3d $\mathcal{N} = 2$ flavoured quiver gauge theories and M2-branes at toric CY_4 cones

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F. Benini, C. Closset, SC, JHEP **1002**, 036 (2010) [arXiv:0911.4127]
SC, arXiv:1007.4562

3d gauge theories and M2: why bother?

- Use gauge theories to understand multiple membranes in M-theory [Bagger, Lambert 07; Gustavsson 07; van Raamsdonk 08; Aharony, Bergman, Jafferis, Maldacena 08; . . .]
- Freund-Rubin AdS_4 vacua of 11d supergravity
  (Warped) AdS_4 vacua of type IIA supergravity
- 3d quiver gauge theories (w/ Chern-Simons terms): toy models for condensed matter systems [Sachdev-Yin 08]

D3 branes at toric CY_3 cones and $\mathcal{N}=1$ SCFTs in 4d

Toric: holomorphic $U(1)^3$ action on the CY_3 cone $Y_6 = C(X_5)$.

Low energy dynamics on a stack of N D3 at the apex of the cone

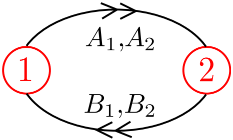
- $\mathcal{N}=1$ superconformal $SU(N)^G$ quiver gauge theory in 4d: chiral superfields X_{ij}^a in bifundamental representations $(\square_i, \overline{\square}_j)$
- Superpotential $W(X)$ satisfying the *toric* condition (superconformal $U(1)_R$, *mesonic* $U(1)^2$)
  F-flatness $\partial W(X) = 0$: monomial=monomial.

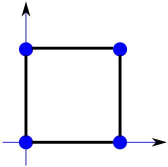
- *Mesonic* branch of the moduli space: $\mathrm{Sym}_N Y_6$.
- Dual to type IIB on $AdS_5 \times X_5$ with $\int_{X_5} F_5 = N$.

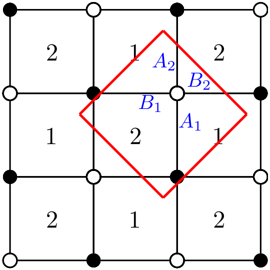
IIB brane tilings & 4d $\mathcal{N} = 1$ toric quiver gauge th's

- Info packaged in a type IIB brane tiling.

  [Hanany, Kennaway 05; Franco, Hanany, Kennaway, Vegh, Wecht 05]







$$W = \mathrm{Tr}\left(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1\right)$$

- We know the quiver gauge theory(/ies) on D3 branes at *any* toric CY_3 Y_6 (inverse algorithm)

M2 branes at toric CY_4 and 3d $\mathcal{N} = 2$ SCFTs

Still far from knowing the 3d $\mathcal{N} = 2$ gauge theory on N M2 at any toric CY_4 $Y_8 = C(X_7)$, dual to M-theory on $AdS_4 \times X_7$ with $\int_{X_7} *G_4 = N$ at large N.

Pre-ABJM: M-theory brane crystals [Lee 06; Lee[2], Park 07; Kim, Lee[2], Park 07]

Proposal for Abelian theories, non-Abelian generalisation unclear.
Missed some partial resolutions.

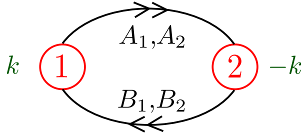
Post-ABJM: 3d $\mathcal{N} = 2$ $U(N)^G$ toric Chern-Simons quiver gauge theories

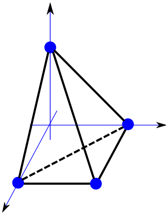
- with 4d parents: derivation by reduction to IIA.
- without 4d parents: crystal-inspired, no stringy derivation.
- with 4d parents, plus fundamental flavours: stringy derivation, replace models without 4d parents.

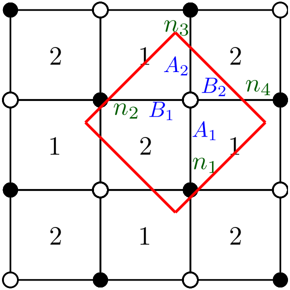
$\mathcal{N} = 2$ SUSY in 3d: dimensional reduction of $\mathcal{N} = 1$ SUSY in 4d.
Abelian vector multiplet contains real scalar σ, complexified by dual photon τ.

IIA brane tilings and 3d theories with 4d parents

[Hanany, Zaffaroni 08; Ueda, Yamazaki 08; Imamura, Kimura 08]







$$W = \mathrm{Tr}\,(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1)$$

$$k = n_1 - n_2 + n_3 - n_4$$

Geometric moduli space of the Abelian theory is a toric CY_4 cone.

Geometric moduli space of $\mathcal{N}=2$ quiver CS theories

$U(1)^G$ case for simplicity. $\sum_{i=1}^{G} k_i = 0.$ [Martelli, Sparks 08; Hanany, Zaffaroni 08]

- **F-flatness**: $\mathcal{Z} = \{X_\alpha,\ \alpha = 1,\dots,E \mid \partial W(X) = 0\} \subset \mathbb{C}^E$
- **D-flatness**: $\mathcal{D}_i = \frac{k_i \sigma_i}{2\pi} \quad \forall_{i=1}^{G}\ , \quad |X_\alpha|^2 \left(\sum_{i=1}^{G} g_i[X_\alpha]\, \sigma_i \right)^2 = 0 \quad \forall_{\alpha=1}^{E}\ ,$
  $\mathcal{D}_i \equiv \sum_{\alpha=1}^{E} g_i[X_\alpha]\, |X_\alpha|^2$

Diagonal photon $A_{diag} \equiv \sum_i A_i$ dualised into a periodic scalar τ.

- **Gauge**: $A_i \to A_i + d\theta_i,\ \ \tau \to \tau + \frac{1}{G} \sum_i k_i\, \theta_i,\ \ X_\alpha \to e^{i\, g_i[X_\alpha]\theta_i}\, X_\alpha$

Branch $\sigma_i = \sigma\ \ \forall_{i=1}^{G}$:

$$\sum_{i=1}^{G} c_i\, \mathcal{D}_i = 0 \quad \forall \{c_i\} \mid \sum_{i=1}^{G} c_i k_i = 0$$
$$\frac{\sigma}{2\pi} = \frac{1}{\|k\|^2} \sum_i k_i\, \mathcal{D}_i\ , \quad \|k\|^2 = \sum_i k_i^2$$

Geometric moduli space of $\mathcal{N} = 2$ quiver CS theories

Geometric moduli space

$$\mathcal{M} = (\mathcal{Z} \,/\!\!/\, U(1)^{G-2})/\mathbb{Z}_q \,, \qquad q = \gcd\{k_i\}$$

Moduli spaces of 3d/4d TQGTs w/ same matter content and $W(X)$

$$\mathcal{M}_{4d}^{mes} = \mathcal{Z} \,/\!\!/\, U(1)^{G-1} = \mathcal{M}_{3d} \,/\!\!/\, U(1)_{\vec{k}}$$

[Jafferis, Tomasiello 08; Martelli, Sparks 08; Hanany, Zaffaroni 08]

Alternatively, use unit flux diagonal monopole operators T, $\tilde{T}$ (instead of σ, τ) and X's to form gauge invariants under $U(1)^G$:

$$\tilde{\mathcal{Z}} = \{X_\alpha, T, \tilde{T} \mid \partial W(X) = 0,\ T\tilde{T} = 1\} \subset \mathbb{C}^{E+2} \,,$$

$$\mathcal{M} = \tilde{\mathcal{Z}} \,/\!\!/\, U(1)^{G-1} \,.$$

Gauge charges induced by CS terms: $\qquad g_i[T] = -g_i[\tilde{T}] = k_i$.

[Benini, Closset, SC 09]

"Stringy derivation" of quiver CS theories [Aganagic 09]

Toric CY_4 cone

S^1 bundle over a 7-manifold, which is a toric CY_3 cone fibred over $\mathbb{R}$.
A Kähler parameter of CY_3 varies linearly along $\mathbb{R}$.

$CY_3 = CY_4 /\!\!/ U(1)_M \longrightarrow$ quiver gauge theory on D2. *(Not quite...)*

For an M2 probe:
- S^1 parametrised by τ
- CY_3 by mesonic $U(1)^G$ invariants of the quiver (Kähler parameters are FI terms)
- $\mathbb{R}$ parametrised by σ

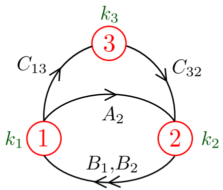
S^1: M-theory circle in the KK reduction to IIA (after $\mathbb{Z}_q$ quotient).

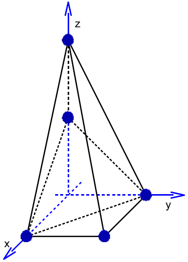
Curvature of the $U(1)_M$ bundle: RR F_2, which induces CS terms on wv of fractional D2 probes.

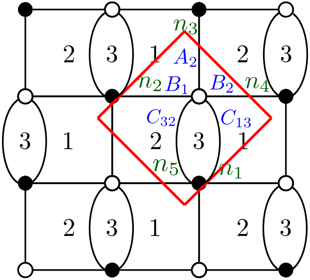
Fibration of CY_3 over $\mathbb{R}$: scalar $\mathcal{N} = 2$ partners of CS terms.

Brane tilings with multiple bonds: no 4d parents

[Hanany, Zaffaroni 08; Hanany, Vegh, Zaffaroni 08; Franco, Hanany, Park, Rodriguez-Gomez 08]







$$W = \mathrm{Tr}\,(C_{13}C_{32}B_1A_2B_2 - C_{13}C_{32}B_2A_2B_1)$$

$$k_1 = n_1 - n_2 + n_3 - n_4$$
$$k_2 = n_2 - n_3 + n_4 - n_5$$
$$k_3 = -n_1 + n_5$$

No stringy derivation of the quiver theory. Actually, let's see...

Rescale proposed CS levels by h:

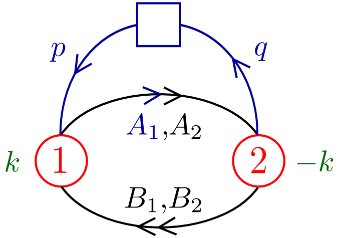
- 3d toric diagram rescaled vertically: $Y_8 \longrightarrow Y_8/\mathbb{Z}_h$
- Non-isolated singularities, locally $\mathbb{C}^2 \times \mathbb{C}^2/\mathbb{Z}_h$
- Dual $SU(h)$ global symmetry not visible in the proposed quiver theory
- h D6 branes in type IIA after KK reduction
  (fixed point sets of $U(1)_M \supset \mathbb{Z}_h$)

Lesson: no new gauge groups, but flavours.

- D2-D6, D6-D2' strings: fund. and antifund. flavours p and q.
- Holom. embedding of D6 in $CY_3 \longrightarrow$ Superpotential term $\delta W = pXq$.
- D6 at the origin of the real line: vanishing real masses for flavours.

Our proposal: quivers with fundamental flavours

[Benini, Closset, SC 09; Jafferis 09]



$$W = \mathrm{Tr}\,(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1) + \sum_{i=1}^{h} p^i A_1 q_i$$

- Quantum corrected geometric branch of moduli space: CY_4 of M-theory.
- Derived by KK reduction to IIA. Manifest flavour symmetries.
- Real masses for flavours: displace D6 along $\mathbb{R}$ (blowup in M-theory).
- Cross D6: $\int F_2$ jump $\Longrightarrow$ CS levels are shifted.

3d toric flavoured quiver gauge theories

Flavouring: couple h_α flavours (p_α, q_α) to bifundamentals X_α

$$W = W_0(X) + \sum_{\alpha=1}^{E} \sum_{i_\alpha=1}^{h_\alpha} (p_\alpha)^{i_\alpha} X_\alpha (q_\alpha)_{i_\alpha}$$

Quantisation of CS levels: $\quad k_i + \frac{1}{2} \sum_\psi \left(g_i[\psi]\right)^2 \in \mathbb{Z}$

We focus on Abelian theories $U(1)^G$ to compute the geometric moduli space.

BPS 't Hooft monopole operators $T^{(n)}$ [Borokhov, Kapustin, Wu 02]

- Local operators (chiral superfields)
- Insert n units of magnetic flux (and background of σ) in each $U(1)$ around puncture in Euclidean theory
- Classically induced electric charges: $\quad n(k_1, k_2, \ldots, k_G)$.
- Quantum induced charges: $\quad \delta Q[T^{(n)}] = \frac{|n|}{2} \sum_\alpha h_\alpha Q[X_\alpha]$
- $T^{(n)} = (T^{(1)})^n \equiv T^n$ if $n > 0$; $\quad T^{(n)} = (T^{(-1)})^{-n} \equiv \tilde{T}^{-n}$ if $n < 0$.

Classically, or in the absence of flavours: $T\tilde{T} = 1$.

In the presence of flavours, according to the quantum induced charges, the chiral ring is modified: $T\tilde{T} = \prod_\alpha X_\alpha^{h_\alpha}$.

Quantum corrected geometric moduli space

Branch: $p_\alpha = q_\alpha = 0 \; \forall \, \alpha$. Same classical F-terms, different quantum relation.

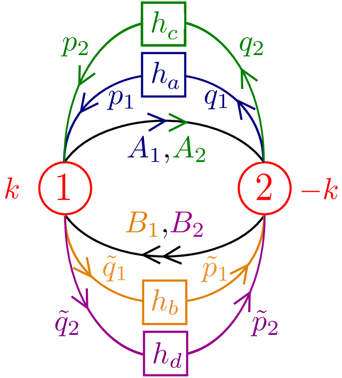
$$\tilde{\mathcal{Z}}_{flav} = \{X_\alpha, T, \tilde{T} \mid \partial W_0(X) = 0, \; T\tilde{T} = \prod_\alpha X_\alpha^{h_\alpha}\} \subset \mathbb{C}^{E+2} ,$$

$$\mathcal{M}_{\mathrm{flav}} = \tilde{\mathcal{Z}}_{flav} \,/\!\!/\, U(1)^{G-1} .$$

Gauge charges of monopoles: $g_i[T^{(\pm 1)}] = \pm k_i + \frac{1}{2} \sum_\alpha h_\alpha g_i[X_\alpha]$

The geometric branch is a toric CY_4 cone , precisely the one that we started from in M-theory when we reduced to type IIA to *derive* the 3d quiver gauge theory.

Examples: torically flavoured ABJ(M)

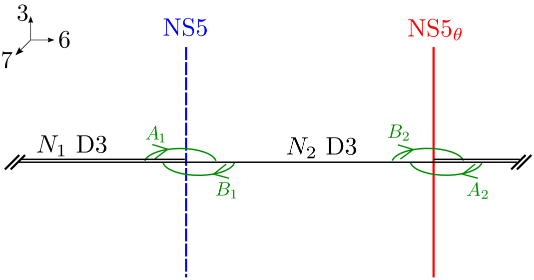


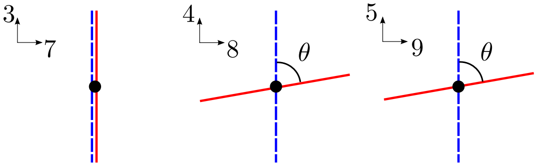
$$W = \mathrm{Tr}\,(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1) +$$
$$+ \sum_{\alpha=1}^{h_a} (p_1)^\alpha A_1 (q_1)_\alpha - \sum_{\beta=1}^{h_b} (\tilde{p}_1)^\beta B_1 (\tilde{q}_1)_\beta - \sum_{\gamma=1}^{h_c} (p_2)^\gamma A_2 (q_2)_\gamma + \sum_{\delta=1}^{h_d} (\tilde{p}_2)^\delta B_2 (\tilde{q}_2)_\delta$$

Dual Type IIB brane configuration

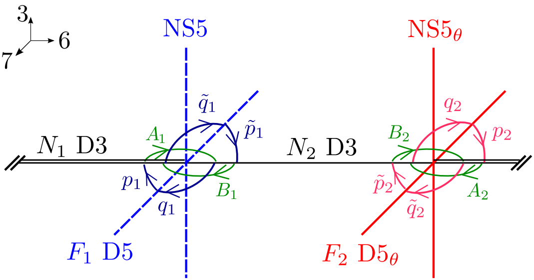
[SC 10]

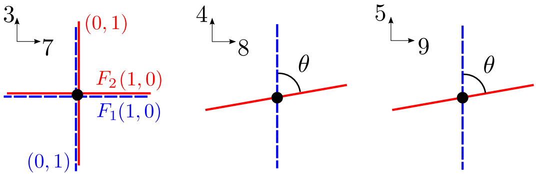
- $k = 0$ and $h_a = h_b = h_c = h_d = 0$ (3d version of KW or KT/KS)



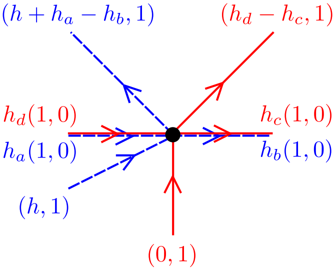


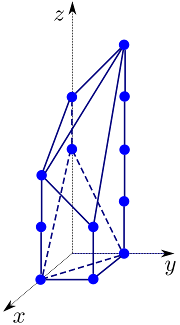
- $k = 0$ and $h_a = h_b = F_1$, $h_c = h_d = F_2$ (vectorlike flavours)





(Torically) flavoured ABJ(M):





$$k = h + \frac{1}{2}(h_a - h_b + h_c - h_d)$$

Thanks to IIB brane construction, results on

- fractional M2 branes and torsion G_4 fluxes
- cascades of 3d Seiberg-like dualities
- interplay between partial resolutions and fractional M2 branes

for (gravity duals of) M2 brane theories at cones over toric SE_7 (including $Q^{1,1,1}$ and $Y^{1,2}(\mathbb{CP}^2)$) and two infinite classes of smooth SE_7).

Outlook

- Added flavours to 3d $\mathcal{N} = 2$ toric quiver gauge theories:
  - New gauge theories for M2-branes at (infinitely many) toric CY_4 singularities
  - Consistent with D6 embedding in KK reduction of M-theory to IIA
  - Conceptual way of generating CS terms by introducing flavours, giving them real masses, integrating out.
- Dual Type IIB brane configuration, when available, allows a more detailed study of the correspondence and confirms the proposal.
- Future directions: generalise the analysis of type IIB brane configurations dual to toric CY_4 and understand field theory interpretation of all partial resolutions.