On three-point correlation functions in gauge/gravity duality

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1008.1070 [hep-th]

With R. Monteiro, J. Santos and D. Zoakos

(overlaps with Zarembo 1008.1059 [hep-th])

Crete Conference on Gauge Theories and The Structure of Spacetime
September 2010

Forward

- Great deal of progress in finding spectrum of planar N=4 SYM at any value of the coupling (AdS/CFT and Integrability) [Kazakov review talk]

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- To solve planar N=4 SYM need to compute n-point functions. For a CFT this means computing structure constants in 3-point functions (OPE)

$$\langle \mathcal{O}_A(0)\mathcal{O}_B(x)\mathcal{O}_C(y)\rangle = \frac{a_{ABC}}{|x|^{\Delta_A+\Delta_B-\Delta_C}|y|^{\Delta_A+\Delta_C-\Delta_B}|x-y|^{\Delta_B+\Delta_C-\Delta_A}}$$

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- Will report on new computations: - at weak coupling using integrability
  - at strong coupling using AdS/CFT

[Zarembo; Roiban,Tseytlin]

[Janik et al]

3-pt coupling from linear deformation

- Deform field theory around conformal fixed point with marginal (or irrelevant) operator $\mathcal{D}$ of dimension Δ

$$S_u = S + u\,\Lambda^{4-\Delta} \int d^4y\,\mathcal{D}(y)$$

$$D = 4$$

$$\Delta \geq 4$$

$$\beta_u = \frac{du}{d\,\ln\Lambda} = (\Delta - 4)u + \cdots$$

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- Will show that effect of deformation on renormalization of operator $\mathcal{O}_A$ is determined by couplings $a_{\mathcal{D}AB}$ [Cardy]

- Consider correlation functions to linear order in u

$$\langle \mathcal{O}_A(x) \cdots \rangle_u = \langle \mathcal{O}_A(x) \cdots \rangle - u \int d^4y \, \langle \mathcal{O}_A(x) \, \mathcal{D}(y) \cdots \rangle$$

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- Need to renormalize $\mathcal{O}_A$ because of divergence when $y \sim x$ in integration

OPE: $\mathcal{D}(y)\,\mathcal{O}_A(x) \sim \sum_B \frac{a_{\mathcal{D}AB}\,\mathcal{O}_B(x)}{|x-y|^{4+\Delta_A-\Delta_B}}$

$$\int \frac{d^4y}{|x-y|^{\Delta+\Delta_A-\Delta_B}} \approx 2\pi^2 \begin{cases} \ln(\Lambda|x|)\,, & \Delta_B = \Delta_A\,, \\ \dfrac{\Lambda^{\Delta_A-\Delta_B}}{\Delta_A-\Delta_B}\,, & \Delta_B < \Delta_A \end{cases}$$

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Finite number of operators $\mathcal{O}_B$ contribute to divergence

$$\Delta_B \leq \Delta_A$$

- Define renormalized operators $\mathcal{O}_A^u$ so that $\langle \mathcal{O}_A(x) \cdots \rangle_u$ finite

$$\mathcal{O}_A^u = \mathcal{O}_A + u \sum_{\Delta_B = \Delta_A} 2\pi^2 a_{\mathcal{D}AB} \left(\ln \Lambda\right) \mathcal{O}_B + u \sum_{\Delta_B < \Delta_A} 2\pi^2 a_{\mathcal{D}AB} \frac{\Lambda^{\Delta_A - \Delta_B}}{\Delta_A - \Delta_B} \mathcal{O}_B$$

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For example $\langle \mathcal{O}_A^u(x)\mathcal{O}_A^u(0)\rangle_u = \dfrac{1}{|x|^{2(\Delta_A+u\,2\pi^2 a_{\mathcal{D}AA})}}$

- Define renormalized operators $\mathcal{O}^u_A$ so that $\langle \mathcal{O}_A(x) \cdots \rangle_u$ finite

$$\mathcal{O}^u_A = \mathcal{O}_A + u \sum_{\Delta_B = \Delta_A} 2\pi^2 a_{\mathcal{D}AB} (\ln \Lambda)\, \mathcal{O}_B + u \sum_{\Delta_B < \Delta_A} 2\pi^2 a_{\mathcal{D}AB} \frac{\Lambda^{\Delta_A - \Delta_B}}{\Delta_A - \Delta_B} \mathcal{O}_B$$

For example $\langle \mathcal{O}^u_A(x) \mathcal{O}^u_A(0) \rangle_u = \dfrac{1}{|x|^{2(\Delta_A + u\, 2\pi^2 a_{\mathcal{D}AA})}}$

- Deformed anomalous dimension matrix

$$\mathcal{O}^u_A = \mathcal{Z}_{AB}(\Lambda, u)\, \mathcal{O}^b_B$$

$\{\mathcal{O}^b_A\}$ diagonalizes $\mathcal{H}$ at critical point

$$\mathcal{H}^u_{AB} = \mathcal{Z}^{-1}_{AC} \frac{d}{d \ln \Lambda} \mathcal{Z}_{CB} = \delta_{AB} \gamma_A + u\, 2\pi^2 a_{\mathcal{D}AB}\, \Lambda^{\Delta^0_A - \Delta^0_B}$$

$\Rightarrow$ C-Z equation

- Can compute couplings $a_{\mathcal{D}AB}$ from knowledge of $\mathcal{H}^u = \mathcal{H}^u(\mathcal{D})$

$$\mathcal{H}^u = \mathcal{H} + u\mathcal{H}' = \delta_{AB}\gamma_A + u\,2\pi^2 a_{\mathcal{D}AB}\,\Lambda^{\Delta_A^0 - \Delta_B^0}$$

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Basic quantum mechanics:

$$\mathcal{O}_A = \Lambda^{\gamma_A}\mathcal{O}^b_A$$

eigenvalues: $\gamma^u_A = \gamma_A + u\gamma'_A$

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$$2\pi^2 a_{\mathcal{D}AA} = \langle \mathcal{O}_A^b|\mathcal{H}'|\mathcal{O}_A^b\rangle = \gamma_A'$$

$$2\pi^2 a_{\mathcal{D}AB} = \Lambda^{\Delta_B^0 - \Delta_A^0}\langle \mathcal{O}_B^b|\mathcal{H}'|\mathcal{O}_A^b\rangle$$

Weak coupling - Planar N=4 SYM

- Simplest example $\mathcal{D} = \mathcal{L}$

$$g^2 = \frac{g_{YM}^2 N}{16\pi^2} = \frac{\lambda}{16\pi^2}$$

$$S = \frac{1}{g_{\text{YM}}^2} \int d^4y \, \text{Tr}\Big(- \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - \dots \Big)$$

$$g^2 \to g^2(1-u)$$

$$\frac{\partial}{\partial u} = -g^2 \frac{\partial}{\partial g^2}$$

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- Consider SU(2) subsector and $\mathcal{O}_A$ made of M magnons

$$\mathcal{O}_A = \sum_x \psi_{p_1, \cdots, p_M}(x_1, \cdots, x_M) \, |x_1, \cdots, x_M\rangle$$

$|x_1, \cdots, x_M\rangle \equiv |Z \cdots ZXZ \cdots ZXZ \cdots\rangle$, with Z, X complex scalars

- Single magnon contribution to anomalous dimension (all orders, neglecting wrapping)

$$\gamma_j(g) = \sqrt{1 + 16g^2 \sin^2 \frac{p_j}{2}} - 1$$

$p_j = p_j(g)$

Bethe equations

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Bethe equations

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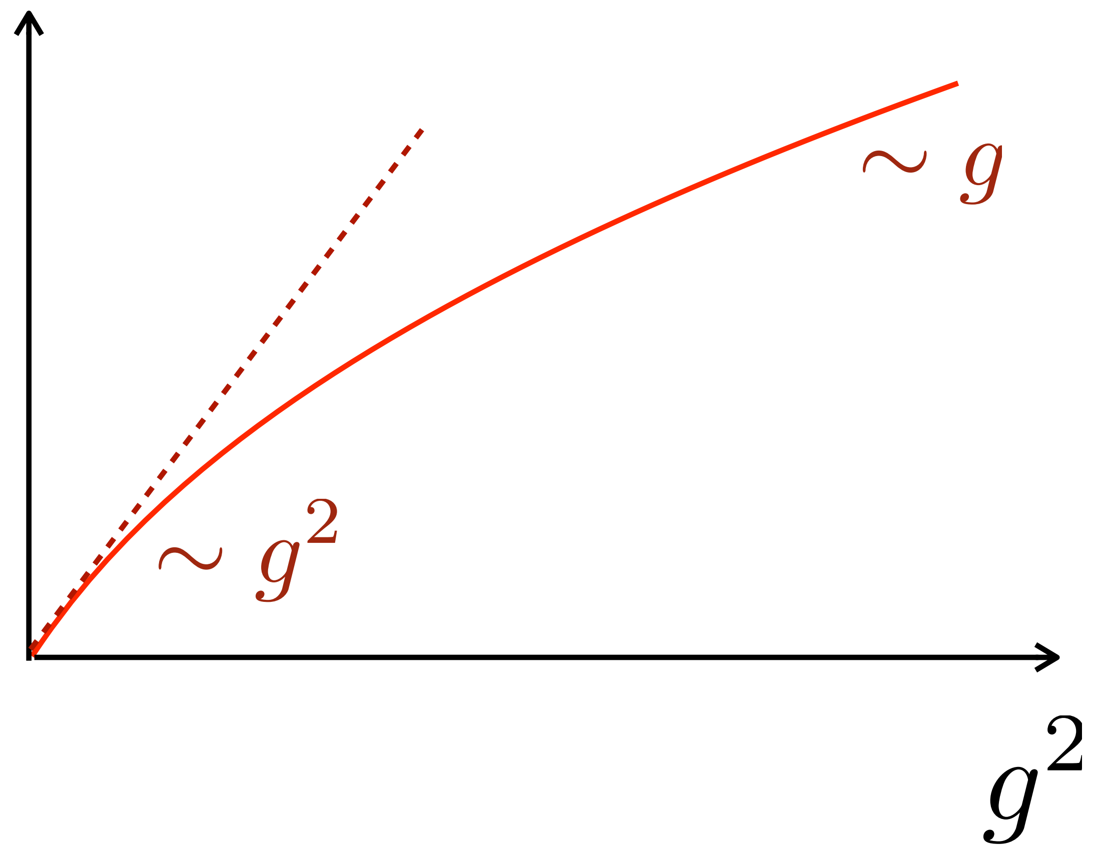
- Dilute limit, $L \gg M \quad \Rightarrow \quad p_j = \frac{2\pi n_j}{L}$



Wick contractions

$$2\pi^2 a^{(j)}_{\mathcal{L}AA} = -8g^2 \frac{\sin^2 \frac{p_j}{2}}{\sqrt{1 + 16 g^2 \sin^2 \frac{p_j}{2}}} = \begin{cases} \boxed{-8g^2 \sin^2 \frac{p_j}{2}} + O(g^4) \\ \boxed{-2g \left|\sin \frac{p_j}{2}\right|} + O(1) \end{cases}$$

Strong coupling, later...

- Different initial and final magnon states: compute $a_{\mathcal{L}AB}$ from $\langle \mathcal{O}_B^b | \mathcal{H}' | \mathcal{O}_A^b \rangle$

$$\mathcal{H} = 2g^2\left(1 - 4g^2\right)\sum_{x=1}^{L}(1 - P_{x,x+1}) + 2g^4\sum_{x=1}^{L}(1 - P_{x,x+2})$$

$$\mathcal{H}' = -\mathcal{H} + 8g^4\sum_{x=1}^{L}(1 - P_{x,x+1}) - 2g^4\sum_{x=1}^{L}(1 - P_{x,x+2})$$

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- Diagonalize $\mathcal{H}^u$ to order g^4 for two magnon states

$$2\pi^2 a_{\mathcal{L}AB} = \frac{\langle \mathcal{O}_B^b | \mathcal{H}' | \mathcal{O}_A^b \rangle}{L(L-1)} = 64g^4\,\frac{e^{\frac{i}{2}(q-p)}}{L-1}\,\sin\frac{p}{2}\,\sin p\,\sin\frac{q}{2}\,\sin q$$

Two magnon operators

$\mathcal{O}_A\,,\qquad p_1 = -p_2 = p$

$\mathcal{O}_B\,,\qquad q_1 = -q_2 = q$

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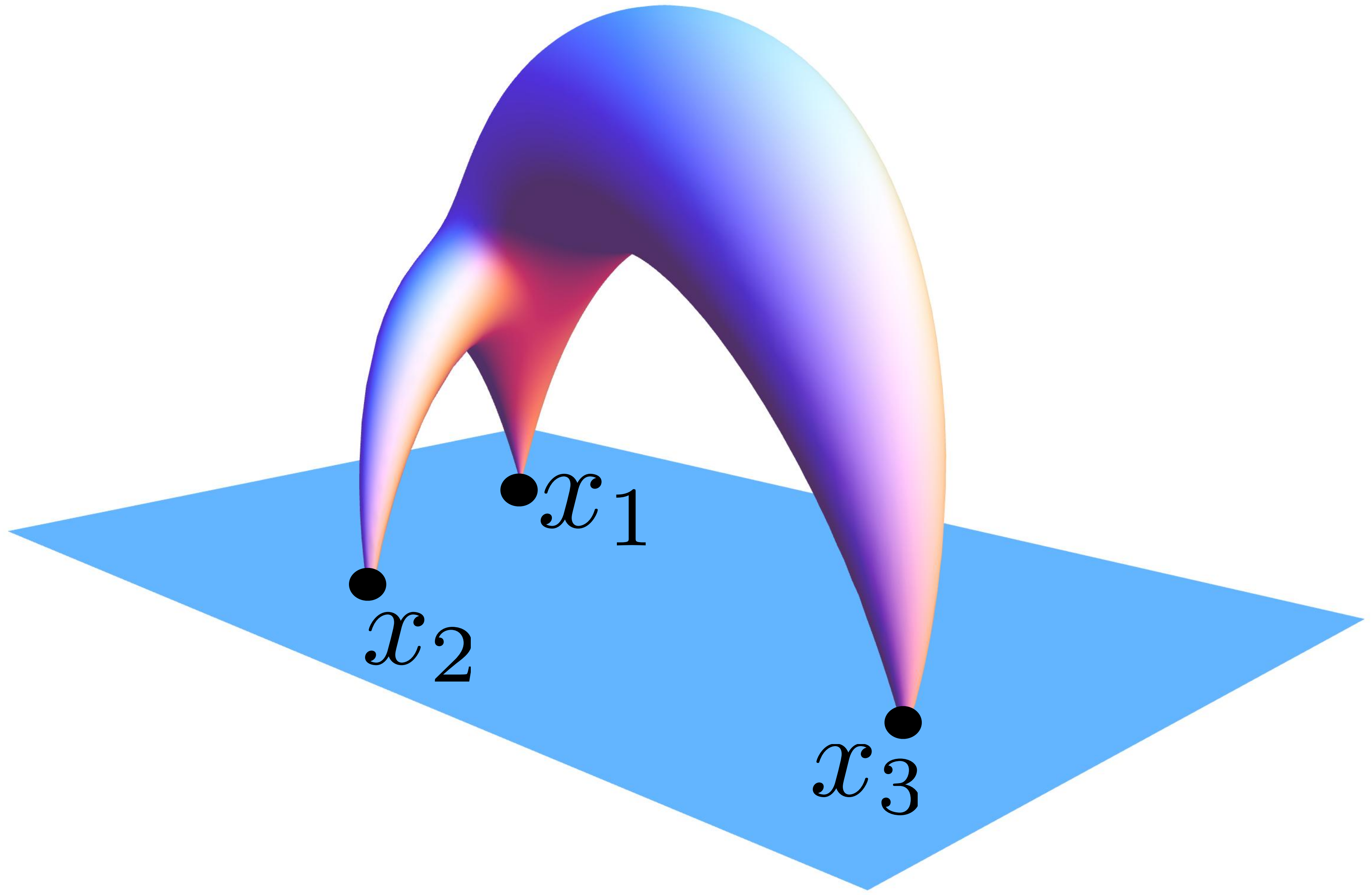
- Can one extend to other operators in a practical way? $\qquad \mathcal{H}' = \mathcal{H}'(\mathcal{D})$

Strong coupling - AdS/CFT

• Operators with very large dimension $\Delta \sim g$ are dual to classical string states. Expect to use semi-classical approximation to string partition function

$$\int DX\, D\gamma\, V_\Phi(x_1) \cdots e^{iS_P[X,\gamma]} \sim e^{iS_P[\bar{X},\bar{\gamma}]}$$



$$\Rightarrow \quad \langle \mathcal{O}_A(x_1)\mathcal{O}_B(x_2)\mathcal{O}_C(x_3)\rangle$$

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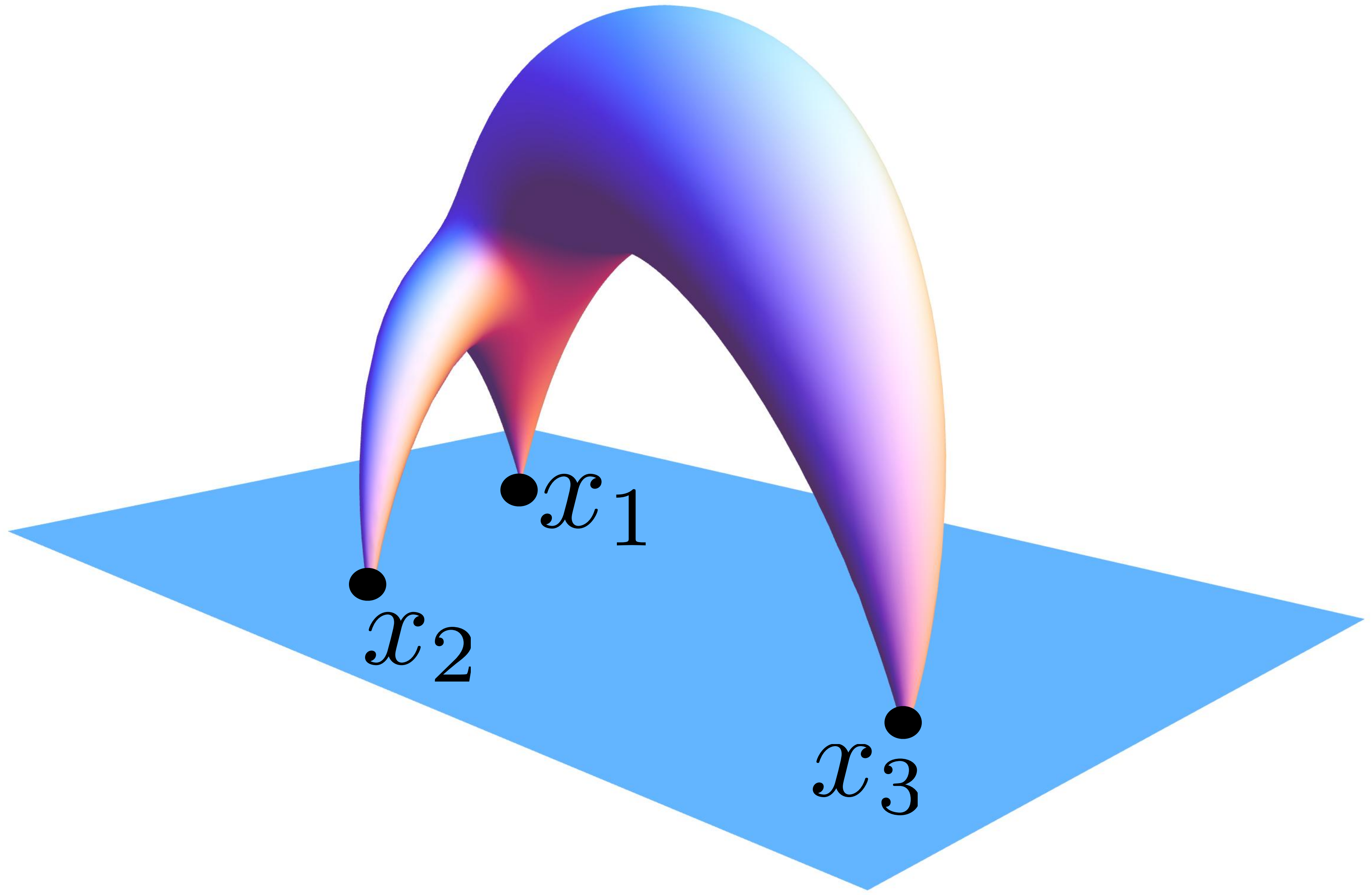
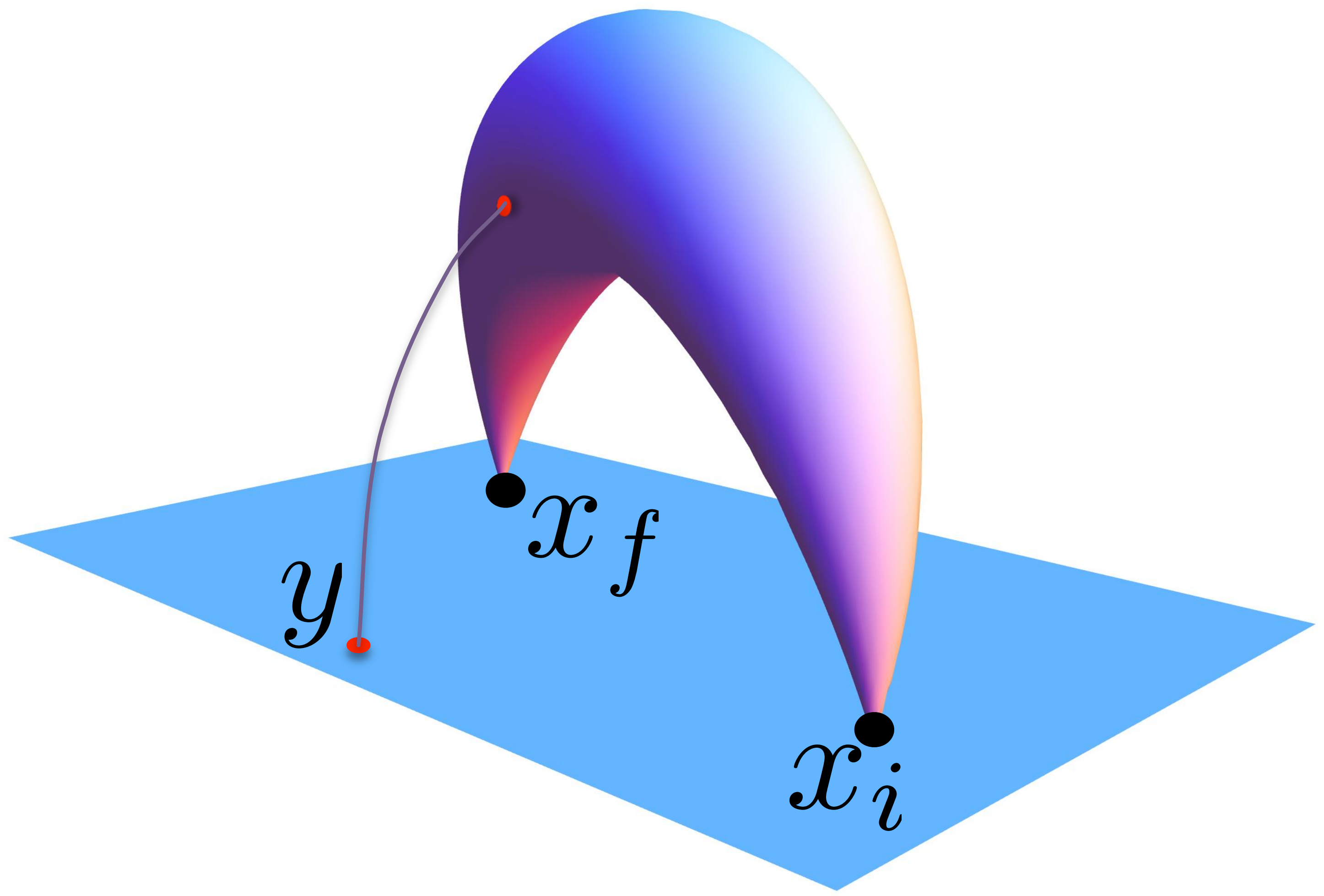


$$\langle \mathcal{O}_A(x_1)\mathcal{O}_B(x_2)\mathcal{O}_C(x_3)\rangle$$

Open problem

Simpler problem

- Two insertions of heavy state $\mathcal{O}_A$ and one insertion of BPS state $\mathcal{D}_\chi$



$$\langle \mathcal{O}_A(x_i)\mathcal{O}_A(x_f)\mathcal{D}_\chi(y)\rangle$$

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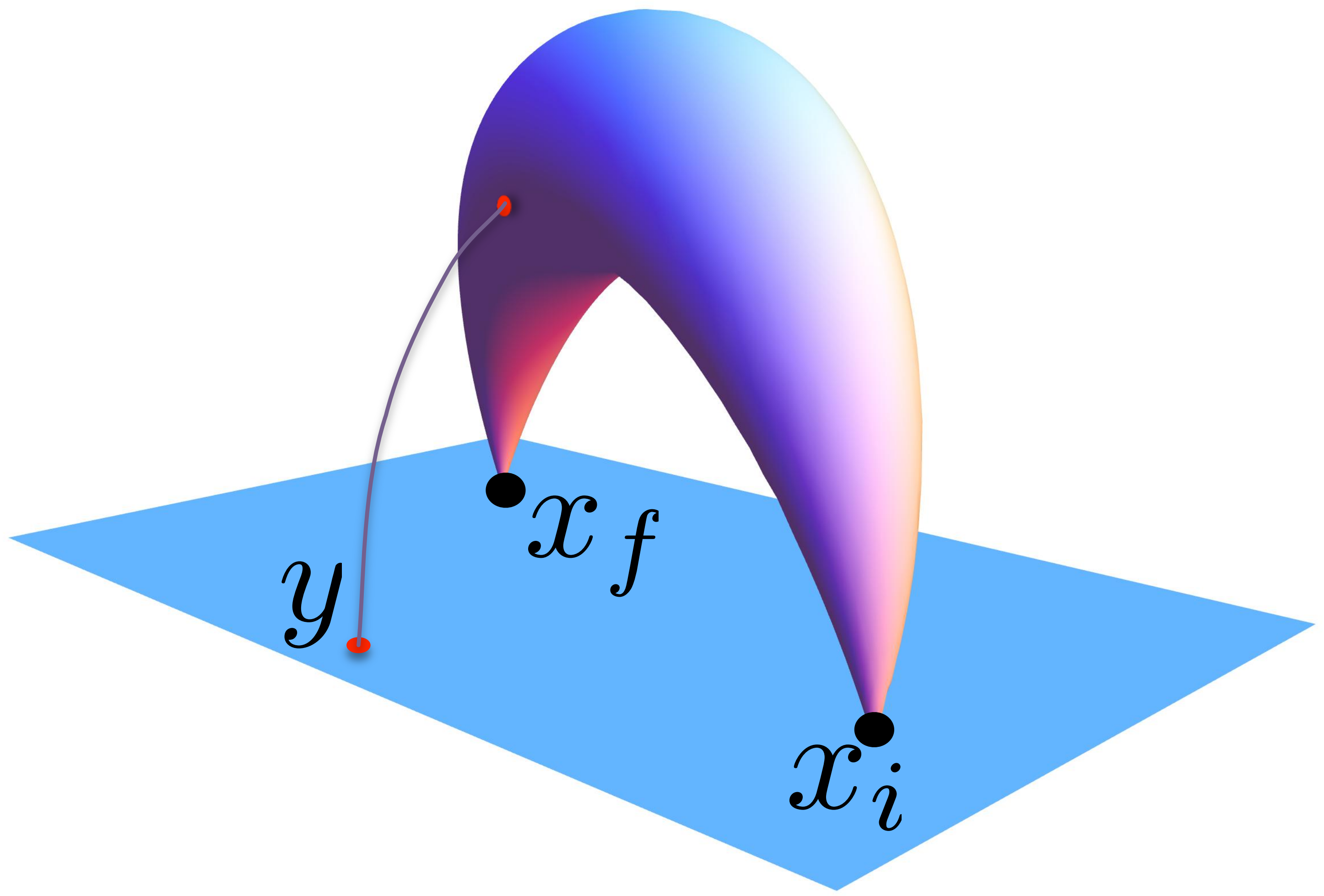

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$$\left\langle \mathcal{O}_A(x_i)\,\mathcal{O}_A(x_f)\,e^{\int d^4y\,\Phi_0(y)\,\mathcal{D}(y)}\right\rangle_{CFT} \approx \int DX\,D\gamma\,D\Phi\,e^{i(S_P[X,\gamma,\Phi]+S_{SUGRA}[\Phi])}$$

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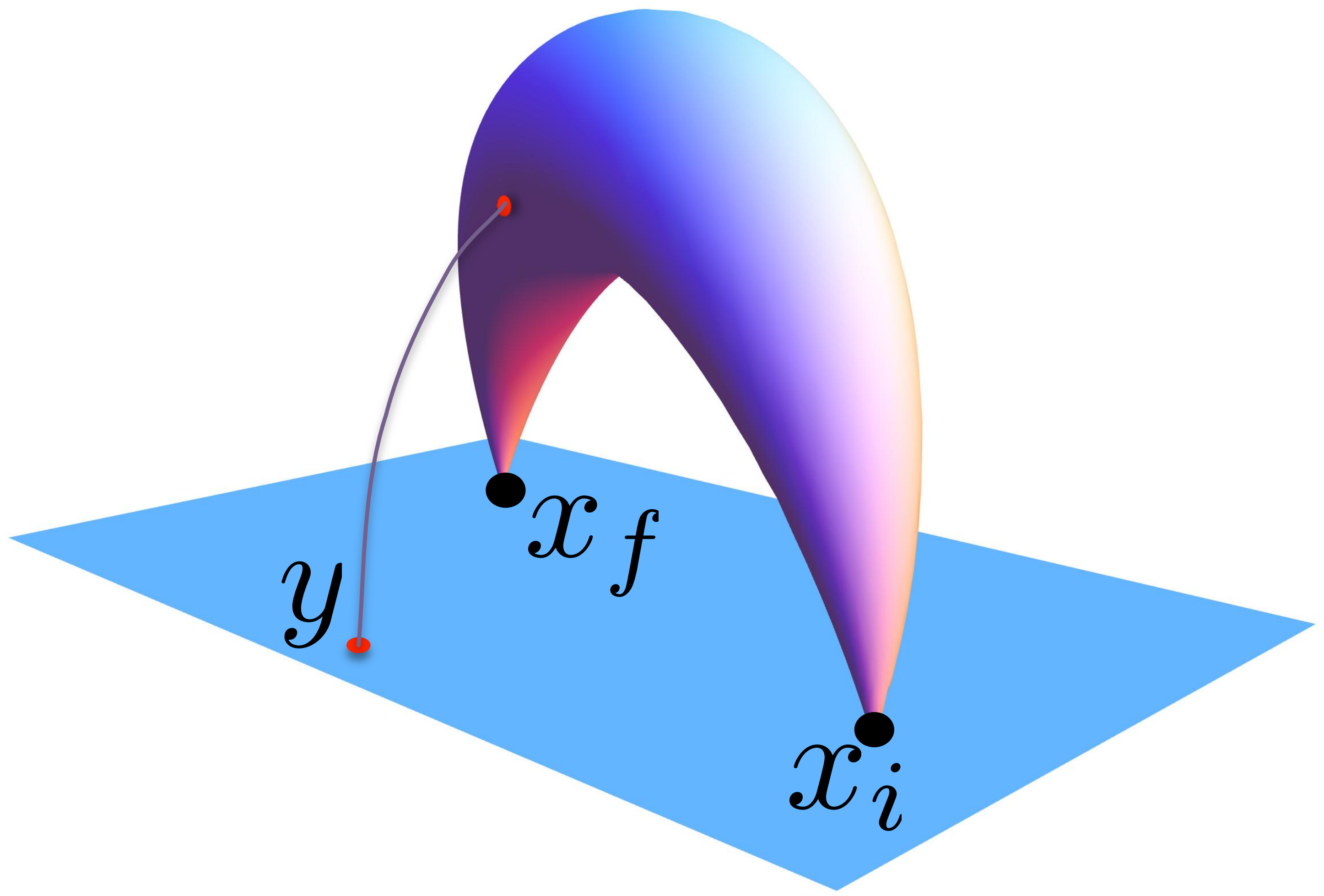

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Boundary conditions string worldsheet:

$$X^a(\tau_i,\sigma) = x_i^a = (x_i^\mu,\epsilon)$$

$$X^a(\tau_f,\sigma) = x_f^a = (x_f^\mu,\epsilon)$$

Boundary condition light field:

$$\Phi(x^\mu,\epsilon) \sim \epsilon^{4-\Delta}\Phi_0(x^\mu)$$

Conformal gauge

$$\int d^2\sigma\,\sqrt{-\gamma}\,\gamma^{\alpha\beta} \;\to\; \int_{-s/2}^{s/2} d\tau \int_0^{2\pi} d\sigma\,\eta^{\alpha\beta}$$

- Heavy string acts as tadpole for light fields

$$\int DX\,ds\,e^{iS_P[X,s,\Phi=0]} \quad \left(\int D\Phi\,e^{i\left(S_{SUGRA}[\Phi]+\int d^2\sigma\,\frac{\delta S_P[X,s,\Phi]}{\delta\Phi}\Big|_{\Phi=0}\Phi+\cdots\right)}\right)$$

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"Legendre" transform

$$S_P \to \tilde{S}_P = S_P - \int \Pi \dot{X}$$

- 2-point function $\langle \mathcal{O}_A(x_i) \mathcal{O}_A(x_f) \rangle$

$$\int DX \, ds \, e^{i\tilde{S}_P[X,s,\Phi=0]} \approx \frac{P}{|x_i - x_f|^{2\Delta_A}}$$

[Janik et al]

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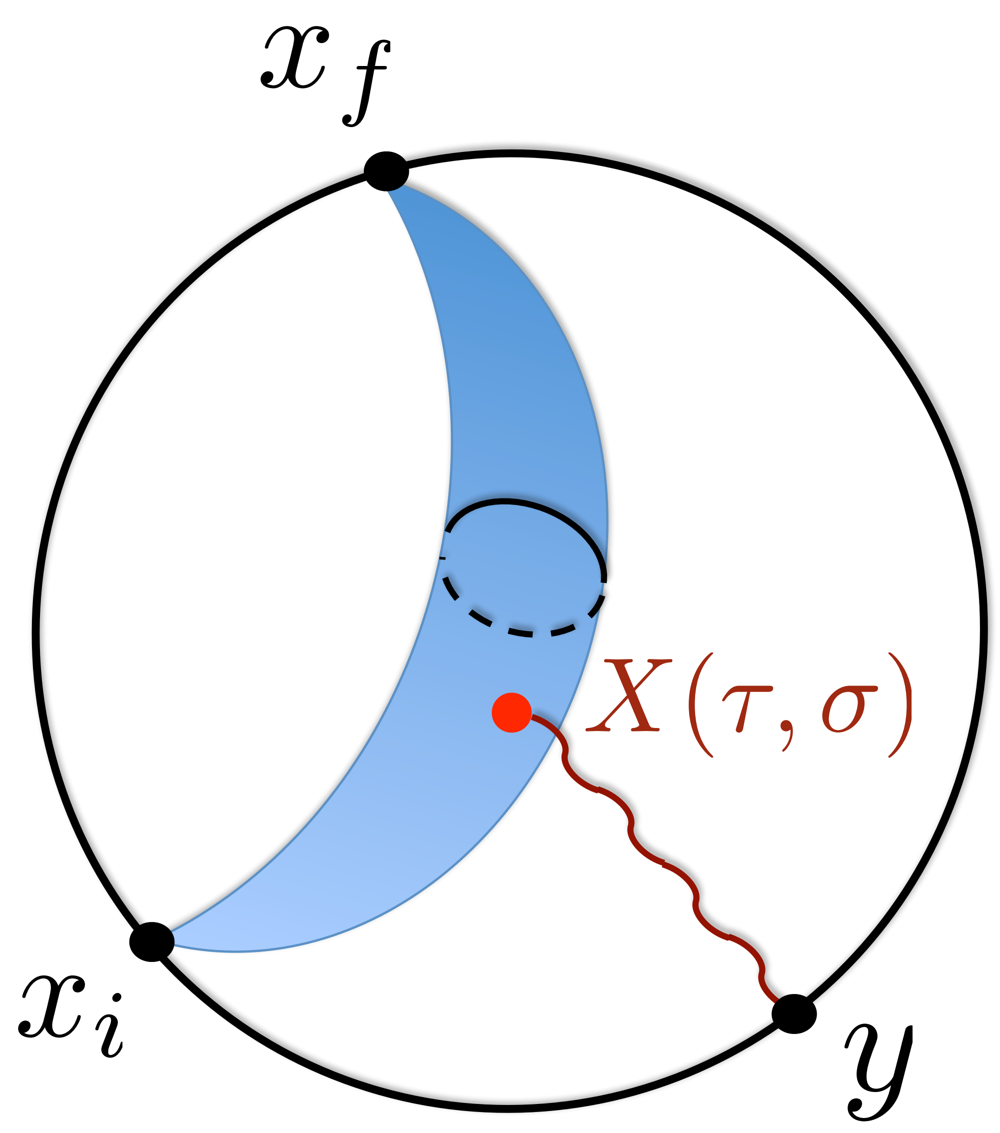
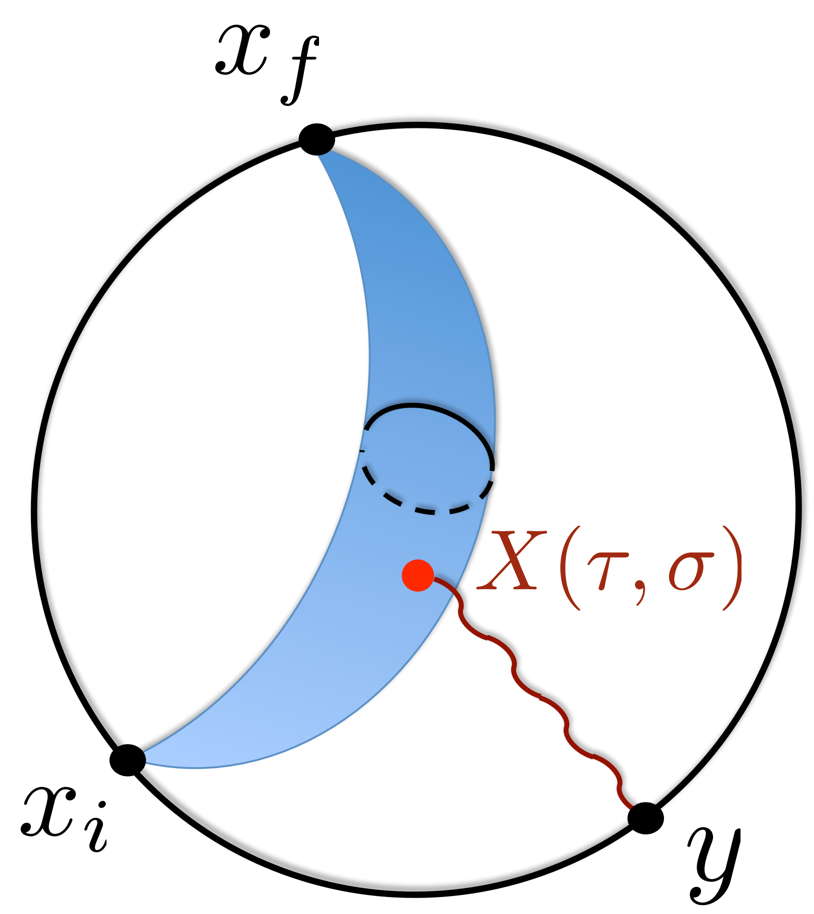
[Janik et al]

Witten diagram with fat string



- 3-point function $\langle \mathcal{O}_A(x_i)\mathcal{O}_A(x_f)\mathcal{D}_\chi(y)\rangle$

$$\int DX\, ds\, e^{i\tilde{S}_P[X,s,\Phi=0]}\, I_\chi[X,s;y]$$



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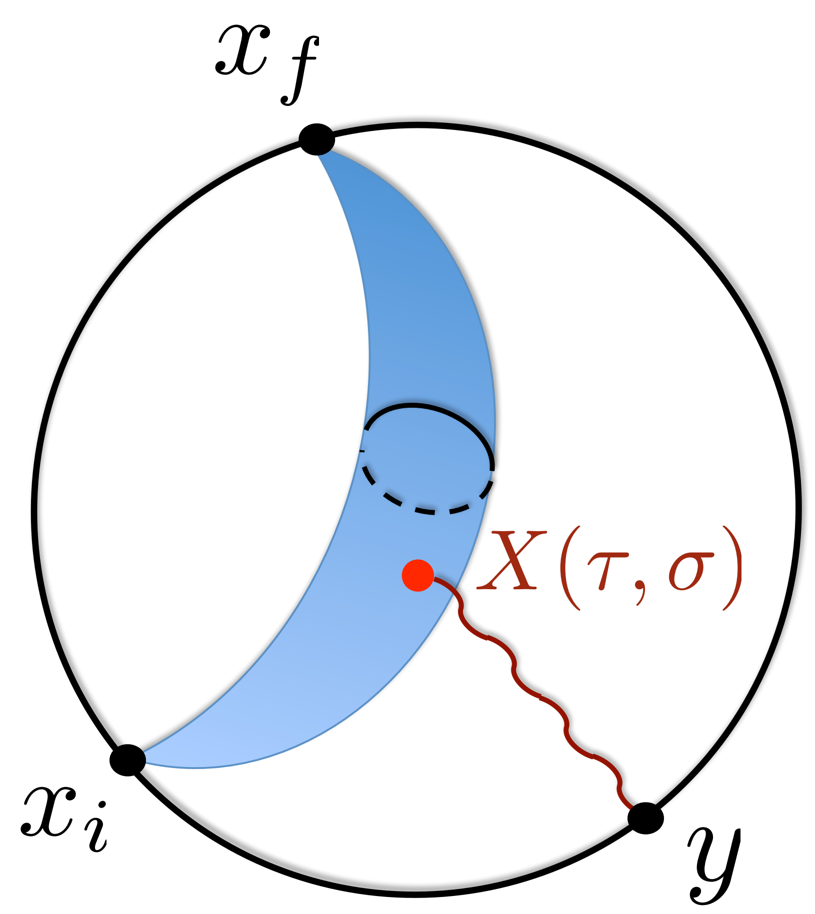
$$=$$

$$\boxed{i\int_{-s/2}^{s/2} d\tau \int_0^{2\pi} d\sigma\, \frac{\delta S_P[X,s,\Phi]}{\delta\chi}\bigg|_{\Phi=0} K_\chi(X(\tau,\sigma);y)}$$



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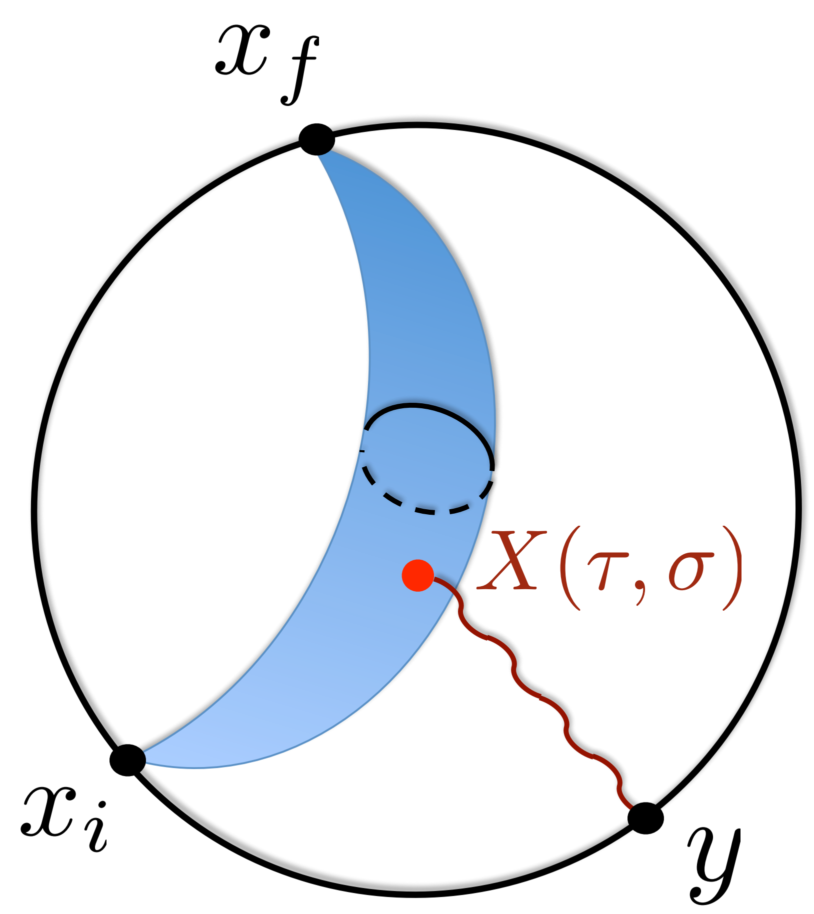

- Leading term in pre-factor is just the same as for 2-point function

$$\langle \mathcal{O}_A(x_i)\mathcal{O}_A(x_f)\rangle \approx \frac{P}{|x_i - x_f|^{2\Delta_A}}$$

$$X(\tau,\sigma) = \bar{X}(\tau,\sigma) + \frac{\delta X(\tau,\sigma)}{\sqrt{g}}$$

$$\boxed{\langle \mathcal{O}_A(x_i)\mathcal{O}_A(x_f)\mathcal{D}_\chi(y)\rangle \approx P\,\frac{I_\chi[\bar{X},\bar{s};y]}{|x_i - x_f|^{2\Delta_A}}}$$

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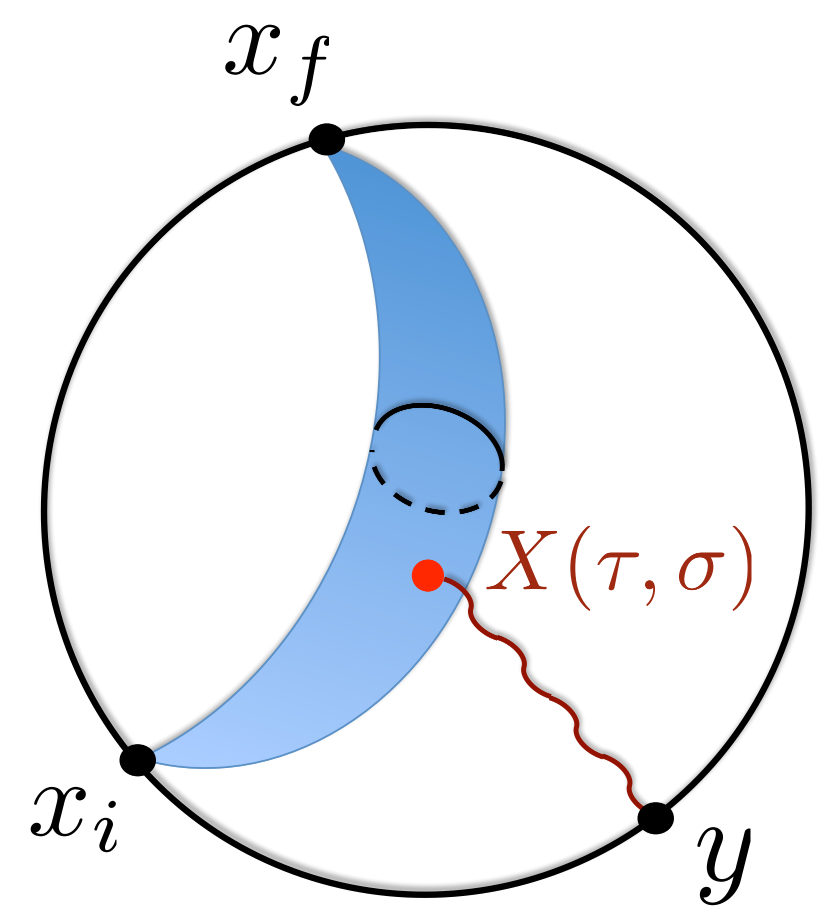
$$\boxed{i\int_{-s/2}^{s/2} d\tau \int_0^{2\pi} d\sigma\, \frac{\delta S_P[X,s,\Phi]}{\delta\chi}\bigg|_{\Phi=0} K_\chi(X(\tau,\sigma);y)}$$



- Leading term in pre-factor is just the same as for 2-point function

$$\langle \mathcal{O}_A(x_i)\mathcal{O}_A(x_f)\rangle \approx \frac{\cancel{P}}{|x_i - x_f|^{2\Delta_A}}$$

$$X(\tau,\sigma) = \bar{X}(\tau,\sigma) + \frac{\delta X(\tau,\sigma)}{\sqrt{g}}$$

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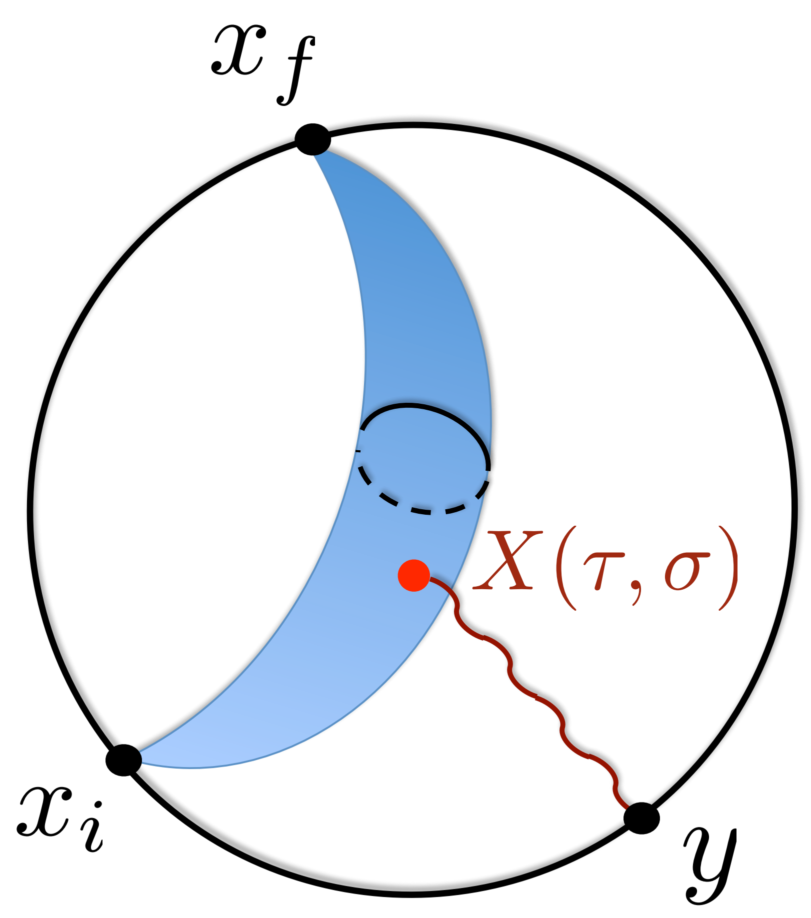

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Should obtain the correct 3-pt function kinematics

Simplest case - point like string

$$S_P[X, s, \Phi] = \frac{1}{2}\int_{-s/2}^{s/2} d\tau\, e^{\phi/4}\left(\dot{X}^a \dot{X}^b g_{ab} - m^2\right)$$

Massive string state

$$\Delta \approx m \sim \sqrt{g}$$

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$$\begin{aligned} x(\tau) &= R\tanh\kappa\tau + R \\ z(\tau) &= \frac{R}{\cosh\kappa\tau} \end{aligned}$$

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$$x(+s/2) = x_f = 2R$$

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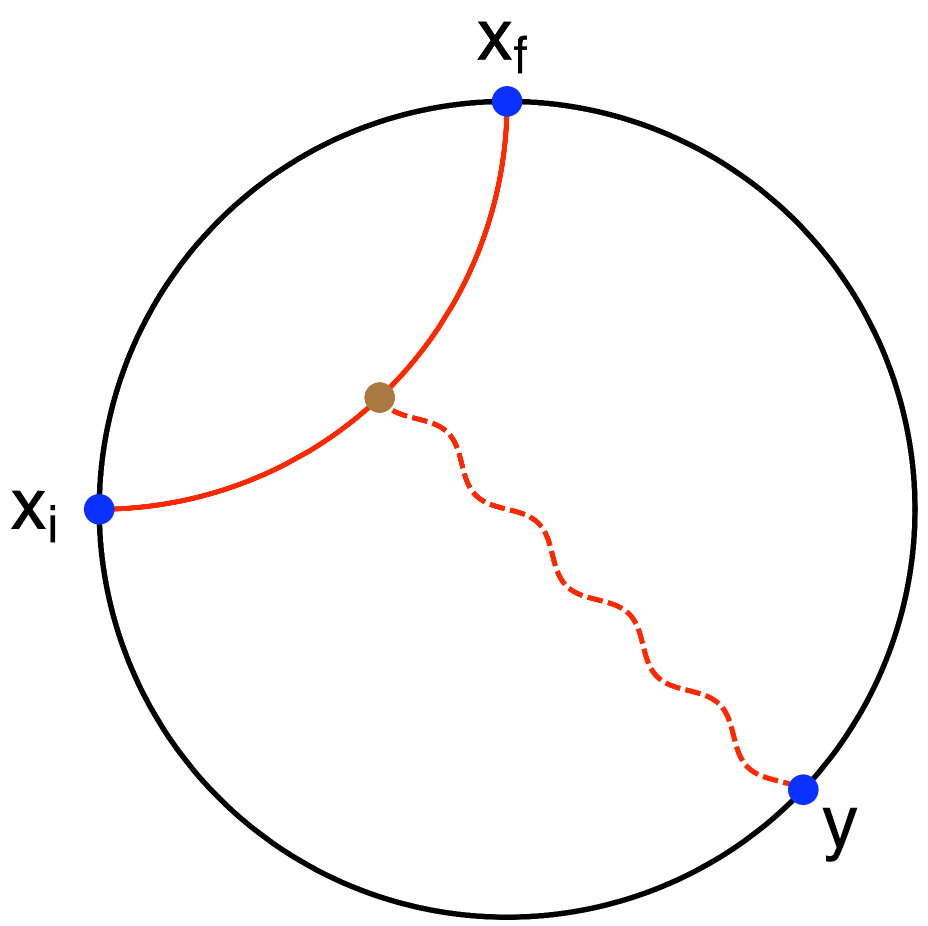
$$\langle\mathcal{O}_A(0)\mathcal{O}_A(x_f)\rangle \approx \int dX ds\, e^{iS_P[X,s]} \approx \int ds\, e^{iS_P[\bar{X},s]} \approx e^{iS_P[\bar{X},\bar{s}]} = \frac{1}{|x_f|^{2\Delta_A}}$$

- Consider 3-point function $\langle \mathcal{O}_A(0)\mathcal{O}_A(x_f)\mathcal{L}(y)\rangle$



$$I_\phi[X,s;y] = i\int_{-s/2}^{s/2} d\tau \left.\frac{\delta S_P[X,s,\Phi]}{\delta\phi}\right|_{\Phi=0} K_\phi(X(\tau,\sigma);y)$$

$$= i\,\frac{3}{4\pi^2}\int_{-s/2}^{s/2} d\tau \left(\frac{\dot{x}^2+\dot{z}^2}{z^2}-m^2\right)\left(\frac{z}{z^2+(x-y)^2}\right)^4$$

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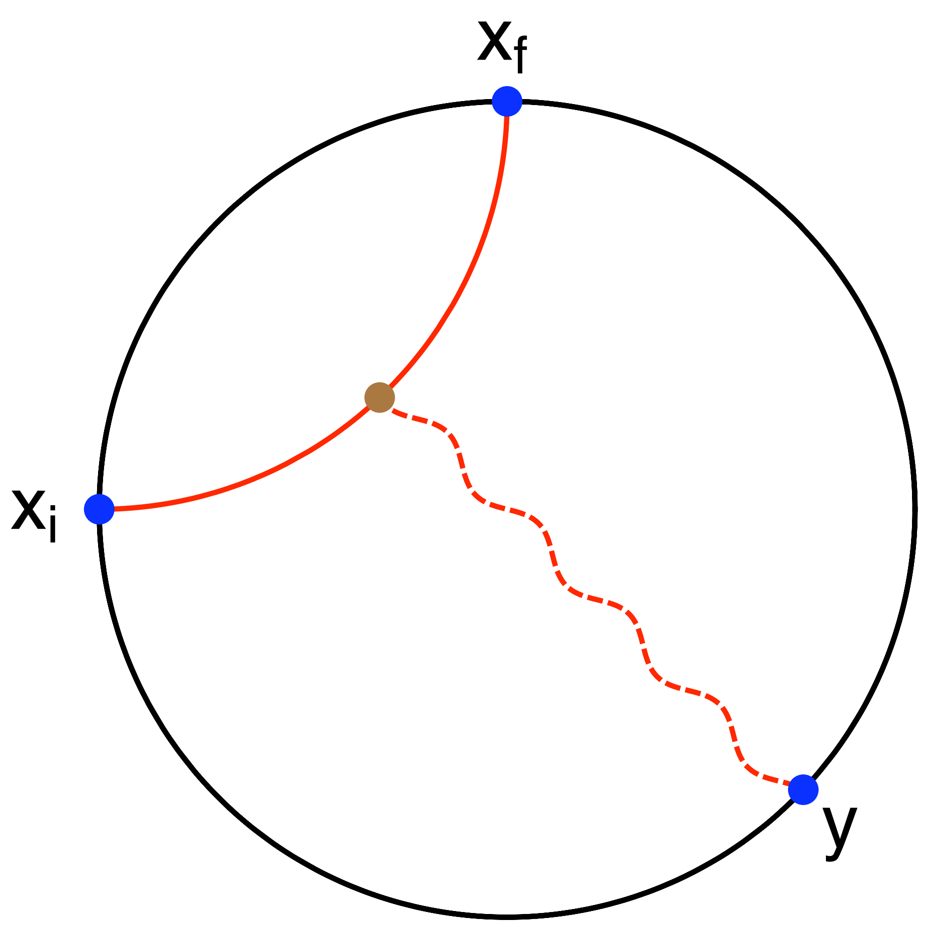

At saddle point $I_\phi[\bar{X},\bar{s};y] = -\frac{m}{8\pi^2}\,\frac{x_f^4}{y^4\,(x_f-y)^4}$

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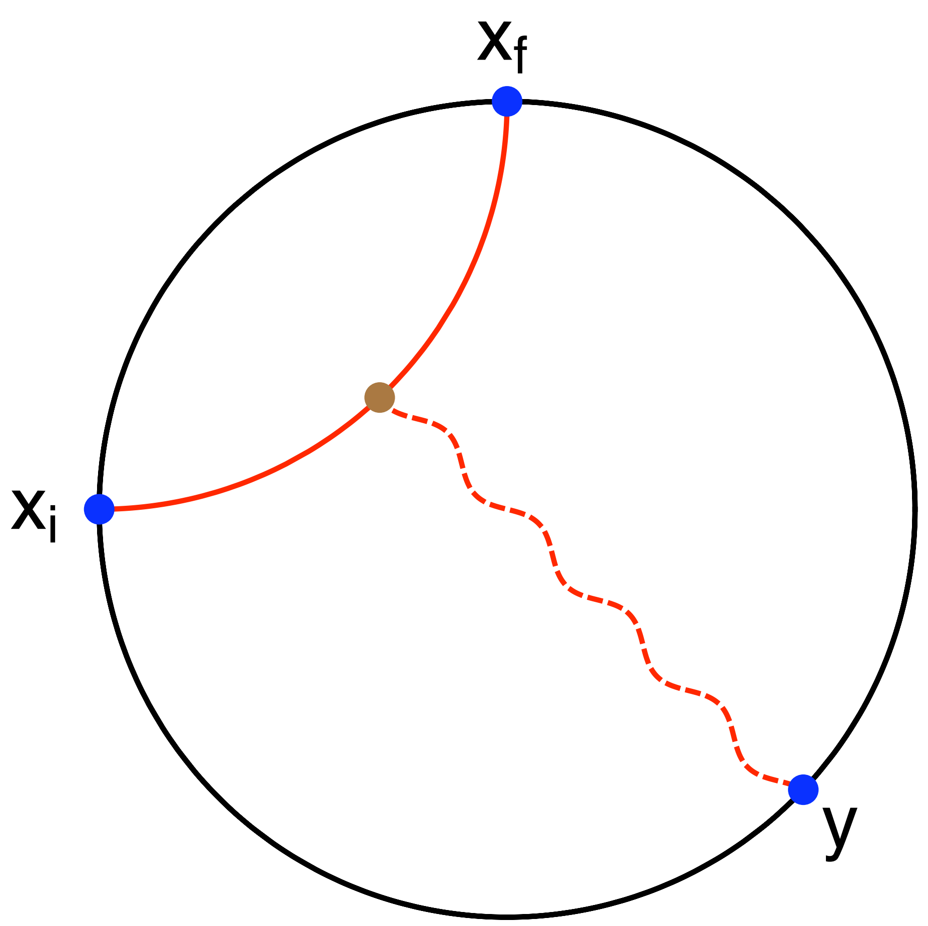

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Correct kinematics and agreement with previous RG argument:

$$2\pi^2 a_{\mathcal{L}AA} = -g^2\frac{\partial\Delta_A}{\partial g^2} \approx -\frac{\Delta_A}{4}$$

Other examples

- Giant magnon

$$\langle \mathcal{O}_A(0)\mathcal{O}_A(x_f)\mathcal{L}(y)\rangle \approx -\frac{g}{\pi^2}\sin\frac{p}{2}\,\frac{1}{x_f^{2\left(J+4g\sin\frac{p}{2}\right)-4}\,y^4\,(x_f-y)^4}$$

As expected: $2\pi^2 a_{\mathcal{L}AA} = -g^2\dfrac{\partial\Delta_A}{\partial g^2} \approx -2g\sin\dfrac{p}{2}$

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- Circular rotating string in S^5

- Spinning string in AdS_5

Final comments

- Extend to other operators, including 3-point functions with non-trivial spin structures

- Compute pre-factors to next order in $1/g$, also checking RG relation between $a_{\mathcal{L}AA}$ and Δ_A

- Perturbative computation, using effect of deformation by other operators on the anomalous dimension matrix and integrability

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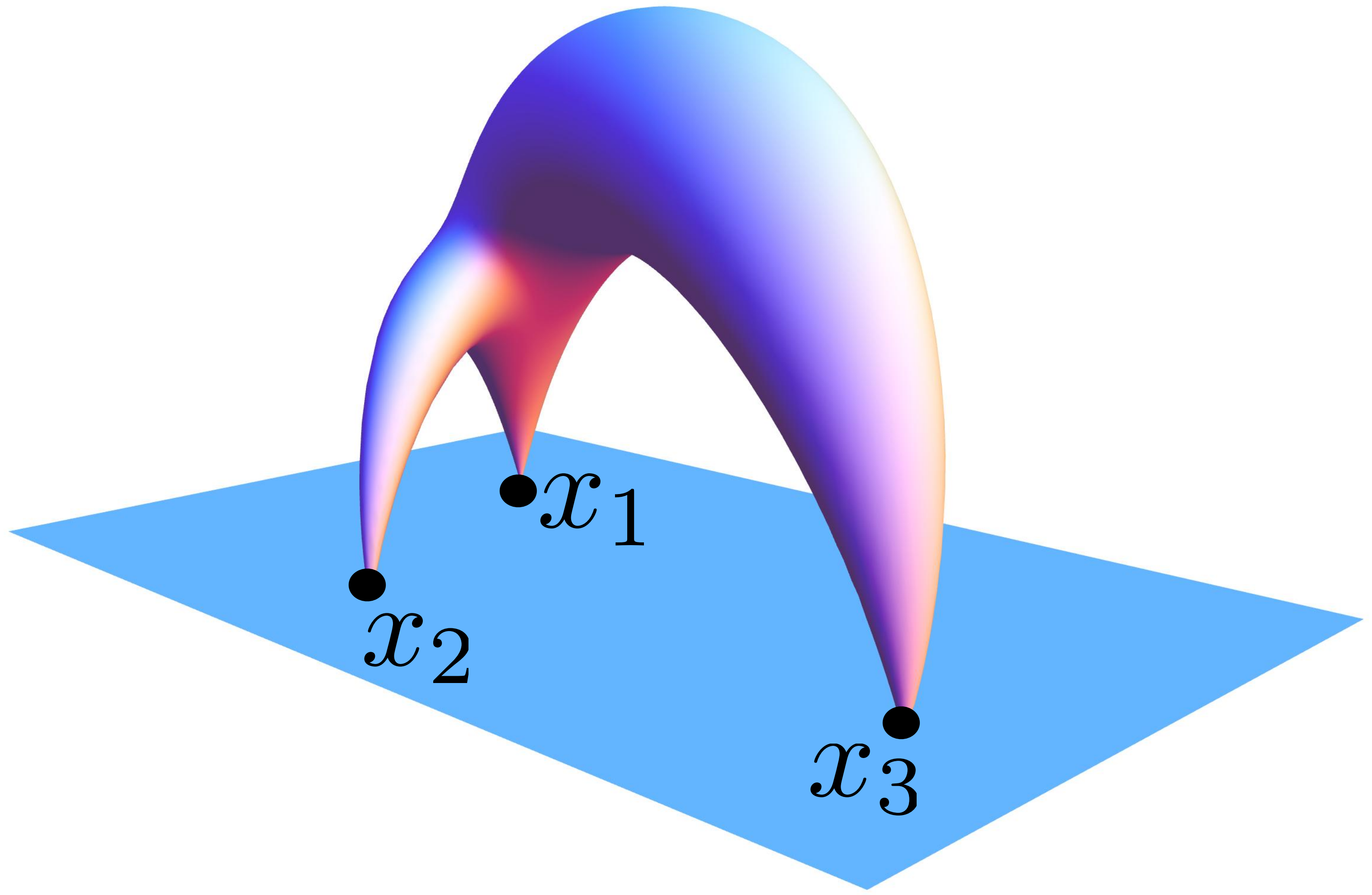
- Perturbative computation, using effect of deformation by other operators on the anomalous dimension matrix and integrability

- More general problem remains. Expect integrability to play a crucial role. (may be some numerics)



THANK YOU