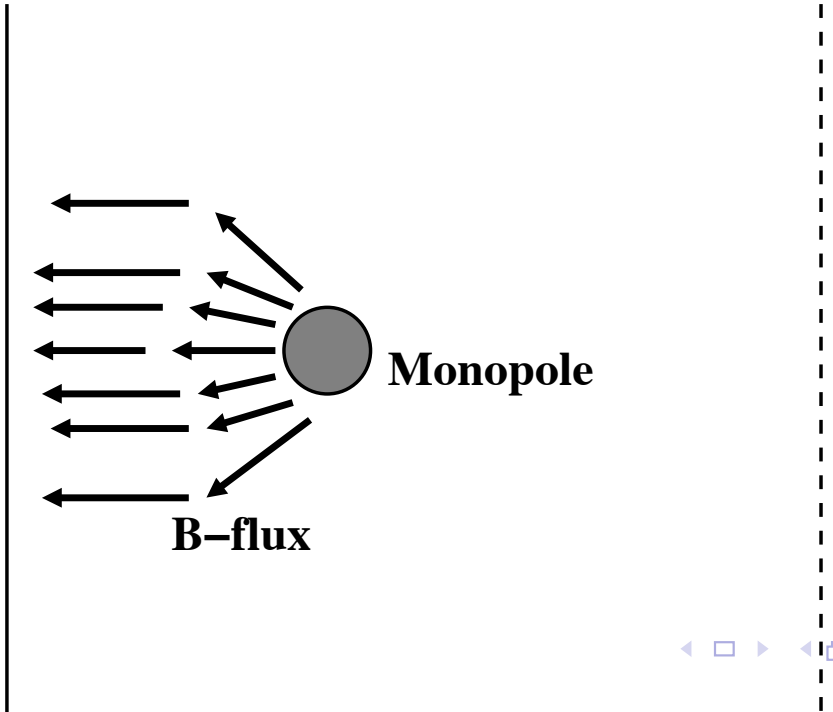
- Electric charge due to Witten effect plus medium (Lee) effect:

$$\rho_e = -\vec{\nabla}\cdot\vec{\Pi} = -\vec{\nabla}\cdot\left(\frac{\vec{E}}{e^2} + \frac{\Theta}{4\pi^2}\vec{B}\right)$$



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- The magnetic monopole is a dyonic object:

$$\vec{\nabla}\cdot\left(\frac{\vec{E}}{e^2}\right)=-Q_b\delta^3(\vec{x}-\vec{x}_0)\,,\ \ \vec{\nabla}\cdot\vec{B}=2\pi\delta^3(\vec{x}-\vec{x}_0)\,.$$

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Conclusion

- We have constructed a holographic dual of QCD_3 with D3/D7.
- Brane realization of parity anomaly: CS level number is the number of D7 branes.
- For even flavors quarks get dynamical mass, breaking $U(2n)$ to $U(n) \times U(n)$.
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