Lifshitz solutions in string/M-theory

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September, 2010

Based on
arXiv:0905.1098, arXiv:1008.2062 with J. P. Gauntlett
and work in progress (to appear) with
J. P. Gauntlett, N. Kim and O. Varella

Motivation

- The AdS/CFT correspondence is a powerful tool to study strongly coupled (conformal) quantum field theories
- Recent interest in application to Condensed Matter Theory
  - One focus has been on systems with strongly coupled "quantum critical points" - phase transition at zero temperature
  - Another focus: superconductivity
    [Gubser; Hartnoll, Herzog, Horowitz]

Non-relativistic critical points

- Some Condensed Matter points non-isotropic scaling invariance

$$t \longrightarrow \lambda^z t, \qquad x_i \longrightarrow \lambda x_i, \qquad z \neq 1$$

where z is the dynamical exponent

- Lifshitz scaling symmetry with dual geometry [Kachru, Liu, Mulligan]

$$ds^2 = -r^{2z}\, dt^2 + r^2\, dx_i dx_i + \frac{dr^2}{r^2}$$
$$t \longrightarrow \lambda^z t, \qquad x_i \longrightarrow \lambda x_i, \qquad r \longrightarrow \lambda^{-1} r$$

Non-relativistic critical points

- Schrödinger symmetry with dual geometry
  [Son; Balasubramanian, McGreevy]

$$ds^2 = -r^{2z}\,dt^2 + r^2\,(2d\xi dt + dx_i dx_i) + \frac{dr^2}{r^2}$$

$$t \to \lambda^z t, \qquad x_i \to \lambda x_i, \qquad r \to \lambda^{-1} r, \qquad \xi \to \lambda^{2-z}\xi$$

  Where the extra coordinate ξ gives

  $\partial_\xi \to$ dual to particle number

  $-t\partial_i + x_i\partial_\xi \to$ dual to non-relativistic boosts

  For $z = 2$ there is also special conformal transformations (Schrödinger algebra)

- For both the above geometries we would also like to construct black holes asymptoting to them describing the dual field theory at finite temperature

Bottom Up

Most work has been carried out in "Bottom Up" models. Find solutions in a simple theory of gravity with a few additional degrees of freedom (vector, a few scalars)

Advantages:

- Simple to construct
- Models should (could) exist in string landscape
- Could capture universal behaviour

Disadvantages:

- Does the model arise in string theory? Is there a well defined dual CFT?
- Viewing a phenomenological model as an approximation to a model to be found in string theory low temperature behavior might not be captured

Top Down

In the alternative approach of "Top Down" explicit solutions are constructed in $d = 10$ or $d = 11$ supergravity

Advantages:

- One is studying bone-fide dual field theories - not all bottom up models might be realized
- One can study small parts of the landscape of solutions

Disadvantages:

- Hard in general!
- Solutions might not be of direct physical relevance

We have been pursuing Top Down constructions

Bottom Up constructions (Lifshitz)

- First Bottom Up model used vector + 2-form with topological coupling [Kachru, Liu, Mulligan]
- Equivalently a (time-like) vector can also be used [M. Taylor]

$$S = \int d^{d+1}x \sqrt{-g} \left(R + \Lambda - \frac{1}{4} \left(F^2 + m^2 A^2 \right) \right)$$

$$\begin{aligned} ds^2 =& \frac{1}{\beta^2} \frac{dr^2}{r^2} - r^{2z}\, dt^2 + r^2\, dx \cdot dx \\ A =& \sqrt{\frac{z-1}{2}} r^z\, dt \\ z =& m^2 / \left(4\beta^2\right), \quad \beta^2 = \Lambda / \left(z^2 + z + 4\right) \end{aligned}$$

- Proposed to study strange metallic holography [Hartnoll, Polchinski, Silverstein, Tong]

Bottom Up constructions (Lifshitz)

In the presence of a charged complex scalar under a $U(1)$ gauge symmetry (Abelian Higgs model) the mass of the vector can be more "dynamical" [Gubser, Nellore]

$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{4} F_{\mu\nu}^2 - \left| \left(\partial_\mu - \imath q A_\mu \right) \psi \right|^2 - V\left(\psi, \psi^*\right) \right]$$

$$ds^2 = - r^{2z} \, dt^2 + r^2 \, \left(dx_1^2 + dx_2^2 \right) + \frac{1}{L^2} \frac{dr^2}{r^2}$$

$$A = v \, r^z dt^2$$

$$\psi = \psi_0 \neq 0$$

- Proposed to describe the IR limit of superconductor ground states at finite chemical potential and $T = 0$

Outline

1. $Sch\,(z)$ and $Lif\;(z=2)$ solutions in type IIB and $d=11$ SUGRA
2. Consistent KK reduction of $d=11$ SUGRA on $\Sigma_3 \times S^4$
3. Comments / Open questions

Schrödinger Solutions

- First examples were Bottom Up
- First Top Down constructions used either duality transformations and/or consistent KK truncations. [Maldacena, Martelli, Tachikawa] [Adam, Balasubramanian, McGreevy] [Herzog, Rangamani, Ross]
- Led to infinite families of supersymmetric solutions [Yoshida, Hartnoll] [AD, J. P. Gauntlett]
- Type IIB: Uses $d = 5$ (Sasaki)-Einstein spaces to construct solutions dual to $d = 3$ field theories with $Sch(z)$ symmetry and $z \geq 3/2$
- $d = 11$: Uses $d = 7$ (Sasaki)-Einstein spaces to construct solutions dual to $d = 2$ field theories with $Sch(z)$ symmetry and $z \geq 5/4$

Lifshitz Solutions

- Bottom Up examples are well studied
- Top Down: Can they be realised in String/M-theory?
- [Li, Nishiola, Takayanagi][Bladäck, Danielsson, Van Riet]: No-go theorem for SUGRA solutions
- [Hartnoll, Polchinski, Silverstein, Tong]: Three constructions but all implicit/schematic
- [AD, J. P. Gauntlett]: New results
  - Infinite $Lif_3(z=2)$ solutions of Type IIB and $d=11$ SUGRA
  - Infinite $Lif_2(z=2)$ solutions of $d=11$ SUGRA
- [AD, J. P. Gauntlett, N. Kim, O. Varela]
  - Single $Lif_3(z \approx 39)$ solution of $d=11$ SUGRA

$Sch\left(z\right)$ solutions in type IIB and $d=11$ SUGRA

A general class of solutions of the type IIB e.o.m.

$$
\begin{aligned}
ds^2_{10} =& \Phi^{-\frac{1}{2}}\left[2dx^+dx^- + hdx^{+2} + 2Cdx^+ + d\mathbf{x}^2\right] + \Phi^{\frac{1}{2}}ds^2\left(M_6\right)\\
F_5 =& dx^+\wedge dx^-\wedge dx_1\wedge dx_2\wedge d\Phi^{-1} + *_{M_6}d\Phi\\
&- dx^+\wedge\left[*_{M_6}dC + d\left(\Phi^{-1}C\right)\wedge dx_1\wedge dx_2\right]\\
G =& dx^+\wedge W\\
P_1 =& g\,dx^+
\end{aligned}
$$

is a solution of the type IIB equations of motion as long as M_6 is Ricci flat and Φ, h, W are defined on M_6 and also depend on x^+ while g depends only on x^+

$$
\begin{aligned}
\nabla^2_{M_6}\Phi &= 0\\
d*_{M_6}dC &= 0\\
dW =0,\ d*_{M_6}W &= 0\\
\nabla^2_{M_6}h &= -\left|W\right|^2_{M_6} - 4g^2\Phi
\end{aligned}
$$

- In general two kinematical supersymmetries are preserved by choosing M_6 to be CY and W $(1,1)+(0,2)$ and primitive
- After fixing the choice of W the function h is fixed up to a harmonic function on M_6
- Supersymmetry is enhanced to include dynamical supersymmetries and also superconformal for the case $z=2$ after a unique choice of h
- For scale invariant solutions we restrict to the case of a cone

$$ds^2\left(M_6\right) = dr^2 + r^2 ds^2\left(E_5\right)$$

with E_5 an Einstein manifold.
- For supersymmetry if M_6 is CY then E_5 is Sasaki-Einstein
- We are also interested in the case where $\Phi = r^{-4}$

- We take $h = r^{2z-2} f$ and $W = d\left(r^z \omega\right)$ where f and ω are defined on the Einstein space E_5

With these choices

$$ds_{10}^2 \; = r^2 \left[2\, dx^- dx^+ + dx_1^2 + dx_2^2\right] + r^{2z} f \, dx^{+2} + \tfrac{dr^2}{r^2} + ds^2\left(E_5\right)$$

$$\triangle \omega = z\left(z+2\right)\omega, \qquad d^\dagger \omega = 0$$
$$\Box_E f + 4\left(z^2 - 1\right) f = -z^2 \left|\omega\right|^2 - \left|d\omega\right|^2$$

The above eigenvalue problem allows for

- $\omega = 0, \quad z \geq 3/2$
- $\omega \neq 0, \quad z \geq 2$
- $z = 2$ when ω is dual to a Killing vector

Lif $(z = 2)$ solutions in IIB and $d = 11$

- To construct Lifshitz invariant solutions we take $h = r^{-2} f$
- Take $x^- = t$ to be the time coordinate

$$ds_{10}^2 = r^2 \left[2\, dx^+ dt + dx_1^2 + dx_2^2\right] + \frac{dr^2}{r^2} + f\, dx^{+2} + ds^2\left(E_5\right)$$

With this choice we need to satisfy

$$dW = 0, \quad d *_E W = 0$$
$$-\Box_E f + 4f - 4\left|g\right|^2 - \left|W\right|_E^2 = 0$$

- Notice that f is unique (No regular solution to the homogeneous equation)
- In this case x^+ becomes spatial if $f > 0$ but t is null $\rightarrow$ perform T-duality along x^+ and uplift to $d = 11$

In eleven dimensions the metric reads

$$
\begin{aligned}
ds_{11}^2 &= f^{1/3}\left[-\frac{r^4}{f}\,dt^2 + r^2\left(dx_1^2+dx_2^2\right)+\frac{dr^2}{r^2}\right] \\
&\quad + f^{-2/3}\left[Dy_1^2+Dy_2^2\right] + f^{1/3}\,ds^2\left(E_5\right) \\
F_4 &= dt\wedge d\left(r^4\,dx_1\wedge dx_2 - \frac{r^2}{f}\,Dy_1\wedge Dy_2\right) \\
Dy_i &= dy_i - A^{(i)}
\end{aligned}
$$

with $W = dA^{(1)} + \imath\, dA^{(2)}$

- For the special choice where $E_5 = Y^{pq}$ we constructed regular solutions with $f > 0$
- E_5 needs to have a two-cycle for a regular W to exist
- For Y^{pq} (or L^{abc}) the topology is $S^2\times S^3$. Non-trivial fibration of an S^1 leads to $Lif\,(z=2)\times S^1\times S^3\times S^3$

Rewriting in a SUSY friendly form [Guantlett, Pakis]

$$
\begin{aligned}
ds_{11}^2 =& -\Delta^2 dt^2 + \Delta^{-1}\left[H_1^{-1}\left(dx_1^2 + dx_2^2\right) + H_2^{-1}\left(D\chi_1^2 + D\chi_2^2\right) + ds^2\left(CY_3\right)\right] \\
G_4 =& dt \wedge d\left[J_{SU(5)}\right] \\
\Delta =& H_1^{-1/3} H_2^{-1/3}, \quad dD\chi_i = -W_i \\
J_{SU(5)} =& H_1^{-1}\, dx_1 \wedge dx_2 + H_2^{-1}\, D\chi_1 \wedge D\chi_2 + J_{CY} \\
\Omega_{SU(5)} =& H_1^{-1/2} H_2^{-1/2}\, \left(dx_1 + \imath dx_2\right) \wedge \left(D\chi_1 + \imath D\chi_2\right) \wedge \Omega_{CY}
\end{aligned}
$$

For the above (electric) ansatz the torsion conditions

$$
\begin{aligned}
d\left(\Delta^{-3} J_{SU(5)}^4\right) =& 0 \\
d\left(\Delta^{-3/2} \Omega_{SU(5)}\right) =& 0
\end{aligned}
$$

imply

$$
\begin{aligned}
\nabla_{CY}^2 H_1 =& 0 \\
\nabla_{CY}^2 H_2 =& -\left|W\right|^2, \quad W = W_1 + \imath W_2
\end{aligned}
$$

KK reduction for $E_5 = T^{(1,1)}$

The case of $T^{(1,1)}$ gives $f = 1$ and can be used to perform a KK reduction down to $d = 4$. A (minimal) consistent reduction ansatz is

$$\begin{aligned} ds_{11}^2 =& e^{-2(\phi - T)} \left[ds_4^2 + e^{-2T} \left(d\sigma + \mathcal{A} \right)^2 + \frac{1}{9} e^{2V} D\psi^2 + \frac{1}{6} e^{2U} \left(d\Omega_1^2 + d\Omega_2^2 \right) \right] \\ &+ e^{4(\phi - T)/3} d\chi^2 \\ F_4 =& 4 e^{T - V - 4U} \mathrm{Vol}_4 + d \left[A \wedge \left(d\sigma + \mathcal{A} \right) \right] \wedge d\chi \\ d \left(D\psi \right) =& J_1 + J_2 \\ d\mathcal{A} =& J_1 - J_2 \end{aligned}$$

Yielding a four dimensional theory with

- A four dimensional metric $g_{\mu\nu}$
- A massive vector field A
- Four scalar fields U, V, T, ϕ

After performing the KK reduction the vector A has the right effective mass to allow for $Lif_4(z=2)$ as a solution to the 4d equations of motion simply as

$$\begin{aligned} ds^2 =& - r^4\, dt^2 + r^2\, \left(dx_1^2 + dx_2^2\right) + \frac{dr^2}{r^2} \\ A =& r^2\, dt \\ U =& V = T = \phi = 0 \end{aligned}$$

$d = 11$ solutions

The corresponding class of general solutions in eleven dimensional supergravity is

$$ds^2_{11} = \Phi^{-2/3}\left[2dx^+dx^- + hdx^{+^2} + 2Cdx^+dx^2\right] + \Phi^{1/3}ds^2\left(E_8\right)$$

$$G = dx^+ \wedge dx^- \wedge dx \wedge d\Phi^{-1} + dx^+ \wedge V + dx^+ \wedge dx \wedge d\left(\Phi^{-1}C\right)$$

with E_8 being Ricci flat and also

$$\nabla^2_{E_8}\Phi = 0, \quad \nabla^2_{E_8}h = -\left|V\right|^2_{E_8}$$

$$d *_{E_8} dC = 0, \quad dV = d *_{E_8} V = 0$$

- Schrödinger solutions with $z \geq 5/4$
- Lifshitz solutions with $z=2$ dual to $1+1$ dimensional field theories

Wrapped M5-branes on SLAG3 manifolds

- Consider a Calabi-Yau M_6, J, Ω with SLAG 3-cycle Σ_3

$$\mathrm{Vol}\,(\Sigma_3) = \mathrm{Re}\,(\Omega)|_{\Sigma_3}$$

- An M5-brane can wrap Σ_3 while preserving supersymmetry
- Worldvolume of wrapped M5-brane becomes $\mathbb{R}^{1,2} \times \Sigma_3$. In the IR one obtains a $d = 3$ QFT with $N = 2$ SUSY
- AdS/CFT suggests that if $\Sigma_3 = H_3/\Gamma$ this QFT is $N = 2$ SCFT dual to $AdS_4 \times H^3/\Gamma \times S^4$

Wrapped M5-branes on SLAG3 manifolds

- One can construct a $D = 11$ solution that interpolates between $AdS_4 \times H^3/\Gamma \times S^4$ in the IR and $AdS_7 \times S^4$ in the UV with

$$ds^2\left(AdS_7\right) = \frac{dr^2}{r^2} + r^2\left[dx_\mu dx_\mu + ds^2\left(H^3/\Gamma\right)\right]$$

  describing a flow across dimensions to the $d = 3$, $N = 2$ SCFT

- Perform a KK reduction down to $d = 4$ to study the $N = 2$ SCFT at finite temperature and charge density with respect to the abelian R-symmetry

Consistent truncation on $H^3/\Gamma \times S^4$

- First perform a reduction on S^4 to obtain $d = 7$ $SO(5)$ gauged SUGRA [Nastase, Vaman, Nieuwenhuizen] [Cvetic, Lu, Pope, Sadrzadeh, Tran]
- Perform a second KK reduction of $d = 7$ $SO(5)$ gauged SUGRA on Σ_3 while keeping breathing mode multiplet
- The four dimensional theory is an $N = 2$ SUGRA with
  - Gauged SUGRA $\rightarrow$ metric + 1 vector
  - 1 Vector multiplet $\rightarrow$ 1 vector +2 scalars
  - 2 Hypermultiplets $\rightarrow$ 8 scalars
- The scalars parametrize the coset $\left(\frac{SU(1,1)}{U(1)}\right)_{VM} \times \left(\frac{G_{2(2)}}{SO(4)}\right)_{HM}$

- Choosing $\Sigma_3 = H^3/\Gamma$ the $d = 4$ theory admits a SUSY AdS_4 solution
- Also a non-SUSY neutral AdS_4, perturbatively stable with respect to the modes kept in our consistent truncation
- This theory admits a Lifshitz solution with $z \approx 39$ which uplifts to $L_4 \times H^3/\Gamma \times S^4$ with the S^4 fibered over both H^3/Γ and the four dimensional Lif_4

- The lower dimensional theory can be used to study the dual $N = 2$ SCFT dual to M5-branes wrapping SLAG H^3/Γ cycles at finite temperature and chemical potential

- Within this KK reduced theory there is a subtruncation describing Einstein-Maxwell theory. This gives a standard $d = 4$ AdS-RN black hole asymptoting to SUSY AdS_4 describing the SCFT at high enough temperature

- Preliminary study of stability issues indicate:
  - Branch of black holes carrying charged scalar hair $\longrightarrow$ new top down holographic superconducting black holes
  - Branch of black holes with no charged scalar hair $\longrightarrow$ top down analogues of dilatonic black holes

Conclusions / Open questions

- Constructed neutral Lifshitz solutions of type IIB and $d = 11$ SUGRA with $z = 2$. Special cases admitting a consistent KK reduction
  - Dual interpretation
  - Construction of finite temperature Lifshitz/Schrödinger solutions
  - Construction on interpolating $Lif \to AdS$ solutions
- New consistent truncation of $d = 11$ SUGRA on $\Sigma_3 \times S^4$ with $\Sigma_3 = S^4, H^3, T^3$ to an $N = 2$, $d = 4$ gauged SUGRA
  - Rich set of solutions
    - $AdS_4 \times H^3/\Gamma \times S^4$ with $N = 2$ SUSY and also $N = 0$
    - Charged $Lif_4 \times H^3/\Gamma \times S^4$ with $N = 0$
  - Are they related via holographic flows?
  - Initiated a study of the $N = 2$ SCFT at finite temperature and chemical potential
  - New branches of black holes $\to$ Numerical construction and study of the phase structure