A new approach to holographic super-conductors

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- Explicit holographic realization Gubser; Hartnoll, Herzog, Horowitz '08
  - Charged AdS BH + Φ
  - A second order normal-to-superfluid transition at T_c, $\langle\Phi\rangle$ order parameter.
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  - A second order normal-to-superfluid transition at T_c, $\langle\Phi\rangle$ order parameter.
  - Enjoy success in reproducing expected behavior in conductivity $\sigma(\omega, T)$ etc.
- Many open issues:
  - No microscopic understanding
  - Weak-strong duality, role of α' corrections ?
  - large-N limit and g_s corrections ?

A different approach

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1. A chain of dualities
   - Superconductors $\Leftrightarrow$ spin-models
     e.g. the XY model of paramagnet-ferromagnet transition, the $O(3)$ spin model etc.
   - Spin-models $\Leftrightarrow$ low-energy effective theory of gauge theories
   - Gauge-theories at strong coupling $\Leftrightarrow$ gravity

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   - Gauge-theories at strong coupling $\Leftrightarrow$ gravity
2. An explicit model based on NCST $\Leftrightarrow$ XY-model of super-fluidity.

Outline

- Lattice gauge theory- Spin model equivalence and the Gravity/Spin-model correspondence
- Continuous Hawking-Page transitions $\Leftrightarrow$ normal-to-superfluid transitions
- A model in $d+1$ NCST: thermodynamics and transport
- Linear dilaton CFT as the world-sheet theory near transition
- Critical exponents and mean-field scaling
- Discussion

Lattice gauge theory and Spin-models

Polyakov '78; Susskind '79

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- Any LGT with *arbitrary* gauge group G in d-dimensions with arbitrary *adjoint matter*
- Integrate out gauge invariant states $\Rightarrow$ generate effective theory for the Polyakov loop
- $Z_{LGT}(T) \sim Z_{SpM}(T^{-1})$
- Ferromagnetic spin model $\mathcal{H} = -J \sum_{\langle ij \rangle} \vec{s}_i \cdot \vec{s}_j + \cdots$ in $d-1$ dimensions with spin symmetry $C = Center(G)$

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- Inversion of temperature:
  Deconfined (high T) phase in LGT $\Leftrightarrow$ Ordered (low T) phase of SpM
  Confined (low T) phase in LGT $\Leftrightarrow$ Disordered (high T) phase of SpM

LGT - SpM equivalence at criticality

Svetitsky and Yaffe ’82

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  The critical phenomena of the gauge theory and the Spin model are in same universality class.
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  $\Rightarrow$ Continuum version of the equivalence
- Examples:
  1. $SU(N)$ with $N > 4$, $d > 3$
     Spin model with Z_N fixed point: $d = 4$ non-trivial $O(2)$ XY model exponents, $d > 4$ mean-field exponents. Includes the $N \to \infty$ limit, where $C \to U(1)$.
  2. $d = 3$, $N > 4$
     Spin model with Z_N fixed point in 2 spatial dimensions: *BKT transition*

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“Gravity/Spin-model correspondence"

- *Continuous* Hawking-Page transitions in gravity $\Leftrightarrow$ normal-to-superfluid transition in the XY model.

Holographic superfluidity, cont'ed

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Holographic superfluidity, cont'ed

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- GR background with Hawking-Page transition:

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\begin{aligned}
ds^2_{TG} &= G_0(r)dr^2 + H_0(r)dt^2 + I_0(r)dK + \cdots; \qquad \overline{\Phi} = \overline{\Phi}_0(r) \\
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- The topological shift symmetry:
  $\psi \to \psi + const; \qquad \psi \equiv \int_M B$
- $U(1)$ symmetry in the winding F-string sector, broken down to Z_N at finite N.

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- $U(1)$ symmetry in the winding F-string sector, broken down to Z_N at finite N.
- Under the correspondence $\langle \mathcal{W}_F \rangle_{gr} \Leftrightarrow \langle P \rangle_{FT} \Leftrightarrow |M| e^{i\psi}$

Holographic superfluidity, cont'ed

F-string expectation value: $\mathcal{W}_F = \int \mathcal{D}X_\mu \mathcal{D}h_{ab}\, e^{-\int (G+iB+\bar{\Phi}R^{(2)})}$

$e^{\int G} = |M|$ and $e^{i\int B} = e^{i\psi}$

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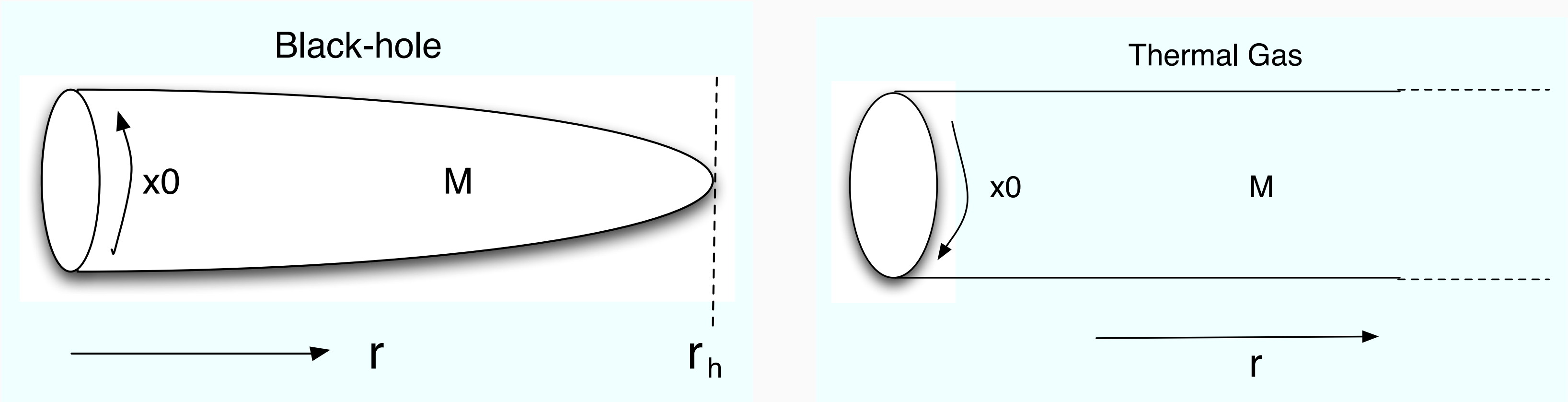

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$|M| \sim \langle \mathcal{W}_F \rangle_{TG} = 0; \qquad normal\ (high\ T)\ phase$

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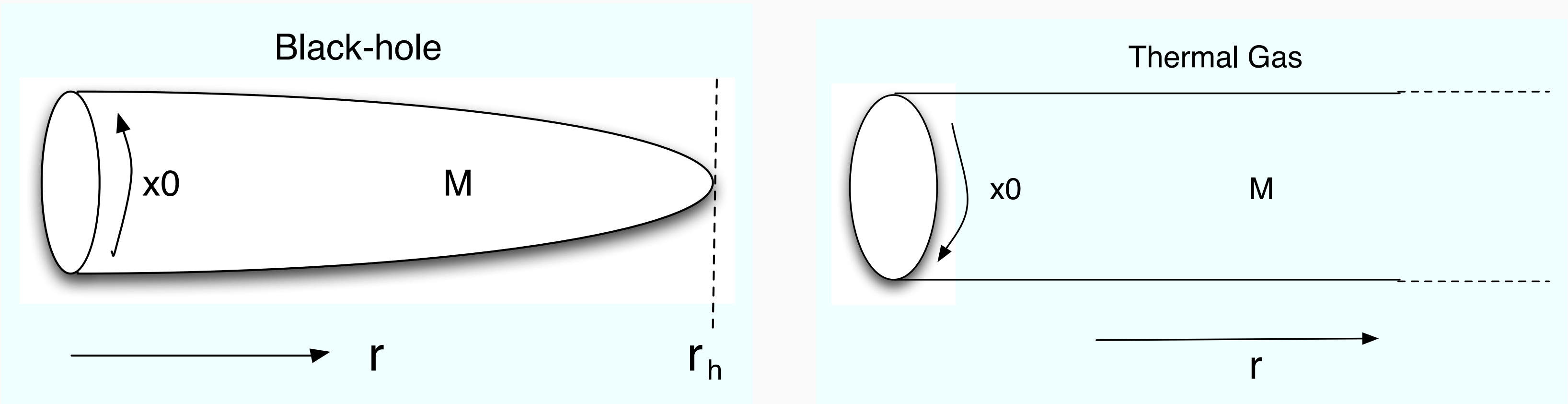

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On the BH, $\langle \mathcal{W}_F \rangle_{BH} \sim \int \mathcal{D}\psi(K) e^{-\mathcal{A}_{gr}[\psi]} \mathcal{W}_F$.

- For $d-1=2$ fluctuations $\delta\psi$ IR divergence, $\langle M \rangle = 0$
- For $d-1>2$ set $\psi = \psi_0$, spontaneous breaking of the $U(1)$.

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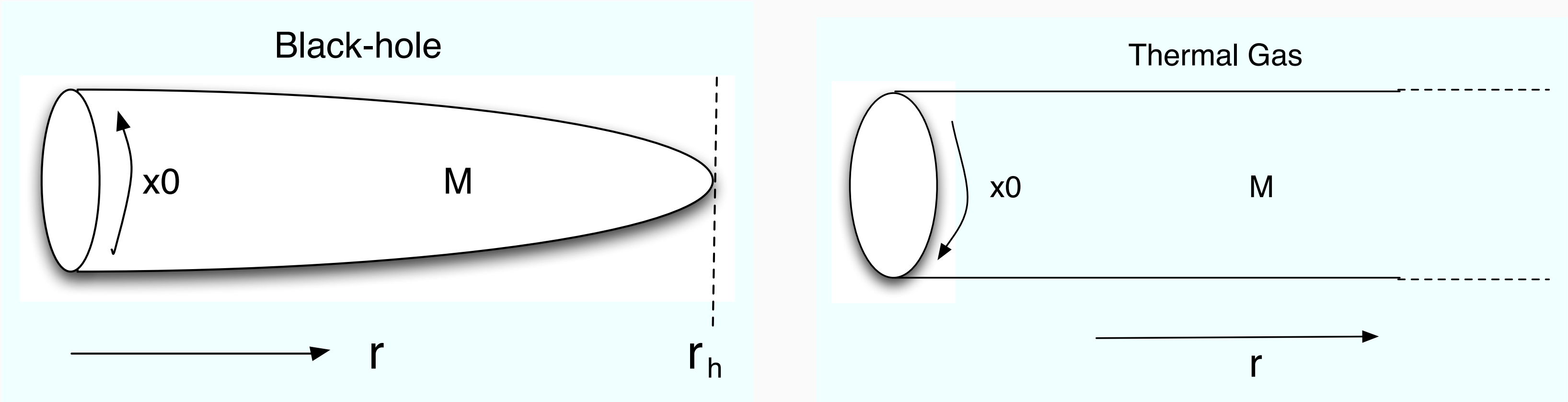

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$\delta\psi$: Goldstone mode of the superfluid

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Gauge theory	Gravity		Spin model
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Another condition for superfluidity:
Second speed $c_\psi \to 0$ as $T \to T_c$ iff a continuous phase transition

Continuous Hawking-Page $\Leftrightarrow$ Normal-to-superfluid transition

Two approaches to GR/SpM correspondence

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Top-bottom approach

- Construct the $SU(N)$ LGT with continuous conf-deconf transition in the continuum
- Construct the dual D-brane set-up
- Take decoupling limit and study the black-brane solution
- Compute thermodynamic observables by bulk objects, critical exponents by probe strings.

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Bottom-up approach

- Construct a dilaton-gravity in $d+1$ dimensions with a continuous HP transition
- Compute observables

The first: a microscopic handle on AdS/CMT

We adopt the latter approach in this talk.

The model

Dual of $SU(N)$ g.t., inspired by NCST U.G., Kiritsis '07; U.G., Kiritsis, Nitti '07

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Action in the string frame

$$\mathcal{A}_s = \frac{1}{g_s^2 \ell_s^{d-1}} \int d^{d+1}x \sqrt{-g_s} e^{-2\bar{\Phi}} \left(R_s + 4(\partial\bar{\Phi})^2 + \frac{\delta c}{\ell_s^2} - \frac{1}{12} H_{(3)}^2 \right) - \frac{1}{2(d+1)!} F_{(d+1)}^2 + \cdots$$

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Replace $F_{(d+1)}$ in the action by the solution $F_{(d+1)} \sim N \epsilon_{(d+1)} F(\Phi)$

Define the rescaled dilaton $\Phi = \bar{\Phi} + \log N$

Action in the Einstein frame $g_{s,\mu\nu} = e^{\frac{4\Phi}{d-1}} g_{\mu\nu}$:

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$$\mathcal{A} = \frac{N^2}{g_s^2 \ell_s^{d-1}} \int d^{d+1}x \sqrt{-g} \left(R_s - \frac{4}{d-1}(\partial\Phi)^2 + V(\Phi) \cdots \right)$$

Dilaton potential $V(\Phi)$: a phenomenological input; contains information on the gauge theory (matter content, beta-function, etc.)

Continuous Hawking-Page transition

U.G. arXiv:1007.0500 $V(\Phi) = V_\infty e^{\frac{4}{d-1}\Phi}\left(1 + V_{sub}(\Phi)\right); \qquad as \;\; \Phi \gg 1$

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We focus on the case $d - 1 = 2, 3$ and $n = 2$ in this talk: For example in 3D $V = V_\infty e^{\frac{4}{3}\Phi}\left(1 + 2e^{2\Phi_0}e^{-2\Phi}\right)$

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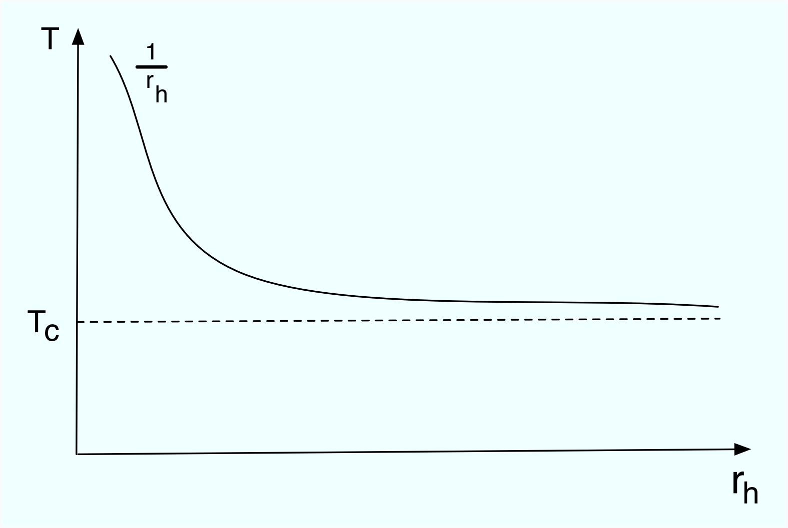
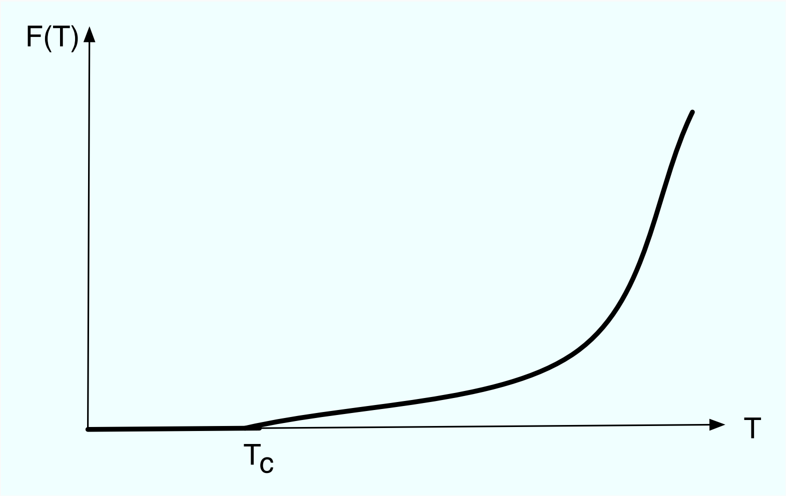
$$ds^2 = e^{-\frac{4}{3}\Phi_0}\frac{\cosh^{\frac{2}{3}}(\frac{3r}{2\ell})}{\sinh^2(\frac{3r}{2\ell})}\left(dt^2 + dx_{d-1}^2 + dr^2\right),$$

$$e^{\Phi(r)} = e^{\Phi_0}\cosh(\frac{3r}{2\ell}).$$

Thermodynamics

U.G. arXiv:1007.0500 A second order transition between the thermal gas and the black hole at $r_h = \infty$, $\phi_h = \infty$:





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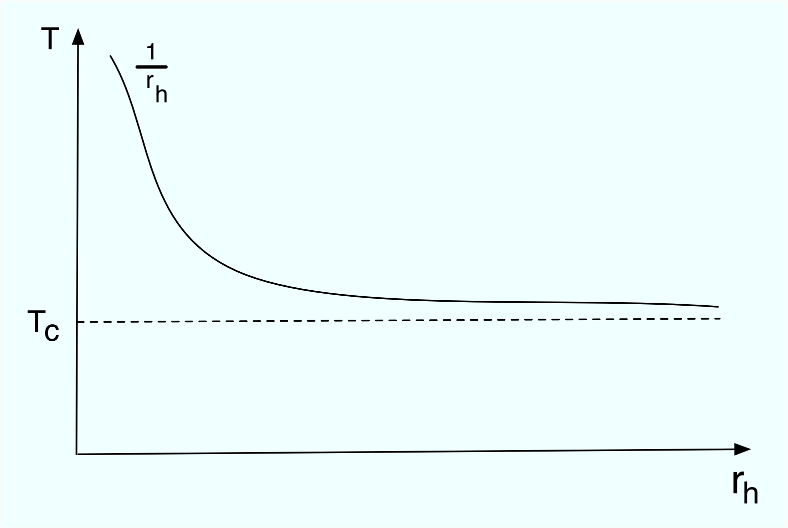

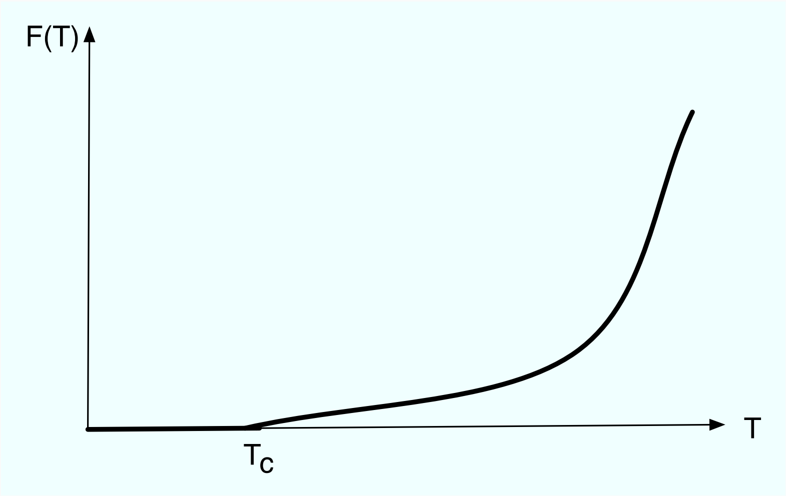

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- Discontinuous specific heat $\Delta C_v(T)$
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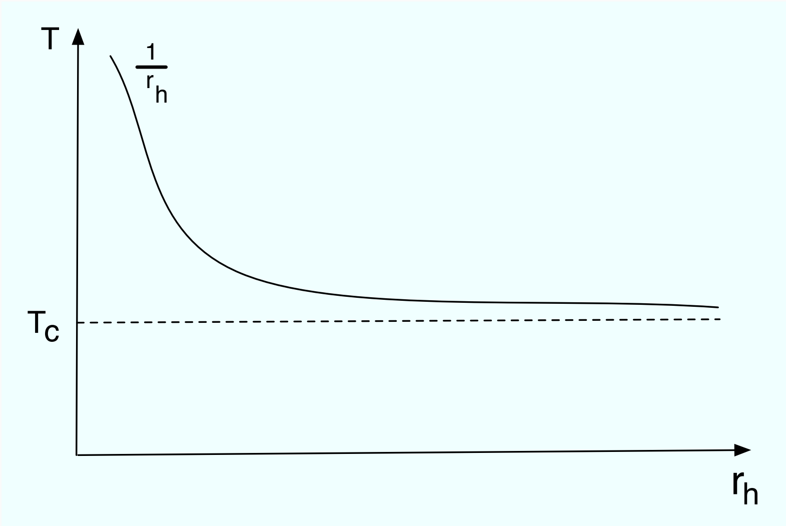

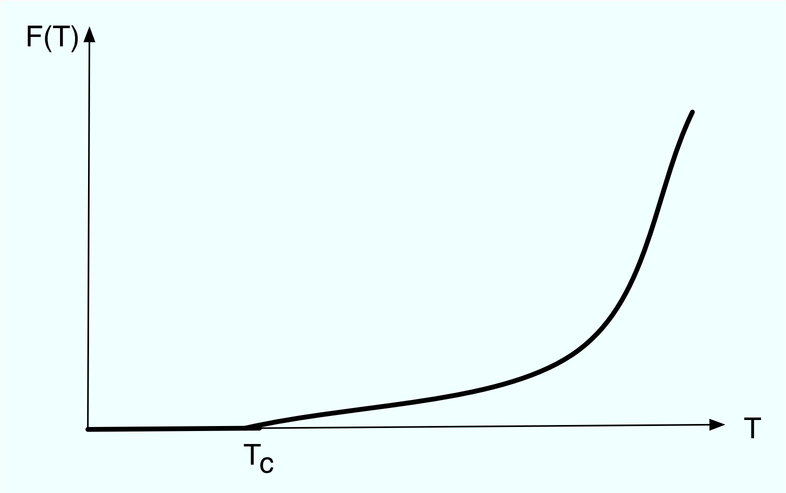

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- Speed of density waves discontinuous $c_s^2 = S/C_v$
- Universality in the bulk viscosity: $\left.\frac{\xi}{s}\right|_{T_c} = \frac{1}{2\pi(d-1)}$.

Large N and string loops

- Boundary value of the dilaton $\bar{\Phi}_0$
- Take $\bar{\Phi}_0 \to -\infty$, $N \to \infty$ such that $e^{\bar{\Phi}_0} N = e^{\Phi_0} = const.$
- String-loop counting:
  $e^{-\frac{1}{4\pi}\int_M \sqrt{h} R^{(2)} \bar{\Phi}} = N^{2(1-g)} e^{-\frac{1}{4\pi}\int_M \sqrt{h} R^{(2)} \Phi}$
- As long as $\int_M \sqrt{h} R^{(2)} \Phi$ is finite string-perturbation expansion well defined
- In the large N limit it is dominated by the sphere diagrams.

Higher derivative corrections

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- Expectation: strong correlations $\Leftrightarrow \alpha'$ corrections suppressed
- The correlation length $\xi \sim t^{-\nu} \to \infty$ near T_c
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for large $1 \ll r \ll r_h$.

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- The continuous HP transition should be governed by a linear-dilaton CFT on the world-sheet!

Second speed of sound

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- Second sound vanishes as $c_\psi^2 \sim |M|^2 \sim (T_c - T)^{2\beta}$
- Gravity/Spin-Model correspondence: $F_L \Leftrightarrow \mathcal{A}_{gr}$ on-shell, at large N
  Expect mean-field scaling $c_\psi^2 \sim (T_c - T)$.

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- Associate $\delta\psi$ with fluctuations of the B-field: $\psi = \int_M B$

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- Equate the Landau free energy and the regulated on-shell action:
  $F_L(T) = \Delta\mathcal{A}(T) = \mathcal{A}_{BH}(T) - \mathcal{A}_{TG}(T)$
- Associate $\delta\psi$ with fluctuations of the B-field: $\psi = \int_M B$
- One finds $c_\psi^2 \propto e^{-\sqrt{V_\infty} r_h} \sim (T - T_c)$.
- Second sound indeed vanishes at T_c with the mean-field exponent!

Critical exponents from probe strings

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$$\begin{aligned} \langle \vec{m}_{||} \cdot \vec{m}_{||} \rangle &\propto \langle RePReP \rangle \\ \langle \vec{m}_{\perp} \cdot \vec{m}_{\perp} \rangle &\propto \langle ImPImP \rangle \end{aligned}$$

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$$|\vec{m}| \Leftrightarrow |\langle P \rangle| \Leftrightarrow \langle e^{\int G + \Phi R} \rangle$$

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Renormalization only affects the sub-leading terms as $r_h \to \infty$

Dilaton piece is finite: $\Phi \sim r_h$, $R \sim e^{-\sqrt{V_\infty} r_h}$

One finds $S_{reg} = \frac{1}{2\pi \ell_s T(r_h)} \int_\epsilon^{r_h} e^{2A_s(r_h)} dr$

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Mean-field result only $V_\infty = \frac{4}{\ell_s^2}$

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Valid for any ℓ/ℓ_s

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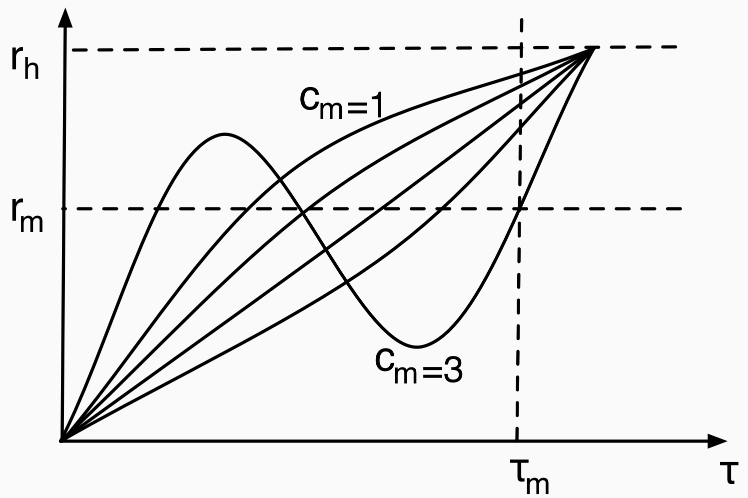
Division of paths: $r \in (0, r_m)$ UV, $r \in (r_m, r_h)$ IR



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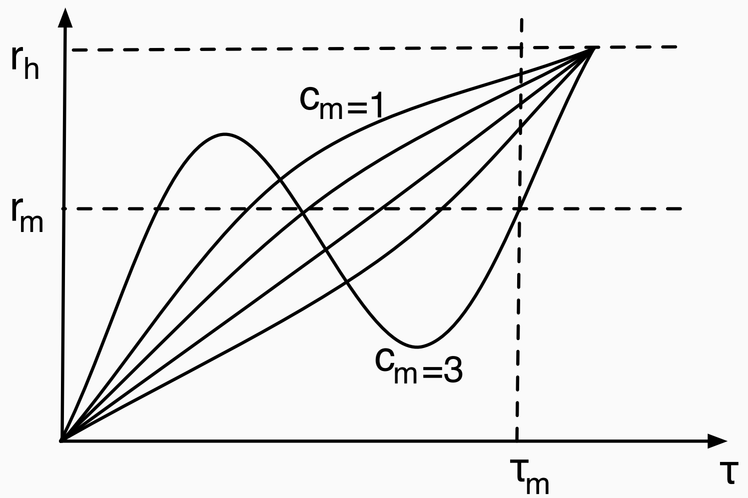

For $r_h \to \infty$ and r_m large enough, the IR region governed by the linear-dilaton CFT.

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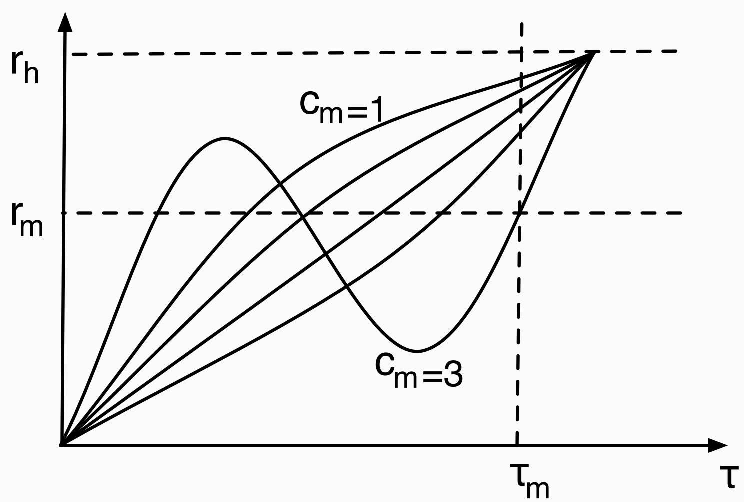

For $r_h \to \infty$ and r_m large enough, the IR region governed by the linear-dilaton CFT.

$\langle \Psi_i | \Psi_f \rangle = \int dX^\mu_m(\sigma) \int_{i,m} \mathcal{D}X^\mu e^{-\mathcal{S}[X]} \int_{m,f} \mathcal{D}X^\mu e^{-\mathcal{S}[X]}$

r_h dependence of $\langle \Psi_i \Psi_f \rangle$ determined by the IR path-integrals:

$M(r_h) \sim PI(\tau_m, \infty)$ approximated by the linear-dilaton CFT.

Linear-dilaton CFT

- Stress-energy tensor $T(z) = -\frac{1}{\alpha'} : \partial X^\mu \partial X_\mu : + v_\mu \partial^2 X^\mu$ with $v_\mu = \frac{\sqrt{V_\infty}}{2} \delta_{\mu,r} \equiv m_0 \delta_{\mu,r}$
- Spectrum: Tachyon for $d+1>2$; Graviton, dilaton and B-field fluctuations (massless); etc.
- Action: $\mathcal{A}_{IR} =$ $\frac{1}{4\pi\alpha'} \int_0^{2\pi} d\sigma \int_{\tau_m}^{\infty} d\tau \sqrt{\hat{h}} \left[\hat{h}^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu} + \frac{\alpha'}{4} v_\mu X^\mu \hat{R} \right].$

One-point function cont'ed

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The IR path-integral in the Hamiltonian language:

$PI_{IR} = \sum_{\chi \in \mathcal{H}_\perp} \langle V_\chi(X_m, \tau_m) V^*_\chi(X_f, \infty) \rangle \Delta_{IR}(\chi)$

The propagator of a state $\chi(p_r, p_\perp, k, w, N, \tilde{N})$:

$\Delta_{IR}(\chi) = \int_{|z|<1} \frac{d^2 z}{|z|^2} z^{L_0(\chi)-1} \bar{z}^{\tilde{L}_0(\chi)-1}$,

The vertex operator $V^*_\chi(X_f, \infty) = e^{-ip_r r_h}(\cdots)$,

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One finds $PI_{IR} \sim \sum_\chi C_\chi e^{-ip^*_r(\chi) r_h}$ with

$$\begin{aligned} p^*_r &= -im_0 \left(1 + \sqrt{1 + \frac{m^2_*(\chi)}{m^2_0}}\right), \\ m^2_* &\equiv \frac{2}{\alpha'}\left(N + \tilde{N} - 2\right) + p^2_\perp + (2\pi k T)^2 + \left(\frac{w}{2\pi T \alpha'}\right)^2. \end{aligned}$$

and level matching $kw + N - \tilde{N} = 0$.

Mean-field scaling from the Tachyon

On the tachyon state: $1 + \frac{m_*^2}{m_0^2} = \frac{1-d}{25-d}$. real contribution to exponent; higher states: imaginary contribution.

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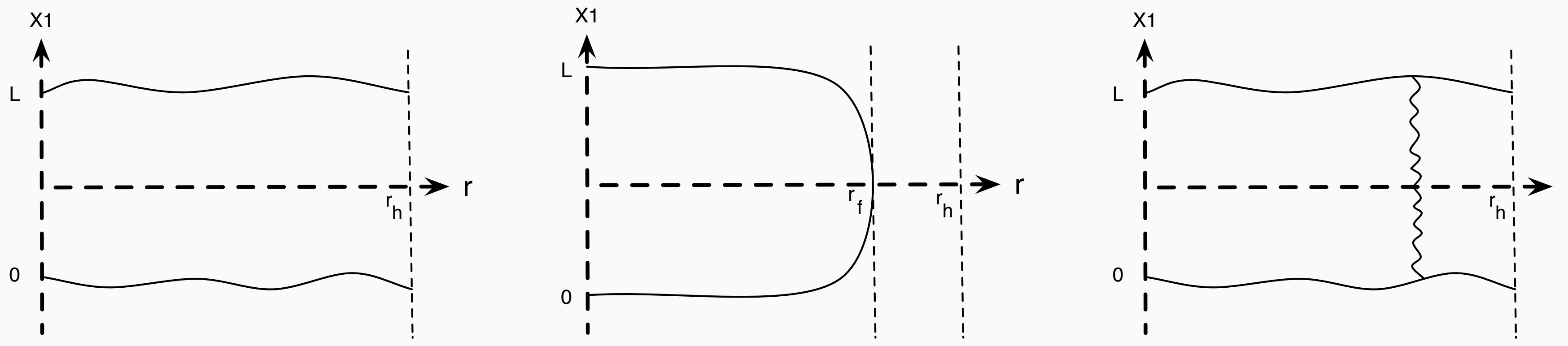
Mean-field scaling arise from the full computation!

Two-point function: classical computation

Three classical saddles:



(a) (b) (c)

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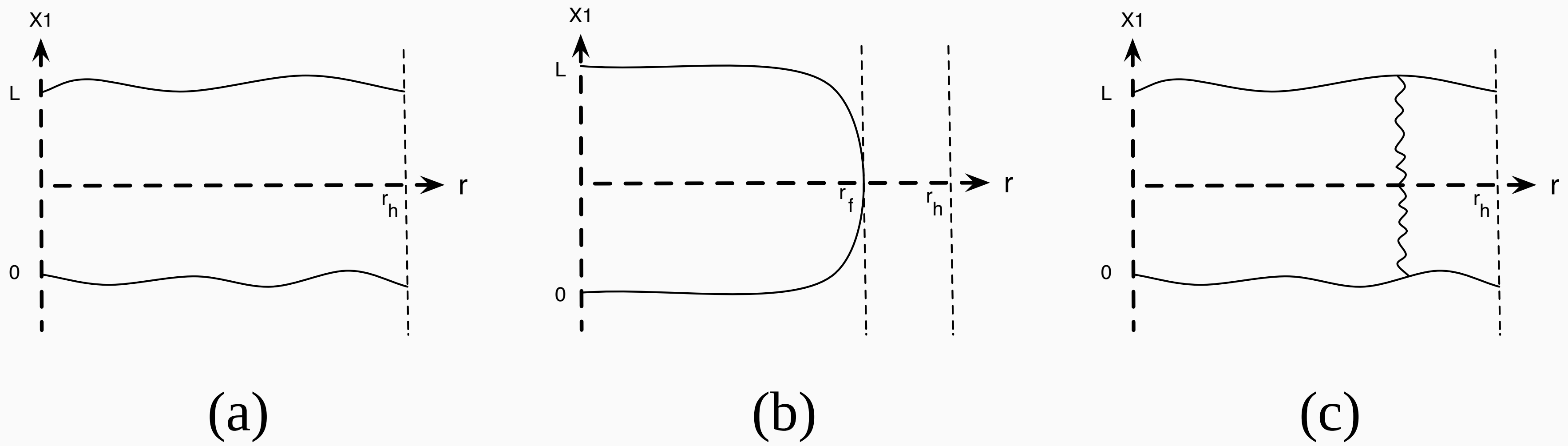

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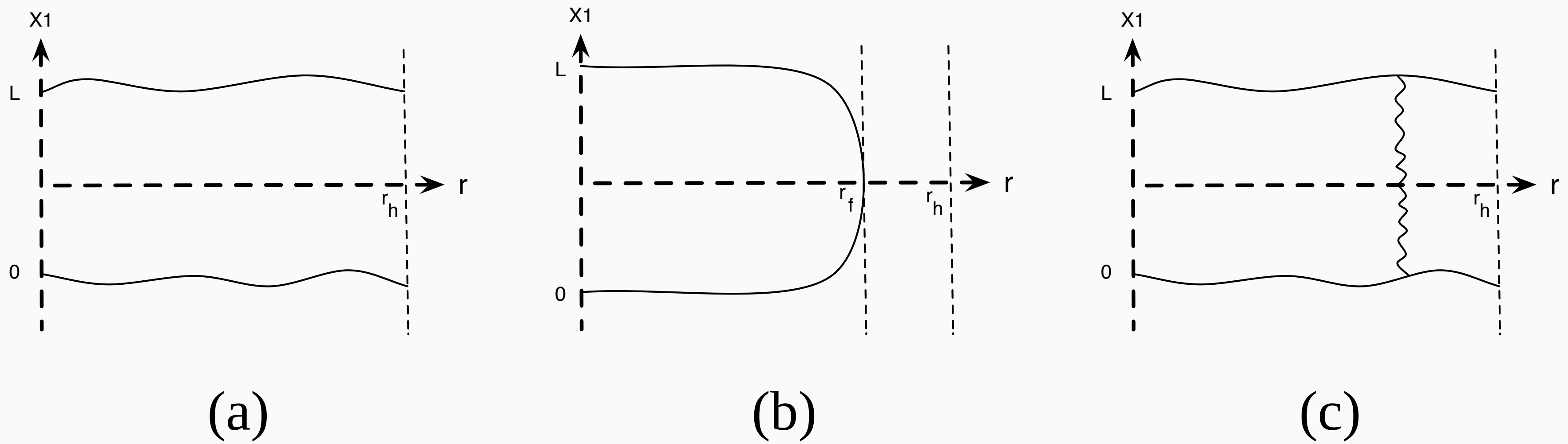

(a) $\langle \vec{m}(L) \cdot \vec{m}(0) \rangle_a = |\vec{m}|^2$. Finite in BH, 0 for TG.

(b) $S_{F1} \to m_T L + \cdots$, as $T \to T_c$ where $m_T = \frac{1}{2\pi \ell_s^2 T_c}$

$\langle \vec{m}(L) \cdot \vec{m}(0) \rangle_b \sim e^{-m_T L + \cdots}$ for $L \gg 1$.

Bulk exchange diagrams

$$\langle \vec{m}_{||}(L) \cdot \vec{m}_{||}(0) \rangle_c \propto \langle ReP[L]ReP[0] \rangle \sim \frac{e^{-m_+ L}}{L^{d-3}}$$
$$\langle \vec{m}_{\perp}(L) \cdot \vec{m}_{\perp}(0) \rangle_c \propto \langle ImP[L]ImP[0] \rangle \sim \frac{e^{-m_- L}}{L^{d-3}}$$

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Precisely the expected behavior from the XY model,

Except that $\xi_{||}^{-1} \to min(m_T, m_+)$ stays finite as $T \to T_c$.

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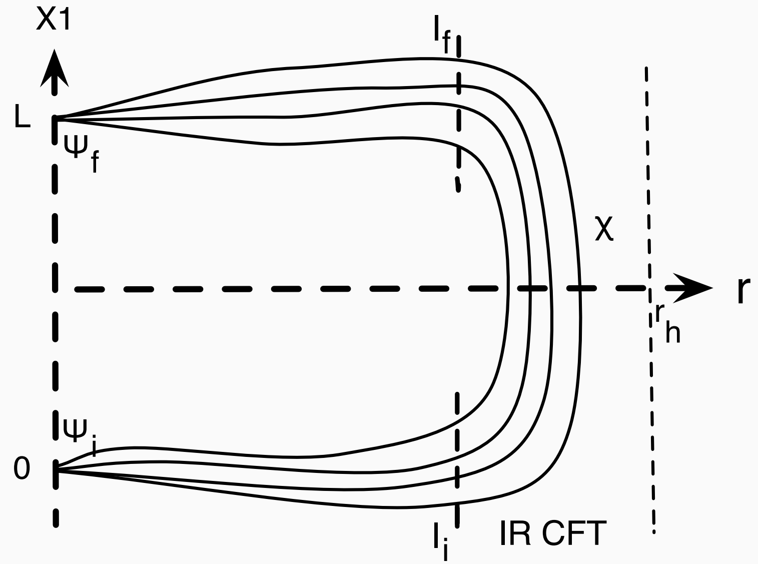

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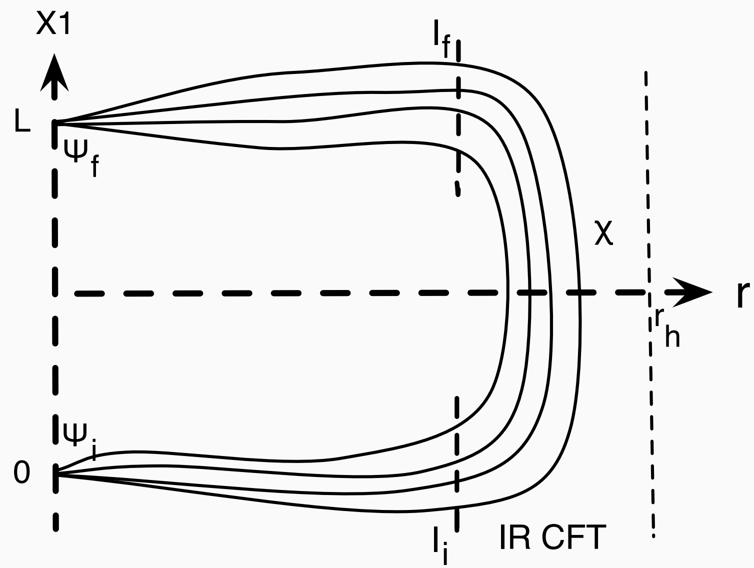

Propagator in the IR: $\Delta_{IR}(\chi) \sim \int dp_r d^{d-2} p_\perp e^{-ip_x^*(\chi)L}$

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On-shell momenta p_x for a winding mode $w = 1$:

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One finds $\xi(T)^{-1} = ip_x^*(\chi)_{min}$. Minimum mode is the "winding tachyon":

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Mean-field scaling again!

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(*) Two-point in the super-fluid phase: $\langle \bar{v}(L) v(0)\rangle_{BH} \sim \frac{1}{L^{p(T)}}$
(as opposed to exponential suppression for $d-1>2$)

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plasma of vortex anti-vortex pairs.

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$\int e^{-2\Phi} \left(R + \frac{1}{2}(\partial\Phi)^2 + V(\Phi) + \frac{1}{2}|DT|^2 - \frac{1}{2}m_T^2|T|^2 + F^2 \right)$
- Generalization to other spin models e.g. discrete center:
3D Ising model from the GR dulal of large-N $Sp(N)$?

Outlook

- Top-down approach to AdS/CMT: D-brane constructions, embedding in *critical* string theory
- Corrections to critical exponents: $1/N$ corrections? beyond the semi-classical approximation?
- A qualitative model based on winding-tachyon:
  $\mathcal{A}_{eff} \sim$
  $\int e^{-2\Phi} \left(R + \frac{1}{2}(\partial\Phi)^2 + V(\Phi) + \frac{1}{2}|DT|^2 - \frac{1}{2}m_T^2|T|^2 + F^2\right)$
- Generalization to other spin models e.g. discrete center:
  3D Ising model from the GR dulal of large-N $Sp(N)$?
- Spin-models with non-Abelian spin symmetry e.g. $O(3)$?
  Enhanced symmetries at special temporal radii?

THANK YOU !