Gravity in Three Dimensions

Eric Bergshoeff

Groningen University

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and with R. Andringa, M. de Roo, J. Rosseel and E. Sezgin

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Outline

The Problem of Quantum Gravity

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4D Quantum Gravity

Consider Einstein gravity as a theory of interacting massless spin 2 particles around a Minkowski space-time background

Problem: This theory is non-renormalizable

$$\mathcal{L} \sim R + a\,({R_{\mu\nu}}^{ab})^2 + b\,(R_{\mu\nu})^2 + c\,R^2$$ is renormalizable

Stelle (1977)

$$\text{propagator} \quad \sim \quad \left(\frac{1}{p^2} + \frac{1}{p^4}\right)_0 + \left(\frac{1}{p^2} + \frac{1}{p^4}\right)_2$$

However: renormalizability: $\surd$ but unitarity: X

Special Cases

- $\mathcal{L} \sim R + R^2$: scalar field coupled to gravity

  unitarity: $\surd$ but renormalizability: X

  propagator $\sim \left(\frac{1}{p^2} + \frac{1}{p^4}\right)_0 + \left(\frac{1}{p^2}\right)_2$

- $\mathcal{L} \sim R + \left(C_{\mu\nu}{}^{ab}\right)^2$: Weyl tensor squared

  propagator $\sim \left(\frac{1}{p^2}\right)_0 + \left(\frac{1}{p^2} + \frac{1}{p^4}\right)_2$

  unitarity: X and renormalizability: X

Why $D = 3$ Dimensions?

- Study the problem of (quantum) gravity in a different setting

- Relation to gravity in $D > 3$ dimensions via dimensional reduction

  Lu, Pope, Sezgin (2010)

- $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence

Outline

3D Einstein-Hilbert Gravity

3D zero Ricci tensor implies 3D zero Riemann tensor

3D spacetime is locally flat outside sources

There are no massless gravitons

Adding higher-derivative terms leads to "massive gravitons"

For 4D massive gravity, see

van Dam, Veltman, Zakharov (1970); Vainshtein (1972);
Dvali, Gabadadze, Porrati (2000); de Rham, Gabadadze, Khoury, Tolley (2010)

Fierz-Pauli

$$\left(\Box - m^2\right)\phi_{\mu\nu} = 0\,, \qquad \phi_{\mu\nu} = \phi_{\nu\mu}\,,\;\; \eta^{\mu\nu}\phi_{\mu\nu} = 0\,,\;\; \partial^{\nu}\phi_{\nu\mu} = 0$$

The number of propagating modes is:

$$\tfrac{1}{2}D(D+1) - 1 - D = \begin{cases} 5 & \text{for } 4D \\ 2 & \text{for } 3D \end{cases}$$

Fierz-Pauli has no non-linear generalization except in 3D...

Non-linear Extension

$$\phi_{\mu\nu} = -\frac{1}{2}\epsilon_{\mu}{}^{\tau_1\rho}\epsilon_{\nu}{}^{\tau_2\sigma}\partial_{\tau_1}\partial_{\tau_2}h_{\rho\sigma} \equiv G^{\text{lin}}_{\mu\nu}(h)\,, \qquad h_{\mu\nu} \;\to\; h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$$

$$\left(\Box - m^2\right)\; G_{\mu\nu}(h) = 0\,, \qquad G(h) = 0$$

$$\text{Non-linear generalization:} \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow$$

$$\mathcal{L} = \sqrt{-g}\left[-R + \frac{1}{m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2\right)\right]$$

"New Massive Gravity" (NMG) : unitary !

cp. to TMG ...

Topological Massive Gravity (TMG)

In 3D you can describe a single helicity mode :

$$\left[\epsilon_{(\mu}{}^{\tau\rho}\delta^{\sigma}_{\nu)}\,\partial_\tau \pm 2m\,\delta^{\rho}_{(\mu}\delta^{\sigma}_{\nu)}\right]\phi_{\rho\sigma} = 0$$

Following the same steps as before leads to

$$\mathcal{L} = -\sqrt{-g}\,R \pm \frac{1}{\mu}\mathcal{L}_{\text{LCS}}$$

"Topological Massive Gravity"

Deser, Jackiw, Templeton (1982)

Equivalence of NMG to Fierz-Pauli

$$\mathcal{L} = \sqrt{-g}\left[-R + f^{\mu\nu}G_{\mu\nu} - \tfrac{1}{4}m^2\left(f^{\mu\nu}f_{\mu\nu} - f^2\right)\right] , \qquad f = g^{\mu\nu}f_{\mu\nu}$$

If we eliminate $f^{\mu\nu}$ by its algebraic e.o.m. we recover NMG

Now we have two fields, $g_{\mu\nu}$ and $f^{\mu\nu}$, to worry about !

$$\mathcal{L}_{\rm quadr} = -\mathcal{L}_{\rm EH}^{\rm (lin)}(h) + \mathcal{L}_{\rm FP}(f)$$

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NMG

Is NMG renormalizable ?

$$\mathcal{L} = \sqrt{-g}\left[\sigma R + \frac{a}{m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2\right) + \frac{b}{m^2}R^2\right] \qquad \sigma = \pm 1$$

$$\text{propagator} \sim \left(\frac{1}{p^2} + \frac{b}{p^4}\right)_0 + \left(\frac{1}{p^2} + \frac{a}{p^4}\right)_2 \quad \Rightarrow \quad ab \neq 0$$

Nishino, Rajpoot (2006)

However, unitarity requires $ab = 0$ $\Rightarrow$

Unitarity and Renormalizability exclude each other !

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Why Supersymmetry (SUSY) ?

- SUSY can soften the ultra-violet divergences

- What is the role of the auxiliary fields in the presence of higher-derivative gravity ?

Auxiliary Fields

"Off-shell" $\mathcal{N} = 1$ SUSY formulation : $\mathcal{L}_{\mathsf{EH}} = R - 2S^2$

Higher-derivative terms lead to extra S^4 , $S\Box S$ and RS^2 terms

$\mathcal{N} = 1$ SUSY NMG can be defined with only S^2 and S^4 terms

Hohm, Rosseel, Sezgin, Townsend + E.B. (2010)

$\mathcal{N} = 1$ SUSY "General Massive Gravity"

$$\mathcal{L}_{\mathrm{bos}} = \sqrt{-\det g}\left\{\sigma\left(R - 2S^2\right) + MS + \frac{1}{m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2 + \frac{1}{6}S^4\right)\right\} + \frac{1}{\mu}\mathcal{L}_{\mathrm{LCS}}$$

cubic S – E.O.M. : $S = \bar{S} \quad \Rightarrow \quad$ bosonic model

$\mathcal{N} > 1$ SUSY

helicity	$+2$	$+3/2$	$+1$	$+1/2$	0	$-1/2$	-1	$-3/2$	-2
$\mathcal{N} = 1$	1	1							
$\mathcal{N} = 2$	1	2	1						
$\mathcal{N} = 3$	1	3	3	1					
$\mathcal{N} = 4$	1	4	6	4	1				
$\mathcal{N} = 5$	1	5	10	10	5	1			
$\mathcal{N} = 6$	1	6	15	20	15	6	1		
$\mathcal{N} = 7$	1	7	21	35	35	21	7	1	
$\mathcal{N} = 8$	1	8	28	56	70	56	28	8	1

Maximal SUSY

NMG with maximal $\mathcal{N} = 8$ SUSY is based on the same supermultiplet as maximal 4D supergravity

Question: has maximal $\mathcal{N} = 8$ NMG similar softened ultraviolet divergencies as maximal 4D SUGRA ?

Outline

Open Questions

- New “unitary” model of (massive) 3D gravity (NMG)
- Not renormalizable
- No unitary boundary conformal field theory

  for TMG see Skenderis, Taylor and van Rees (2009)
- Applications to condensed matter ?