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Holographic d-wave superconductors

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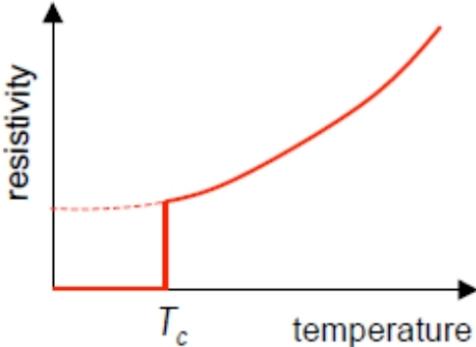
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Outline

- Holographic superconductors
- d-wave superconductors
charged massive spin-2 fields
- Fermionic operators and spectral function
- d-wave gap, Dirac nodes, Fermi arcs
- Future directions

Real superconductors

- Superconductor: a system characterized by a transition to a state with zero resistivity (below T_c)



It can be modeled by spontaneous breaking of U(1) e.m.

- BCS: weakly coupled description. E.g. cuprates look strongly coupled
- Phenomenology depends on the nature of the order parameter
- Cuprates: **d-wave superconductors** (spin-2 order parameter)
- Interesting phenomenology, ARPES & STM:
d-wave gap, Dirac nodes, Fermi arcs, pseudo-gap, ...
- Motivation:
how far can we go without using the details of the atomic structure,
but only "symmetries" and basic features?

Holographic Superconductors

- **Holographic superfluid:** a field theory (CFT) at temperature $T \geq 0$ and finite chemical potential ρ , with $U(1)$ global symmetry and charged order parameter ψ
- **Holographic superconductor:** weakly gauge the $U(1)$ (photon)
- Study the CFT at strong coupling via AdS/CFT
- Study the behavior of extra operators (not directly involved in condensation), e.g. fermionic operators
- Bottom-up approach: focus on subset of fields

d-wave

- The order parameter is d-wave
→ massive charged spin-2 field in the bulk
(graviton: massless neutral)
- Various problems could arise:
wrong number of d.o.f.
ghosts
causality violations ($v > c$) on non-trivial background
- What action?

Spin-2 fields

- E.g.: $L = -\partial_\rho \varphi_{\mu\nu} \partial^\rho \varphi^{\mu\nu} - m^2 \varphi_{\mu\nu} \varphi^{\mu\nu}$ in $\mathbb{R}^{d,1}$
- Number of d.o.f.: symmetric $\varphi_{\mu\nu}$ has $(d+1)(d+2)/2$ components, while massive spin-2 particle has $(d+2)$ less: need *constraints*. The extra modes contain *ghosts*.

- Fierz-Pauli action (unique quadratic and 2 derivatives):

$$L_{FP} = -|\partial_\rho \varphi_{\mu\nu}|^2 + 2|\varphi_\mu|^2 - 2\varphi^\mu \partial_\mu \varphi + |\partial_\mu \varphi|^2 - m^2 (|\varphi_{\mu\nu}|^2 - \varphi^2)$$
$$\varphi_\mu \equiv \partial^\nu \varphi_{\nu\mu} \quad \varphi \equiv \varphi_\mu^\mu$$

get the equations:

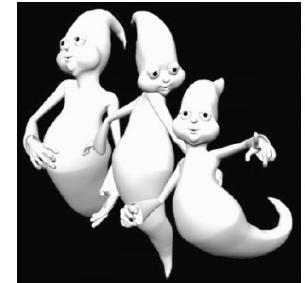
$$\begin{aligned} 0 &= (\square - m^2) \varphi_{\mu\nu} \\ 0 &= \varphi_\nu \\ 0 &= \varphi \end{aligned} \quad \left. \right\} d+2 \text{ constraints}$$

Correct number of d.o.f., no ghosts, causal propagation

Charged Spin-2 field on background

- Covariant derivative: $\partial_\mu \rightarrow D_\mu = \partial_\mu + \Gamma_\mu - iq A_\mu$
 → get the problems back!

Solved by coupling to curvatures $R_{\mu\nu\rho\lambda}$ & $F_{\mu\nu}$



- Metric: fully consistent action on Einstein background
 → **probe limit** Buchbinder Gitman Pershin
- E.m.: # d.o.f. & no ghosts Federbush
- Action (up to dim d+1):

$$L_{\text{spin 2}} = -|D_\rho \varphi_{\mu\nu}|^2 + 2|\varphi_\nu|^2 + |D_\mu \varphi|^2 - (\varphi^{*\nu} D_\nu \varphi + \text{c.c.}) - m^2 (|\varphi_{\mu\nu}|^2 - |\varphi|^2) \\ + 2R_{\mu\nu\rho\lambda} \varphi^{*\mu\rho} \varphi^{\nu\lambda} - \frac{1}{d+1} R |\varphi|^2 - iq F_{\mu\nu} \varphi^{*\mu\lambda} \varphi_\lambda^\nu$$

$$L_{\text{tot}} = R - \Lambda - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + L_{\text{spin 2}}$$

- **Probe limit:** large q and small ρ
 matter & gauge fields do not backreact on metric

Charged Spin-2 field on background

$$\begin{aligned} L_{\text{spin } 2} = & -|D_\rho \varphi_{\mu\nu}|^2 + 2|\varphi_\nu|^2 + |D_\mu \varphi|^2 - (\varphi^{*\nu} D_\nu \varphi + \text{c.c.}) - m^2 (|\varphi_{\mu\nu}|^2 - |\varphi|^2) \\ & + 2R_{\mu\nu\rho\lambda} \varphi^{*\mu\rho} \varphi^{\nu\lambda} - \frac{1}{d+1} R |\varphi|^2 - i q F_{\mu\nu} \varphi^{*\mu\lambda} \varphi_\lambda^\nu \end{aligned}$$

- $F_{\mu\nu} \rightarrow$ new problem: faster than light signals

$$v_{\max} \simeq \left(1 + \frac{q |F_{\mu\nu}|}{m^2} \right)$$



- No full answer is known \rightarrow work with

$$\frac{q |F_{\mu\nu}|}{m^2} \ll 1 \quad \rightarrow \quad \Delta \gg 1$$

- Argyres-Nappi: causal action on flat spacetime & constant $F_{\mu\nu}$.
Each term is non-linear function of $F_{\mu\nu}$.

AdS/CFT

- Field/operator correspondence: $\varphi_{\mu\nu} \leftrightarrow O_{mn}$

$$\varphi_{mn} \underset{z \rightarrow 0}{\sim} \langle O_{mn} \rangle z^{\Delta-2} + O_{mn}^{(s)} z^{d-\Delta-2} \quad m^2 L^2 = \Delta(\Delta-d) \quad \Delta > d$$

- Ansatz:
$$ds^2 = \frac{L^2}{z^2} \left(-f(z) dt^2 + d\vec{x}_d^2 + \frac{dz^2}{f(z)} \right) \quad f(z) = 1 - \left(\frac{z}{z_h} \right)^d$$

$$A = \phi(z) dt$$

$$\varphi_{xy} = \frac{L^2}{2z^2} \psi(z) \quad \Rightarrow D^\mu \varphi_{\mu\nu} = \varphi = 0$$

- Equations: same as s-wave!

- Boundary conditions: chemical potential ρ
→ critical temperature T_c

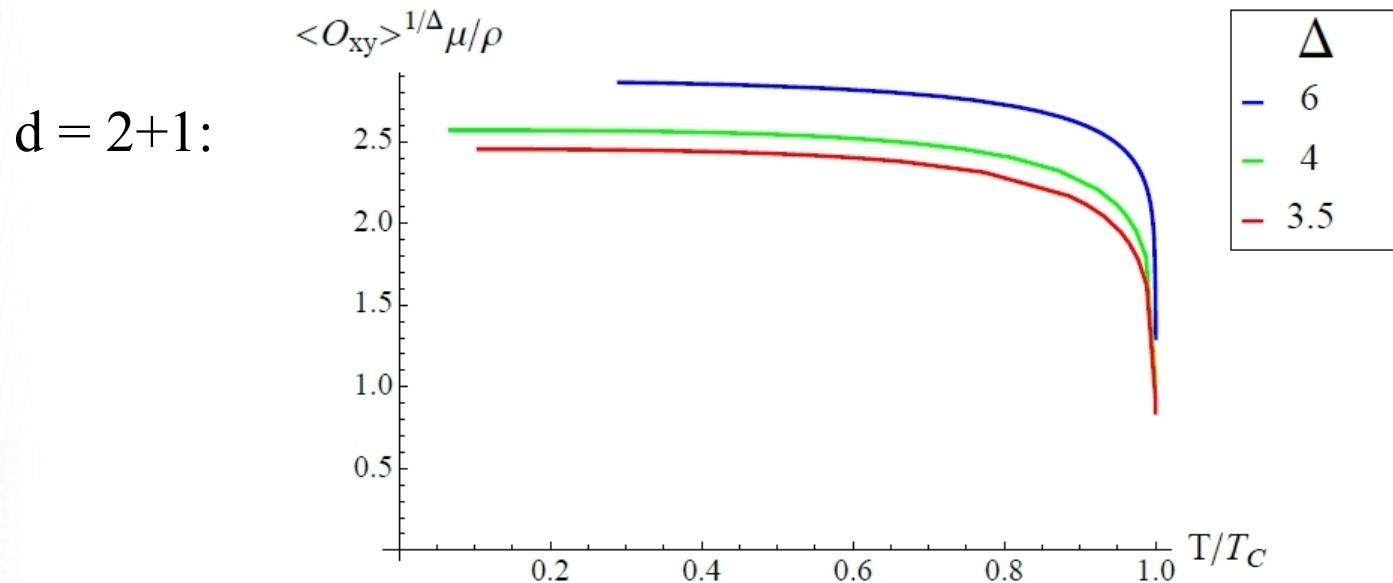
AdS/CFT

- There exist a critical temperature T_c

- $T > T_c$: normal state (charged BH)

$$A = \mu \left[1 - \left(\frac{z}{z_h} \right)^{d-2} \right] dt \quad \varphi_{\mu\nu} = 0$$

- $T < T_c$: superconducting phase (condensate) $\varphi_{xy} \neq 0$



Compute conductivity σ_{mn} (isotropic at leading order)

Fermions

- In AdS/CFT only gauge-invariant operators: study fermionic operators

bulk spinor $\Psi \leftrightarrow$ composite fermionic operator O_Ψ
(from p.o.w of weakly gauged U(1) "composite electron")

- Compute retarded Green's function & spectral function:

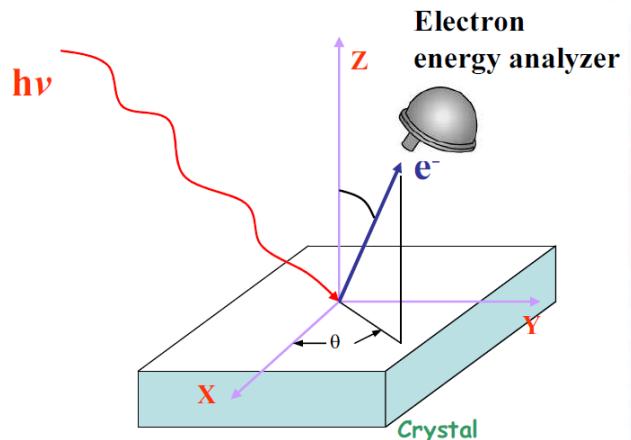
$$G_R(t, \vec{x}) = i\Theta(t)\langle\{O_\Psi(t, \vec{x}), O_\Psi^+(0)\}\rangle$$

$$\rho(\omega, \vec{k}) = \text{Tr Im } G_R(\omega, \vec{k})$$

- Direct connection with ARPES

- Green's function used to detect Fermi surface in normal phase

Liu, McGreevy, Vegh
Cubrovic, Zaanen, Schalm



in the following $d = 3$

Fermionic action

- What action?

Write down all terms up to dimension 5 (on background):

$$L_\Psi = i \bar{\Psi} (\Gamma^\mu D_\mu - m_\zeta) \Psi + \eta^* \varphi_{\mu\nu}^* \bar{\Psi}^c \Gamma^\mu D^\nu \Psi + \text{h.c.} + i |\varphi_{\mu\nu}|^2 \bar{\Psi} (c_3 + i c_4 \Gamma_5) \Psi \\ \varphi_{\mu\nu}^* \varphi^{*\mu\nu} \bar{\Psi}^c (c_1 + c_2 \Gamma_5) \Psi + \text{h.c.}$$

- Majorana-like term:
considered for s-wave
it gives rise to a **gapped** Fermi surface

Faulkner, Horowitz, McGreevy, Roberts, Vegh

Retarded Green's function

- 2-point function: $G_R(t, \vec{x}) = i\Theta(t)\langle\{O_\Psi(t, \vec{x}), O_\Psi^+(0)\}\rangle$
Response function

EOM
probe
asymptotic
infalling b.c.

$$0 = (\Gamma^\mu D_\mu - m_\zeta) \zeta + 2i\eta \varphi_{\mu\nu} \Gamma^\mu D^\nu \zeta^c$$

$$\zeta = e^{-i\omega t + i\vec{k} \cdot \vec{x}} \zeta^{(\omega, \vec{k})}(z) + e^{i\omega t - i\vec{k} \cdot \vec{x}} \zeta^{(-\omega, -\vec{k})}(z)$$

$$\zeta = \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix} \quad \zeta_\alpha \underset{z \rightarrow 0}{\sim} \begin{pmatrix} O(z) \\ (\sigma_1 S)_\alpha \end{pmatrix} e^{-m_\zeta L} + \begin{pmatrix} R_\alpha \\ O(z) \end{pmatrix} e^{m_\zeta L}$$

$$R_\alpha^{(\omega, \vec{k})} = M_\alpha{}^\beta S_\beta^{(\omega, \vec{k})} + \tilde{M}_\alpha{}^\beta S_\beta^{(-\omega, -\vec{k})c} \quad G_R(\omega, \vec{k}) = -i M \gamma^t$$

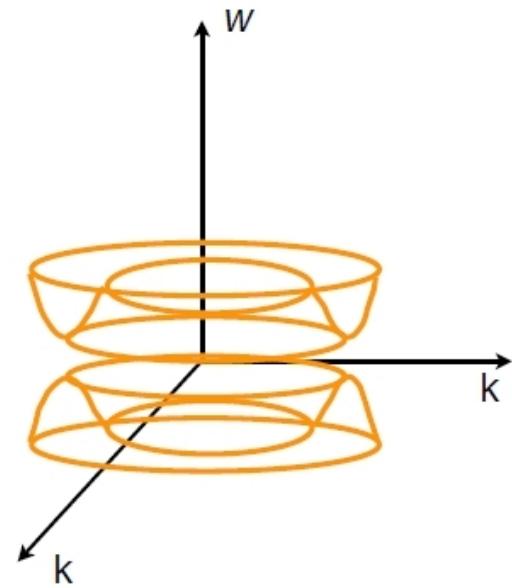
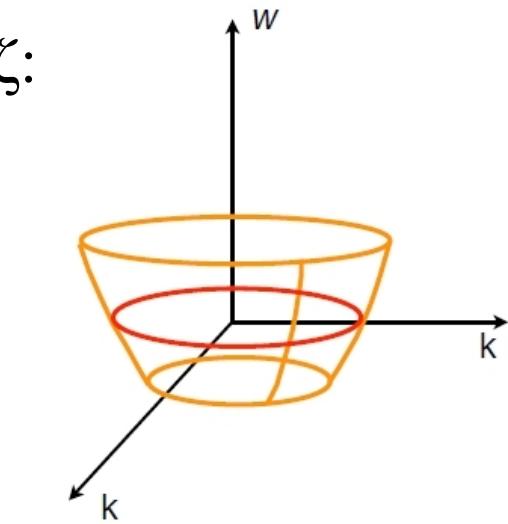
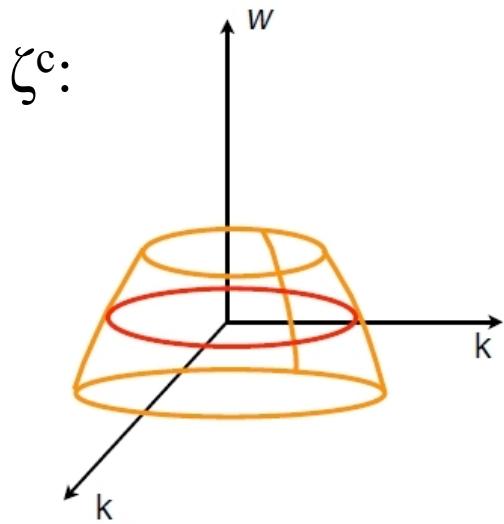
- Spectral function (density of states): $\rho(\omega, \vec{k}) = \text{Tr Im } G_R(\omega, \vec{k})$
sharp peaks \rightarrow dispersion relation $\omega(\vec{k})$ of quasi-normal modes

The gap – E.g. s-wave

- Peaks in spectral func. $\rightarrow \omega(k)$ quasi-normal modes

- $\eta \neq 0$: gap

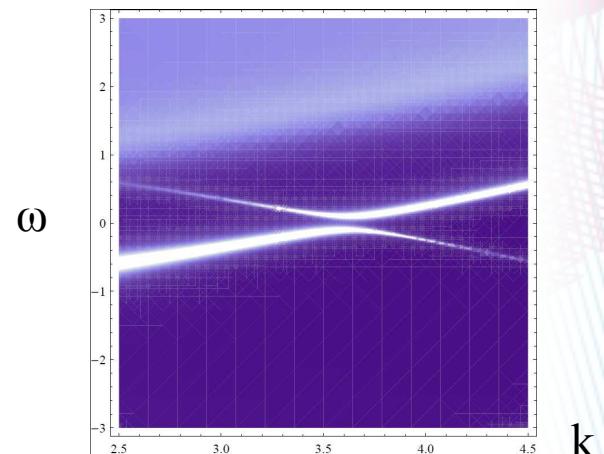
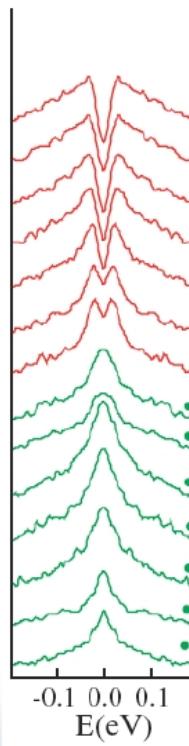
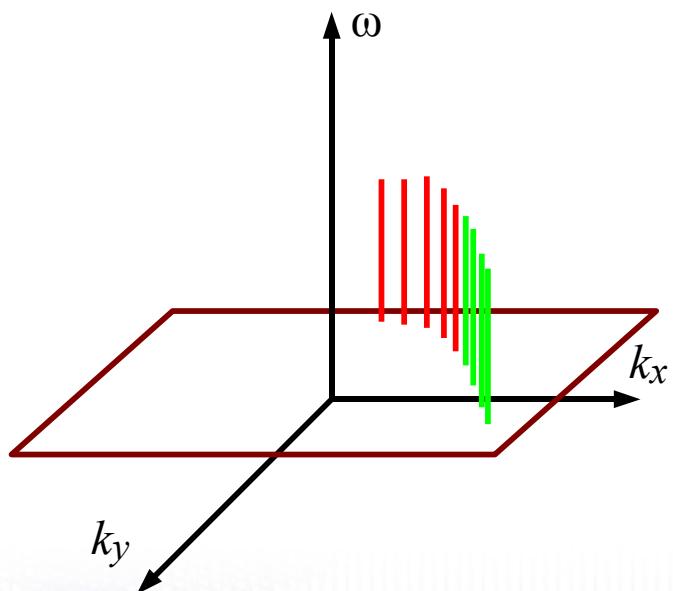
$$\begin{aligned} 0 &= D_{(1)}\zeta_1 + \eta\zeta_2^* \\ 0 &= D_{(2)}\zeta_2 + \eta\zeta_1^* \end{aligned} \Rightarrow \begin{aligned} \zeta: \quad \omega &= E(k) \\ \zeta^c: \quad \omega &= -E(-k) \end{aligned}$$



d-wave spectral function

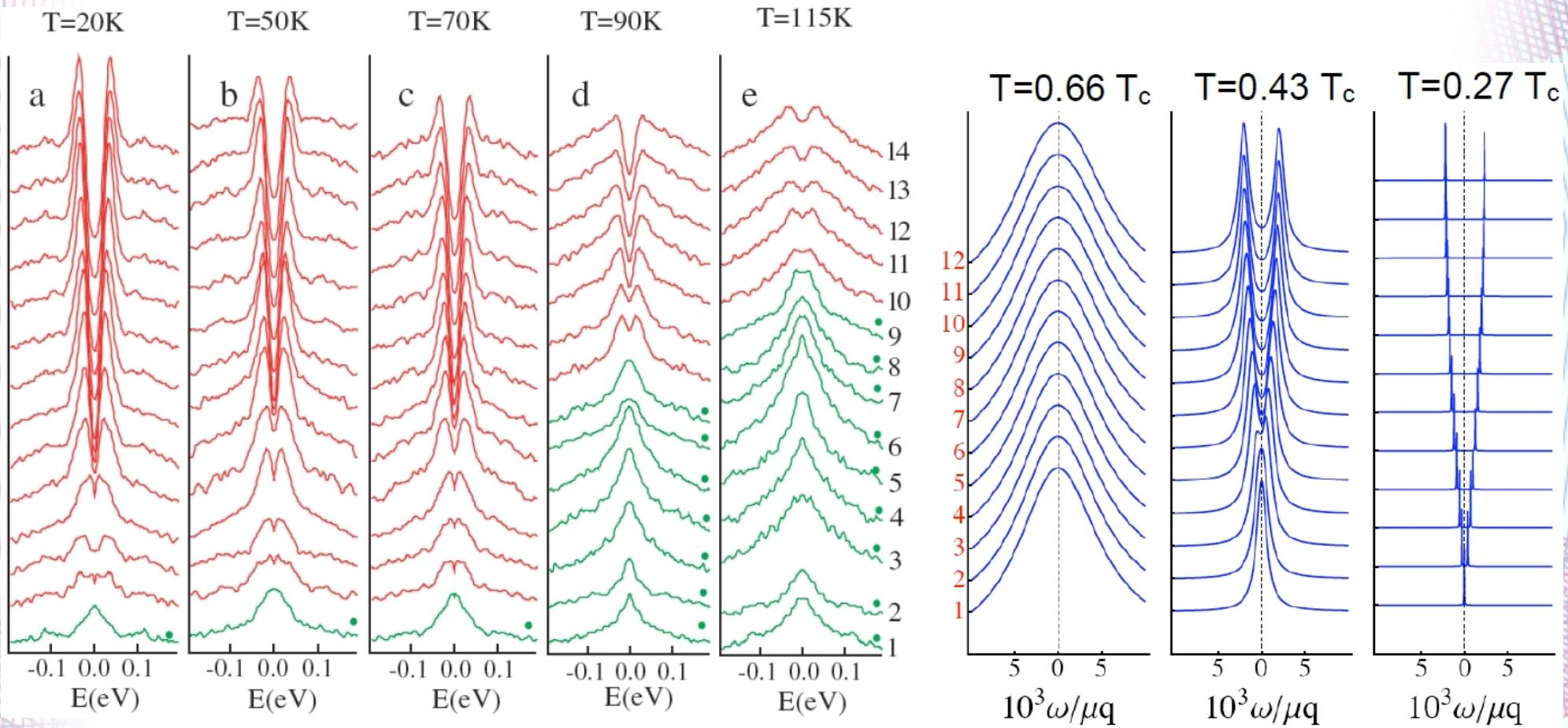
- E.g.: spectral func. at $\theta = \text{fixed}$

- Exp procedure: for every θ
 - 1) identify Fermi momentum
 - 2) draw EDC



EDC's

- Compare Energy Distribution Curves with exp's:

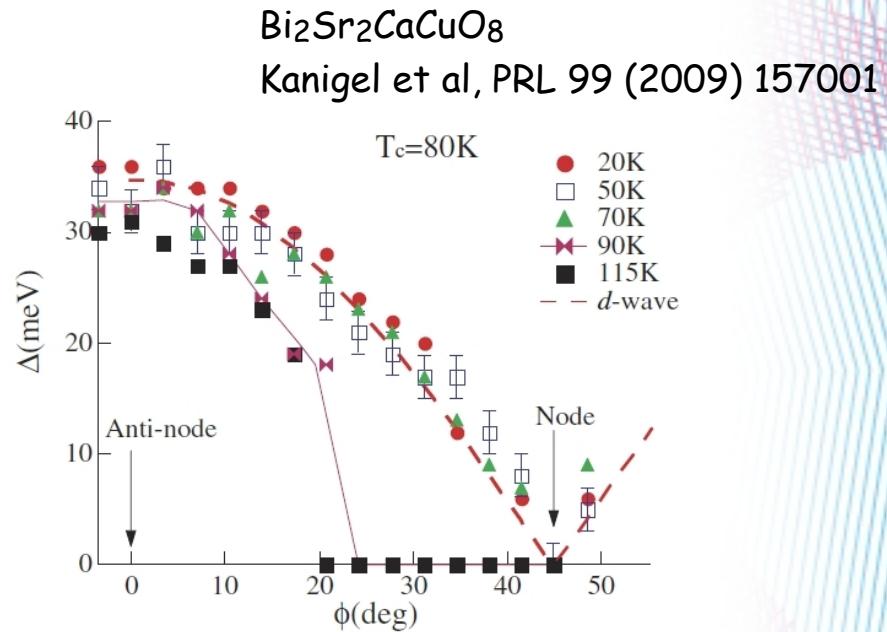
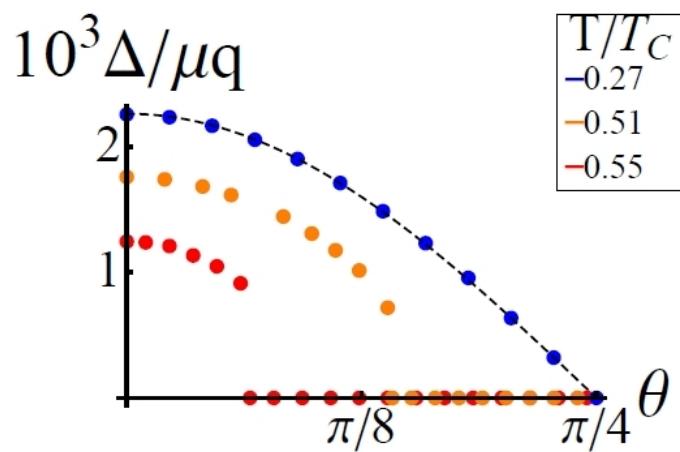


Kanigel et al, PRL 99 (2009) 157001

Underdoped $\text{Bi}_2\text{Sr}_2\text{Ca}\text{CuO}_8$

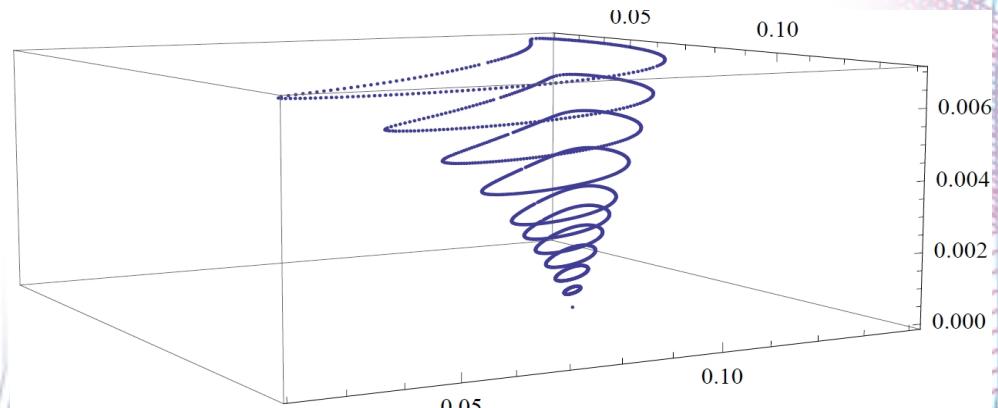
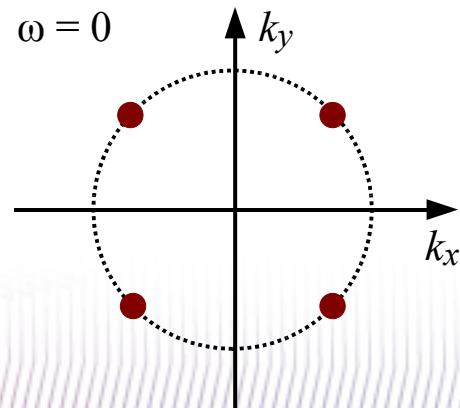
d-wave gap and Dirac cones

- $T < T_{\text{gap}}$: Fermi surface gapped everywhere but at four **nodes** $\theta = \pi/4$.



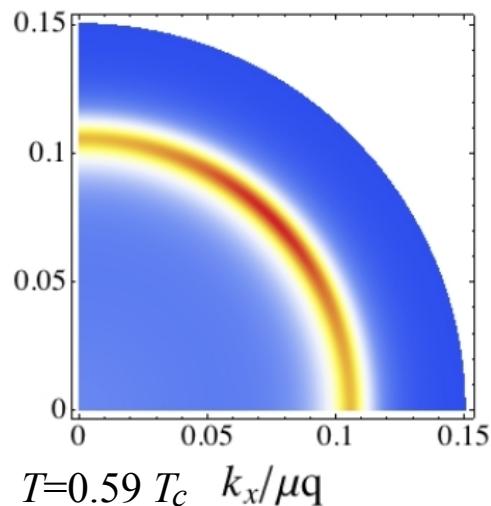
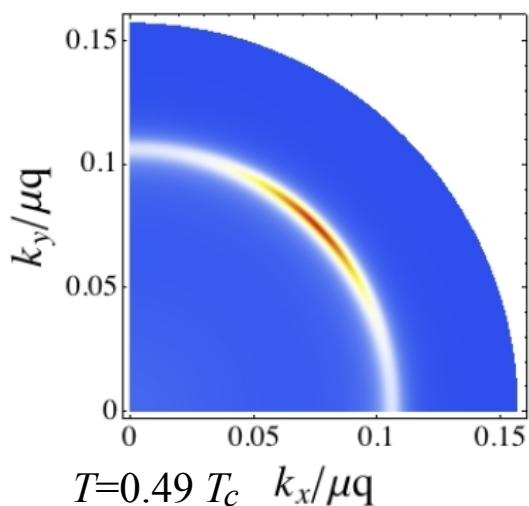
In both cases, gap fit by $\Delta(\theta) = \Delta_0 |\cos(2\theta)|$

- Nodes:
Dirac cones
exp observed

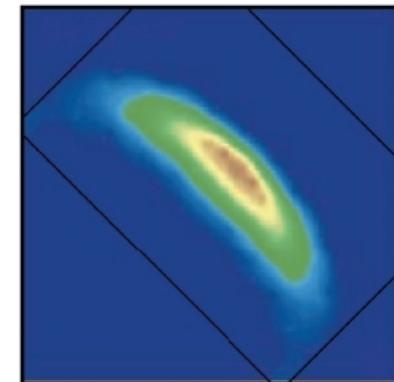


Fermi arcs

- $T_{\text{gap}} < T < T_{\text{arc}}$: Fermi arcs

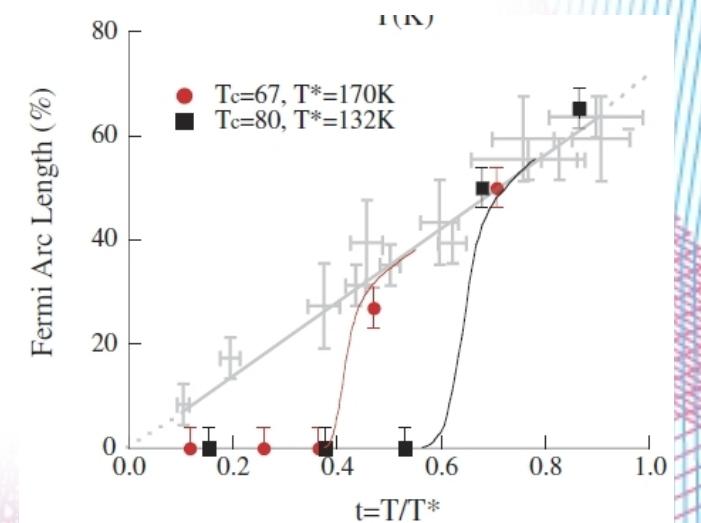
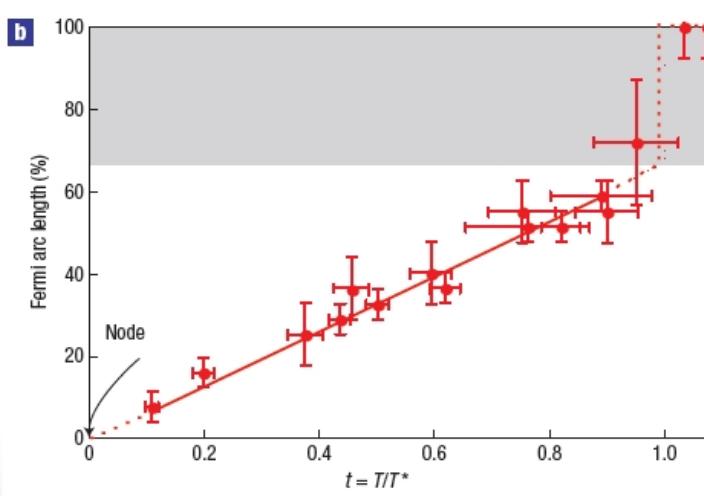
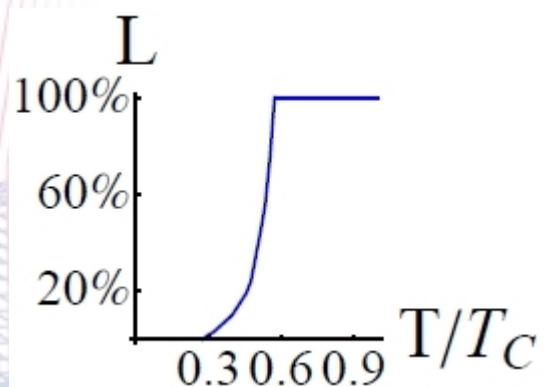


$\omega = 0$



Na-CCOC

Shen et al, Science 307 (2005) 901



Kanigel et al, Nature Phys 2 (2006) 447

Future directions

- Improve the action (maybe along Argyres-Nappi)
- Fully consistent model (beyond probe approx):
KK decomposition, e.g. $\text{AdS}_d \times S^1$
- Arcs from inhomogeneities (Vegh). Explicit realization and temperature dependence of arc length
- Pseudo-gap
- Complex ansatz (Hall conductivity,
chiral $d+id$ superconductivity, boundary currents)