condensed matter + holography

Sean Hartnoll

Harvard University

September 2010 – Crete

Plan of talk

Breakdown of Landau's Fermi liquid theory

The holographic approach

Electron stars

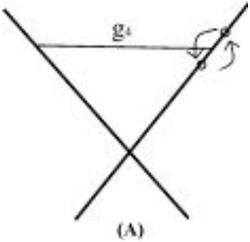
Effective field theory for metals

- In condensed matter physics we know the UV.
- UV is electrons + Coulomb interactions. Strongly interacting.
- The challenge is to describe the IR.
- ‘Opposite’ of e.g. quantum gravity.
- For conventional metals the IR turns out to be simple.

Landau's Fermi liquid theory

[as re-interpreted by Polchinski and Shankar (∼1993)]

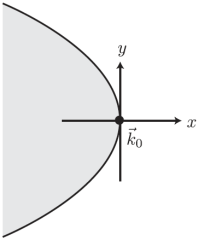
- A metal has a Fermi surface.
- Implies existence of gapless particle-hole excitations.



- Step 1: Write down a free action for these excitations.
- Step 2: Consider all possible interactions.

Landau's Fermi liquid theory continued

- Zoom in to a point on the Fermi surface



- Free action for excitations at that point

$$S_\psi \sim \int d^3x \psi^\dagger \left(\frac{\partial}{\partial \tau} - i v_F \frac{\partial}{\partial x} - \frac{\kappa}{2} \frac{\partial^2}{\partial y^2} \right) \psi \,.$$

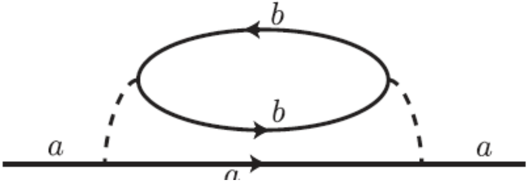
Landau's Fermi liquid theory continued

- Lowest order nontrivial interaction is

$$S_{\psi^4} \sim u \int d^3x \, \psi^\dagger \psi^\dagger \psi \, \psi \, .$$

- Interaction is irrelevant under: $x \to s^2 x \, , y \to sy \, , \tau \to s^2 \tau$.

- Effect on lifetime of quasiparticle excitation



- Find at low energies – Stable

$$\text{Im}\, \Sigma(\omega) \sim \omega^2 \log \omega \ll \omega \, .$$

Landau's Fermi liquid theory continued

- There are (almost) no relevant operators.
- Generic stable free IR fixed point.
- Very robust predictions, for instance DC resistivity

$$\rho(T) \sim \operatorname{Im} \Sigma(T) \sim T^2 \,.$$

- What could go wrong?

Breakdown of Fermi liquid theory

- In e.g. heavy fermion compounds, high temperature superconductors or organic superconductors one observes.

$$\rho(T) \sim T \,.$$

- Bad omen — suggests (very naively)

$$\mathrm{Im}\,\Sigma(\omega) \sim \omega \log \omega \sim \omega \,.$$

- But this means quasiparticle is not stable anymore — effective theory unlikely to be weakly interacting.

- Loophole in Landau's argument: additional gapless fields?

Emergent gapless bosons

- There are two scenarios in which gapless bosons appear:
- Tune system to a quantum critical point.
- Protect gaplessness by a ‘deconfined’ gauge symmetry: critical phase.
- In first case, action for order parameter, e.g.

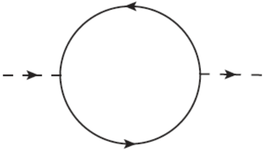
$$S_\phi \sim \int d^3x \left[\phi\left(\frac{\partial^2}{\partial\tau^2} + c^2\frac{\partial^2}{\partial x^2} + c^2\frac{\partial^2}{\partial y^2} + r\right)\phi + \frac{u}{24}\phi^4\right] .$$

- Coupling to fermions

$$S_{\phi\psi^2} \sim \lambda \int d^3x \phi\psi^\dagger\psi \,.$$

One loop physics

- The boson propagator is 'Landau damped' by the fermions



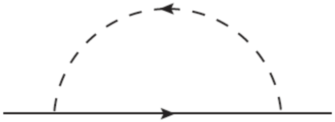
- Leading to the low energy momentum space action

$$S_\phi \sim \int d^3p \left[\phi \left(k_y^2 + \gamma \frac{|\omega|}{|k_y|} \right) \phi \right] .$$

- This leads to modified IR scaling: $x \to s^2 x \, , y \to sy \, , \tau \to s^3 \tau$.
- Under this scaling the 'Yukawa' coupling is classically marginal.

One loop physics continued

- The bosons in turn scatter the fermions



- Leading to destruction of the electronic quasiparticle!

$$\text{Im}\,\Sigma(\omega) \sim \omega^{2/3} \gg \omega\,.$$

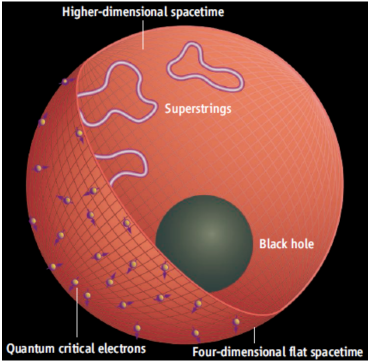
- Perturbation theory is rendered uncontrolled, the whole system is believed to run to a strongly interacting IR fixed point.

- Dynamical critical exponent renormalised away from $z = 3$ at 3 loops.

Statement of the challenge

We would like to have a controlled theory that:

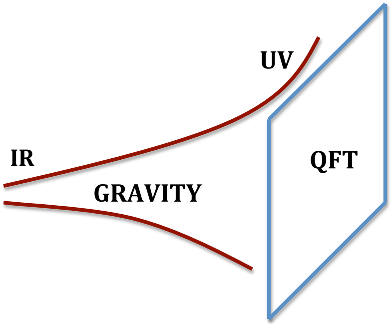
- Has an emergent dynamical scaling symmetry in the IR due to a finite density (of fermions?) in the UV.
- Has a Fermi surface but no stable quasiparticles.



Let us see what we can do with holography!

The holographic method of inquiry

- Holography geometrises the RG flow



- Finite charge density $\Rightarrow$ A $U(1)$ flux at infinity.

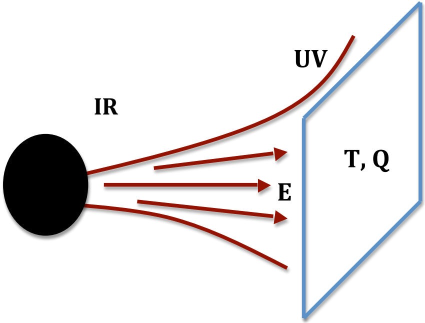
- Given a bulk theory, and a flux at infinity, what is the IR geometry?

Old time favorite: extremal Reissner-Nordstrom

- The simplest theory with the necessary ingredients

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)-\frac{1}{4g^2}F^2\right] .$$

- Unique $T = 0$ finite density planar solution. Extremal RN-AdS



- IR is $AdS_2 \times R^2$. Emergent scaling! Are we done?

The trouble with $AdS_2 \times R^2$, Part I.

- $AdS_2 \times R^2$ is the degenerate limit of the Lifshitz metric

$$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{dx^2 + dy^2}{r^2} \, .$$

  when $z \to \infty$.

- A $z = \infty$ theory is almost certainly an artifact of the large N limit (seen before in conventional condensed matter physics).

- Pathological properties. From dimensional analysis

$$S \sim T^{2/z} \, .$$

  When and only when $z = \infty$ there is a finite entropy at $T = 0$.

Aside: Why people like $AdS_2 \times R^2$

- When $z = \infty$, time scales but space does not.
- Therefore, the scaling behavior is visible at all momenta, not just low momenta.
- Fermionic excitations live at finite $k \sim k_F$.
- Therefore, by coupling a $z = \infty$ theory to fermions, one can easily obtain non-Fermi liquid behavior.

An attempt to avoid $AdS_2 \times R^2$

- Add a dilaton to the theory

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) + \frac{1}{2}(\nabla\phi)^2 - \frac{1}{4g^2}e^{-\alpha\phi}F^2 + \cdots\right] .$$

- IR metric is now Lifshitz with finite z (depending on α).

- Why? Because the dilaton violates the naive Gauss law, 'screening' the electric field. In the IR, not enough flux to support AdS_2.

- But scalar has a runaway behavior

$$\phi \sim \pm \log \Lambda r \, .$$

- Have not reached the true IR, there is still a coupling flowing.

- Classical action breaks down as gradients $|\nabla\phi|^n$ becomes large.

The trouble with $AdS_2 \times R^2$, Part II.

- For planar black holes in AdS, extremality bound is parametrically above the BPS bound.
- Therefore, no reason to expect extremal black holes to be the ground state.
- If light charged matter is present, can be favorable for matter to carry the charge, rather than black holes.
- Versions of 'superradiance instabilities' in GR.
- Thus
  - Charged scalars $\rightarrow$ Hololgraphic superconductors.
  - Charge fermions $\rightarrow$ Electron stars.

Electron stars

- Neutron star is a $T = 0$ gravitating ideal fluid of fermions.
- AdS held a chemical potential has solutions given by a gravitating charged ideal fluid of fermions: electron stars.
- Action for (irrotational) gravitating charged ideal fluid at $T = 0$

$$S = \int d^4x\sqrt{-g}\left(\mathcal{L}_{\text{Eins.}-\text{Maxwl.}} + \mathcal{L}_{\text{fluid}}\right) ,$$

where

$$\mathcal{L}_{\text{fluid}} = -\rho(\sigma) + \sigma u^a(\partial_a\phi + A_a) + \lambda(u^a u_a + 1) .$$

- $\rho(\sigma)$ is equation of state.

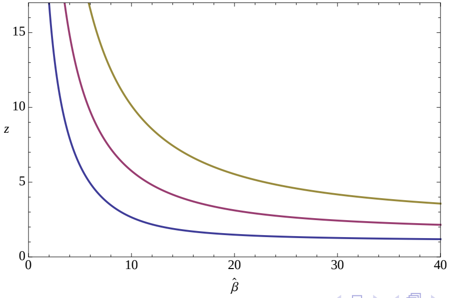
IR emergent scaling

- Find an exact IR Lifshitz solution if mass m of fermion satisfies (charge $q = 1$ WLOG)

$$\kappa m < g \,.$$

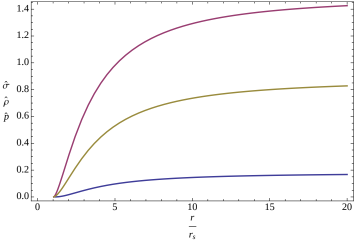
- Scaling exponent z depends on mass and on ratio

$$\hat{\beta} = \frac{g^4 L^2}{\kappa^2} \,.$$

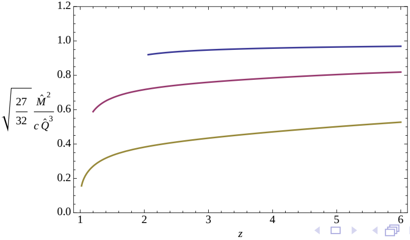


Properties of electron stars

- Numerically integrate out towards the boundary:



- Compare energy to RN-AdS black hole



Comments on electron stars

- Things I like about electron stars
  - Emergent IR scaling from UV charge density.
  - Charge apparently manifestly carried by fermions.
- Some things to understand better
  - Finite temperature and magnetic field (in progress).
  - Realisation in string theory? Probe branes?
  - Effect of interactions: pairing instabilities? Jeans instabilities?
  - Condensed matter phenomenology.

Assortment of concluding questions

- Can one make sharp connections between the UV and IR in general at finite density? Cf. Luttinger's theorem.
- Are there new ways of getting Lifshitz in the IR?
- Does the runaway dilaton lead to controllable stringy or quantum gravity physics?
- Ideal fluids underexplored in holography...
- Connection between non-Fermi liquid behavior and superconducting or other instabilities?
- Phase transition between Fermi liquid and non-Fermi liquid behavior?